Indian Institute of Space Science and Technology
AV336 - Digital Signal Processing Lab
Department of Avionics

Labsheet 7

For this labsheet, whereever you are asked to plot the magnitude response of low pass filters, you should plot the magnitude normalized with the gain at discrete frequency 0 in dB scale.

1. Given the following requirements on a filter in continuous time, manually derive the desired ideal frequency response $H_d(e^{j\omega})$ in the discrete frequency domain.

   - sampling frequency $= 8kHz$, and,
   - pass all signals below $1kHz$ with a gain of 1, and,
   - cutoff all signals above $1kHz$ (or the cutoff frequency is $\Omega_c = 1kHz$).

   Also derive the corresponding impulse response $h_d[n]$.

   - In general, what is the ideal lowpass filter response $H_d(e^{j\omega})$ that would also have linear phase for a cutoff frequency of ω_c?
   - What is the corresponding impulse response $h_d[n]$?

2. Let $w[n]$ be a rectangular window of length $M+1$. That is

$$w[n] \quad = \quad \begin{cases} 1, & \text{for } n \in \{0, 1, \ldots, M\}, \\ 0, & \text{otherwise.} \end{cases}$$

   (a) Plot the magnitude (in dB) and phase response of the window (i.e., $20log_{10}|W(e^{j\omega})|$ and $\angle W(e^{j\omega})$) for $M = 10, 50$ and 100. What do you observe for different values of M?

   (b) Demonstrate the effect of using rectangular windows with $M = 10, 50$ and 100 on the $h_d[n]$ obtained in Task 1. That is, for each M, obtain $h[n] = h_d[n]w[n]$ and plot the corresponding magnitude and phase plots of the DTFT of $h[n]$, i.e., $H(e^{j\omega})$. (Magnitude plot should be in dB scale). What do you observe? What are the values of the peak overshoots and undershoots for each value of M. Do they change as M changes? What do you need to do to ensure that for each value of M the phase response is linear?

3. Study what the following inbuilt Matlab window functions do:

   (a) bartlett
   (b) hamming
   (c) hanning
   (d) blackman
   (e) kaiser

4. In Task 3 above, you would have found that each function can be used to generate windows $w[n]$ of any needed length. If length $M+1$ windows are generated, then $w[n]$ has symmetry around $(M+1)/2$.

(a) For each window function above, plot the magnitude (in dB) and phase response of the window (i.e., $20log_{10}|W(e^{j\omega})|$ and $\angle W(e^{j\omega})$) for $M = 10, 50$ and 100. Observe and tabulate the maximum sidelobe amplitude and the width of the main lobe for each window and for each M.

(b) For each M and for each window function above, i.e., a $w[n]$, plot the magnitude (in dB) of the filter that would be obtained when using the FIR response $h[n] = w[n]h_d[n]$, where $h_d[n]$ is the desired response derived in Task 1. What is the peak approximation error (in dB) that you obtain for each window and for each M? Again make sure that each filter has a linear phase response.

5. Suppose one desires to design the following low pass filter (this is a specification of the desired response $H_d(e^{j\omega})$).

$$|H_d(e^{j\omega})| \text{ is } \begin{cases} \in [1-0.01, 1+0.01], \text{ for } 0 \le |\omega| \le 0.25\pi, \\ \in [0, \delta], \text{ for } |\omega| \ge 0.3\pi. \end{cases}$$

The transition band is $(0.25\pi, 0.3\pi)$.

(a) Obtain a complete specification of $H_d(e^{j\omega})$ so that we have a filter with linear phase response.

(b) Design a filter which meets the above specifications using either Hamming, Hanning, or Blackman windows separately for the cases $\delta = 0.01$ and $\delta = 0.001$. Use the width of the main lobe and the peak approximation error that you have found in Task 4 above for this (or you can refer to the table containing the main lobe width and peak approximation error that we had discussed in class).

(c) For each δ, plot the desired magnitude plot along with the magnitude plot of the filters that you have designed and comment on the differences. Also check whether the designed filters have linear phase responses.

6. Suppose one desires to design a filter using the Kaiser window method. We will use the $H_d(e^{j\omega})$ defined in Task 5 as the desired frequency response.

(a) Design a linear phase low pass filter which meets the above specifications using Kaiser window for the cases $\delta = 0.01$ and $\delta = 0.001$. Please use the design formulae (formulae for β and M) that we have studied in class.

(b) For each δ, plot the desired magnitude plot along with the magnitude plot of the filters that you have designed and comment on the differences. Also check whether the designed filters have linear phase responses.

7. Study what the Matlab inbuilt function “fir1” does. Use “fir1” to obtain a fir filter matching the desired response in Task 6.

8. Generate a signal $x[n] = cos(0.1\pi n) + 2cos(0.5\pi n)$ for $n \in \{0, 1, \ldots, 5M^*\}$ where M^* is the maximum length of the filters that you have designed in Tasks 5, 6, 7. For each of the filters that you have designed above in Tasks 5,6, and 7, obtain the signal $y[n]$ which results when $x[n]$ is passed through the filter. Plot $x[n]$ and $y[n]$ for all cases. Also plot their DTFT magnitudes, i.e., $|X(e^{j\omega})|$ and $|Y(e^{j\omega})|$. What do you observe?