2017-1 Computer Algorithms Homework #4 (Midterm Exam)

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April 17, 2017

(Deadline : April 24)

1. If $n \in \mathbb{N}$ then $(n+4)^4 + 4$ is not in PRIME. Prove or disprove.
2. Prove by mathematical induction that for any integer $n > 23$ there exist nonnegative integers x and y such that
$$n = 7x + 5y$$
3. Prove by mathematical induction that for $n > 1$
$$1 \times 1! + 2 \times 2! + 3 \times 3! + \cdots + n \times n! = (n+1)! - 1$$

4. Using your favorite computer programming language (but C/C++, Java, Python, C# recommended), write programs that calculate the following.

   Mina is playing a piano imported from wonderland. The piano from wonderland is very different from the pianos in the Earth.

   It has only white keys, each of which is numbered from 1 to N. (i.e. N is the number of keys)

   When Mina presses one key, e.g. key 8, then the key 8 disappears. And the keys adjacent to the pressed key 8, that is 7 and 9, will move down. After that, the keys adjacent to the downed keys 7 and 9, therefore 6 and 10, will move down. This will be repeated until there is no adjacent keys because the key number is 1, or N, or the adjacent key has been disappeared. Therefore, Mina has to push each of the downed keys up, in order to press the other keys.

   For example, if there are 10 keys, and Mina wants to press 5 and 3.

   When she press the key 5, the key 5 disappears. 4 and 6 move down. 3 and 7 move down. 2 and 8 move down. 1 and 9 move down. Finally, 10 moves down. Mina pushes up nine keys, (1,2,3,4,6,7,8,9, and 10) back to their original positions.

   Then she presses the key 3. The key 3 disappears. 2 and 4 move down. After that only key 1 moves down because the key 5 is not there. (It disappeared before.) Mina pushes up three keys, (1,2, and 4) back to their original positions.

   Therefore, if she presses 5 and 3 from 10 keys, then she has to push up 12 (=9+3) keys. But, if she presses 3 and 5 from 10 keys, then she has to push up 15 (=9+6) keys.

   To help Mina, given the total number of keys (N) and keys to press (K and K numbers), write a program that calculate the minimum number of pushes. The running time of the program should be less than 2 seconds. $1 \leq N \leq 10,000$, and $1 \leq K \leq 100$

```
Input
10
2
3 5

Output
12

Input
10000
100
6428 1760 7536 901  7983 2452 8818 1241 4174 5426
274  1685 4666 5599 3485 6977 8181 8131 2965 4746
9193 4275 6063 1498 7192 824  4068 1990 9423 754
184  641  5695 6995 3449 8393 6605 6665 700  2518
5566 8677 4309 520  4467 8123 6756 4768 4905 5187
6494 6236 2404 9400 8492 5138 2820 6205 9763 5941
6489 8629 2695 8958 4473 5568 2696 7112 8121 8077
8156 4251 4361 3712 8014 5522 4430 6377 1441 4641
5954 9233 1906 565  3702 9951 5862 25   3822 3519
7634 8509 7264 4924 4032 6062 1702 541  7316 6870

Output
62842
```

5. Using your favorite computer programming language (but C/C++, Java, Python, C# recommended), write programs that calculate the following.

   Given $n \times n, (1 \leq n \leq 30)$ matrix X and $k, (1 \leq k \leq 10^9)$, calculate $R = X + X^2 + \cdots + X^k$ and print R.

   To avoid overflow problem, instead of printing each entry in r_{ij} of R, print the remainder of dividing r_{ij} by 32767.

   The running time of the program should be less than 2 seconds.

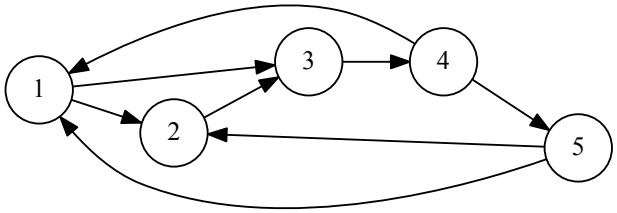
```
Input
n=3
k=4
X={{1,2,3},{4,5,6},{7,8,9}}

Output
R={{8059, 9902, 11745},{18250, 22424, 26598},{28441, 2179, 8684}}
```

6. Using your favorite computer programming language (but C/C++, Java, Python, C# recommended), write programs that calculate the following.

   Given a directed graph $G = (V, E)$, where $1 \leq |V| \leq 100$, calculate the number of all distinct paths with length k, where $1 \leq k \leq 10^9$.

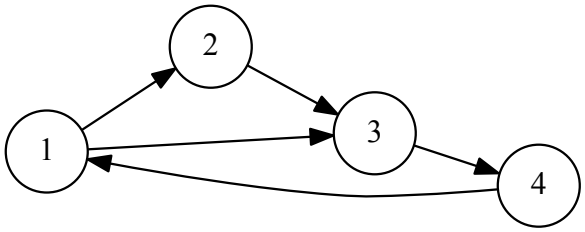
   The running time of the program should be less than 2 seconds.

   For example, for the following graph and $k = 3$:




```
Output
17
```

   Another example for the following graph and $k = 2$:




```
Output
6
```

7. Using your favorite computer programming language (but C/C++, Java, Python, C# recommended), write programs that calculate the following.

   One Korean chaebol company tries to hire $1 \leq A \leq 10,000$ number of men and $1 \leq B \leq 10,000$ number of women. It costs \$10,000 to hire one person.

   There is intimacy $1 \leq d \leq 9999$ between man and woman. So if there is intimacy d between a man and a woman and if one is hired, then the other can be hired with \$10,000$-d$. One person can have intimacies with multiple people, but we can use only one intimacy when hiring that particular person. The total number of intimacies D can be $1 \leq D \leq 50,000$.

   Given A, B, D, and D number of (a, b, d)'s where a is a man, b is a woman, and d is their intimacy, calculate the minimum cost to hire all A men and B women.

```
Input
5 5 8
0 0 6590
0 1 3073
1 3 4573
1 3 2149
2 2 789
3 3 975
4 2 204
4 3 631

Output
83165

Input
5 5 10
0 4 4372
2 0 16
2 0 146
2 4 326
2 4 2133
2 4 9220
3 1 8364
3 2 6336
3 4 8833
4 1 2339

Output
60390
```

8. Using your favorite computer programming language (but C/C++, Java, Python, C# recommended), write programs that calculate the following.

   In a certain near future, Korea is united and we can drive to Europe from Korea. The straight line distance from Busan to Paris is about 9,307 km.

   Mina is considering moving from Busan to a certain city. She is thinking of renting a truck for moving. The fuel price is very high, so she wants to minimize the fuel price for the movement.

   For easy calculation, let's assume that the truck consumes 1 km/l, and the truck can contain upto 200 liters of fuel. When Mina rents the truck at Busan, the truck tank has a half of its capacity. When she returns the truck at the destination, she has to fill the fuel as much as the half of its capacity, otherwise she has to pay a lot of money to the rental company for the fuel handling charge. Mina wants to minimize the fuel cost, but doesn't want to run out of fuel during the trip.

   Input will have the distance in kilometers ($\leq 10,000$) in the first line. The second line is the number of filling stations (≤ 100) . From the third line, we have information about the filling stations, sorted by the distance from Busan. The information has two integers: the distance from Busan ($\leq 10,000$) and fuel price per liter in Korean Won ($\leq 2,000$).

   Output will be the cost for the minimum fuel needed. If it is impossible to go to the city because of input condition, output "impossible".

   The running time of the program should be less than 2 seconds.

```
Input
500
9
100 1200
150 1400
200 1300
250 1000
300 1100
350 1400
400 1300
450 1000
500 1399

Output
530000
```

9. (a) What is an optimal Huffman code for the following set of frequencies, based on the first 8 Fibonacci numbers?

   `a:1 b:1 c:2 d:3 e:5 f:8 g:13 h:21`

   (b) Generalize the answer for the optimal code when the frequencies are the first n Fibonacci numbers.

   What is a Huffman code for the kth character of n characters?

   (c) Using pseudo code, present Huffman algorithm for binary codes.

   (d) Using pseudo code, present Huffman algorithm for ternary codes.

10. The diameter of a tree $T = (V, E)$ is given by $\max_{u,v \in V} \delta(u, v)$, where $\delta(u, v)$ is the shortest-path distance from u and v.

    Therefore the diameter of a tree is the largest if all shortest-path distances in the tree.

    Provide an efficient algorithm to computer a diameter of a tree, and analyze the running time of your algorithm.