Today

- Cache organization and operation
- **Performance impact of caches**
  - The memory mountain
  - Rearranging loops to improve spatial locality

The Memory Mountain

- **Read throughput (read bandwidth)**
  - Number of bytes read from memory per second (MB/s)

- **Memory mountain: Measured read throughput as a function of spatial and temporal locality.**
  - Compact way to characterize memory system performance.

Memory Mountain Test Function

```
long data[MAXELEMS];  /* Global array to traverse */

/* test - Iterate over first "elems" elements of
 *        array "data" with stride of "stride", using
 *        using 4x4 loop unrolling.
 */
int test(int elems, int stride) {
    long i, sx2=stride*2, sx3=stride*3, sx4=stride*4;
    long acc0 = 0, acc1 = 0, acc2 = 0, acc3 = 0;
    long length = elems, limit = length - sx4;

    /* Combine 4 elements at a time */
    for (i = 0; i < limit; i += sx4) {
        acc0 = acc0 + data[i];
        acc1 = acc1 + data[i+stride];
        acc2 = acc2 + data[i+sx2];
        acc3 = acc3 + data[i+sx3];
    }

    /* Finish any remaining elements */
    for (; i < length; i++) {
        acc0 = acc0 + data[i];
    }
    return ((acc0 + acc1) + (acc2 + acc3));
}
                                   mountain/mountain.c
```

Call `test()` with many combinations of `elems` and `stride`.

For each `elems` and `stride`:

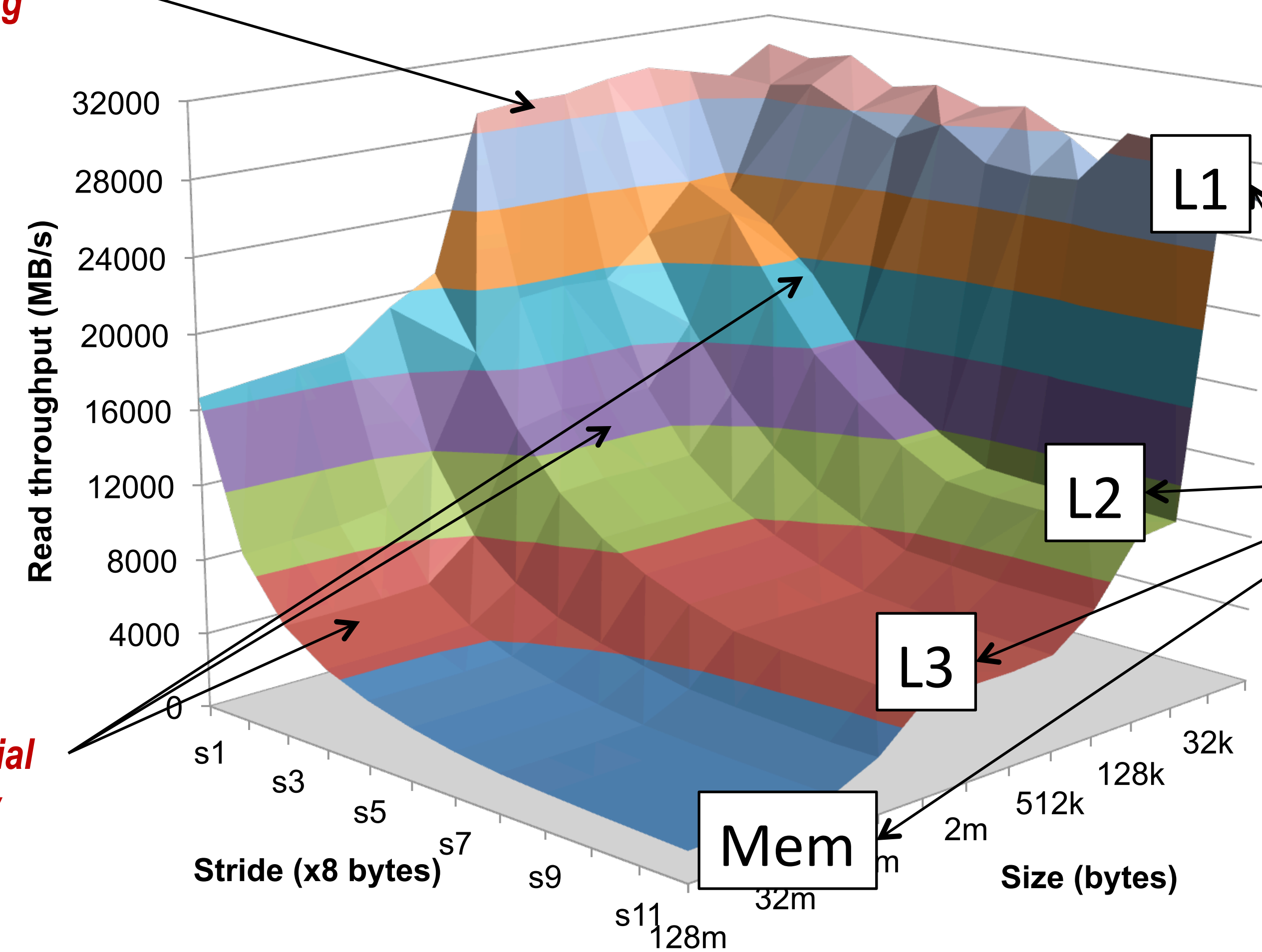
1. Call `test()` once to warm up the caches.

2. Call `test()` again and measure the read throughput(MB/s)

The Memory Mountain

Core i5 Haswell
3.1 GHz
32 KB L1 d-cache
256 KB L2 cache
8 MB L3 cache
64 B block size

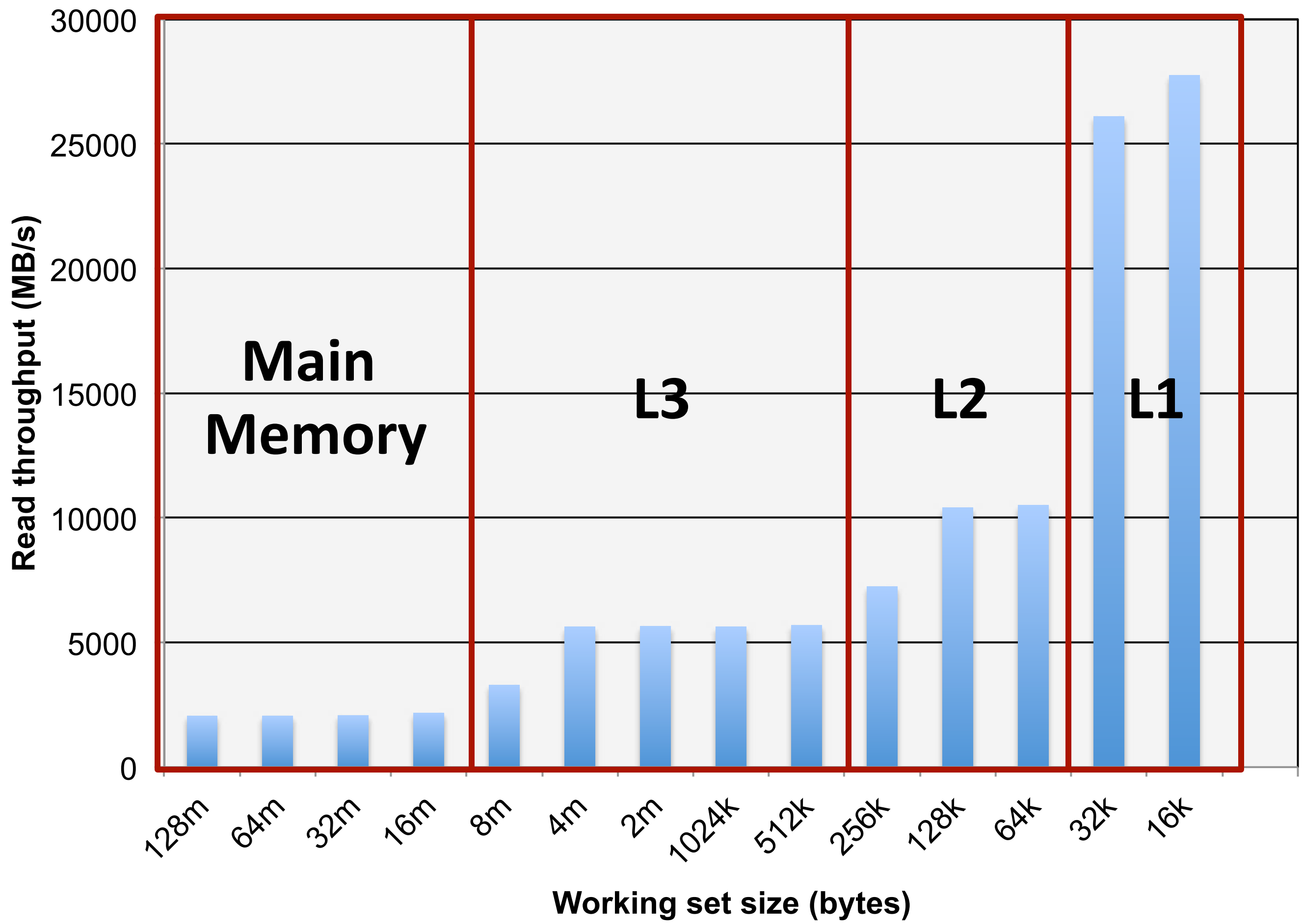
Aggressive prefetching



Ridges of temporal locality

Slopes of spatial locality

Cache Capacity Effects from Memory Mountain

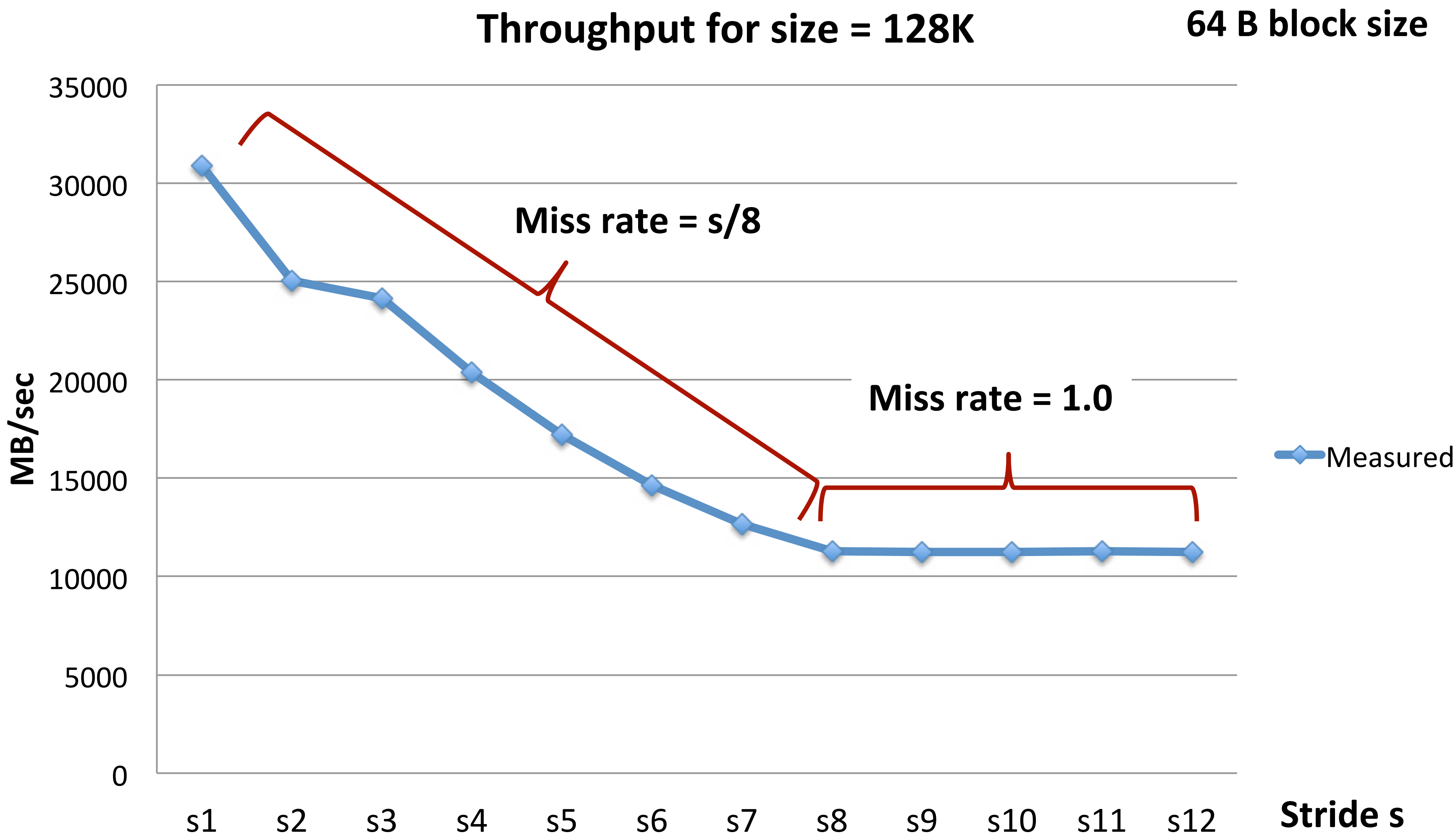


Core i7 Haswell
3.1 GHz
32 KB L1 d-cache
256 KB L2 cache
8 MB L3 cache
64 B block size

Slice through memory mountain with stride=8

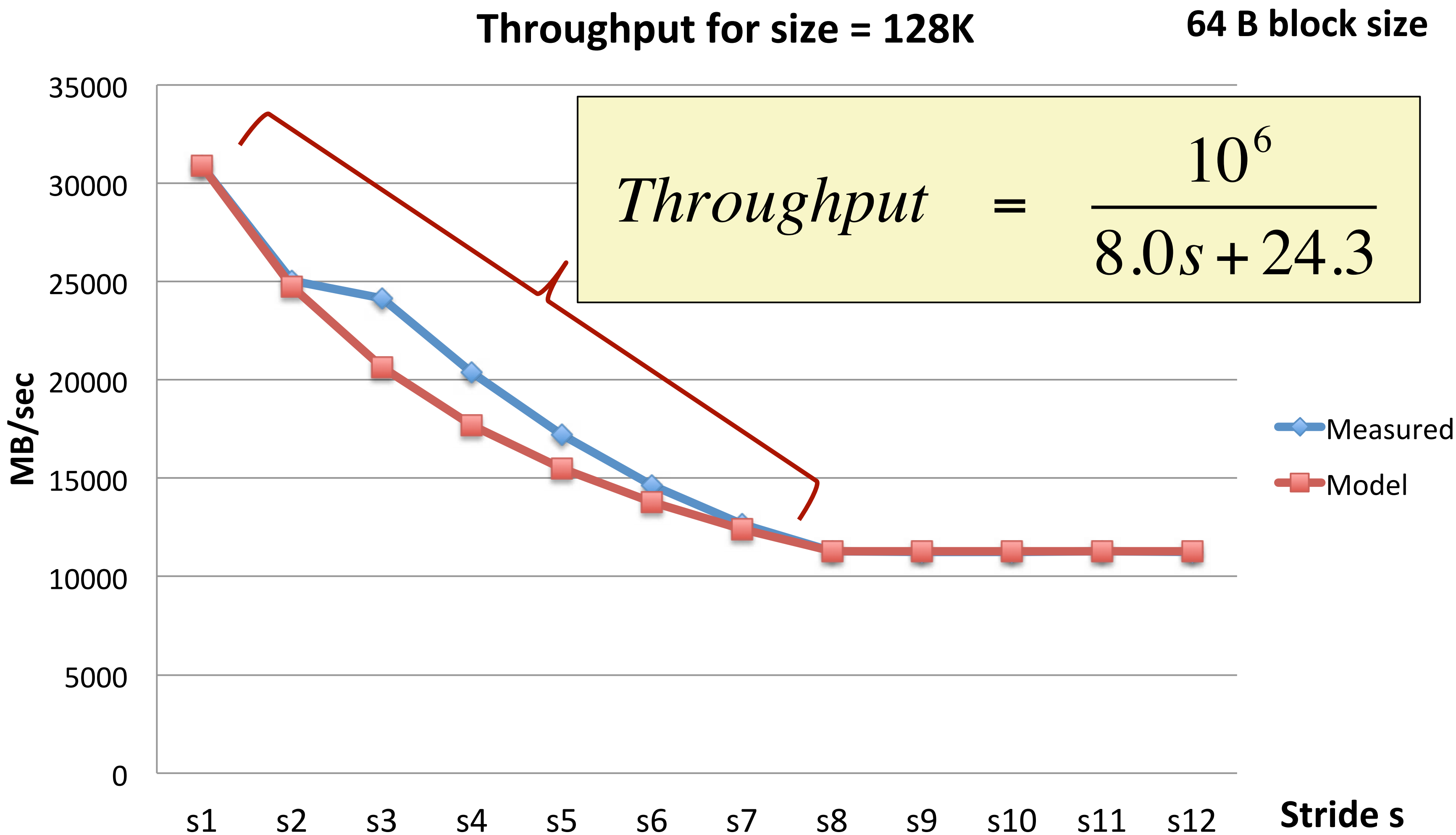
Cache Block Size Effects from Memory Mountain

Core i7 Haswell
2.26 GHz
32 KB L1 d-cache
256 KB L2 cache
8 MB L3 cache
64 B block size



Modeling Block Size Effects from Memory Mountain

Core i7 Haswell
2.26 GHz
32 KB L1 d-cache
256 KB L2 cache
8 MB L3 cache
64 B block size



$$Throughput = \frac{10^6}{8.0s + 24.3}$$

Today

- Cache organization and operation
- Performance impact of caches
  - The memory mountain
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Matrix Multiplication Example

- **Description:**
  - Multiply *N* x *N* matrices
  - Matrix elements are doubles (8 bytes)
  - $2N^3$ total FP operations
  - *N* reads per source element
  - *N* values summed per destination
    - but may be able to hold in register

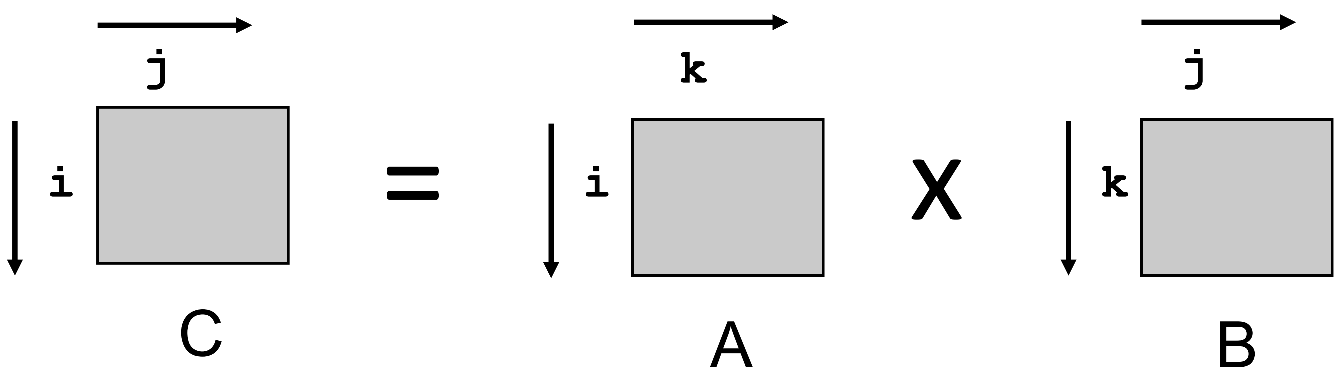
Variable ***sum*** *held in register*

```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

matmult/mm.c

Miss Rate Analysis for Matrix Multiply

- **Assume:**
  - Block size = 64B (big enough for four doubles)
  - Matrix dimension (N) is very large
    - Approximate 1/N as 0.0
  - Cache is not even big enough to hold multiple rows
- **Analysis Method:**
  - Look at access pattern of inner loop



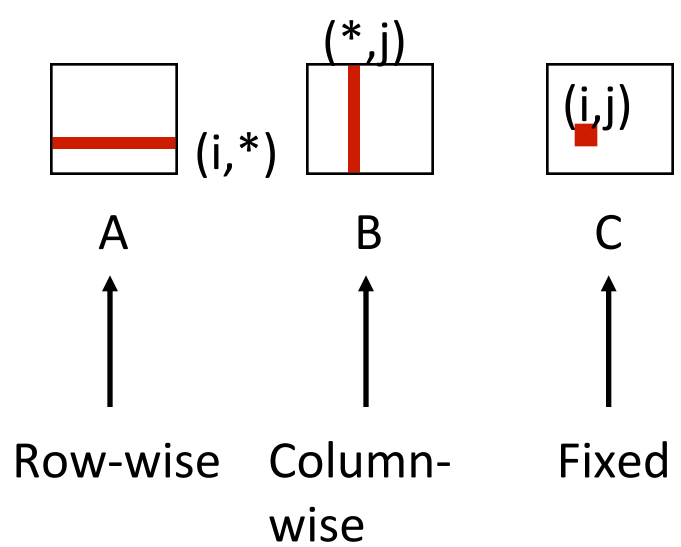
Layout of C Arrays in Memory (review)

- **C arrays allocated in row-major order**
  - each row in contiguous memory locations
- **Stepping through columns in one row:**
  - ```
    for (i = 0; i < N; i++)
      sum += a[0][i];
    ```
  - accesses successive elements
  - if block size (B) > sizeof(a_{ij}) bytes, exploit spatial locality
    - miss rate = sizeof(a_{ij}) / B
- **Stepping through rows in one column:**
  - ```
    for (i = 0; i < n; i++)
      sum += a[i][0];
    ```
  - accesses distant elements
  - no spatial locality!
    - miss rate = 1 (i.e. 100%)

Matrix Multiplication (`ijk`)

```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}                           matmult/mm.c
```

Inner loop:



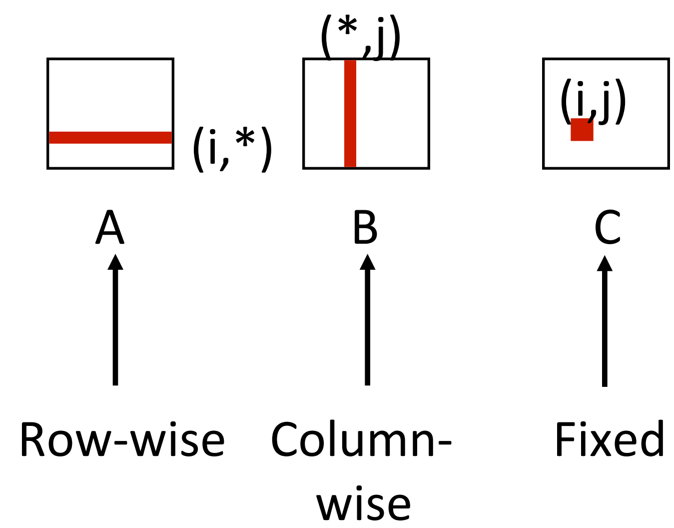
Misses per inner loop iteration:

A	B	C
0.125	1.0	0.0

Matrix Multiplication (`jik`)

```
/* jik */
for (j=0; j<n; j++) {
  for (i=0; i<n; i++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum
  }
}                        matmult/mm.c
```

Inner loop:



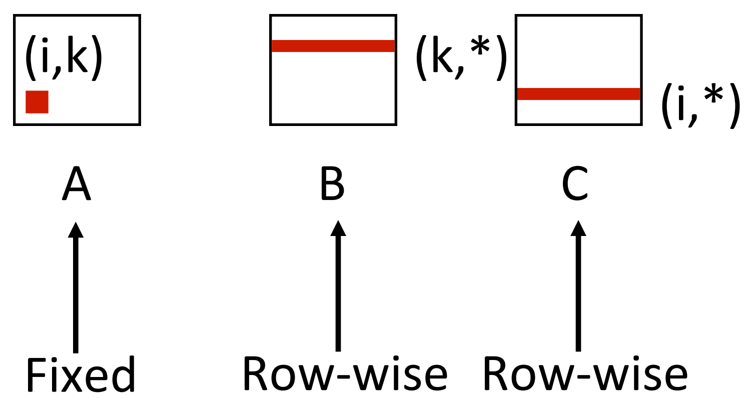
Misses per inner loop iteration:

A	B	C
0.125	1.0	0.0

Matrix Multiplication (`kij`)

```
/* kij */
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```
matmult/mm.c

Inner loop:

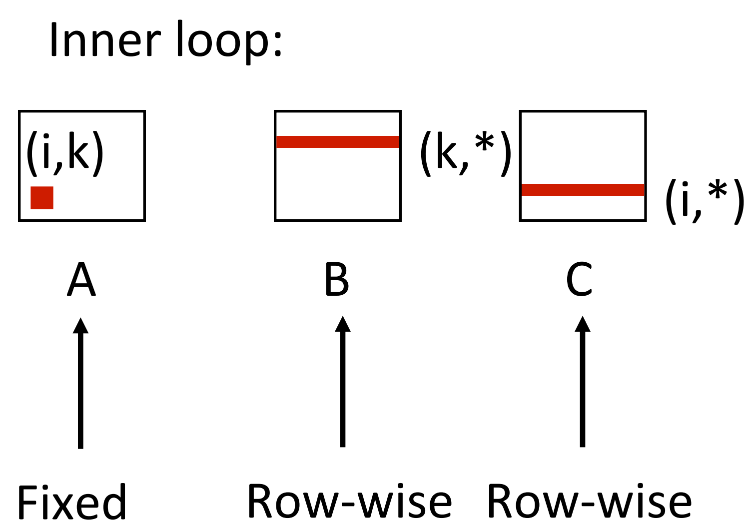


Misses per inner loop iteration:

A	B	C
0.0	0.125	0.125

Matrix Multiplication (ikj)

```
/* ikj */
for (i=0; i<n; i++) {
  for (k=0; k<n; k++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
                              matmult/mm.c
```



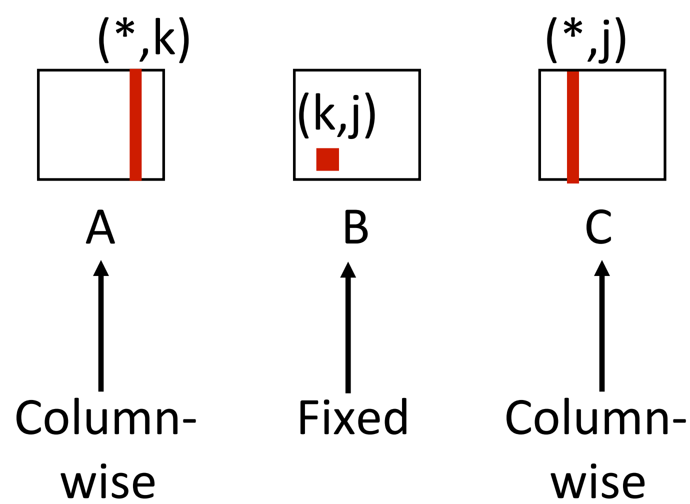
Misses per inner loop iteration:

A	B	C
0.0	0.125	0.125

Matrix Multiplication (`jki`)

```
/* jki */
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
                              matmult/mm.c
```

Inner loop:



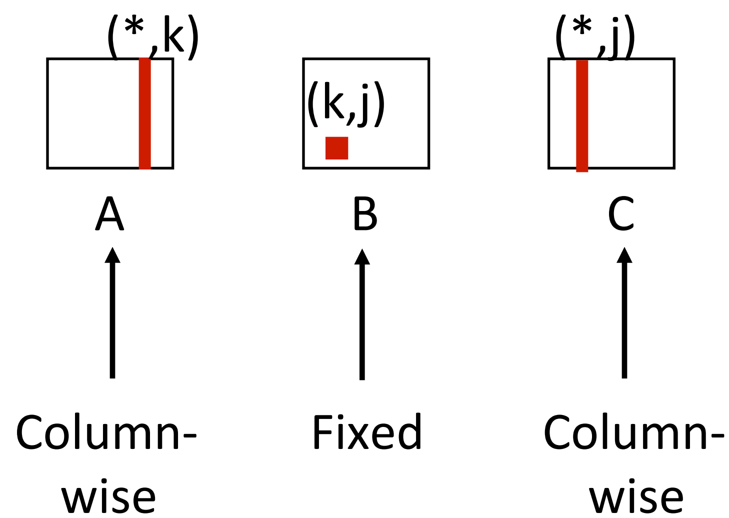
Misses per inner loop iteration:

A	B	C
1.0	0.0	1.0

Matrix Multiplication (`kji`)

```
/* kji */
for (k=0; k<n; k++) {
  for (j=0; j<n; j++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}                               matmult/mm.c
```

Inner loop:



Misses per inner loop iteration:

A	B	C
1.0	0.0	1.0

Summary of Matrix Multiplication

```
for (i=0; i<n; i++) {
  for (j=0; j<n; j++) {
   sum = 0.0;
   for (k=0; k<n; k++)
     sum += a[i][k] * b[k][j];
   c[i][j] = sum;
 }
}
```

ijk (& jik):
- 2 loads, 0 stores
- misses/iter = **1.125**

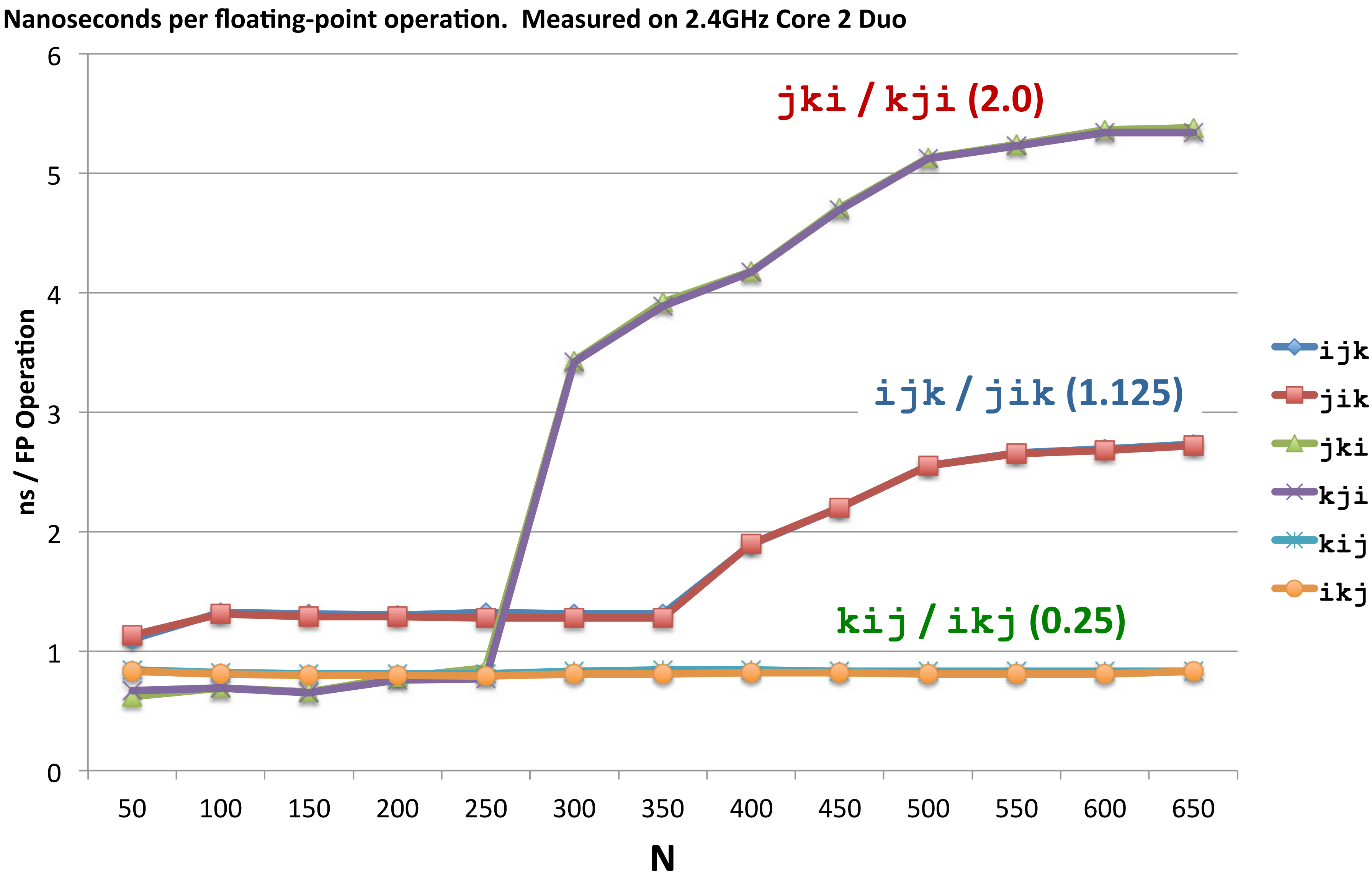
```
for (k=0; k<n; k++) {
 for (i=0; i<n; i++) {
  r = a[i][k];
  for (j=0; j<n; j++)
   c[i][j] += r * b[k][j];
 }
}
```

kij (& ikj):
- 2 loads, 1 store
- misses/iter = **0.25**

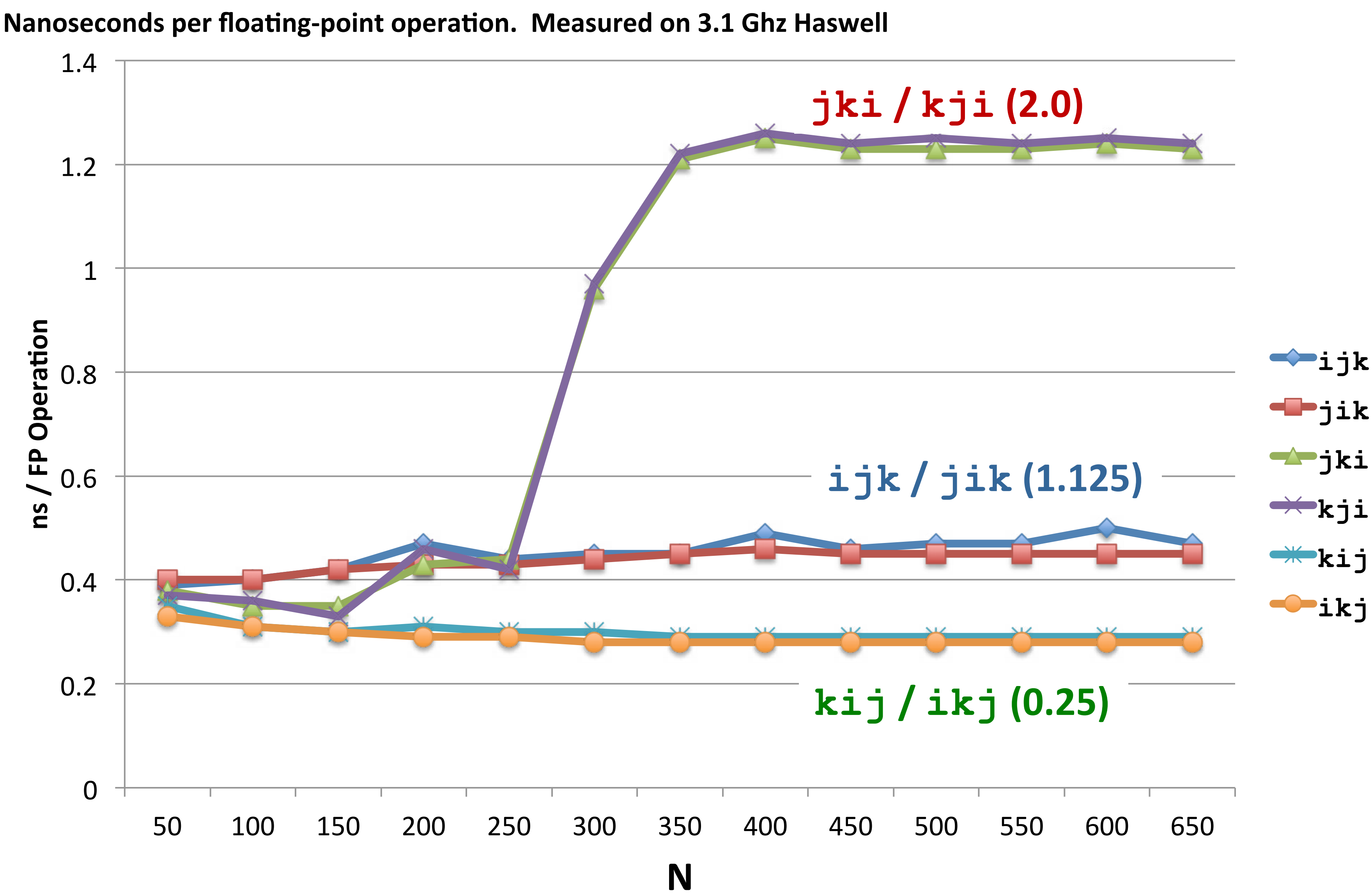
```
for (j=0; j<n; j++) {
 for (k=0; k<n; k++) {
   r = b[k][j];
   for (i=0; i<n; i++)
    c[i][j] += a[i][k] * r;
 }
}
```

jki (& kji):
- 2 loads, 1 store
- misses/iter = **2.0**

2008-era Matrix Multiply Performance

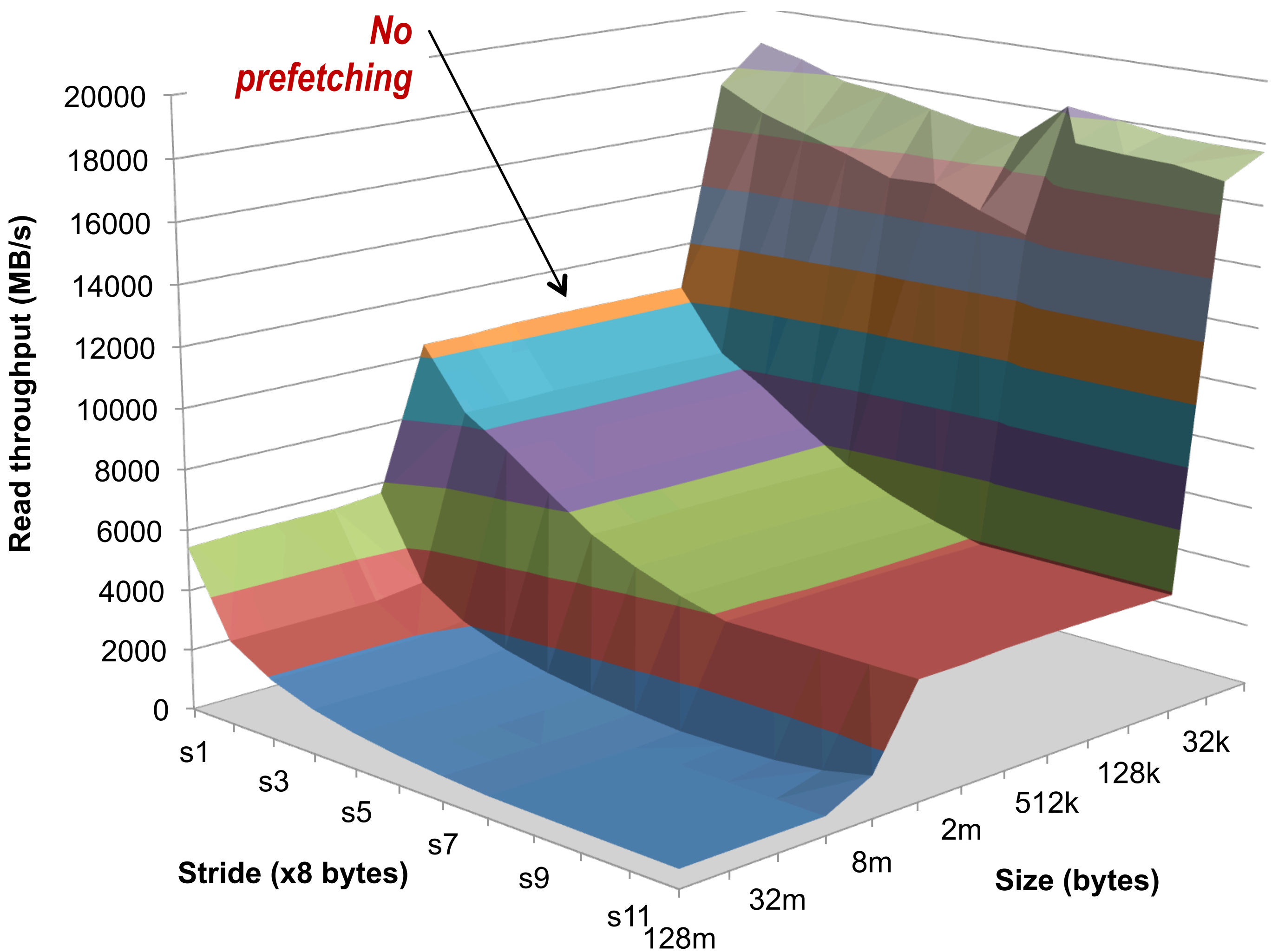


2014-era Matrix Multiply Performance



2008 Memory Mountain

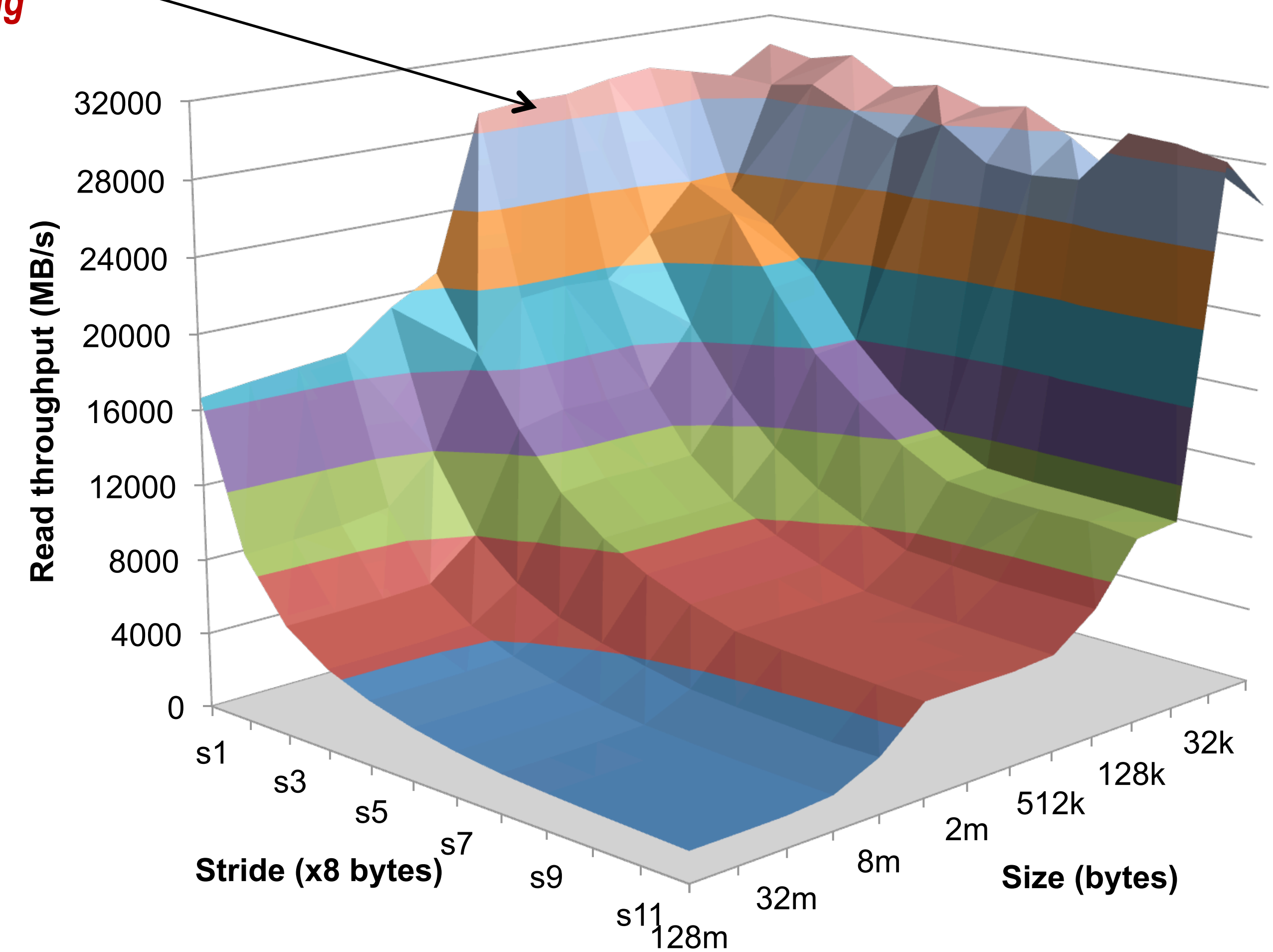
Core 2 Duo
2.4 GHz
32 KB L1 d-cache
6MB L2 cache
64 B block size



2014 Memory Mountain

Core i5 Haswell
3.1 GHz
32 KB L1 d-cache
256 KB L2 cache
8 MB L3 cache
64 B block size

Aggressive prefetching



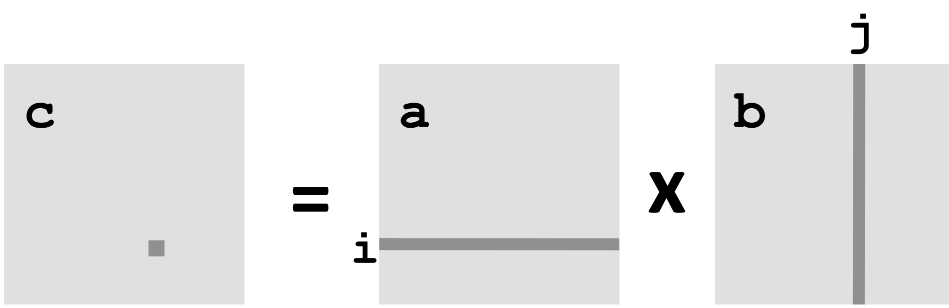
EXTRA SLIDES

Today

- Cache organization and operation
- Performance impact of caches
  - The memory mountain
  - Rearranging loops to improve spatial locality
  - Using blocking to improve temporal locality

Example: Matrix Multiplication

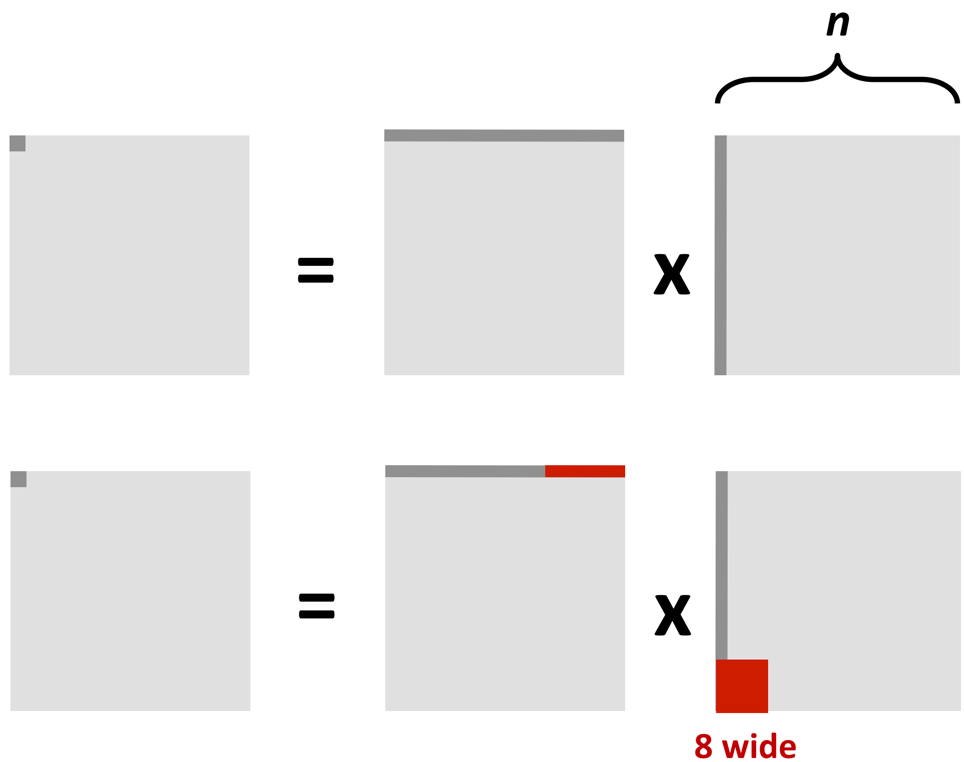
```
c = (double *) calloc(sizeof(double), n*n);

/* Multiply n x n matrices a and b  */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i++)
        for (j = 0; j < n; j++)
            for (k = 0; k < n; k++)
                c[i*n + j] += a[i*n + k] * b[k*n + j];
}
```



Cache Miss Analysis

- **Assume:**
  - Matrix elements are doubles
  - Cache block = 8 doubles
  - Cache size C << *n* (much smaller than *n*)

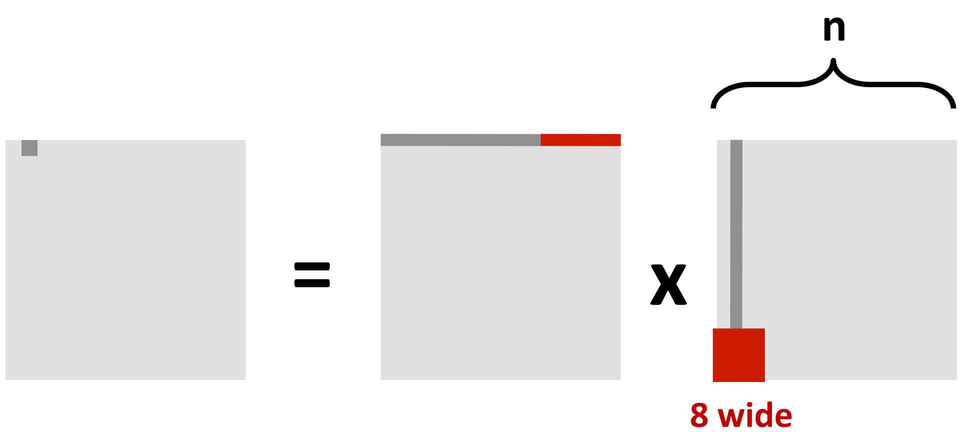
- **First iteration:**
  - *n*/8 + n = 9*n*/8 misses
  - Afterwards in cache: (schematic)



Cache Miss Analysis

- **Assume:**
  - Matrix elements are doubles
  - Cache block = 8 doubles
  - Cache size C << *n* (much smaller than *n*)

- **Second iteration:**
  - Again:
    $n/8 + n = 9n/8$ misses

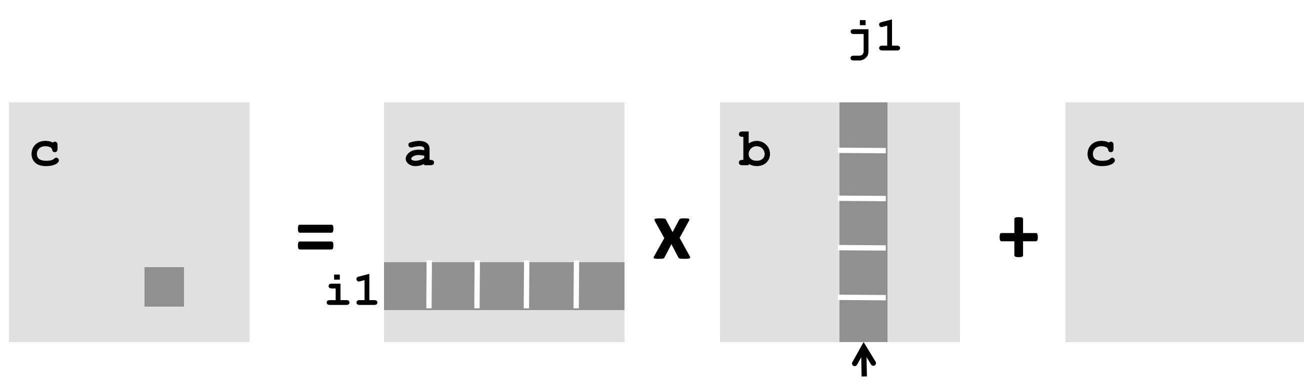


- **Total misses:**
  - $9n/8\ n^2 = (9/8)\ n^3$

Blocked Matrix Multiplication

```
c = (double *) calloc(sizeof(double), n*n);

/* Multiply n x n matrices a and b  */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i+=B)
        for (j = 0; j < n; j+=B)
             for (k = 0; k < n; k+=B)
                 /* B x B mini matrix multiplications */
                  for (i1 = i; i1 < i+B; i1++)
                      for (j1 = j; j1 < j+B; j1++)
                          for (k1 = k; k1 < k+B; k1++)
                              c[i1*n+j1] += a[i1*n + k1]*b[k1*n + j1];
}
```

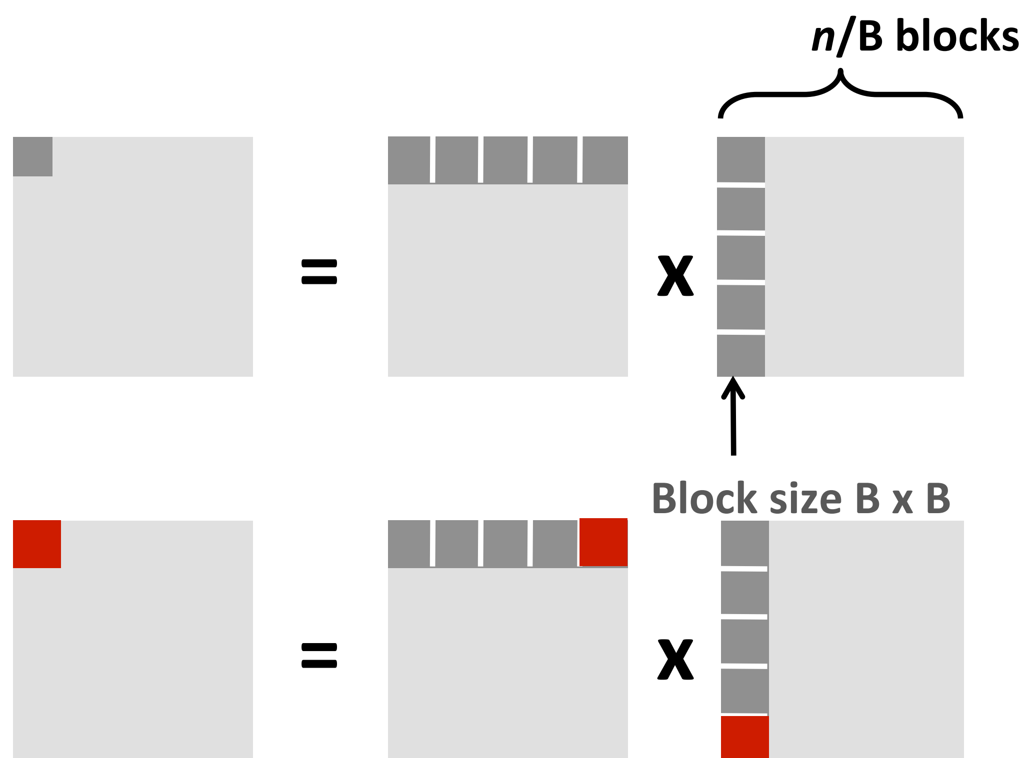
matmult/bmm.c



Block size B x B

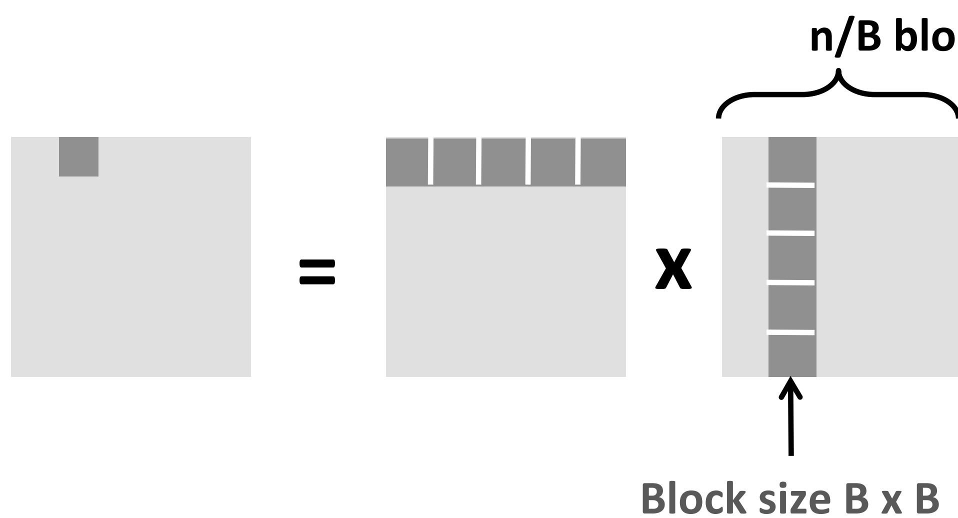
Cache Miss Analysis

- **Assume:**
  - Cache block = 8 doubles
  - Cache size C << *n* (much smaller than n)
  - Three blocks fit into cache: $3B^2 < C$
- **First (block) iteration:**
  - $B^2/8$ misses for each block
  - $2n/B \times B^2/8 = nB/4$ (omitting matrix c)
  - Afterwards in cache (schematic)



Cache Miss Analysis

- **Assume:**
  - Cache block = 8 doubles
  - Cache size C << *n* (much smaller than n)
  - Three blocks fit into cache: $3B^2 < C$
- **Second (block) iteration:**
  - Same as first iteration
  - $2n/B \times B^2/8 = nB/4$



- **Total misses:**
  - $nB/4 * (n/B)^2 = n^3/(4B)$

Blocking Summary

- **No blocking: (9/8) n^3**
- **Blocking: 1/(4B) n^3**

- **Suggest largest possible block size B, but limit $3B^2 < C$!**

- **Reason for dramatic difference:**
  - Matrix multiplication has inherent temporal locality:
    - Input data: $3n^2$, computation $2n^3$
    - Every array elements used $O(n)$ times!
  - But program has to be written properly

Cache Summary

- **Cache memories can have significant performance impact**

- **You can write your programs to exploit this!**
  - Focus on the inner loops, where bulk of computations and memory accesses occur.
  - Try to maximize spatial locality by reading data objects with sequentially with stride 1.
  - Try to maximize temporal locality by using a data object as often as possible once it's read from memory.