MAD3703, Numerical Analysis I, MWF, 2:30-3:20, room Love 102
Lecture February 23:

Newton's method in class group work :

$$
\begin{aligned}
f(x) &= 0 \\
f(x) = x^4 - 10 \quad f'(x) &= 4x^3 \\
p_{n+1} = p_n &- \frac{f(p_n)}{f'(p_n)}
\end{aligned}
$$

Suppose $p_0 = 1.0$:

$$
\begin{aligned}
p_1 &= 1 - \frac{1^4 - 10}{4(1)^3} = 3.25 \\
p_2 &= 3.25 - \frac{3.25^4 - 10}{4(3.25)^3} = 2.51033 \\
p_3 &= 2.51033 - \frac{2.51033^4 - 10}{4(2.51033)^3} = 2.04078 \\
p_4 &= 2.04078 - \frac{2.04078^4 - 10}{4(2.04078)^3} = 1.82472 \\
p_5 &= 1.82472 - \frac{1.82472^4 - 10}{4(1.82472)^3} = 1.78002
\end{aligned}
$$

Polynomial interpolation or piecewise polynomial interpolation (spline interpolation) :

given (x_i, y_i), $i = 0, \dots, n$ find $P_n(x)$ such that $P_n(x_i) = y_i$, $i = 0, \dots, n$. Polynomial interpolation is used to either (i) replace a given $f(x)$ with a simpler function to handle, or (ii) provide compact representation of available data (this latter case is more appropriately called "approximation").

Lagrange basis polynomial :

$$
L_{n,i}(x) \equiv \Pi_{j=0, j\neq i}^{n} \frac{x - x_j}{x_i - x_j}
$$

Lagrange interpolating polynomial :

$$
f(x) \approx P_n(x) = \sum_{i=0}^{n} f(x_i) L_{n,i}(x)
$$

Note: (Theorem 3.2) polynomial interpolation is unique.
Note: One can use matrix technique to find interpolant.

example: find $P_n(x)$ if $f(x) = e^x$ and x_i given :

Error in polynomial interpolation :

$$f(x) = P_n + R_n(x)$$
$$R_n(x) = \frac{f^{(n+1)}(\xi(x))}{(n+1)!} \Pi_{j=0}^{n} (x - x_j)$$

Example: bound on error for linear interpolation? quadratic? :
use $f(x) = e^x$ as test function. What if test function was discontinuous?