MAD3703, Numerical Analysis I, MWF, 2:30-3:20, room Love 102
Lecture Feburary 6:

BISECTION METHOD

example $f(x) = x^2 - 2$, $a = 0$, $b = 2$

$$a_1 = 0 \quad b_1 = 2 \quad p_1 = 1 \tag{1}$$
$$a_2 = 1 \quad b_2 = 2 \quad p_2 = 1.5 \tag{2}$$
$$a_3 = 1 \quad b_3 = 1.5 \quad p_3 = 1.25 \tag{3}$$
$$a_4 = 1.25 \quad b_4 = 1.5 \quad p_4 = 1.375 \tag{4}$$
$$\ldots \tag{5}$$

example $f(x) = (x+2)(x+1)^2 x(x-1)^3(x-2)$, $a = -1.5$, $b = 2.5$

$$a_1 = -1.5 \quad f(a_1) = (+)(+)(-)(-)(-) < 0 \tag{6}$$
$$b_1 = 2.5 \quad f(b_1) = (+)(+)(+)(+)(+) > 0 \tag{7}$$
$$p_1 = 0.5 \quad f(p_1) = (+)(+)(+)(-)(-) > 0 \tag{8}$$
$$a_2 = -1.5 \quad f(a_1) = (+)(+)(-)(-)(-) < 0 \tag{9}$$
$$b_2 = 0.5 \quad f(b_2) = (+)(+)(+)(-)(-) > 0 \tag{10}$$
$$p_2 = -0.5 \quad f(p_2) = (+)(+)(-)(-)(-) < 0 \tag{11}$$
$$a_3 = -0.5 \quad f(a_3) = (+)(+)(-)(-)(-) < 0 \tag{12}$$
$$b_3 = 0.5 \quad f(b_3) = (+)(+)(+)(-)(-) > 0 \tag{13}$$
$$\ldots \tag{14}$$

root converges to $p = 0$. Why?

example: "how big n in order to guarantee $|p - p_n| < \epsilon$?" :

1. Using method of induction, one can show that $|b_n - a_n| = |b_1 - a_1|/2^{n-1}$.
2. Since $p_n = (a_n + b_n)/2$, one must have $|p - p_n| \le |b_n - a_n|/2$.
3. $$|p - p_n| \le |b_n - a_n|/2 = |b_1 - a_1|/2^n \tag{15}$$
4. Suppose hypothetically that $f(x) = x^2 - 2$, $a = 0$, $b = 2$, and $\epsilon = 10^{-10}$. Then solve the following equation for n:

$$|b - a|/2^n \le \epsilon \tag{16}$$
$$2/2^n \le 10^{-10} \tag{17}$$
$$2^{n-1} \ge 10^{10} \tag{18}$$
$$n - 1 \ge \log(10^{10})/\log(2) \tag{19}$$
$$n \ge 37 \tag{20}$$

FIXED POINT METHOD The root finding problem $f(x) = 0$ can be transformed into an equivalent fixed point problem $x = g(x)$ in which the fixed point algorithm is applied:

$$p^{(k+1)} = g(p^{(k)}). \tag{21}$$

As long as g is a contraction mapping on some interval $[a, b]$, g maps this interval onto itself, and $p^{(0)}$ in the interval, then the fixed point method is guaranteed to converge.

Example application of fixed point method $f(x) = 0$ where $f(x) = x^3-2$ is the same as $g(x) = x$ where $g(x) = (2/3)x + 2/(3x^2)$.

In class group work for February 6 How many iterations of the bisection method guarantee that the root $x^{(k)}$ is within ϵ of the actual root for the problem $f(x) = 0$ where $f(x) = x^5 - 2$ and a valid bracketing interval is the starting interval.