MAD3703, Numerical Analysis I, MWF, 2:30-3:20, room Love 102
Lecture February 11:

fixed point method :

1. p is a fixed point of g if $g(p) = p$.
2. example: find the fixed points of
$$g(x) = x - \frac{x^3 - 2}{3x^2} \tag{1}$$
3. theorem 2.3: if g maps an interval $[a, b]$ onto itself, then a fixed point exists. If g is furthermore a contraction mapping, i.e. $|g'(x)| \leq k < 1$, then the fixed point is unique.
4. example: show (using theorem 2.3) that a unique fixed point for $g(x) = x - \frac{x^3-2}{3x^2}$ is guaranteed for the interval $[1, 2]$. $[1, 3]$? $[1, 4]$?
5. theorem 2.4: the fixed point iteration,
$$p_n = g(p_{n-1}) \tag{2}$$
is guaranteed to converge if g maps $[a, b]$ onto itself, g is a contraction mapping on $[a, b]$ and p_0 is from $[a, b]$.
6. Corollary 2.5: $|p_n - p| \leq k^n \max(p_0 - a, b - p_0)$.
7. example: how many iterations guarantee an accuracy of 10^{-10} for $g(x) = x - \frac{x^3-2}{2x}$ and $p_0 = 2$?

In class group work for Feb 11 :

Do 4 iterations of the fixed point method in order to approximate $3^{1/3}$. Find an interval $[a, b]$ in which g is a contraction mapping on the interval and maps the interval onto itself. How many iterations needed to get an accuracy of 10^{-8}?