Module 2: Wave mechanics (Quantum mechanics)

1. An electron has speed $4.8 \times 10^5 m/s$ accurate to 0.012% with what accuracy can be located the position of electron?

Given data, The minimum percentage of error in the measurement of speed of an electron = 0.012%

Then the minimum uncertainty (error) involved in the measurement of

speed = $$\Delta v = \frac{4.8 \times 10^5 \times 0.012}{100} = 57.6\, m/s$$

To Find, $\Delta x = ?$

$$\Delta \boldsymbol{x}\, \Delta \boldsymbol{p} \geq \frac{\boldsymbol{h}}{\boldsymbol{4\pi}}$$

$$\Delta \boldsymbol{x} \geq \frac{\boldsymbol{h}}{\boldsymbol{4\pi}\, \Delta \boldsymbol{p}}$$

$$\Delta \boldsymbol{x} \geq \frac{\boldsymbol{h}}{\boldsymbol{4\pi} \times \boldsymbol{m} \times \Delta \boldsymbol{v}} \qquad \Delta \boldsymbol{x} \geq \frac{\mathbf{6.625 \times 10^{-34}}}{\boldsymbol{4\pi} \times \mathbf{9.1 \times 10^{-31} \times 57.6}}$$

$$\Delta \boldsymbol{x} = \mathbf{1 \times 10^{-6}}\, \boldsymbol{m}$$

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```
import math

# Constants
h = 6.625e-34   # Planck constant
m = 9.1e-31   # Mass of electron
v = 4.8e5    # velocity of electron
dv = (v * 0.012)/100  # Maximum uncertainty in speed

# Calculations

dx = h / (4 * math.pi * m * dv)  # Uncertainty in position

# Output
print("Uncertainty in position of the electron:", dx, "m")
```

2. In a measurement that involved a maximum uncertainty of 0.003% the speed of an electron was found to be 800 m/s. Calculate the corresponding uncertainty involved in determining its position

Given data, The percentage of error in the measurement of speed of an electron = 0.003%

Then the uncertainty (error) involved in the measurement of speed =

$$\Delta v = \frac{800 \times 0.003}{100} = 0.024\, m/s$$

$$\Delta \boldsymbol{x}\, \Delta \boldsymbol{p} \geq \frac{\boldsymbol{h}}{\boldsymbol{4\pi}} \qquad \Delta \boldsymbol{x} = \frac{\boldsymbol{h}}{\boldsymbol{4\pi} \times \boldsymbol{m} \times \Delta \boldsymbol{v}}$$

$$\Delta \boldsymbol{x} \times \boldsymbol{m} \times \Delta \boldsymbol{v} \geq \frac{\boldsymbol{h}}{\boldsymbol{4\pi}} \qquad \Delta \boldsymbol{x} = \mathbf{2.4}\, \boldsymbol{mm}$$

Python code

```
import math

# Constants
h = 6.626e-34   # Planck constant
m = 9.109e-31   # Mass of electron
dv = (800 * 0.003)/100  # Maximum uncertainty in speed

# Calculations

dx = h / (4 * math.pi * m * dv)  # Uncertainty in position

# Output
print("Uncertainty in position of the electron:", dx, "m")
```

3. An electron is confined in a box of length $10^{-8}m$. Calculate minimum uncertainty in the velocity of the electron.

Given data, Then the maximum uncertainty (error) involved in the measurement of position of an electron, $\Delta \boldsymbol{x} = 10^{-8}m$

To Find, minimum uncertainty in the velocity of the electron, $\Delta v = ?$

$$\Delta \boldsymbol{x}\, \Delta \boldsymbol{p} \geq \frac{\boldsymbol{h}}{\boldsymbol{4\pi}}$$

$$\Delta \boldsymbol{x} \times \boldsymbol{m} \times \Delta \boldsymbol{v} \geq \frac{\boldsymbol{h}}{\boldsymbol{4\pi}}$$

$$\Delta \boldsymbol{v} \geq \frac{\boldsymbol{h}}{\boldsymbol{4\pi} \times \Delta \boldsymbol{x} \times \boldsymbol{m}}$$

$$\Delta \boldsymbol{v} = \frac{\mathbf{6.625} \times \mathbf{10^{-34}}}{\boldsymbol{4\pi} \times \mathbf{10^{-8}} \times \mathbf{9.1} \times \mathbf{10^{-31}}}$$

$$\Delta \boldsymbol{v} = \mathbf{5.8}\ \boldsymbol{km/s}$$

4. A cricket ball of mass 250g moves with a velocity of 100 m/s. if its velocity is measured with an accuracy of 1%. What is the accuracy of a simultaneous measurement of its position?

$$\Delta \boldsymbol{x}\, \Delta \boldsymbol{p} \geq \frac{\boldsymbol{h}}{\boldsymbol{4\pi}}$$

$$\Delta \boldsymbol{x} \times \boldsymbol{m} \times \Delta \boldsymbol{v} \geq \frac{\boldsymbol{h}}{\boldsymbol{4\pi}}$$

$$\Delta \boldsymbol{x} = \frac{\boldsymbol{h}}{\boldsymbol{4\pi} \times \boldsymbol{m} \times \Delta \boldsymbol{v}}$$

$$\Delta \boldsymbol{x} = \mathbf{2.1} \times \mathbf{10^{-34}}\boldsymbol{m}$$

5. In a gamma decay process, the life time of decaying nuclei is found to be 2 ns. Compute the uncertainty in the energy of gamma rays emitted.

Given data,

The uncertainty in the life time of decaying nuclei is, $\Delta t = 2 \times 10^{-9} s$

T find, The uncertainty in the energy of gamma rays emitted , $\Delta E = ?$

$$\Delta E \, \Delta t \geq \frac{h}{4\pi}$$

$$\Delta E = \frac{h}{4\pi \times \Delta t}$$

$$\Delta E = 2.635 \times 10^{-26} J$$

6. Uncertainty in time of an excited atom is about $10^{-8}s$. What are the uncertainty in energy and in frequency of the radiation?

Given data, The uncertainty in the time of excited atom is, $\Delta t = 10^{-8}s$

To find, The uncertainty in the energy, $\Delta E = ?$

The uncertainty in the frequency of the radiation, $\Delta \nu = ?$

$$\Delta E\, \Delta t \geq \frac{h}{4\pi}$$

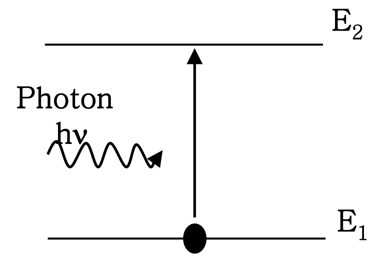
$$\Delta E = \frac{h}{4\pi \times \Delta t}$$

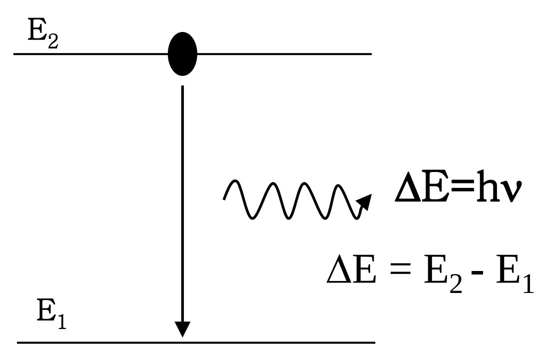
$$\Delta E = 5.27 \times 10^{-27} J$$

$$\Delta E = h \times \Delta \nu$$

$$\Delta \nu = \frac{\Delta E}{h}$$

$$\Delta \nu = 7.9\ MHz$$





7. The inherent uncertainty in the measurement of time spent by Iridium -191 nuclei in its excited state is found 1.4×10^{-10} s. Estimate the uncertainty that results in its energy excited state.

$$\Delta E = 2.35 \times 10^{-6} eV$$

8. The position and momentum of 1 KeV electron are simultaneously determined and if its position is located within 1 Å. What is the percentage of uncertainty in its momentum?

Given data, The uncertainty (error) involved in the measurement of position of an electron, $\Delta \boldsymbol{x} = 10^{-10} m$

$E_k = 1\ KeV$

To find, The percentage of uncertainty in its momentum , $\frac{\Delta p}{p} \times 100 = ?$

$$\Delta \boldsymbol{x}\ \Delta \boldsymbol{p} \geq \frac{\boldsymbol{h}}{\boldsymbol{4\pi}}$$

$$\Delta \boldsymbol{p} = \frac{\boldsymbol{h}}{\boldsymbol{4\pi} \times \Delta \boldsymbol{x}}$$

$$\Delta \boldsymbol{p} = \mathbf{0.53} \times \mathbf{10^{-24}} \boldsymbol{Kg\ m/s}$$

$$\boldsymbol{p} = \sqrt{\mathbf{2}\boldsymbol{mE_k}}$$

$$\boldsymbol{p} = \mathbf{1.707} \times \mathbf{10^{-23}} \boldsymbol{Kg\ m/s}$$

$$\frac{\Delta \boldsymbol{p}}{\boldsymbol{p}} = \times \mathbf{100} = \mathbf{3.1}\%$$

9. Compute the first 3 permitted energy values for an electron in a box of width $4 \times 10^{-10} m$.

Given data, $a = 4 \times 10^{-10} m$

To find, $E_1, E_2, E_3 = ?$

$$E_n = \frac{n^2 h^2}{8\, m\, a^2}$$

$$E_1 = \frac{h^2}{8\, m\, a^2}$$

Zero point energy

$$E_1 = 2.35\ eV$$

$$E_n = n^2 E_1$$

$E_2 = 4E_1$ $E_2 = 9.41\ eV$ $E_3 = 9E_1$ $E_3 = 21.17\ eV$

```
import math

# Constants
h = 6.625e-34   # plancks constant
m = 9.1e-31    # mass of electron
#variables
A = 1e-10
n = input("Enter quantum number value")   # quantum number
n = int(n)
a = input("Enter width of the box in angstrom")   # width
a = int(a)
# Calculations

E_n = (n**2*h**2)/(8*m*(a*A)**2)   # Energy of nth excite state

E_n_eV = E_n/(1.602e-19)

# Output
print("Uncertainty in position of the electron:", E_n, "J")

print("Uncertainty in position of the electron:", E_n_eV, "eV")
```

10. An electron is bound in one dimensional potential well of width 0.12 nm calculate the energy values and de-Broglie wavelength for first 3 permitted energy states.

Given data, $a = 0.12 \times 10^{-9} m$ To find, $E_1, E_2, E_3 = ?$

$\lambda_1, \lambda_2, \lambda_3 = ?$

$$E_n = \frac{n^2 h^2}{8\, m\, a^2}$$

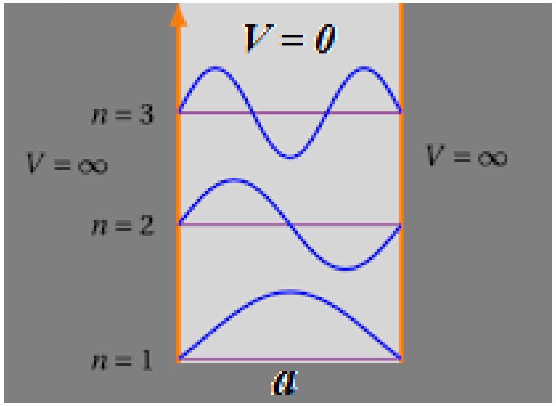
$$E_1 = \frac{h^2}{8\, m\, a^2}$$

$$E_1 = 26.14\ eV$$

$$E_n = n^2 E_1$$

$$E_2 = 104.5\ eV$$

$$E_3 = 235.2\ eV$$



$$TE = KE + PE$$

$$TE = E_k + V$$

$$TE = E_k + 0$$

$$TE = E_k$$

$$\lambda = \frac{h}{\sqrt{2mE_K}}$$

$$\lambda_1 = 2.4\ \text{Å}$$

$$\lambda_2 = 1.2\ \text{Å}$$

$$\lambda_3 = 0.8\ \text{Å}$$

11. Calculate the zero point energy for an electron in a box of width 10Å.

$$E_1 = 6.02 \times 10^{-20} J$$

$$E_1 = 0.376\ eV$$

12. The ground state energy of an electron in an infinite potential well is 5.6×10^{-3} eV what will be the ground state energy if the width of the well is doubled?

Given data, $E_1 = 5.6 \times 10^{-3} eV$

$$E_n = \frac{n^2 h^2}{8\, m\, a^2}$$

$$E_1 = \frac{h^2}{8\, m\, a^2}$$

$$E_{1(a=2a)} = \frac{h^2}{8\, m\, (2a)^2}$$

$$E_{1(a=2a)} = \frac{h^2}{8\, m\, 4\, a^2}$$

$$E_{1(a=2a)} = \frac{E_1}{4} \qquad E_{1(a=2a)} = 1.4 \times 10^{-3} eV$$

13. The 2nd excited state Eigen value of an electron confined to a one dimensional box is 77.eV. Find the size of the box.

Given data, $E_3 = 77\ eV$ To find, $a =?$

$$E_n = \frac{n^2 h^2}{8\ m\ a^2}$$

$$E_3 = \frac{9h^2}{8\ m\ a^2}$$

$$a = \sqrt{\frac{9h^2}{8\ m\ E_3}}$$

$$a = 0.2\ nm$$

14. An eigen value of an electron confined to a one dimensional box of size 0.2nm is 151 eV. What is the order of the existed state?

$$n = 4$$

15. The zero point energy of an electron in a one dimensional potential box of 2Å is 16 eV. Calculate its energy in the third existed state.

$$E_n = \frac{n^2 h^2}{8\, m\, a^2}$$

$$E_n = n^2 E_1$$

$$E_4 = 16 E_1$$

$$E_4 = 256\ eV$$

```
import math

# Constants
E_1 = 16*1.602e-19   # zero point energy
n = 4   # quntum number

# Calculations

E_4 = n**2*E_1  # Energy of third excite state

E_4_eV = E_4/(1.602e-19)

# Output
print(“Energy of third excited state:", E_4, "J")

print(“Energy of third excited state:", E_4_eV, "eV")
```

```
import math

# Constants
E_1 = 16*1.602e-19   # zero point energy
n = input("Enter quantum number value")   # quntum number
n = int(n)

# Calculations

E_4 = n**2*E_1  # Energy of third excite state

E_4_eV = E_4/(1.602e-19)

# Output
print("Uncertainty in position of the electron:", E_4, "J")

print("Uncertainty in position of the electron:", E_4_eV, "eV")
```

Question Bank-Module 2-Wave mechanics

1. **State and explain uncertainty principle. Give its physical significance.**
2. **Show that electrons cannot be existed inside the nucleus based on uncertainty principle.**
3. **What is wave function? Give its physical significance.**
4. **What is wave function? Explain the properties of wave function.**
5. **What is physics significance of wave function? Discuss the nature of Eigen values and Eigen functions.**
6. **Set up time-independent one dimensional Schrodinger's wave equation.**
7. **Deduce time-independent Schrodinger's wave equation for a particle in one dimensional potential well of infinite height and discuss the solutions.**
8. **Explain stationary states and linear combination of stationary states**
9. **What is Gaussian wave packet? Explain spreading of Gaussian wave packet**