Lecture January 7:

Taylor series expansion

$$f(x) = P_n(x) + R_n(x) \tag{1}$$

$$P_n(x) = \sum_{i=0}^{n} \frac{f^{(i)}(x_0)}{i!}(x - x_0)^i \tag{2}$$

$$R_n(x) = \frac{f^{(n+1)}(\xi(x))}{(n+1)!}(x - x_0)^{n+1} \tag{3}$$

find bound on error over interval $0.5 < x < 1.5$

$$f(x) = (x-1)\ln x \qquad x_0 = 1 \tag{4}$$

$$f'(x) = \ln x + 1 - 1/x \tag{5}$$

$$f''(x) = 1/x + 1/x^2 \tag{6}$$

$$f'''(x) = -1/x^2 - 2/x^3 \tag{7}$$

$$f^{(4)}(x) = 2/x^3 + 6/x^4 \tag{8}$$

$$P_3(x) = 0 + 0(x-1) + 2(x-1)^2/2 - 3(x-1)^3/6 = \tag{9}$$

$$(x-1)^2 - (x-1)^3/2 \tag{10}$$

$$R_3(x) = \frac{f^{(4)}(\xi(x))}{4!}(x-1)^4 \tag{11}$$

$$g(x) = 2/x^3 + 6/x^4 \tag{12}$$

$$g(1/2) = 16 + 96 = 112 \tag{13}$$

$$g(3/2) = 1.7778 \tag{14}$$

$$g'(x) = -6/x^4 - 24/x^5 = 0 \tag{15}$$

$$-x - 4 = 0 \quad x = -4 \tag{16}$$

$$\max_{1/2<x<3/2} |R_3(x)| = (112/24)(3/2-1)^4 = 0.292 \tag{17}$$

find bound on error at a single point $0.5 < x < 1.5$**; e.g.** $x = 3/4$

$$R_3(x) = \frac{f^{(4)}(\xi(x))}{4!}(x-1)^4 \tag{18}$$

$$|R_3(x)| \leq \max_{3/4 \leq x \leq 1} |\frac{f^{(4)}(x)}{4!}(3/4-1)^4| \tag{19}$$

At $x = 3/4$ the actual error is

$$|f(3/4) - P_3(3/4)| = |(3/4-1)\ln(3/4) - 1/16 - 1/128| = 0.0016 \tag{20}$$

The error bound is:

$$|R_3(x)| \leq (80/81)(3/4-1)^4 = 0.0039 \tag{21}$$