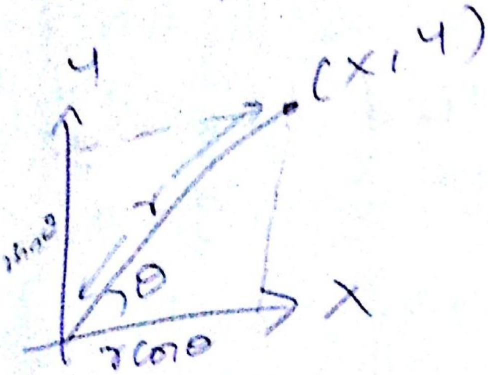
8/08/14

→ Transformation from radical co-ord. to polar co-ord.



$$(x, y) \longrightarrow (r, \theta)$$

$$r = \sqrt{x^2 + y^2}$$

$$x = r\cos\theta$$

$$y = r\sin\theta.$$

→ Legrangia, $L = \underset{(KE)}{T} - \underset{(PE)}{V(r)}$

$$T = \frac{1}{2} m v^2$$

$$\frac{\partial L}{\partial v} = \frac{\partial T}{\partial v} - \frac{\partial V}{\partial v}$$

→ func^n of r

= 0

→ vel.

$$\Rightarrow \frac{\partial L}{\partial v} = \frac{\partial T}{\partial v}.$$

momentum = P

$$\boxed{P = \frac{\partial L}{\partial \dot{r}}} \quad \text{— (1)}$$

$$\dot{r} = \frac{\partial r}{\partial t}.$$

$$F = -\frac{\partial V}{\partial r}$$

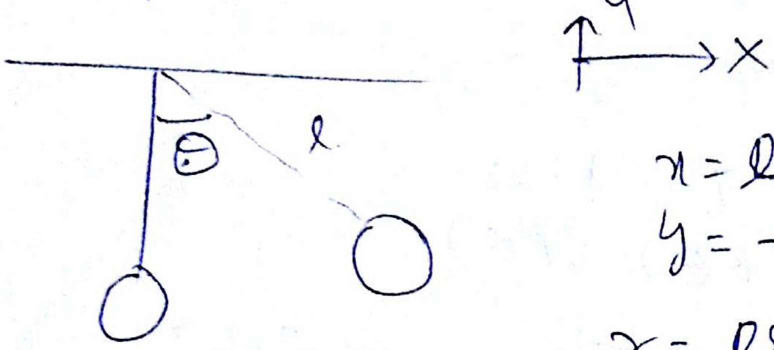
$$\boxed{F = \frac{\partial L}{\partial R}} \quad \text{— (2)}$$

From Newton's law,

$$\boxed{F = \frac{dP}{dt}} \longrightarrow (3).$$

$$\boxed{\frac{\partial L}{\partial r} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right)}$$ is called Lagrange equation

→ Simple Pendulum:



$$x = l\sin\theta \Rightarrow \dot{x} = l\cos\theta \cdot \frac{d\theta}{dt}$$

$$y = -l\cos\theta \Rightarrow \dot{y} = l\sin\theta \cdot \frac{d\theta}{dt}$$

$$r = l\sin\theta\,\hat{i} - l\cos\theta\,\hat{j}$$

$$T = \frac{1}{2}mv^2 = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2)$$

$$= \frac{1}{2}m(l^2\cos^2\theta \cdot \dot{\theta}^2 + l^2\sin^2\theta \cdot \dot{\theta}^2)$$

$$= \frac{1}{2}ml^2\dot{\theta}^2$$

$$V = mgy = -mgl\cos\theta.$$

$$L = \frac{1}{2}ml^2\dot{\theta}^2 + mgl\cos\theta.$$

From Lagrangie's eqn.,

$$\frac{\partial L}{\partial \dot{\theta}} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right)$$

$\frac{\partial L}{\partial \theta}$ ~~$ml^2\ddot{\theta} + mgl \frac{\partial \cos\theta}{\partial \theta}$~~

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right)$

$ml^2\ddot{\theta} + mgl\sin\theta = 0$

$\ddot{\theta} + k\sin\theta = 0.$

$$\boxed{\ddot{\theta} + \frac{g}{L}\sin\theta = 0}$$

→ I have a set of eqn's

$y_1' = f_1(t, y_1, \dots, y_n)$

$\vdots$

$y_n' = f_n(t, y_1, \dots, y_n).$

generally,

$y' = f(t, y_1, \dots, y_n).$

$y' = Ay + g,\ A = \begin{bmatrix} a_{11}(t) & \cdots & a_{1n}(t) \\ \vdots & & \\ \cdot & \cdots & a_{nn}(t) \end{bmatrix},\ g = \begin{bmatrix} g_1(t) \\ \vdots \\ g_n(t) \end{bmatrix}$

$y = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$

homogeneous $g = 0$

$y' = cy \Rightarrow y = ke^{\lambda t}$

Assuming homogeneous

$y = xe^{\lambda t}$ $\quad y' = Ay$

$y' = \lambda x e^{\lambda t}$

$Axe^{\lambda t} = \lambda x e^{\lambda t}$

$$\boxed{Ax = \lambda x}$$

eq. $\dot{M} =$ ______

eq. $\dot{N} =$ ______

x, y

$$\begin{bmatrix} \frac{\partial M}{\partial x} & \frac{\partial M}{\partial y} \\ \frac{\partial N}{\partial x} & \frac{\partial N}{\partial x} \end{bmatrix}$$

$y^{(1)} = x^{(1)} e^{\lambda_1 t}$

$y^{(n)} = x^{(n)} e^{\lambda_n t}$

If there are only 2 eqn's

$$y = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}$$

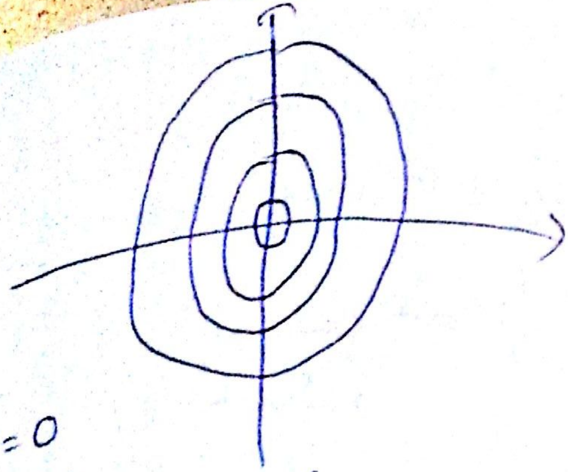
$$\det(A - \lambda I) = \begin{bmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{bmatrix}$$

p = Sum of principal diagonal elements of A

$= a_{11} + a_{22}$

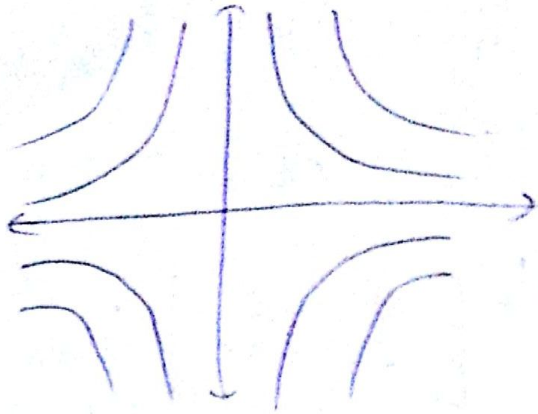
$q = \det(A)$

$\Delta = p^2 - 4q$

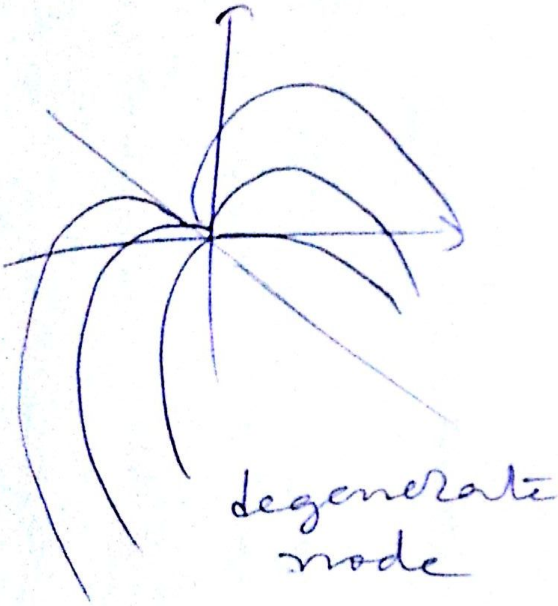


$p = 0$

$q > 0$ center



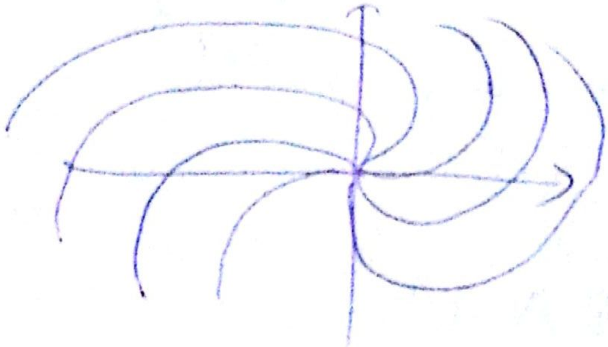
$q < 0$ Saddle point



degenerate node

$q > 0$

$\Delta \geq 0$



spiral $p \neq 0$

$\Delta < 0$

Example: $\ddot{\theta} + \frac{g}{L} \sin\theta = 0$.

$\theta = y_1$ & $\dot{\theta} = y_2$. $\frac{g}{L} = K$.

$\dot{y}_2 + K \sin y_1 = 0$. — (1) $\sin\theta \approx \theta$

$y_2 = \dot{y}_1$

$\Rightarrow \dot{y}_2 + K y_1 = 0$

$\dot{y}_2 = -ky_1$

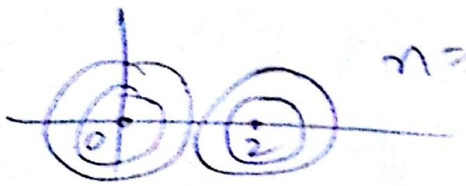
$\dot{y}_1 = y_2$

$y' = \begin{bmatrix} 0 & 1 \\ -k & 0 \end{bmatrix} y$

$p = 0$

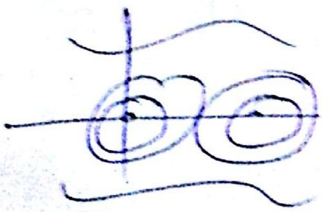
$q = k$

$n = 0, 2\pi, \ldots$



$n = \pi, 3\pi, \ldots$

$\Rightarrow p = 0 \;\&\; q = -k$



→ **Damped Pendulum**:

$\ddot{\theta} + k\sin\theta + c\dot{\theta} = 0$

$\Rightarrow \dot{y}_1 = y_2$ — ①

$\dot{y}_2 + ky_1 + cy_2 = 0$ — ②

y

$\dot{y}_2 + k\sin y_1 = 0$

(critical soln is when $y_1 = n\pi$

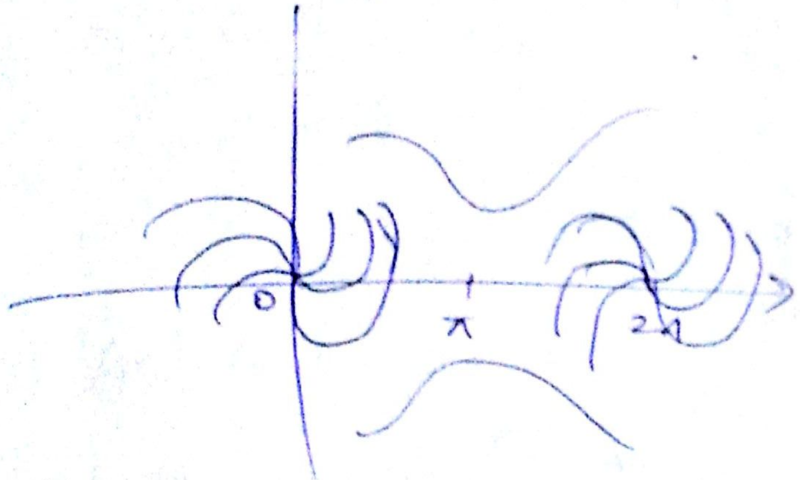
$n = 0, \pm 2, \pm 4, \ldots$

$\Rightarrow A = \begin{bmatrix} 0 & 1 \\ -K & c \end{bmatrix}$ $\quad \Delta = c^2 + K$

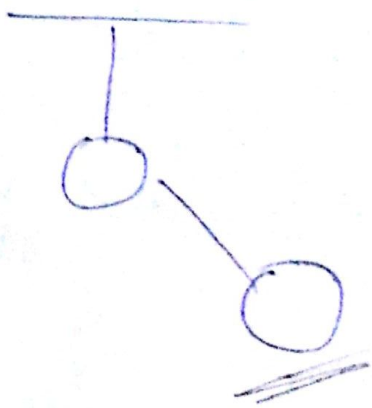
$$\left.\begin{array}{l} p = c \\ q = K \end{array}\right\} 0, \pm 2\pi, \ldots$$

$$\left.\begin{array}{l} p = c \\ q = K \end{array}\right\} \pm\pi, \pm 3\pi, \ldots$$

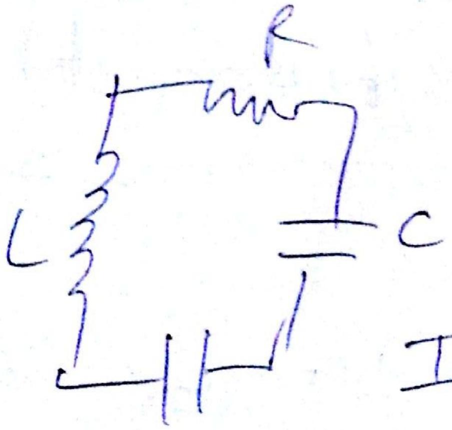
$\Delta = c^2 + 4K$



→ Google for



→ RLC circuits



$I = \dot{Q}$

$LI'' + RI + \frac{1}{C}Q = E_0 \sin(\omega t)$

$LI'' + RI' + \frac{1}{C}I = K$ — ②

Let $K = 0$

Sol^n is I_h, $I_h = ce^{\lambda t}$ — ①

put ① in ②

$L\lambda^2 + R\lambda + \frac{1}{C} = 0$

λ_1, λ_2

$\boxed{I_h = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}}$ λ's are distinct & real

Let $\lambda_1 = a$

$\boxed{(c_1 + c_2 t)e^{-at}}$ λ's are equal

if $\lambda = a \pm ib$

$\boxed{e^{-at}(A\cos\omega t + B\sin\omega t)}$ damped harmonic motion

Since $K != 0$

$LI'' + RI' + \frac{1}{C}I = E_0\omega\cos(\omega t)$.

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$\Rightarrow I_p = a\cos(\omega t) + b\sin(\omega t)$

$I = I_h + I_p$.

we get, $\omega L - \frac{1}{\omega C} = S$ (reactance).

$a = \frac{E_0 S}{R^2+S^2}$, $b = \frac{E_0 R}{R^2+S^2}$

R^2+S^2 = Impedance

As $t \to \infty$, $I_h \to 0$ (λ_1 & λ_2 are −ve).

After steady state we only have I_p.

first

read

REFERENCE: Kreyzig 9th Edition

adv. in (4, 2.9).

Maths 4.5

harmoni

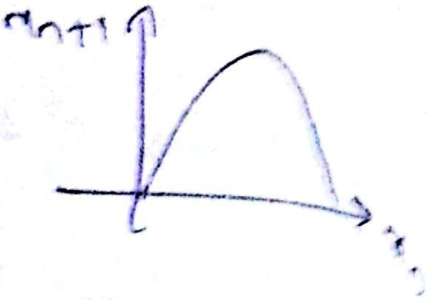
motion

28th August

Wrongskian & Hessian.

Sept-1:

→ Logistic Map:



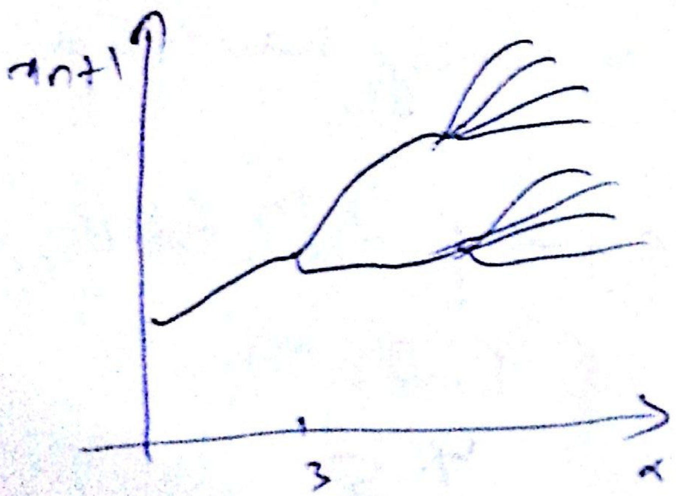
$$x_{n+1} = \alpha x_n (1 - x_n)$$
$$= \alpha x_n - \alpha x_n^2$$

How x_{n+1} varies with α?

till $\alpha = 3$, graph is fine

$3.2 < \alpha < 1+\sqrt{6}$ bifurcations arise

$1+\sqrt{6} < \alpha < 3.4$



Mid Sem

1. 2-body Pendulum
2. Lotka-Volt[illegible]
3. Logistic Map
4. phase diag

(wiki)

(saddle point)

←

eg: $U(x) = ax^4 - bx^2$

$F(x) = -4ax^3 + 2bx$ $\quad F = m\frac{dv}{dt}$

$v = \frac{dx}{dt}$

$\frac{dv}{dx} = \frac{dv/dt}{dx/dt} = \frac{-4ax^3 + 2bx}{v}$

$\int v\,dv = \int -4ax^3\,dx + \int 2bx\,dx$

→ Eigen Values: $AV = \lambda V$.

for algebraic func's,

eg: $e^{\lambda t}$ $\quad A = \frac{\partial}{\partial t}$

$\therefore \frac{\partial}{\partial t}(e^{\lambda t}) = \lambda e^{\lambda t} = \lambda V$.

Jacobean Matrix