FACULTY OF
ENGINEERING
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Module - 1: Oscillations Mumericals

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Numerical 1: A man weighing 600N steps on a spring scale machine. The spring in the machine is compressed by 1 cm. Find the force constant of the spring. Write a python code for solving this

Given, $W = 600N, \quad x = -1\ cm$ To find, k=?

$$F = -kx \qquad k = -\frac{F}{x} = -\frac{W}{x} = \frac{-600}{-0.01} = 6 \times 10^4 N/m$$

```
import math

# Define the variables
F = 600 # Force (N)
x = 0.01 # Compression (m)

# Calculate the force constant
k = -F/-x

# Print the result
print("The force constant of the spring is", k, "N/m")
```

```
import math

# Define the constant
k = 60000 # Force constant N/m
# Define the variable
F = input("Enter Force constant value")
F = int(F)
# Calculate the compression
x = -F/-k
# Print the result
print("The force constant of the spring is", x, "N/m")
```

```
import math

# Define constants
F = 600 # Force (N)
x = 0.01 # Compression (m)

# Calculate the force constant
k = -F/-x

# Define the variable
F_var = input("Enter Force constant value")
F_var = int(F_var)

# Calculate the compression
x = -F_var/-k

# Print the result
print("The force constant of the spring is", k, "N/m")
print("The compression produced is", x, "m")
```

Numerical 2: A mass of 5kg is suspended from the free end of a spring. When set for vertical oscillations, the system executes 100 oscillations in 40 sec. Calculate the force constant of the spring. Write a python code to calculate time period as variable keeping k as constant.

Given, $N = 100, t = 40\ sec$ $T = 0.4\ sec$ To find, k=?

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad k = 4\pi^2\frac{m}{T^2} \qquad k = 1233.7\ N/m$$

```
import math

# Define the constant
k = 1233.7 # Force constant N/m

# Define the variable
m = input("Enter mass of spring")
m = int(m)

# Calculate the Time period
T = 2*math.pi*math.sqrt(m/k)

# Print the result
print("The time period of the oscillating spring is", T, "s")
```

3. Given the force constant as 9.8 N/m for a spring, estimate the number of oscillations it would complete in 1 minute if it is set for oscillations with a load of 89.37g.

Ans: f= 1.667 Hz (Oscillations/s) Oscillation/minute=100

```
import math

# Define the constant
k = 9.8 # Force constant N/m

# Define the variable
m = float(input("Enter mass of spring"))

# Calculate the Time period
T = 2*math.pi*math.sqrt(m/k)
f = 1/T
O_pm = f*60

# Print the result
print("The time period of the oscillating spring is", T, "s")
print("The frequency of the oscillating spring is", f, "s")
print("The oscillating per minute is", O_pm,)
```

Numerical 4: A mass of 0.5kg causes an extension 0.03 m in a spring and the system is set for oscillations. Calculate force constant of the spring, angular frequency and time period of the resultant oscillations. Write a python code to calculate the angular frequency and time period of the resultant oscillations keeping mass as variable input.

Given, $x = -0.03\, m,\qquad m = 0.5\, kg$ To find, $k, \omega, T = ?$

$$Force\ acting, F = mg = 0.5 \times 9.8N$$

$$Restoring\ force, F = 4.9N$$

$$k = -\frac{F}{x} = \frac{-4.9}{-0.03} = 163.3\ N/m \qquad \omega = \sqrt{\frac{k}{m}} = 18.1\, rad/s$$

$$f = \frac{\omega}{2\pi} = 2.877\ Hz \qquad T = \frac{1}{f} = 0.35s$$

```
import math

# calculate force constant
m = 0.5 # kg
k = ((m*9.8)/0.03)

w = math.sqrt(k/m)  # angular frequency
f = w/(2*math.pi)  # frequency of oscillation
T = (1/f) # time period of oscillation

# Print the results
print("Force constant", k, "N/m")
print("Angular frequency", w, "rad/s")
print("Frequency of oscillation", f, "Hz")
print("Time period", T, "s")
```

```
import math

# calculate force constant
m = 0.05 # kg
k = ((m*9.8)/0.05)

# Define the variable
m_var = float(input("Enter mass of spring"))

w = math.sqrt(k/ m_var )  # angular frequency
f = w/(2*math.pi)  # frequency of oscillation
T = (1/f) # time period of oscillation

# Print the results
print("Force constant", k, "N/m")
print("Angular frequency", w, "rad/s")
print("Frequency of oscillation", f, "Hz")
print("Time period", T, "s")
```

5. A spring undergoes an extension of 5 cm for a load of 50g. Find its frequency of oscillations and time period if it is set for vertical oscillations with a load of 200g attached to its bottom using python code.

Numerical 6: An electric motor weighing 50 kg is mounted on 4 springs each of which has a spring constant 2 x 10^3 N/m. The motor moves only in vertical direction. Find the natural frequency of the system. (Try: Write a python code to give variable input of spring constant and calculate the natural frequency of the system)

Given: m=50 kg on 4 springs, k = 2 x 10 ^{3} N/m.   To find : f=?

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k_{eff}}{m}} = 2Hz \qquad k_{eff(p)} = k_1 + k_2 + k_3 + \ldots\ldots + k_n$$

7. A mass of 10kg is suspended from free end of spring when set for oscillations, the systems executes 100 oscillations in 5 mins. Calculate the force constant.

Ans: 2.27 N/m

8. A car has a spring system that supports the in-built mass 1000 kg. When a person with a weight 980 N sits at the C of G, the spring system sinks by 2.8 cm. When the car hits a bump, it starts oscillating vertically. Find the period and frequency of oscillation.

Data: x = 0.028 m, In-built mass of car m = 1000 kg. Person's weight, W = 980 N. To find : *T & f.*

Solution: When the person sits, the weight acting is 980 N.

$$k = -\frac{F}{x} = \frac{-980}{-0.028} = 3.5 \times 10^4\ N/m$$

Person mass, $m = \frac{F}{g} = \frac{980}{9.8} = 100kg$

Total mass= 1000+100=1100 kg

$$T = 2\pi\sqrt{\frac{m}{k}} = 1.11\ sec$$

$$f = \frac{1}{T} = 0.9Hz$$

9. A free particle is executing simple harmonic motion in a straight line. The maximum velocity it attains during any oscillations is 62.8 m/s. Find the frequency of the oscillations its amplitude is 0.5m.

Given, Vmax = 62.8m/s a = 0.5m, To find, f = ?

$$\boldsymbol{v = \omega\sqrt{a^2 - x^2}}$$

$$\boldsymbol{v_{max} = \omega\sqrt{a^2 - 0}}$$ y = 0 at V_{max} $$\boldsymbol{v = \omega\, a}$$ $$\boldsymbol{\omega = \frac{v}{a} = 125.6\, rad/s}$$

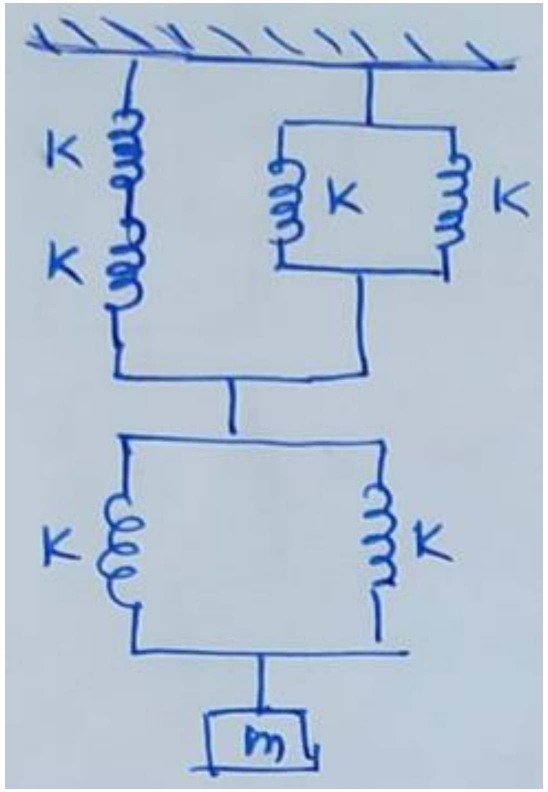
(maximum)

$$\boldsymbol{\omega = 2\pi f}$$ $$\boldsymbol{f = \frac{\omega}{2\pi} = 20Hz}$$

10. Find the frequency of oscillations of a free particle executing SHM of amplitude 0.35m, if the maximum velocity it can attain is 220m/s.

Ans: f = 100Hz

11. An arrangement of identical springs is shown in the figure. If the spring constant of each spring is 100 N/m. calculate the effective spring constant of the combination. Also calculate the frequency of oscillations of the systems when a mass of 1 kg is attached.



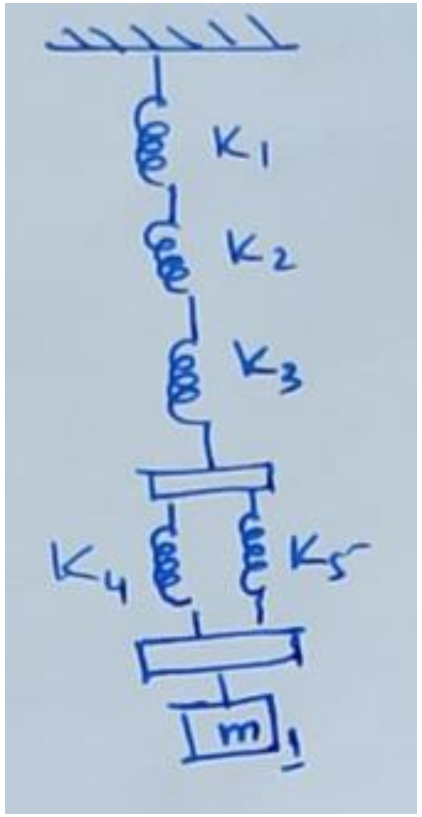
$$\boldsymbol{k_s = \frac{k_1 k_2}{k_1 + k_2}}$$

$$\boldsymbol{k_p = k_1 + k_2}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k_{eff}}{m}}$$

$$f = 1.667\ Hz$$

12. In the two mass spring systems shown in the figures, k1 =2000 N/m, k2 = 1500 N/m k3 = 3000 N/m k4 = k5 = 500 N/m. Find the mass 'm' such that the systems has a natural frequency of 10Hz in each of the cases.



$$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$$

$$k_s = 667\ N/m$$

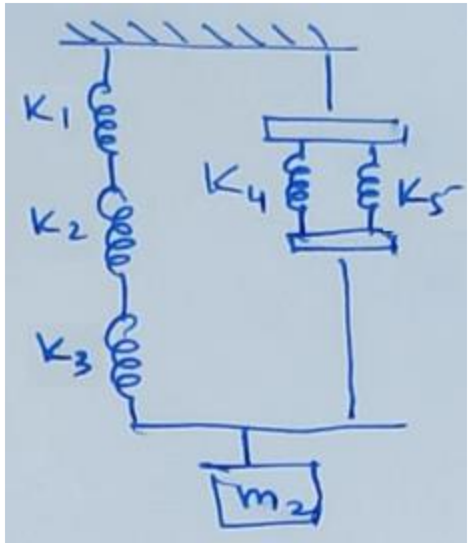
$$k_p = 1000\ N/m$$

$$\frac{1}{k_{set\ 1}} = \frac{1}{k_s} + \frac{1}{k_p}$$

$$k_{set\ 1} = 400\ N/m$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k_{eff}}{m}}$$

$$m1 = 0.1kg$$



$$k_{set\ 2} = k_s + k_p$$

$$k_{set\ 2} = 1667\ N/m$$

$$m2 = 0.422kg$$

13. Write Python code for plot displacement of SHM (Free oscillations)

```
import numpy as np
import matplotlib.pyplot as plt

A = int(input("enter A Value "))
f = int(input("enter f Value "))

phi = 0
sr = 100 # sampling rate

time = np.arange(0, 2, 1/sr) # 0.001 sec

x = A* np.sin(2*np.pi*f*time + phi)

plt.figure(figsize=(10,4))
plt.plot(time,x)
plt.title("Differential equation for Free Oscillations")
plt.xlabel("time(s)")
plt.ylabel("Displacement")
plt.show()
```

Question Bank

1. Define simple harmonic motion. Give the characteristics of SHM.

2. Derive differential equation of SHM or free oscillation or spring mass simple harmonic oscillator.

3. Derive the expressions for equivalent force constant for two springs in series and parallel combination.

4. What are damped oscillations? Obtain differential equation for damped oscillations.

5. Solve the differential equation to get the solution for displacement of damped oscillations.

6. Discuss the solution of damped oscillation for weakly, critical and heavy damping with graphical representation.

7. Derive an equation for energy decay of damped oscillations.

8. What are forced oscillations? Obtain the equations for amplitude and phase of the forced oscillations.

9. What is resonance? Obtain the condition for resonance.

10. Discuss the dependence of amplitude and phase on the frequency of applied periodic force of forced oscillations.