# Lecture 16: Confidence intervals

## Criminology 1200


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# How to interpret p-values correctly

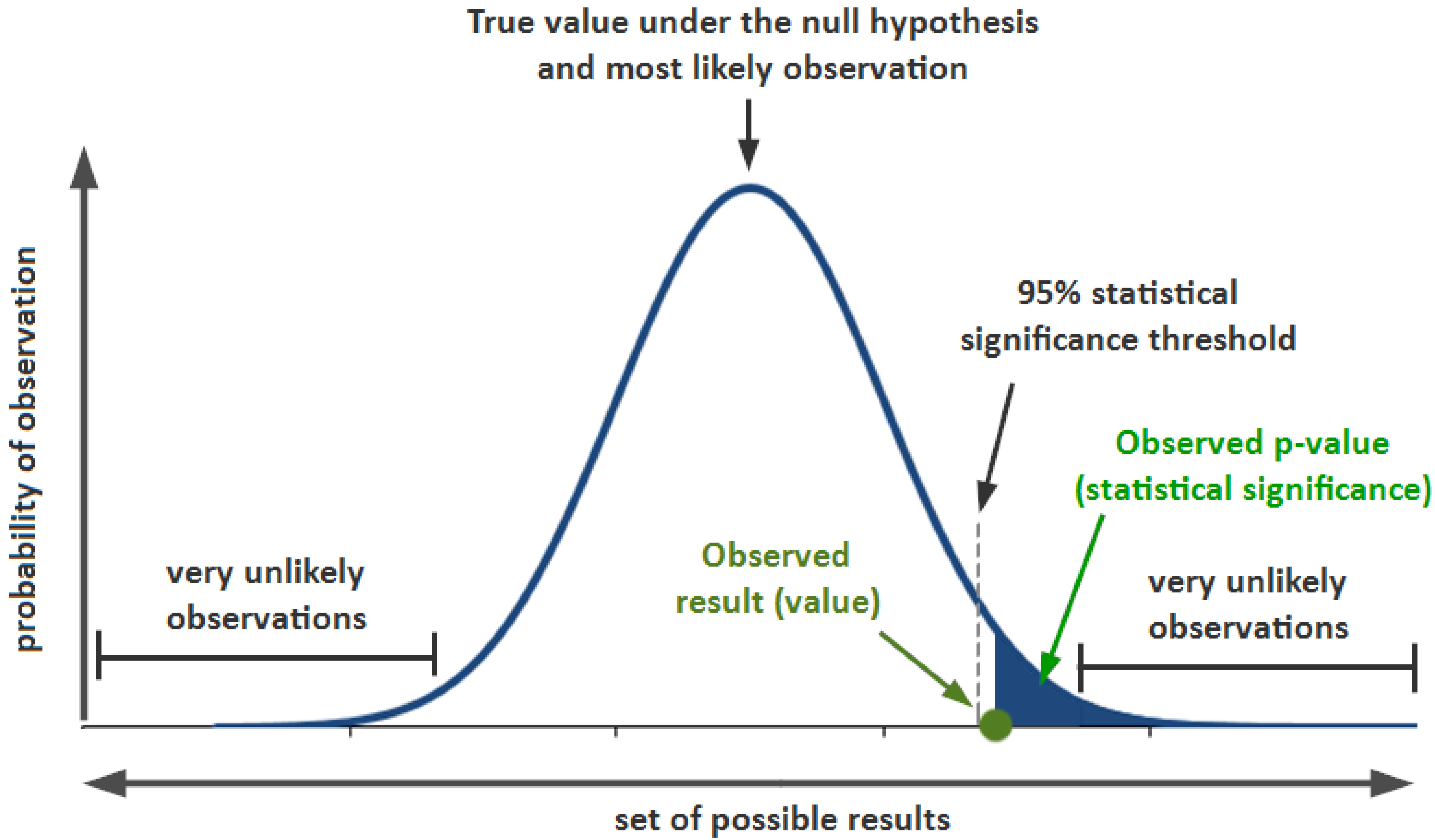
Correct:

- "If the model assumptions are satisfied, there is evidence that we can reject the null hypothesis in favor of the alternative at the 5% level."
- "If the model assumptions are satisfied, if we assume that there is no effect, you'd obtain the observed coefficient (or effect) or larger in 3% of studies because of random sample error.
- How probable are your sample data if the null hypothesis is correct? That's the only question that p-values answer.

Incorrect:

- $p<0.05$: "We can state that the observed result was not due to random chance with 95% confidence."
- $p=0.03$: "There's a 3% chance of making a mistake by rejecting the null hypothesis."
- The idea that p-values are the probability of making a mistake is WRONG! They are also not error rates.

# p-value


True value under the null hypothesis
and most likely observation
probability of observation
95% statistical
significance threshold
Observed p-value
(statistical significance)
very unlikely
observations
Observed
result (value)
very unlikely
observations
set of possible results

# Confidence intervals definition

Besides p-values, another way to express what the evidence of an experiment is telling us is to compute confidence intervals (CI):

$$CI = \text{Estimate} \pm \text{Margin of error} = (\text{Estimate} - ME,\ \text{Estimate} + ME).$$

# Confidence interval definiton for mean

- First: difference between a *parameter* (e.g. $\mu$) and a *sample estimate* (e.g. $\overline{X}$.

Definition (for a mean):

A (1- $\alpha$ )100% (e.g. 95%) confidence interval for the population parameter of the mean, $\mu$, is given by

$$\overline{X} \pm z_{\alpha/2} \times \frac{\sigma}{\sqrt{n}}$$

A 95% CI has $z_{\alpha/2} = 1.96 \approx 2$:

$$\overline{X} \pm 2 \times \frac{\sigma}{\sqrt{n}}$$

# Confidence interval definiton for proportion

Definition (for a proportion):

A (1- $\alpha$ ) 100% (e.g. 95%) confidence interval for the population parameter of a proportion, $p$, is given by

$$\hat{p} \pm 2\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}.$$

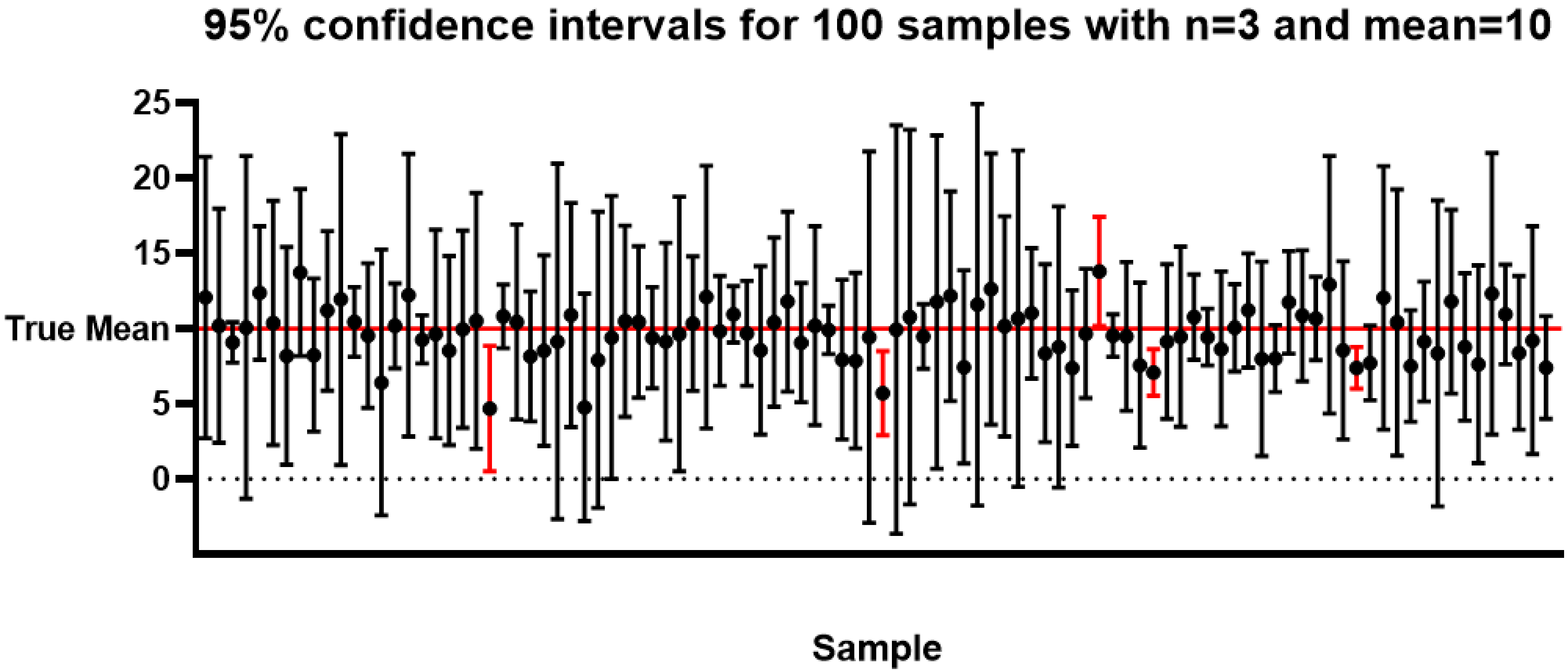
# Example of confidence interval

- Example: Every day, newspapers report opinion polls. For example, they might say that "83 percent of the population favor arming pilots with guns." Usually, you will see a statement like "this poll is accurate to within 4 points 95 percent of the time." They are saying that $83 \pm 4$ is a 95 percent confidence interval for the true but unknown proportion $p$ of people who favor arming pilots with guns. If you form a confidence interval this way every day for the rest of your life, 95 percent of your intervals will contain the true parameter. This is true even though you are estimating a different quantity (a different poll question) every day. (Wasserman, All of Statistics.)

# How to understand the meaning of 95% CI?

- Interpretation: If all the assumptions are met, and if we repeat the experiment many times, the *random* interval that we compute each time will contain the single, fixed, true parameter value 95% of the time.
- But we probably won't repeat the same experiment over and over, so this interpretation is pretty useless.
- Instead, it's also correct to say that if you use the same method of computing CIs, and you calculate CIs for many different samples of different datasets, then 95% of your intervals will trap the true parameter value. There is no need to introduce the idea of repeating the same experiment over and over. (Wasserman, All of Statistics.)

# Visualizing a confidence interval



# Confidence intervals from lm model

You can use R to translate the p-values from a linear model into confidence intervals:

```
dat <- read.csv(file = 'sim.data.csv')
reg.output <- lm(po.brut ~ funds, dat)
confint(reg.output)
```

```
##                  2.5 %     97.5 %
## (Intercept) 39.9859678 41.1001708
## funds       -0.3759656 -0.3582317
```

# How to interpret confidence intervals correctly

Correct:

- "95% of samples of this size will produce confidence intervals that capture the true proportions."
- "We are 95% confident that the true population parameter lies in our interval."

Incorrect:

- 95% CI (A to B), there is a 95% probability that the true population mean lies between A and B. (Actually, the mean is either in the interval or it's not. What is random is the interval itself.)
- Everything we have seen in this class and these slides so far is frequentist (or classical) statistics. There is also another branch of statistics that is Bayesian.
- Bayesian credible intervals are what we wish frequentist intervals did: "The probability that $\mu$ is in the confidence interval, given the data, is 95%." Note: Bayesian intervals will not, in general, trap the parameter 95% of the time.

# Relationship between p-values and confidence intervals

- CIs and p-values are closely related although they provide different information. While p-values are the outcome of hypothesis tests and indicate whether or not the sample data provide sufficient evidence to reject the null hypothesis, CIs indicate whether the estimate is a precise one or only a very “rough” estimate.
- Usually CIs are more informative than p-values.
- How are they calculated? If the underlying data are normally distributed (or have a large sample size for a sum or mean, so we can call the CLT), then a confidence interval for a statistic is the statistic plus or minus the appropriate "multiplier" times the esimated standard error of the quantity.
- The multiplier depends on the desired confidence level (e.g., 95, 90), and the degrees of freedom for the standard error.
- The extent of the interval on either side of the estimate is called the margin of error (ME).