

ASSIGNMENT-1

Deadline - 6th September 2014

Submission details will be informed later.

1. Adams can go to work by either bus, car or train. If he goes by car, he has a 50% chance of being late. If he goes by bus, he has a 20% chance of being late. If he goes by train, there is only 1% chance of being late. If Adams is late, what is the probability that he came by car, assuming that there is equal possibility of him coming by bus, car and train?
2. In front of you are two identical looking coins. One is fair and other comes up head 60% of the time. If you chose a coin at random and flip it, what is the probability that it is the fair coin given that head turns up?
3. A factory production line is manufacturing bolts using three machines, A, B and C. Of the total output, machine A is responsible for 25%, machine B for 35% and machine C for the rest. It is known from previous experience with the machines that 5% of the output from machine A is defective, 4% from machine B and 2% from machine C. A bolt is chosen at random from the production line and found to be defective. What is the probability that it came from (a) machine A (b) machine B (c) machine C?
4. Given a deck of 52 cards, a card is drawn at random and is discarded. Then, another card is drawn at random from the remaining 51 cards. It is known that the second card is a spade, what is the probability that the first card is also a spade?
5. While watching a game of Champions League football in a cafe, you observe someone who is clearly supporting Manchester United in the game. What is the probability that they were actually born within 25 miles of Manchester ? Assume that

- the probability that a randomly selected person in a typical local bar environment is born within 25 miles of Manchester is $1/20$.
- The chance that a person born within 25 miles of Manchester actually supports United is $7/10$.
- the probability that a person not born within 25 miles of Manchester supports United with probability $1/10$.

6. $f(z) = y - z_{xy} + i(-x + x^2 - y^2)$ and $z = x + iy$. For what values of z , $f(z)$ exists?

7. Write $|\exp(2z + i)|$ and $|\exp(iz^2)|$ in terms of x and y . Then show that:
 $|\exp(2z + i) + \exp(iz^2)| \leq e^{2x} + e^{-2xy}$ where $\exp(a) = e^a$

8. Calculate the values of m such that the roots x_1 and x_2 of $x^2 - 2mx + m = 0$ satisfy the condition $x_1^3 + x_2^3 = x_1^2 + x_2^2$. Calculate the roots for those m values and check the condition.

9. Is $f(z) = (\bar{z} + 1)^3 - 3\bar{z}$ analytic? Show why / why not?

10. Show that the function

$f(z) = f(x + iy) = e^{-y}\cos x + i e^{-y}\sin x$ is differentiable for all $z = x + iy$ and find its derivative.