

## Assignment

### Hypothesis Testing

A textile factory consumes on average  $1000\text{m}^3$  of water per day. If a sample of 100 days is drawn randomly to test if the mean daily water intake remains  $1000\text{m}^3$  against the alternative that the mean water consumption has increased.

Assume that the sample mean equals  $\bar{x}$  ( $x$ -bar) =  $1005\text{m}^3$

and the sample variance is  $s^2 = 400\text{m}^6$ .

- a) Define the null and alternative hypotheses for this test
- b) Perform the test at the significance level  $\alpha = 0.05$ .
- c) What is the smallest value of  $\alpha$  for which the null hypothesis can be rejected?
- d) Calculate the power of the test in two scenarios: If the true mean water consumption equals  $\mu_1 = 1000\text{m}^3$  and  $\mu_1 = 1008\text{m}^3$ .
- e) Indicate which of the following statements are true/false and justify your answer:
  1. If we reject the hypothesis at the level  $\alpha = 0.05$ , we can also reject  $H_0$  at the level  $\alpha = 0.1$
  2. The Type-I error is the probability to reject the null hypothesis when  $H_1$  is true
  3. If the p-value equals 0.15, we can reject the null hypothesis at the level 10%