Obrada informacija

4. Laboratorijska vježba: Multivarijatni financijski vremenski nizovi

Prosinac 2023.

Upute

U ovoj bilježnici dana je priprema sa svim uputama za 4. laboratorijsku vježbu iz predmeta Obrada informacija - uz bilježnicu su dostupni i podatci u datoteci `prices.csv`.

Vaš zadatak je u bilježnicu na odgovarajuća mjesta dopisati kod Vašeg rješenja,.

Riješenu bilježnicu potrebno je predati kao izvještaj u .pdf formatu na *Moodle* najkasnije do 14.1.2024. u 23:59h. Datoteka koju predajete se mora zvati *PrezimeImeJMBAG.pdf*.

Uvod

U laboratorijskoj vježbi razmatra se dinamika cijena vrijednosnica na financijskim tržištima. Dane su povijesne dnevne cijene 24 ETF-a (eng. *exchange traded fund*) koji prate određene dioničke, obvezničke ili druge indekse.

Oznaka	Naziv	Klasa imovine	
SPY	SPDR S&P 500 ETF Trust	Equity: U.S. - Large Cap	
IEFA	iShares Core MSCI EAFE ETF	Equity: Developed Markets Ex-U.S. - Total Market	
VWO	Vanguard FTSE Emerging Markets ETF	Equity: Emerging Markets - Total Market	
EWJ	iShares MSCI Japan ETF	Equity: Japan - Total Market	
XLF	Financial Select Sector SPDR Fund	Equity: U.S. Financials	
XLK	Technology Select Sector SPDR Fund	Equity: U.S. Technology	
XLV	Health Care Select Sector SPDR Fund	Equity: U.S. Health Care	
XLY	Consumer Discretionary Select Sector SPDR Fund	Equity: U.S. Consumer Cyclicals	
XLP	Consumer Staples Select Sector SPDR Fund	Equity: U.S. Consumer Non-cyclicals	
XLU	Utilities Select Sector SPDR Fund	Equity: U.S. Utilities	
XLI	Industrial Select Sector SPDR Fund	Equity: U.S. Industrials	
XLE	Energy Select Sector SPDR Fund	Equity: U.S. Energy	
XLC	Communication Services Select Sector SPDR Fund	Equity: U.S. Telecommunications	
XLRE	Real Estate Select Sector SPDR Fund	Equity: U.S. Real Estate	
XLB	Materials Select Sector SPDR Fund	Equity: U.S. Basic Materials	
BND	Vanguard Total Bond Market ETF	Fixed Income: U.S. - Broad Market	Broad-based Investment Grade
LQD	iShares iBoxx USD Investment Grade Corporate Bond ETF	Fixed Income: U.S. - Corporate	Broad-based Investment Grade

Oznaka	Naziv	Klasa imovine	
BNDX	Vanguard Total International Bond ETF	Fixed Income: Global Ex-U.S. - Broad Market	Broad-based Investment Grade
SHV	iShares Short Treasury Bond ETF	Fixed Income: U.S. - Government	Treasury Investment Grade Ultra-Short Term
HYG	iShares iBoxx USD High Yield Corporate Bond ETF	Fixed Income: U.S. - Corporate	Broad-based High Yield
GLD	SPDR Gold Trust	Commodities: Precious Metals Gold	
SLV	iShares Silver Trust	Commodities: Precious Metals Silver	
PDBC	Invesco Optimum Yield Diversified Commodity Strategy No K-1 ETF	Commodities: Broad Market	
USO	United States Oil Fund LP	Commodities: Energy Crude Oil	

Pri modeliranju zajedničkog kretanja i rizika vrijednosnica, koristit ćemo aritmetičke povrate:

$$R(t) = \frac{S(t)-S(t-1)}{S(t-1)},$$

gdje je $S(t)$ cijena vrijednosnice u danu t. U sklopu ove laboratorijske vježbe cilj je analizirati kretanje danih ETF-ova i izračunati glavne komponente (PCA) koje utječu na njihovu dinamiku. Laboratorijsku vježbu je potrebno riješiti unutar ove bilježnice i predati riješenu bilježnicu kao izvještaj.

In [ ]:

```python
import pandas as pd

prices = pd.read_csv('prices.csv')
prices.set_index('Time', inplace=True)
prices.index = pd.to_datetime(prices.index)

prices.head()
```

Out[ ]:

Time	SPY	IEFA	VWO	EWJ	XLF	XLK	XLV	XLY	XLP	XLU	...	XLB	BND	LQD	BNDX	SHV	HYG	GLD	SLV	PDBC	
2019-01-02	231.492233	48.000053	33.417080	47.497765	21.776472	59.029892	78.483353	95.259102	44.591763	44.784126	...	46.068901	69.980576	97.039024	48.517841	101.234894	63.253788	121.330002	14.56	8.545244	78.8
2019-01-03	225.968170	47.684727	32.893154	47.227894	21.286907	56.050797	76.889748	93.196533	44.335598	44.775520	...	44.762505	70.253906	97.081886	48.517841	101.271584	63.285011	122.430000	14.75	8.607411	79.5
2019-01-04	233.537125	49.129986	33.932262	48.828522	21.994057	58.534962	79.183434	96.280815	45.280781	45.438545	...	46.522518	70.042305	97.021873	48.473194	101.299133	64.347198	121.440002	14.73	8.731748	81.4
2019-01-07	235.378525	49.366474	34.002121	48.949490	22.021255	59.058445	79.487411	98.457954	45.218948	45.128563	...	46.685822	69.936508	97.030441	48.357071	101.299133	64.971985	121.860001	14.67	8.799566	82.3
2019-01-08	237.589920	49.716839	34.194221	49.182148	22.039383	59.553371	80.104607	99.546509	45.634129	45.688255	...	47.175724	69.848312	97.244827	48.321350	101.289948	65.393723	121.529999	14.69	8.873038	84.0

5 rows × 24 columns

Zadatak 1 - Računanje korelacijske matrice i matrice kovarijance povrata

1.1. U prvom zadatku ove laboratorijske vježbe potrebno je prvo iz danih cijena (gore učitanih u Pandas DataFrame) izračunati dnevne povrate za sve pojedine vrijednosnice (prateći formulu danu u uvodu).

Izračunajte srednje povrate i volatilnost (standardnu devijaciju povrata) za svaku pojedinu vrijednosnicu. Pri analizi srednjih povrata i volatilnosti, te se brojke često *anualiziraju* - to znači da se srednji povrati pomnože s 252 (cca. broj trgovinskih dana u godini), a volatilnost s $\sqrt{252}$.

Izračunajte anualizirane srednje povrate i volatilnosti. Sve ETF-ove prikažite u dijagramu raspršenja s volatilnošću na x-osi i srednjim povratom na y-osi.

Razmislite - koji se ETF-ovi ističu po odnosu povrata i rizika (posebno dobri ili posebno loši kao investicije)?

In [ ]:

```
import numpy as np
import matplotlib.pyplot as plt

returns = prices.pct_change()
returns = returns.dropna()
annualized_returns = returns.mean() * 252
annualized_volatility = returns.std() * np.sqrt(252)

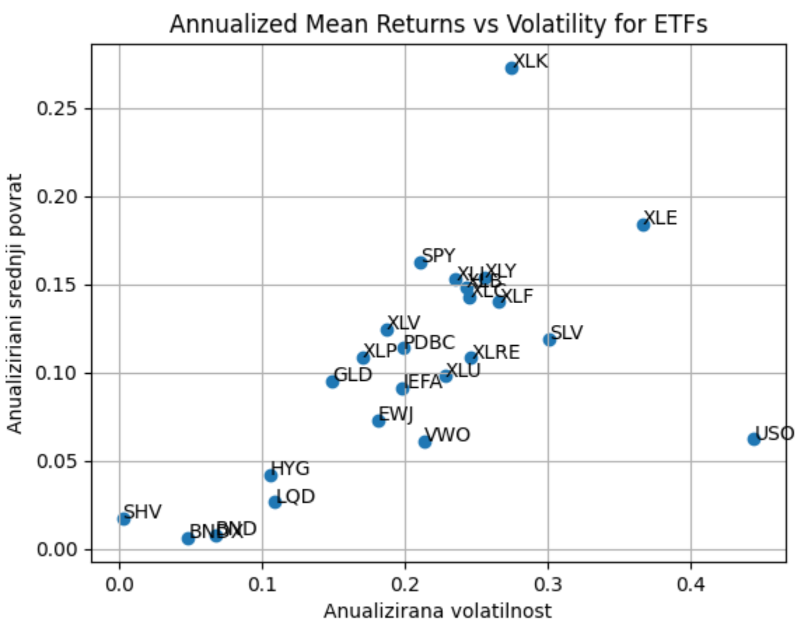
plt.scatter(annualized_volatility, annualized_returns)
plt.title('Annualized Mean Returns vs Volatility for ETFs')
plt.xlabel('Anualizirana volatilnost')
plt.ylabel('Analiziriani srednji povrat')
plt.grid(True)
for i, txt in enumerate(annualized_returns.index):
    plt.annotate(txt, (annualized_volatility[i], annualized_returns[i]))
plt.show()


# ETF-ovi s niskom volatilnošću i visokim povratom mogu biti atraktivni za konzervativnije investitore.
# ETF-ovi s visokom volatilnošću i niskim povratom mogu biti manje privlačni i mogu nositi veći rizik za gubitak kapitala.

# Kao potencijalno zanimljivi ETF-ovi mogao bih se izdvojiti XLK zato jer ima najbolji odnos povrata i volatilnosti.
# Ukolik smo zbog manjeg rizika spremni na 40% manji povrat, zanimljiv nam postaje i SPY

# S druge strane, USO se ističe kao ETF s najlošijim omjerom povrata i volatilnosti te bi ga trebalo izbjegavati.
```

```
C:\Users\Karlo\AppData\Local\Temp\ipykernel_18624\3305585970.py:15: FutureWarning: Series.__getitem__ treating keys as positions is deprecated. In a future version, integer keys will always be treated as labels (consistent with DataFrame behavior). To access a value by position, use `ser.iloc[pos]`
  plt.annotate(txt, (annualized_volatility[i], annualized_returns[i]))
```



1.2. Kovarijancu i korelaciju moguće je iz podataka izračunati koristeći Pandas, ali i NumPy ili neke druge biblioteke.

Koristeći dnevne povrate, izračunajte matricu kovarijance Σ i matricu korelacije C povrata svih ETF-ova. Matrice ispišite u konzolu ili vizualizirajte.

Proučite strukturu matrice i razmislite o tome koje zajedničke komponente u podatcima možete očekivati.

In [ ]:
```
import seaborn as sns

covariance_matrix = returns.cov()

correlation_matrix = returns.corr()

print("Covariance Matrix (Σ):")
print(covariance_matrix)
print("\nCorrelation Matrix (C):")
print(correlation_matrix)

plt.figure(figsize=(15, 10))
plt.title('Correlation Matrix of ETF Returns')
sns.heatmap(correlation_matrix, annot=True, fmt='.2f', cmap='coolwarm')
plt.show()
```

```
Covariance Matrix (Σ):
               SPY          IEFA           VWO           EWJ           XLF  \
SPY   1.761574e-04  1.457528e-04  1.376812e-04  1.170823e-04  1.918401e-04
IEFA  1.457528e-04  1.563030e-04  1.409613e-04  1.259424e-04  1.728926e-04
VWO   1.376812e-04  1.409613e-04  1.815550e-04  1.112717e-04  1.534720e-04
EWJ   1.170823e-04  1.259424e-04  1.112717e-04  1.299253e-04  1.357587e-04
XLF   1.918401e-04  1.728926e-04  1.534720e-04  1.357587e-04  2.808551e-04
XLK   2.158161e-04  1.701803e-04  1.706754e-04  1.387625e-04  2.036297e-04
XLV   1.318726e-04  1.061152e-04  9.604616e-05  8.357183e-05  1.406268e-04
XLY   1.934884e-04  1.614538e-04  1.574461e-04  1.310982e-04  1.994529e-04
XLP   1.107683e-04  8.944329e-05  7.605146e-05  7.151245e-05  1.225315e-04
XLU   1.274866e-04  1.049761e-04  8.575088e-05  8.290778e-05  1.461856e-04
XLI   1.753387e-04  1.551730e-04  1.373171e-04  1.224394e-04  2.245905e-04
XLE   1.907531e-04  1.835906e-04  1.738178e-04  1.417821e-04  2.810847e-04
XLC   1.796809e-04  1.441639e-04  1.424863e-04  1.176312e-04  1.748918e-04
XLRE  1.628937e-04  1.385278e-04  1.222450e-04  1.077890e-04  1.868533e-04
XLB   1.757895e-04  1.626840e-04  1.479333e-04  1.284975e-04  2.209945e-04
BND   1.032848e-05  1.106674e-05  9.632471e-06  9.707243e-06  5.550302e-06
LQD   3.073254e-05  3.035281e-05  2.706990e-05  2.332045e-05  2.590348e-05
BNDX  6.836683e-06  6.458734e-06  5.747600e-06  5.987623e-06  2.102074e-06
SHV  -1.782225e-07 -1.215006e-07 -1.699283e-07  2.821937e-08 -4.059010e-07
HYG   7.076129e-05  6.390899e-05  5.976081e-05  5.006845e-05  8.011112e-05
GLD   1.307499e-05  2.218935e-05  2.325321e-05  1.778782e-05 -1.998805e-06
SLV   6.410859e-05  7.962121e-05  8.086376e-05  5.718553e-05  5.123144e-05
PDBC  5.730408e-05  5.856160e-05  6.037230e-05  4.461771e-05  7.280871e-05
USO   1.171177e-04  1.116622e-04  1.196180e-04  7.435808e-05  1.564442e-04

               XLK           XLV           XLY           XLP           XLU  \
SPY   2.158161e-04  1.318726e-04  1.934884e-04  1.107683e-04  1.274866e-04
IEFA  1.701803e-04  1.061152e-04  1.614538e-04  8.944329e-05  1.049761e-04
VWO   1.706754e-04  9.604616e-05  1.574461e-04  7.605146e-05  8.575088e-05
EWJ   1.387625e-04  8.357183e-05  1.310982e-04  7.151245e-05  8.290778e-05
XLF   2.036297e-04  1.406268e-04  1.994529e-04  1.225315e-04  1.461856e-04
XLK   2.996448e-04  1.510928e-04  2.403027e-04  1.222616e-04  1.316509e-04
XLV   1.510928e-04  1.385254e-04  1.271010e-04  9.811664e-05  1.172870e-04
XLY   2.403027e-04  1.271010e-04  2.591421e-04  1.078307e-04  1.199366e-04
XLP   1.222616e-04  9.811664e-05  1.078307e-04  1.154083e-04  1.193987e-04
XLU   1.316509e-04  1.172870e-04  1.199366e-04  1.193987e-04  2.069732e-04
XLI   1.910086e-04  1.299066e-04  1.859044e-04  1.148908e-04  1.390192e-04
XLE   1.834210e-04  1.329837e-04  1.816440e-04  1.091994e-04  1.356845e-04
XLC   2.298729e-04  1.227429e-04  2.057247e-04  1.007367e-04  1.094027e-04
XLRE  1.805967e-04  1.317512e-04  1.722832e-04  1.208262e-04  1.756801e-04
XLB   1.937529e-04  1.310538e-04  1.861967e-04  1.143999e-04  1.386152e-04
BND   1.330583e-05  7.160052e-06  1.404303e-05  8.633379e-06  1.322661e-05
LQD   3.825942e-05  2.096342e-05  3.828408e-05  1.900692e-05  3.080993e-05
BNDX  8.851570e-06  5.695455e-06  9.282111e-06  5.575668e-06  1.193018e-05
SHV  -1.452970e-07 -9.666214e-08 -1.939843e-07 -4.617104e-08  4.848319e-08
HYG   8.294049e-05  5.245450e-05  7.940941e-05  4.330333e-05  5.912695e-05
GLD   1.719533e-05  1.085658e-05  1.310897e-05  1.405772e-05  2.878039e-05
SLV   7.808044e-05  4.390190e-05  7.235107e-05  4.033467e-05  6.391738e-05
PDBC  5.823096e-05  3.552949e-05  5.444496e-05  2.980149e-05  3.136934e-05
USO   1.189163e-04  7.579423e-05  1.034852e-04  6.289437e-05  5.387372e-05

      ...           XLB           BND           LQD          BNDX  \
SPY   ...  1.757895e-04  1.032848e-05  3.073254e-05  6.836683e-06
IEFA  ...  1.626840e-04  1.106674e-05  3.035281e-05  6.458734e-06
VWO   ...  1.479333e-04  9.632471e-06  2.706990e-05  5.747600e-06
EWJ   ...  1.284975e-04  9.707243e-06  2.332045e-05  5.987623e-06
```

XLF	...	2.209945e-04	5.550302e-06	2.590348e-05	2.102074e-06
XLK	...	1.937529e-04	1.330583e-05	3.825942e-05	8.851570e-06
XLV	...	1.310538e-04	7.160052e-06	2.096342e-05	5.695455e-06
XLY	...	1.861967e-04	1.404303e-05	3.828408e-05	9.282111e-06
XLP	...	1.143999e-04	8.633379e-06	1.900692e-05	5.575668e-06
XLU	...	1.386152e-04	1.322661e-05	3.080993e-05	1.193018e-05
XLI	...	2.047923e-04	7.666242e-06	2.831092e-05	5.608365e-06
XLE	...	2.420647e-04	1.351107e-06	2.017134e-05	-8.716908e-07
XLC	...	1.612733e-04	1.230099e-05	3.346059e-05	7.892247e-06
XLRE	...	1.730627e-04	1.730387e-05	4.088237e-05	1.311175e-05
XLB	...	2.339992e-04	8.019766e-06	2.815773e-05	5.579961e-06
BND	...	8.019766e-06	1.804064e-05	2.506508e-05	9.805009e-06
LQD	...	2.815773e-05	2.506508e-05	4.741936e-05	1.392387e-05
BNDX	...	5.579961e-06	9.805009e-06	1.392387e-05	9.331558e-06
SHV	...	-2.004156e-07	1.783662e-07	1.881198e-07	1.431022e-07
HYG	...	7.358275e-05	1.137993e-05	2.675037e-05	7.607734e-06
GLD	...	2.226311e-05	1.504004e-05	2.333186e-05	1.010811e-05
SLV	...	8.964691e-05	1.956764e-05	3.791384e-05	1.520044e-05
PDBC	...	7.391043e-05	9.697108e-07	9.178127e-06	-1.070061e-06
USO	...	1.377812e-04	-1.954240e-06	7.810558e-06	-3.598700e-06

	SHV	HYG	GLD	SLV	PDBC \
SPY	-1.782225e-07	7.076129e-05	1.307499e-05	6.410859e-05	5.730408e-05
IEFA	-1.215006e-07	6.390899e-05	2.218935e-05	7.962121e-05	5.856160e-05
VWO	-1.699283e-07	5.976081e-05	2.325321e-05	8.086376e-05	6.037230e-05
EWJ	2.821937e-08	5.006845e-05	1.778782e-05	5.718553e-05	4.461771e-05
XLF	-4.059010e-07	8.011112e-05	-1.998805e-06	5.123144e-05	7.280871e-05
XLK	-1.452970e-07	8.294049e-05	1.719533e-05	7.808044e-05	5.823096e-05
XLV	-9.666214e-08	5.245450e-05	1.085658e-05	4.390190e-05	3.552949e-05
XLY	-1.939843e-07	7.940941e-05	1.310897e-05	7.235107e-05	5.444496e-05
XLP	-4.617104e-08	4.330333e-05	1.405772e-05	4.033467e-05	2.980149e-05
XLU	4.848319e-08	5.912695e-05	2.878039e-05	6.391738e-05	3.136934e-05
XLI	-2.654281e-07	7.304679e-05	7.898453e-06	6.205533e-05	6.888892e-05
XLE	-6.132212e-07	7.971625e-05	1.550908e-05	9.618807e-05	1.797461e-04
XLC	-1.412257e-07	7.091742e-05	1.449226e-05	6.546367e-05	4.964330e-05
XLRE	-4.647225e-08	7.650127e-05	2.367132e-05	7.822725e-05	4.649864e-05
XLB	-2.004156e-07	7.358275e-05	2.226311e-05	8.964691e-05	7.391043e-05
BND	1.783662e-07	1.137993e-05	1.504004e-05	1.956764e-05	9.697108e-07
LQD	1.881198e-07	2.675037e-05	2.333186e-05	3.791384e-05	9.178127e-06
BNDX	1.431022e-07	7.607734e-06	1.010811e-05	1.520044e-05	-1.070061e-06
SHV	3.290195e-08	3.012192e-08	3.364304e-07	3.191481e-07	-2.125596e-07
HYG	3.012192e-08	4.433736e-05	1.293071e-05	3.729614e-05	2.542381e-05
GLD	3.364304e-07	1.293071e-05	8.868473e-05	1.389859e-04	2.891826e-05
SLV	3.191481e-07	3.729614e-05	1.389859e-04	3.588115e-04	8.087417e-05
PDBC	-2.125596e-07	2.542381e-05	2.891826e-05	8.087417e-05	1.573913e-04
USO	-8.205055e-07	4.886216e-05	2.067511e-05	1.013434e-04	2.905185e-04

	USO
SPY	1.171177e-04
IEFA	1.116622e-04
VWO	1.196180e-04
EWJ	7.435808e-05
XLF	1.564442e-04
XLK	1.189163e-04
XLV	7.579423e-05
XLY	1.034852e-04
XLP	6.289437e-05
XLU	5.387372e-05

```
XLI   1.424165e-04
XLE   3.886086e-04
XLC   1.016578e-04
XLRE  9.185401e-05
XLB   1.377812e-04
BND  -1.954240e-06
LQD   7.810558e-06
BNDX -3.598700e-06
SHV  -8.205055e-07
HYG   4.886216e-05
GLD   2.067511e-05
SLV   1.013434e-04
PDBC  2.905185e-04
USO   7.829798e-04

[24 rows x 24 columns]

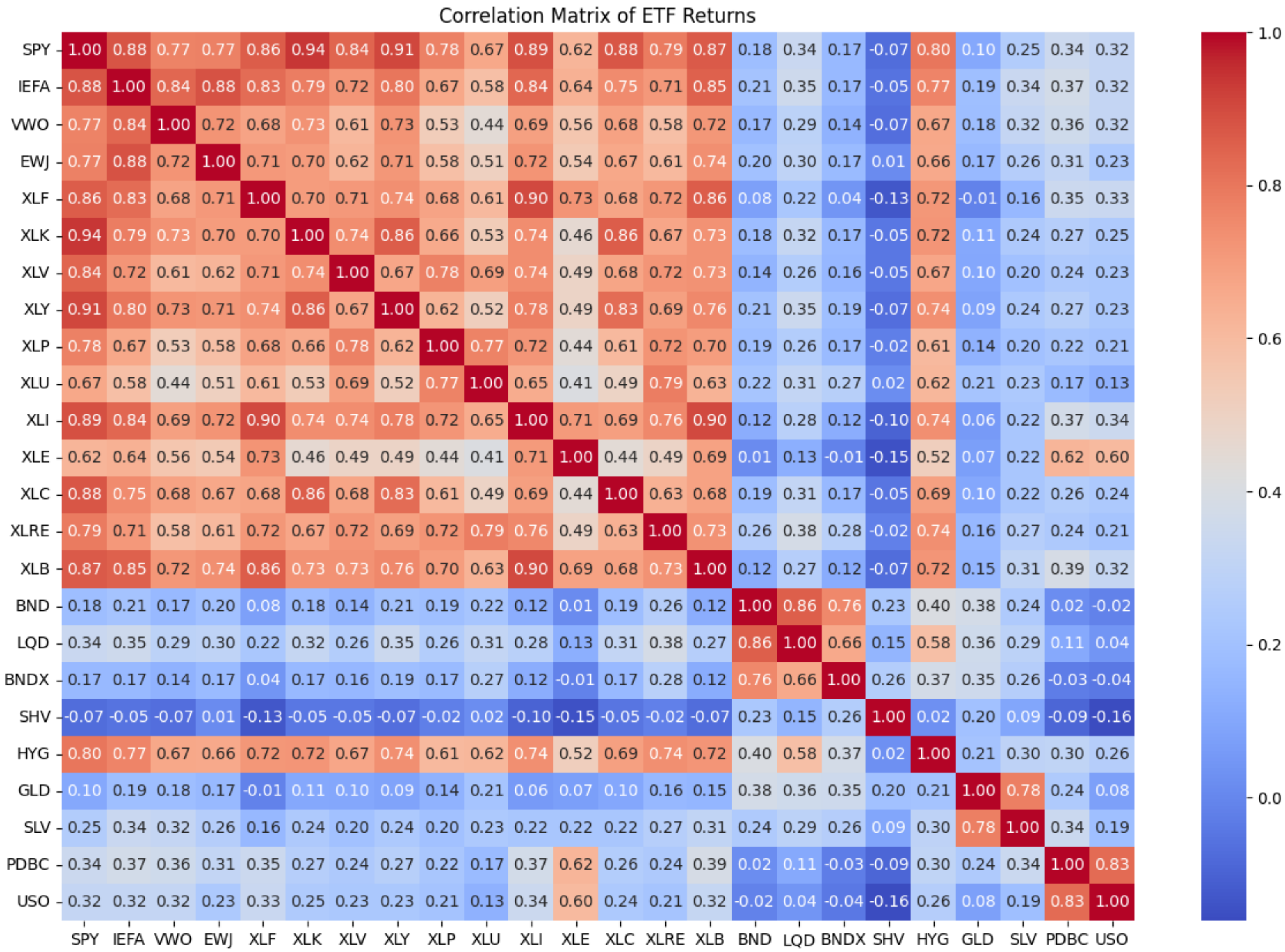
Correlation Matrix (C):
           SPY      IEFA       VWO       EWJ       XLF       XLK       XLV  \
SPY   1.000000  0.878381  0.769875  0.773916  0.862477  0.939356  0.844188
IEFA  0.878381  1.000000  0.836782  0.883774  0.825186  0.786362  0.721156
VWO   0.769875  0.836782  1.000000  0.724492  0.679648  0.731751  0.605635
EWJ   0.773916  0.883774  0.724492  1.000000  0.710689  0.703270  0.622943
XLF   0.862477  0.825186  0.679648  0.710689  1.000000  0.701934  0.712955
XLK   0.939356  0.786362  0.731751  0.703270  0.701934  1.000000  0.741610
XLV   0.844188  0.721156  0.605635  0.622943  0.712955  0.741610  1.000000
XLY   0.905598  0.802224  0.725870  0.714465  0.739316  0.862356  0.670835
XLP   0.776867  0.665956  0.525394  0.584005  0.680594  0.657458  0.775996
XLU   0.667663  0.583646  0.442362  0.505582  0.606326  0.528644  0.692673
XLI   0.890781  0.836904  0.687169  0.724299  0.903636  0.744034  0.744235
XLE   0.622738  0.636284  0.558951  0.538962  0.726742  0.459124  0.489573
XLC   0.876503  0.746577  0.684653  0.668155  0.675663  0.859777  0.675201
XLRE  0.790579  0.713748  0.584411  0.609142  0.718209  0.672044  0.721077
XLB   0.865835  0.850656  0.717719  0.736954  0.862051  0.731708  0.727910
BND   0.183215  0.208406  0.168309  0.200504  0.077974  0.180972  0.143227
LQD   0.336256  0.352564  0.291746  0.297107  0.224460  0.320965  0.258654
BNDX  0.168623  0.169117  0.139639  0.171961  0.041061  0.167394  0.158411
SHV  -0.074029 -0.053578 -0.069527  0.013649 -0.133527 -0.046275 -0.045277
HYG   0.800683  0.767703  0.666082  0.659678  0.717905  0.719579  0.669319
GLD   0.104608  0.188468  0.183254  0.165711 -0.012665  0.105483  0.097950
SLV   0.254996  0.336211  0.316823  0.264854  0.161385  0.238125  0.196918
PDBC  0.344148  0.373370  0.357144  0.312011  0.346300  0.268139  0.240621
USO   0.315353  0.319189  0.317261  0.233134  0.333613  0.245506  0.230142

           XLY       XLP       XLU  ...       XLB       BND       LQD  \
SPY   0.905598  0.776867  0.667663  ...  0.865835  0.183215  0.336256
IEFA  0.802224  0.665956  0.583646  ...  0.850656  0.208406  0.352564
VWO   0.725870  0.525394  0.442362  ...  0.717719  0.168309  0.291746
EWJ   0.714465  0.584005  0.505582  ...  0.736954  0.200504  0.297107
XLF   0.739316  0.680594  0.606326  ...  0.862051  0.077974  0.224460
XLK   0.862356  0.657458  0.528644  ...  0.731708  0.180972  0.320965
XLV   0.670835  0.775996  0.692673  ...  0.727910  0.143227  0.258654
XLY   1.000000  0.623527  0.517876  ...  0.756129  0.205384  0.345359
XLP   0.623527  1.000000  0.772545  ...  0.696145  0.189207  0.256930
XLU   0.517876  0.772545  1.000000  ...  0.629864  0.216454  0.310997
XLI   0.778689  0.721124  0.651570  ...  0.902713  0.121703  0.277217
XLE   0.488918  0.440439  0.408655  ...  0.685659  0.013783  0.126923
XLC   0.827407  0.607115  0.492349  ...  0.682585  0.187506  0.314599
```

XLRE	0.689391	0.724493	0.786606	...	0.728765	0.262427	0.382428
XLB	0.756129	0.696145	0.629864	...	1.000000	0.123432	0.267308
BND	0.205384	0.189207	0.216454	...	0.123432	1.000000	0.856969
LQD	0.345359	0.256930	0.310997	...	0.267308	0.856969	1.000000
BNDX	0.188756	0.169903	0.271464	...	0.119412	0.755692	0.661919
SHV	-0.066433	-0.023694	0.018579	...	-0.072229	0.231513	0.150607
HYG	0.740829	0.605366	0.617225	...	0.722410	0.402373	0.583401
GLD	0.086472	0.138954	0.212430	...	0.154545	0.376009	0.359788
SLV	0.237270	0.198211	0.234546	...	0.309382	0.243209	0.290661
PDBC	0.269587	0.221121	0.173803	...	0.385131	0.018198	0.106239
USO	0.229739	0.209227	0.133827	...	0.321890	-0.016443	0.040535

	BNDX	SHV	HYG	GLD	SLV	PDBC	USO
SPY	0.168623	-0.074029	0.800683	0.104608	0.254996	0.344148	0.315353
IEFA	0.169117	-0.053578	0.767703	0.188468	0.336211	0.373370	0.319189
VWO	0.139639	-0.069527	0.666082	0.183254	0.316823	0.357144	0.317261
EWJ	0.171961	0.013649	0.659678	0.165711	0.264854	0.312011	0.233134
XLF	0.041061	-0.133527	0.717905	-0.012665	0.161385	0.346300	0.333613
XLK	0.167394	-0.046275	0.719579	0.105483	0.238125	0.268139	0.245506
XLV	0.158411	-0.045277	0.669319	0.097950	0.196918	0.240621	0.230142
XLY	0.188756	-0.066433	0.740829	0.086472	0.237270	0.269587	0.229739
XLP	0.169903	-0.023694	0.605366	0.138954	0.198211	0.221121	0.209227
XLU	0.271464	0.018579	0.617225	0.212430	0.234546	0.173803	0.133827
XLI	0.123795	-0.098669	0.739707	0.056554	0.220897	0.370256	0.343185
XLE	-0.012364	-0.146484	0.518735	0.071358	0.220025	0.620802	0.601757
XLC	0.167273	-0.050409	0.689556	0.099635	0.223753	0.256196	0.235216
XLRE	0.276487	-0.016503	0.740074	0.161916	0.266021	0.238749	0.211453
XLB	0.119412	-0.072229	0.722410	0.154545	0.309382	0.385131	0.321890
BND	0.755692	0.231513	0.402373	0.376009	0.243209	0.018198	-0.016443
LQD	0.661919	0.150607	0.583401	0.359788	0.290661	0.106239	0.040535
BNDX	1.000000	0.258261	0.374019	0.351373	0.262691	-0.027922	-0.042101
SHV	0.258261	1.000000	0.024939	0.196952	0.092886	-0.093407	-0.161657
HYG	0.374019	0.024939	1.000000	0.206212	0.295696	0.304344	0.262248
GLD	0.351373	0.196952	0.206212	1.000000	0.779136	0.244770	0.078460
SLV	0.262691	0.092886	0.295696	0.779136	1.000000	0.340319	0.191200
PDBC	-0.027922	-0.093407	0.304344	0.244770	0.340319	1.000000	0.827577
USO	-0.042101	-0.161657	0.262248	0.078460	0.191200	0.827577	1.000000

[24 rows x 24 columns]



Zadatak 2 - Analiza glavnih komponenti

2.1. Za analizu glavnih komponenti potrebno je izračunati svojstvenu dekompoziciju, koju možete pronaći u sklopu biblioteke NumPy https://numpy.org/doc/stable/reference/generated/numpy.linalg.eig.html.

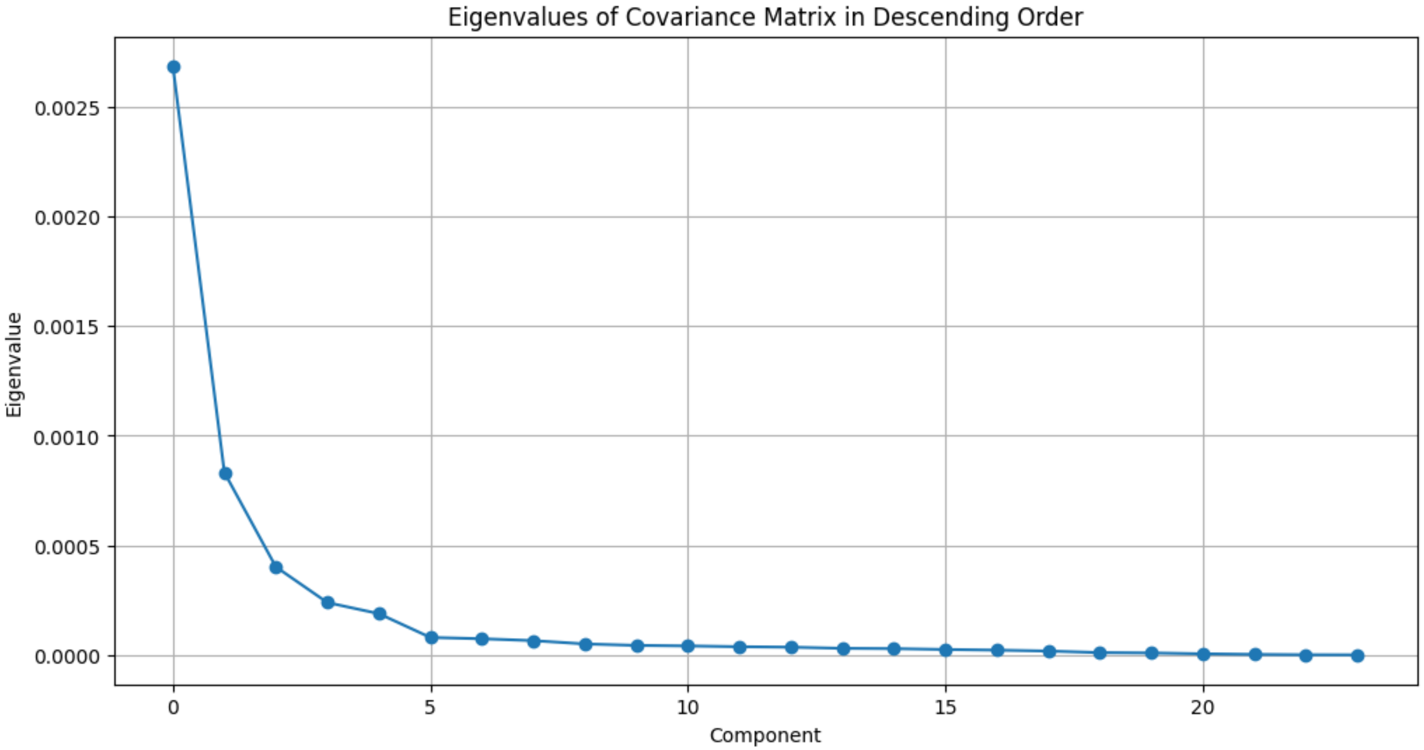
Izračunajte svojstvene vektore i pripadajuće svojstvene vrijednosti matrice kovarijance povrata Σ. Poredajte komponente padajući po svojstvenim vrijednostima i prikažite svojstvene vrijednosti grafički.

In [ ]:
```
# Calculate eigenvalues and eigenvectors of the covariance matrix
eigenvalues, eigenvectors = np.linalg.eig(covariance_matrix)

# Sort eigenvalues and eigenvectors in descending order
sorted_indices = np.argsort(eigenvalues)[::-1]
sorted_eigenvalues = eigenvalues[sorted_indices]
sorted_eigenvectors = eigenvectors[:, sorted_indices]

# Visualize the eigenvalues
plt.figure(figsize=(12, 6))
plt.plot(sorted_eigenvalues, marker='o')
plt.title('Eigenvalues of Covariance Matrix in Descending Order')
plt.xlabel('Component')
plt.ylabel('Eigenvalue')
plt.grid(True)
plt.show()

sorted_eigenvalues, sorted_eigenvectors
```



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```

2.2. **Izračunajte koliki udio varijance objašnjavaju prve tri komponente?**

In [ ]:
```
total_variance = sum(sorted_eigenvalues)
variance_explained_first_three = sum(sorted_eigenvalues[:3]) / total_variance

print("Variance Explained by First Three Components: {:.2%}".format(variance_explained_first_three))
```

```
Variance Explained by First Three Components: 79.71%
```

2.3. Komponente PCA će u financijama često opisivati neke zajedničke faktore u podatcima, što je moguće analizirati promatranjem pojedinih elemenata svojstvenih vektora. Ako je neki element određenog svojstvenog vektora velik po magnitudi (pozitivan ili negativan), to znači da ta komponenta opisuje odgovarajuću vrijednosnicu i objašnjava njenu varijancu, za razliku od slučaja kad je element blizu 0, što znači da razmatrana vrijednosnica ne ovisi previše o toj komponenti.

Prikažite grafički (npr. stupčastim dijagramom za svaku komponentu posebno) koeficijente prve 3 glavne komponente (elemente prva tri svojstvena vektora).

S obzirom na to koje vrijednosnice opisuju prve tri komponente, razmislite možete li zaključiti kakve zajedničke faktore u tržištu opisuju razmatrane komponente?

NAPOMENA: pripazite na to što vraća funkcija koju koristite i u kojoj se dimenziji (stupac ili red) nalaze svojstveni vektori.

In [ ]:
```
# Extract the first three eigenvectors
first_three_eigenvectors = sorted_eigenvectors[:, :3]

# Create bar plots for the coefficients of the first three principal components
plt.figure(figsize=(15, 6))

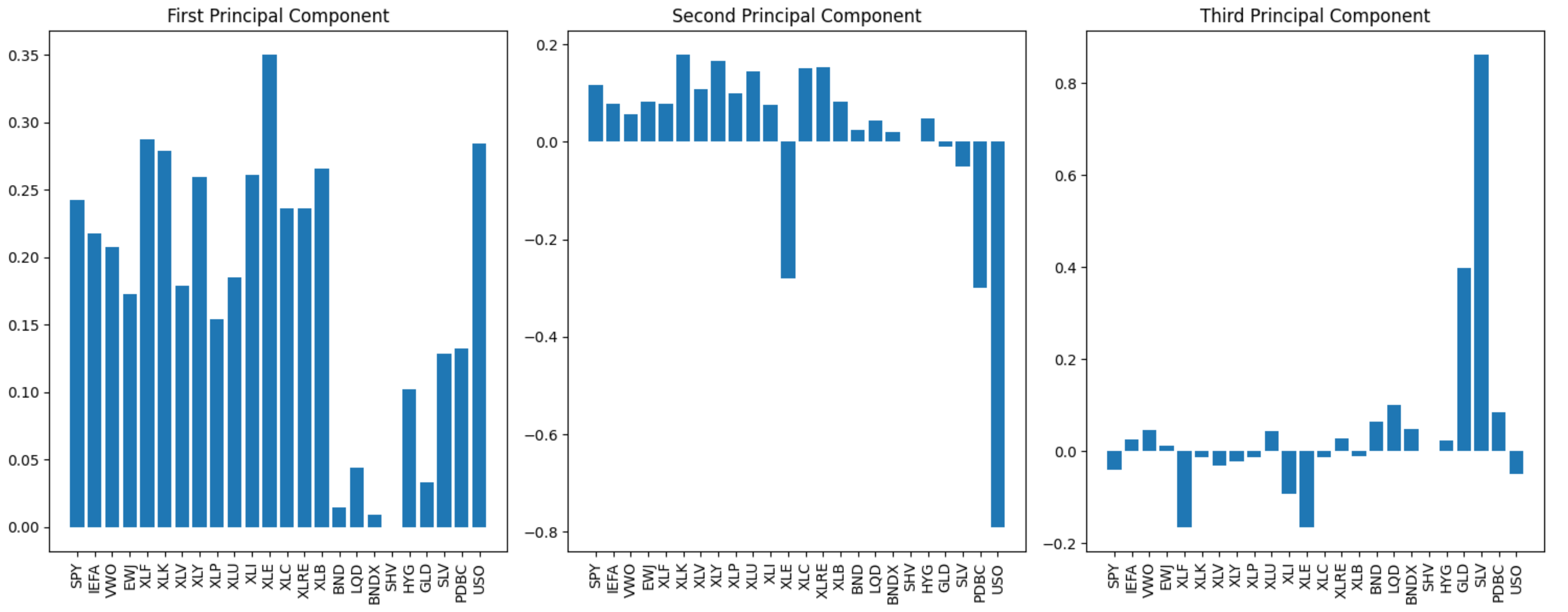
# Plot for the first principal component
plt.subplot(1, 3, 1)
plt.bar(range(len(first_three_eigenvectors)), first_three_eigenvectors[:, 0], tick_label=returns.columns)
plt.title('First Principal Component')
plt.xticks(rotation=90)

# Plot for the second principal component
```

```python
plt.subplot(1, 3, 2)
plt.bar(range(len(first_three_eigenvectors)), first_three_eigenvectors[:, 1], tick_label=returns.columns)
plt.title('Second Principal Component')
plt.xticks(rotation=90)

# Plot for the third principal component
plt.subplot(1, 3, 3)
plt.bar(range(len(first_three_eigenvectors)), first_three_eigenvectors[:, 2], tick_label=returns.columns)
plt.title('Third Principal Component')
plt.xticks(rotation=90)

plt.tight_layout()
plt.show()
```



Prva komponenta: Ova komponenta ima dosta velike koeficijente za mnoge ETF-ove, što sugerira da ona hvata neki opći trend ili faktor koji utječe na većinu tržišta. To bi moglo biti nešto kao opće kretanje tržišta ili ekonomska klima koja utječe na sve ili većinu dionica.

Druga komponenta: Ovdje vidimo da određeni ETF-ovi imaju veće koeficijente, ali ne toliko kao u prvoj komponenti. Ovo bi moglo značiti da druga komponenta hvata nešto specifičnije, možda vezano za određene sektore ili regije. Drugim riječima, ova komponenta bi mogla predstavljati faktore koji utječu na određene vrste dionica, ali ne nužno na cijelo tržište. Također, pokazuje neke ETF-ove s velikim negativnim koeficijentima, što je zanimljivo. Ovo zapravo znači da ti ETF-ovi imaju jaku obrnutu vezu s onim što ta komponenta predstavlja.

Treća komponenta: Slično kao i druga, ali s još specifičnijim skupom ETF-ova koji imaju visoke koeficijente. Ovo bi moglo značiti da treća komponenta prikazuje još uži skup faktora, možda vezanih za određene vrste rizika, strategije ili čak specifične industrije.

2.4. **Ponovite prethodnu analizu za matricu korelacije povrata C (prikažite svojstvene vrijednosti, udio varijance i koeficijente pojedinih komponenti za prve tri komponente).**

Usporedite rezultate - mijenjaju li se interpretacije komponenti?

In [ ]:
```python
# Calculate eigenvalues and eigenvectors for the correlation matrix
correlation_eigenvalues, correlation_eigenvectors = np.linalg.eig(correlation_matrix)

# Sort eigenvalues and eigenvectors in descending order
sorted_correlation_indices = np.argsort(correlation_eigenvalues)[::-1]
sorted_correlation_eigenvalues = correlation_eigenvalues[sorted_correlation_indices]
sorted_correlation_eigenvectors = correlation_eigenvectors[:, sorted_correlation_indices]

# Calculate the total variance explained by the first three components of the correlation matrix
total_correlation_variance = sum(sorted_correlation_eigenvalues)
variance_explained_first_three_correlation = sum(sorted_correlation_eigenvalues[:3]) / total_correlation_variance

# Extract the first three eigenvectors for the correlation matrix
first_three_correlation_eigenvectors = sorted_correlation_eigenvectors[:, :3]

# Visualize the eigenvalues for the correlation matrix
plt.figure(figsize=(12, 6))
plt.plot(sorted_correlation_eigenvalues, marker='o')
plt.title('Eigenvalues of Correlation Matrix in Descending Order')
plt.xlabel('Component')
plt.ylabel('Eigenvalue')
plt.grid(True)
plt.show()

# Create bar plots for the coefficients of the first three principal components of the correlation matrix
plt.figure(figsize=(15, 6))

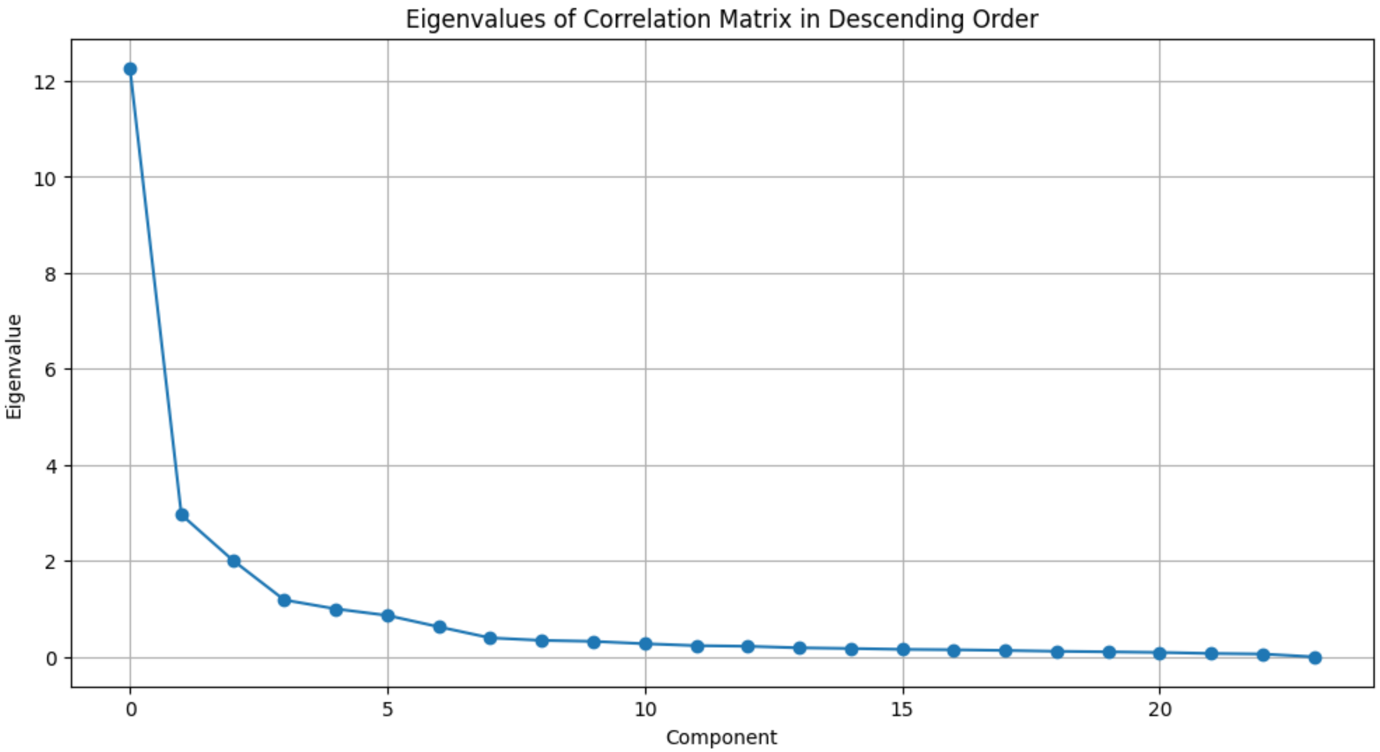
# Plot for the first principal component
plt.subplot(1, 3, 1)
plt.bar(range(len(first_three_correlation_eigenvectors)), first_three_correlation_eigenvectors[:, 0], tick_label=returns.columns)
plt.title('First Principal Component (Correlation Matrix)')
plt.xticks(rotation=90)

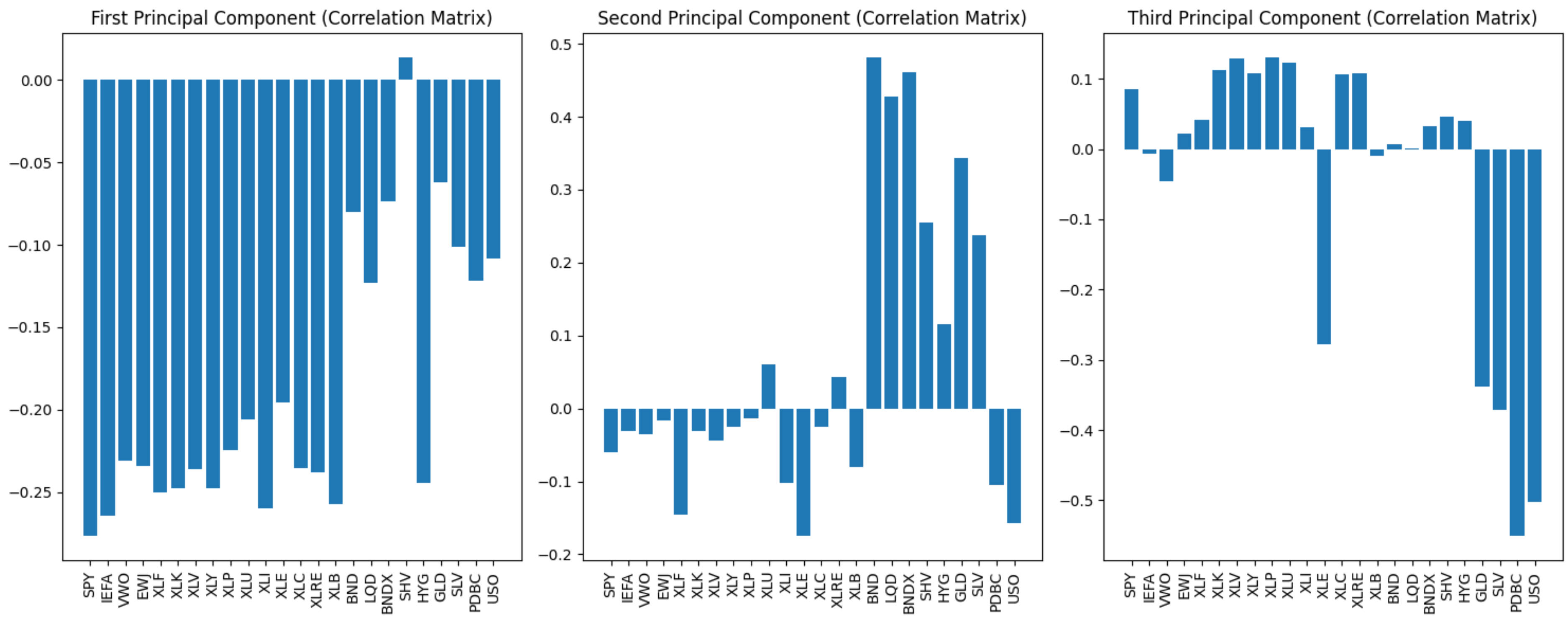
# Plot for the second principal component
plt.subplot(1, 3, 2)
plt.bar(range(len(first_three_correlation_eigenvectors)), first_three_correlation_eigenvectors[:, 1], tick_label=returns.columns)
plt.title('Second Principal Component (Correlation Matrix)')
plt.xticks(rotation=90)

# Plot for the third principal component
plt.subplot(1, 3, 3)
plt.bar(range(len(first_three_correlation_eigenvectors)), first_three_correlation_eigenvectors[:, 2], tick_label=returns.columns)
plt.title('Third Principal Component (Correlation Matrix)')
plt.xticks(rotation=90)

plt.tight_layout()
plt.show()

variance_explained_first_three_correlation, sorted_correlation_eigenvalues, sorted_correlation_eigenvectors
```


Eigenvalues of Correlation Matrix in Descending Order
Eigenvalue
Component
0
2
4
6
8
10
12
0
5
10
15
20



First Principal Component (Correlation Matrix)
Second Principal Component (Correlation Matrix)
Third Principal Component (Correlation Matrix)
SPY IEFA VWO EWJ XLF XLK XLV XLY XLP XLU XLI XLE XLC XLRE XLB BND LQD BNDX SHV HYG GLD SLV PDBC USO

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```

Usporedbom rezultata PCA analize izvedene na matrici kovarijance i matrici korelacije, uočavamo da se interpretacije komponenti mogu razlikovati. S obzirom na to da matrica korelacije normalizira podatke (tako da svaka vrijednosnica ima jednaku varijancu), moguće je da ona bolje otkriva odnose koji nisu izravno povezani s razlikama u ukupnoj varijanci pojedinih vrijednosnica. To znači da matrica korelacije može pružiti uvid u suptilnije, međusobno neovisne tržišne faktore koji utječu na povrate ETF-ova. Primjerice, dok matrica kovarijance može naglasiti odnose vođene zajedničkim trendovima i ukupnom tržišnom volatilnošću, matrica korelacije može istaknuti odnose koji su više povezani s relativnim kretanjima među vrijednosnicama. Stoga, iako oba pristupa pružaju vrijedne informacije, matrica korelacije može biti korisnija za razumijevanje suptilnijih, specifičnih tržišnih dinamika.

Zadatak 3 - Svojstveni portfelji

U primjeni PCA i svojstvenoj dekompoziciji kovarijance u financijama, svojstveni vektori se često zovu i tzv. svojstveni portfelji.

Općenito, portfelj je vektor $w = [w_1, \dots, w_N]$ u kojem svaki element predstavlja težinu ili udio kapitala u određenoj vrijednosnici. Same težine svojstvenih portfelja mogu biti rotirane i skalirane u odnosu na elemente svojstvenih vektora.

U ovoj analizi ćemo pomnožiti njihove težine s predznakom njihove sume - na taj način zapravo samo "okrećemo" predznak svojstvenog vektora tako da mu je suma pozitivna (konačni PCA rastav je i dalje isti ako svojstveni vektor pomnožimo s -1). Također, dobro je i skalirati svojstvene portfelje sa sumom njihovih apsolutnih vrijednosti:

$\tilde{w}_i = \frac{w_i}{\sum_j^N |w_j|}$.

Na taj način se osigurava da visoke magnitude pojedinih elemenata ne uzrokuju velike razlike u volatilnostima svojstvenih portfelja.

Ukoliko znamo povrate $R \in \mathbb{R}^{T \times N}$ (gdje je $R_i \in \mathbb{R}^T$ vektor povrata za vrijednosnicu i) za N vrijednosnica u nekom vremenskom periodu od T dana, povrate portfelja w u tom istom periodu možemo izračunati kao:

$R_p = \sum R_i w_i = R \cdot w$.

Izračunajte skalirane svojstvene portfelje $\tilde{w}$ koji proizlaze iz prve tri glavne komponente dobivene iz matrice kovarijance Σ. Za ta tri svojstvena portfelja izračunajte povijesne povrate kroz razmatrani period. Grafički prikažite vremensko kretanje njihovih vrijednosti (njihove povrate "vratite" natrag u cijene, s tim da početna cijena bude jednak za oba portfelja, npr. 100).

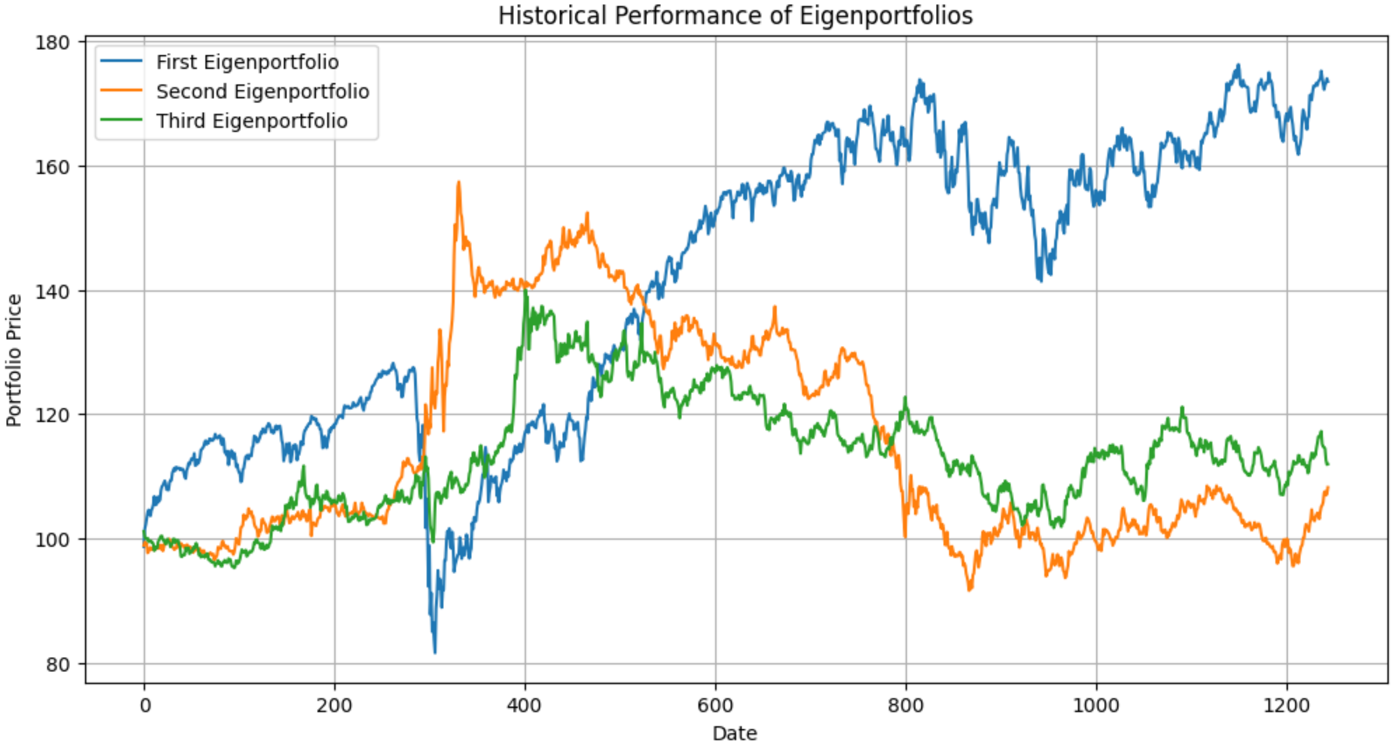
In [ ]:
```
#Vaš kod ide ovdje

# Normalize the eigenvectors to create scaled eigenportfolios
scaled_eigenportfolios = first_three_eigenvectors / np.sum(np.abs(first_three_eigenvectors), axis=0)

# Calculate the historical returns for each eigenportfolio
historical_returns = np.dot(returns.fillna(0), scaled_eigenportfolios)

# Convert returns to prices (starting at 100)
initial_price = 100
portfolio_prices = np.cumprod(1 + historical_returns, axis=0) * initial_price

# Create a plot for the historical performance of each eigenportfolio
plt.figure(figsize=(12, 6))
plt.plot(portfolio_prices, label=['First Eigenportfolio', 'Second Eigenportfolio', 'Third Eigenportfolio'])
plt.title('Historical Performance of Eigenportfolios')
plt.xlabel('Date')
plt.ylabel('Portfolio Price')
plt.legend(loc='upper left')
plt.grid(True)
plt.show()
```



Ako usporedite dobivene rezultate s kretanjem cijena originalnih vrijednosnica, vidjet ćete sličnosti između vrijednosnica koje pripadaju određenim klasama imovina i pojedinih svojstvenih portfelja. Svojstveni portfelji dakle predstavljaju niže-dimenzionalan prostor tzv. sintetičkih vrijednosnica (u našem slučaju 3 umjesto originalnih 24) koje najbolje opisuju cijeli razmatrani skup podataka. Dobra procjena tih komponenti je ključna u razumijevanju zajedničkog kretanja većih skupova dionica i upravljanju financijskim rizikom.

Razmislite što to znači za tržište koje smo analizirali - koji su glavni izvori rizika prevladavali u razmatranom periodu?

Uspoređujući dobivene rezultate svojstvenih portfelja s kretanjem cijena originalnih vrijednosnica, možemo vidjeti sličnosti između određenih klasa imovina i pojedinih svojstvenih portfelja. Ovo ukazuje na to da svojstveni portfelji efektivno predstavljaju niže-dimenzionalan prostor sintetičkih vrijednosnica (u ovom slučaju 3, umjesto originalnih 24), koje najbolje opisuju cjelokupni skup podataka. Precizna procjena ovih komponenti ključna je za razumijevanje zajedničkog kretanja većih skupova dionica i za upravljanje financijskim rizikom.

Razmatrajući što ovo znači za tržište koje smo analizirali, zaključujemo da su glavni izvori rizika koji su prevladavali u razmatranom periodu vjerojatno vezani uz faktore koje su opisivale prve tri glavne komponente. Ovi faktori mogu uključivati specifične tržišne trendove, sektorske utjecaje, regionalne faktore, ili čak šire ekonomske indikatore u koje se u ovom trenutku ne razumijem previše. Zasigurno, prepoznavanje ovih izvora rizika može pomoći u boljem razumijevanju tržišnih uvjeta i trendova, te u stvaranju strategija za diverzifikaciju i smanjenje rizika.