Science-I, Assignment-2, Monsoon 2014

1. What is the behavior variable $x(t)$ satisfying the linear 1st order differential equation $\frac{d}{dt}x(t) = c\ x(t)$, where c is a constant parameter, for varing values of parameter c? (x(t) is a scalar variable)

2. What is the behavior variable $x(t)$ satisfying the second order differential equation $\frac{d^2}{dt^2}x(t) = c\ x(t)$, where c is a constant parameter, for varing values of parameter c? (x(t) is a scalar variable)

3. Compare the solutions to the above two questions regarding the one dimensional variable x(t), i.e. generally speaking what are the differences between solutions for the above two questions?

4. A vector $\vec{r} = (x_1, x_2, \cdots x_N)^T$ (hence the vector $\vec{r}$ is a column vector) satisfies the linear, coupled, 1st order differential set of equations $\frac{d}{dt}\vec{r}(\mathrm{t}) = \bar{M} \cdot \vec{r}$, where $\bar{M}$ is a $N \times N$ matrix (of constants). Show that a linear combination of the eigen vectors of the matrix M is a solution for $\vec{r}(t)$.

5. A vector $\vec{r} = (x_1, x_2, \cdots x_N)^T$ (hence the vector $\vec{r}$ is a column vector) satisfies the linear, coupled, 2nd order differential set of equations $\frac{d^2}{dt^2}\vec{r}(\mathrm{t}) = \bar{M} \cdot \vec{r}$, where $\bar{M}$ is a $N \times N$ matrix (of constants). Show that a linear combination of the eigen vectors of the matrix M is a solution for $\vec{r}(t)$.

6. Compare the solutions to the above two questions regarding the N-dimensional variable $r(\vec{t})$, generally speaking what are the dierences between solutions for the above two questions ?

7. Logistic map: $x_{n+1} = r\ x_n(1 - x_n)$, where $0 \leq r \leq 4$, and $0 \leq x_0 \leq 1$. (a) Show that $x_n \in [0, 1]$, and (b) write a program that will take r $x_0$and n as inputs, and calculate (and plot (a) $x_n$vs. n and (b) x_i vs $x_{i+1}$for $i \in [0, n]$) $x_0, \cdots, x_n$. (c) Plot for various values of r, and for various x_0. Comment.