Project 2 solutions

1a.

$$452 - 2^8 = 196 \quad 196 - 2^7 = 68 \quad 68 - 2^6 = 4$$
$$4 - 2^2 = 0$$
$$452 = 111000100 = 1.11000100 \times 2^8$$
$$c = 135 \quad f = .1100010000 \quad s = 0$$
$$135 - 2^7 = 7 \quad 7 - 2^2 = 3 \quad 3 - 2^1 = 1$$
$$c = 10000111$$

1b.

$$0.00321 - 2^{-9} = 0.001256875 \quad 0.001256875 - 2^{-10} = 0.0002803125$$
$$0.0002803125 - 2^{-12} = 0.0000361719 \quad 0.0000361719 - 2^{-15} = 0.0000056543$$
$$0.0000056543 - 2^{-18} = 0.0000018396$$
$$0.0000018396 - 2^{-20} = 0.00000088593$$
$$0.00321 = 1.10100100101 \times 2^{-9}$$
$$c - 127 = -9 \quad c = 118 \quad f = .10100100101 \quad s = 0$$
$$118 - 2^6 = 54 \quad 54 - 2^5 = 22 \quad 22 - 2^4 = 6$$
$$6 - 2^2 = 2 \quad 2 - 2^1 = 0$$
$$c = 01110110$$

1c.

$$21.66 - 2^4 = 5.66 \quad 5.66 - 2^2 = 1.66 \quad 1.66 - 2^0 = 0.66$$
$$0.66 - 2^{-1} = 0.16 \quad 0.16 - 2^{-3} = 0.035 \quad 0.035 - 2^{-5} = 0.00375$$
$$0.00375 - 2^{-9} = 0.001796875$$
$$-21.66 = -1.0101101010001 \times 2^4$$
$$c = 131 \quad f = .0101101010001 \quad s = 1$$
$$131 - 2^7 = 3 \quad 3 - 2^1 = 1 \quad 1 - 2^0 = 0$$
$$c = 10000011$$

2.

$$\lim_{h \to 0} \frac{\ln(x+1+h) - \ln(x+1)}{h} = \tag{1}$$
$$\lim_{h \to 0} \frac{\frac{1}{1+x+h}}{1} = \frac{1}{1+x} \tag{2}$$

$$N(0.01) = \frac{\ln(2.01) - \ln(2.00)}{0.01} = \frac{0.698 - 0.693}{0.01} = 0.500 \tag{3}$$

$$P_3(h) = 0 + \frac{1}{1+x}h - \frac{1}{(x+1)^2}\frac{h^2}{2} + \frac{2}{(x+1)^3}\frac{h^3}{6} \quad (4)$$

$$N(h) \approx \frac{1}{1+x} - \frac{1}{(x+1)^2}\frac{h}{2} + \frac{2}{(x+1)^3}\frac{h^2}{6} = 0.500 - 0.00125 + 4.2E-6 \approx 0.498 \quad (5)$$

Relative error without Taylor series trick:

$$|0.500 - 0.4987541511|/0.4987541511 = 0.0025 \quad (6)$$

Relative error with Taylor series trick:

$$|0.498 - 0.4987541511|/0.4987541511 = 0.0015 \quad (7)$$

3.

$$M(0) = \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} =$$
$$\lim_{h\to 0} \frac{(1/2)(x+h)^{-1/2}}{1} = \frac{1}{2\sqrt{x}}$$

$$fl(M(h)) \approx \frac{\sqrt{x+h}-\sqrt{x}}{h} + \frac{2\epsilon}{h} \approx$$
$$\frac{1}{2\sqrt{x}} - \frac{1}{8}x^{-3/2}h + \frac{2\epsilon}{h}$$
$$|fl(M(h)) - M(0)| \approx |-\frac{1}{8}x^{-3/2}h + \frac{2\epsilon}{h}| \leq \frac{Mh}{2} + \frac{2\epsilon}{h}$$

since $x = 1$,

$$M \approx \frac{1}{4}1^{-3/2} = \frac{1}{4}$$

Define $g(h) = \frac{Mh}{2} + \frac{2\epsilon}{h}$.

$$g'(h) = \frac{M}{2} - \frac{2\epsilon}{h^2} = 0$$
$$\epsilon = \frac{Mh_{crit}^2}{4}$$