Algorithms

Assignment 1
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Due: 26-AUGUST-2014, 830 AM, in the drop box at CSTAR

Problem 1. Prove or disprove the following statements.

- $f(n) = O(g(n))$ implies $2^{f(n)} = O(2^{g(n)}$.
- $f(n) + g(n) = \Theta(\min\{f(n), g(n)\})$.
- $f(n) \neq O(g(n))$ implies $g(n) = O(f(n))$.
- $\min\{f(n), g(n)\} \in O(f(n) + g(n))$.

(4 Points)

Problem 2. Solve the following recurrence relations.

- $T(n) = 3T(n/2) + n \log n$
- $T(n) = T(n-2) + 2 \log n$
- $T(n) = 5T(n/3) + n^{4/3}$
- $T(n) = T(n-1) + \log n$

(4 Points)

Problem 3. Write the recurrence relations for the selection algorithm when the input is divided into groups of 3 and groups of 13. In which of the above two cases does the algorithm run in linear time? Justify your answer. **(5 Points)**

Problem 4. In an array A define a pair (i, j) to be an inversion if $i < j$ and $A[i] > A[j]$. Answer the following questions.

- Given $A = (3, 5, 2, 9, 6, 1)$ find the array B consisting of elements of A but with the maximum number of inversions.
- What is the relationship between the number of inversions in an array A and the runtime of insertion sort on A. Justify your answer.
- Use the divide-and-conquer strategy to count the number of inversions in a given array of n elements.

(1+2+4=7 Points)