Project 3 solutions

1. Let $f(x) = (x-3)^4(x-2)^3(x-1)x^2(x+1)(x+2)^4(x+3)^3$. To which zero of f does the bisection method converge when applied on the interval $[-2.3, 3.1]$?

$$\begin{aligned}
p_1 &= (-2.3+3.1)/2 = 0.4 \\
f(0.4) &= (+)(-)(-)(+)(+)(+)(+) = (+) \\
f(-2.3) &= (+)(-)(-)(+)(-)(+)(+) = (-) \\
f(3.1) &= (+)(+)(+)(+)(+)(+)(+) = (+) \\
p_2 &= (-2.3+0.4)/2 = -0.95 \\
f(-0.95) &= (+)(-)(-)(+)(+)(+)(+) = (+) \\
f(0.4) &= (+)(-)(-)(+)(+)(+)(+) = (+) \\
f(-2.3) &= (+)(-)(-)(+)(-)(+)(+) = (-) \\
p_3 &= (-2.3-0.95)/2 = -1.625 \\
f(-1.625) &= (+)(-)(-)(+)(-)(+)(+) = (-) \\
f(-2.3) &= (+)(-)(-)(+)(-)(+)(+) = (-) \\
f(-0.95) &= (+)(-)(-)(+)(+)(+)(+) = (+)
\end{aligned}$$

The only root in the interval $[-1.625, -0.95]$ is $p = -1$. The bisection method will converge to $p = -1$.

2. Let

$$\begin{aligned}
g(x) &= \frac{4x}{5} + \frac{2}{5x^4} \\
x &= \frac{4x}{5} + \frac{2}{5x^4} \\
x/5 &= \frac{2}{5x^4} \\
x^5 &= 2 \\
x &= 2^{1/5}
\end{aligned}$$

try $[a, b] = [1, 2]$,

$$\begin{aligned}
g(1) &= 4/5 + 2/5 = 6/5 \in [1, 2] \\
g(2) &= 8/5 + 1/40 = 65/40 = 13/8 \in [1, 2] \\
g'(x) &= 4/5 - 8/(5x^5) = 0 \\
4/5 &= 8/(5x^5) \\
x^5 &= 2 \\
x &= 2^{1/5} \\
g(2^{1/5}) &= 2^{1/5} \in [1, 2]
\end{aligned}$$

$$g'(1) = 4/5 - 8/5 = -4/5$$
$$g'(2) = 4/5 - 1/20 = 15/20 = 3/4$$
$$g''(x) = 40/(5x^6) \neq 0$$
$$|g'(x)| \leq 4/5 = k < 1$$
$$|p_n - p| \leq k^n \max(p_0 - a, b - p_0)$$
$$(4/5)^n(1) \leq 10^{-10}$$
$$10^{10} \leq (5/4)^n$$
$$\log(10^{10}) \leq n \log(5/4)$$
$$n \geq \log(10^{10})/\log(5/4) = 103.2$$