MAD3703, Numerical Analysis I, MWF, 2:30-3:20, room Love 102
Lecture March 27:

1. From last class:

a. find the cubic Hermite interpolant that interpolates $f(x) = x^4$ at the points $x_0 = 0$ and $x_1 = h$:

$$f(0) = 0 \quad f'(0) = 0$$
$$f(h) = h^4 \quad f'(h) = 4h^3$$

z_k	$f[z_k]$	$f[z_k, z_{k+1}]$	$f[z_k, \ldots, z_{k+2}]$	$f[z_k, \ldots, z_{k+3}]$
0.0	0.0			
0.0	0.0	0.0		
h	h^4	h^3	h^2	
h	h^4	$4h^3$	$3h^2$	$2h$

$$H(x) = 0 + 0(x-0) + h^2(x-0)^2 + 2h(x-0)^2(x-h)$$

b. find a bound on the error for using $H(x)$ to approximate $f(x)$ and determine how small h must be in order that the error be bounded by 10^{-10}:

$$R(x) = \frac{f^{(4)}(\xi(x))}{4!}(x-0)^2(x-h)^2$$

$$|R(x)| \leq \max_{0 \leq x \leq h} \frac{|f^{(4)}(x)|}{24} \max_{0 \leq x \leq h} |(x-0)^2(x-h)^2|$$

Define $g(x) = (x-0)^2(x-h)^2$, then,

$$g(0) = g(h) = 0$$
$$g'(x) = 2x(x-h)^2 + 2(x-h)x^2 = 2x(x-h)(x-h+x) = 0$$
$$2x = h \quad x = h/2$$
$$g(h/2) = h^4/16$$

Error bound is:

$$\frac{Mh^4}{384} \quad M = \max_{0 \leq x \leq h} |f^{(4)}(x)|$$

Since $f^{(4)}(x) = 24$, this means $M = 24$ and a bound on the error is:

$$\frac{Mh^4}{384} = h^4/16$$

The error is guaranteed less than 10^{-10} if

$$h^4/16 < 10^{-10}$$
$$h < ((16)10^{-10})^{1/4} = 0.0063$$

2. parameterization for a square:

$$x(t) = \begin{cases} t & 0 \le t < 1 \\ 1 & 1 \le t < 2 \\ 3-t & 2 \le t < 3 \\ 0 & 3 \le t \le 4 \end{cases}$$

$$x(t+4) = x(t)$$

$$y(t) = \begin{cases} 0 & 0 \le t < 1 \\ t-1 & 1 \le t < 2 \\ 1 & 2 \le t < 3 \\ 4-t & 3 \le t \le 4 \end{cases}$$

$$y(t+4) = y(t)$$

3. parameterization for a circle?

4. in class group work: find the parameterization for a particle that first moves from $(0,0)$ to $(0,1)$, then moves counter clockwise in a circular arc of radius one until the particle reaches the point $(1,0)$, then moves back to the origin $(0,0)$.