Problem 1a

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$$\ell = \sum_{w \in V} \sum_{c \in V} \#(w,c)\left(\log(\sigma(v_w^T v_c)) + k\mathbb{E}_{c_n}[\log(\sigma(-v_w^T v_{c_n}))]\right).$$

↳ Break them apart to 2 terms.

$$\ell = \sum_{w \in V} \sum_{c \in V} \#(w,c)\log(\sigma(v_w^T v_c)) + \underbrace{\sum_{w \in V} \sum_{c \in V} \#(w,c)}_{\#(w) = \sum_{c' \in V} \#(w,c')} k\, \mathbb{E}_{c_n}[\log(\sigma(-v_w^T v_{c_n}))].$$

$$= \sum_{w \in V} \sum_{c \in V} \#(w,c)\log(\sigma(v_w^T v_c)) + \sum_{w \in V} k\#(w)\underbrace{\mathbb{E}_{c_n}[\log(\sigma(-v_w^T v_{c_n}))]}_{\sum_{c_n} \underbrace{P(c_n)}_{\frac{\#(c_n)}{|D|}} \log(\sigma(-v_w^T v_{c_n}))}$$

$$= \sum_{w \in V} \sum_{c \in V} \#(w,c)\log(\sigma(v_w^T v_c)) + \sum_{w \in V} k\#(w) \underbrace{\sum_{c_n} \frac{\#(c_n)}{|D|}\log(\sigma(-v_w^T v_{c_n}))}_{\text{from here, we're pulling an arbitrary } \hat{c}.}$$

$$\boxed{= \sum_{w \in V} \sum_{c \in V} \#(w,c)\log(\sigma(v_w^T v_c)) + \sum_{w \in V} k\#(w)\left(\frac{\#(\hat{c})}{|D|}\log(\sigma(-v_w^T v_{\hat{c}})) + \sum_{c_n \in V - \{\hat{c}\}} \frac{\#(c_n)}{|D|}\log(\sigma(-v_w^T v_{c_n}))\right)}$$

Problem 1a cont'd.

for specific (w, c) pair $w=1,\ c=1$

don't need to $\sum_{w \in V} \sum_{c \in V}$, & plus into equation that we got for global

$$l = \#(w,c)\log(\sigma(v_w^T v_c)) + k\,\#(w)\Big(\frac{\#(c)}{|D|}\log(\sigma(-v_w^T v_c)) + \underbrace{\sum_{c_n \in V - c} \frac{\#(c_n)}{|D|}\log(\sigma(-v_w^T v_{c_n}))}_{}\Big)$$

doesn't exist for specific pair. because we have only 1 pair we're looking @.

$$\boxed{l = \#(w,c)\log(\sigma(v_w^T v_c)) + \frac{k\,\#(w)\,\#(c)}{|D|}\log(\sigma(-v_w^T v_c))}$$

题 1 (b) & (c) together.

$x = v_w^T \cdot v_c$.

$$\ell(w,c) = \underbrace{\#(w,c)}_{A} \log(\sigma(x)) + \underbrace{\frac{k\,\#(w)\,\#(c)}{|D|}}_{B} \log(\sigma(-x))$$

$$= A \log(\sigma(x)) + B \log(\sigma(-x))$$

$$\frac{\partial \ell}{\partial x} = A \frac{1}{1+e^x} + B \frac{-e^x}{1+e^x}$$

$$= \frac{A}{1+e^x} - \frac{B e^x}{1+e^x} = 0.$$

$$\Rightarrow A - Be^x = 0 \Rightarrow e^x = A/B.$$

$$x = \log\left(\frac{A}{B}\right) = \log\left(\frac{\#(w,c)}{k\,\#(w)\,\frac{\#(c)}{|D|}}\right)$$

$$= \log\left(\frac{\#(w,c)\,|D|}{(\#(w)\,\#(c))} \cdot \frac{1}{k}\right)$$

$$= \log\left(\frac{\#(w,c)\,|D|}{\#(w)\,\#(c)}\right) - \log k$$

so

$$\boxed{x = v_w^T v_c = \log\left(\frac{\#(w,c)\,|D|}{\#(w)\,\#(c)}\right) - \log(k)}$$