Project Notes:

Project Title: Identifying, Measuring, and Improving Partisan Gerrymandering
Name: Karthik Seetharaman

Note Well: There are NO SHORT-cuts to reading journal articles and taking notes from them. Comprehension is paramount. You will most likely need to read it several times so set aside enough time in your schedule.

Contents:

Knowledge Gaps:

This list provides a brief overview of the major knowledge gaps for this project, how they were resolved and where to find the information.

Knowledge Gap	Resolved By	Information is located	Date resolved
Knowledge of Mathematical Metrics	Reading a review article explaining the different mathematical metrics commonly used in modeling gerrymandering.	Articles 5,9	Resolved 9/30, reinforced 10/10
In-depth knowledge of declination	Reading a note on the intuition and motivation behind the declination function.	Article 12	Resolved 10/15
How computer simulations for modeling work	Reading a lower-level article that goes in-depth on how computer simulations were performed and the algorithms that were used.	Article 2	Resolved 9/10
How to measure plans by different metrics	Reading an article that detailed the process on how to change metrics for scoring plans in GerryChain	Article 14	Resolved 11/23

Literature Search Parameters:

These searches were performed between (Start Date of reading) and XX/XX/2019.
List of keywords and databases used during this project.

Database/search engine	Keywords	Summary of search
Google Scholar	Mathematical modeling, gerrymandering	Was able to find several papers with full analyses on how mathematical modeling of voting maps was carried out, which was very helpful for getting a feel for the processes involved.
Free-Patents-Online	Gerrymandering	Was able to find both patents relating to redistricting (of which there aren't many available).
Google Scholar	Declination, Gerrymandering	Was able to find article #12, which gave a good background on the declination as well as the motivation for its construction.
Google Scholar	Efficiency gap, Gerrymandering	Found several articles contesting the use of efficiency gap, as well as several reviewing many different mathematical metrics commonly used in gerrymandering.
Google Scholar	District Compactness, redistricting compactness	Found an article detailing discrete measures of compactness, which is a new area of research that tackles a lot of problems with current compactness measures. Worth playing with in my project.

Article #1 Notes: Neutral redistricting using a multi-level weighted graph partitioning algorithm

Article notes should be on separate sheets

Source Title	Neutral redistricting using a multi-level weighted graph partitioning algorithm
Source citation (APA Format)	Magleby, D. B. and Mosesson, D. B. (2018, 29 November). Neutral redistricting using a multi-level weighted graph partitioning algorithm. Patent Number: 0342030
Original URL	https://patentimages.storage.googleapis.com/c9/21/ca/29c446a86602ff/US20180342030A1.pdf
Source type	Patent
Keywords	Gerrymandering, redistricting, graph, partitioning, algorithm,
Summary of key points (include methodology)	Steps of algorithm: - Simplify given voting data into a graph - Simplify the graph further through a series of iterative steps - Divide the graph into the desired number of districts - Projects this information back onto the original data - Refines the districts so they meet the set of given criteria - Repeats until a sufficiently neutral set of districts has been created This technology can be used to create optimal maps based on a given set of criteria. This technology avoids the natural bias present in most other redistricting methods, making it a favorable choice for mathematical analysis. The proposed algorithm is not just limited to redistrictings, but can be applied to anything that requires equal partitioning (youth sports teams and transportation routes were two given examples).
Research Question/Problem/ Need	The need was to create an algorithm to generate neutral districts, in order to determine whether or not a given voting map was truly gerrymandered.
Important Figures	The following figures show the algorithm working on a simple state,

which is imperative to seeing how it really works. The captions presented in the patent sufficiently explain what is going on in each figure:

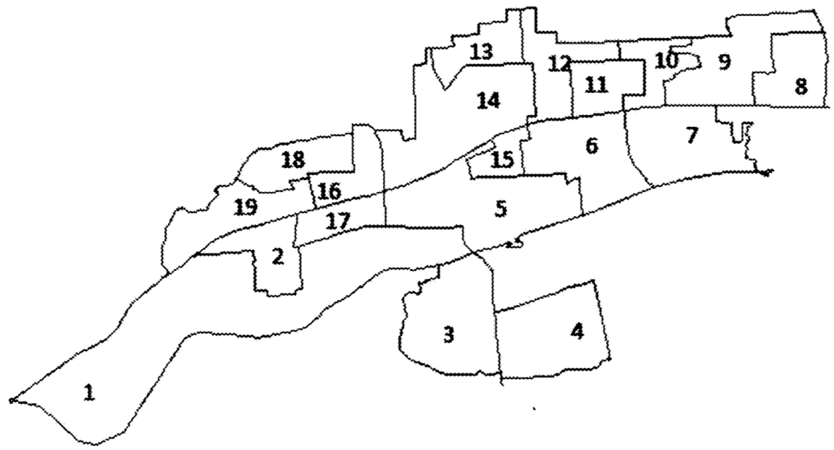


Fig. 1A

Initial Map

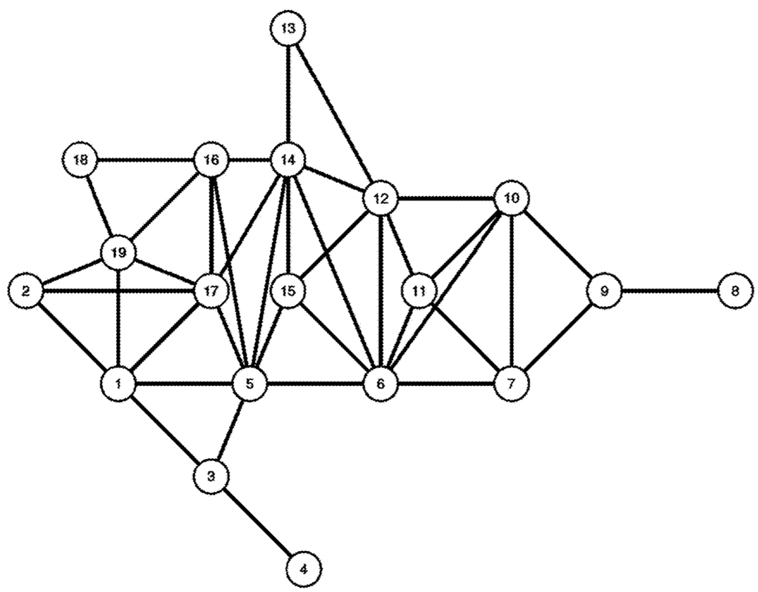


Fig. 1B

Simplified Graph

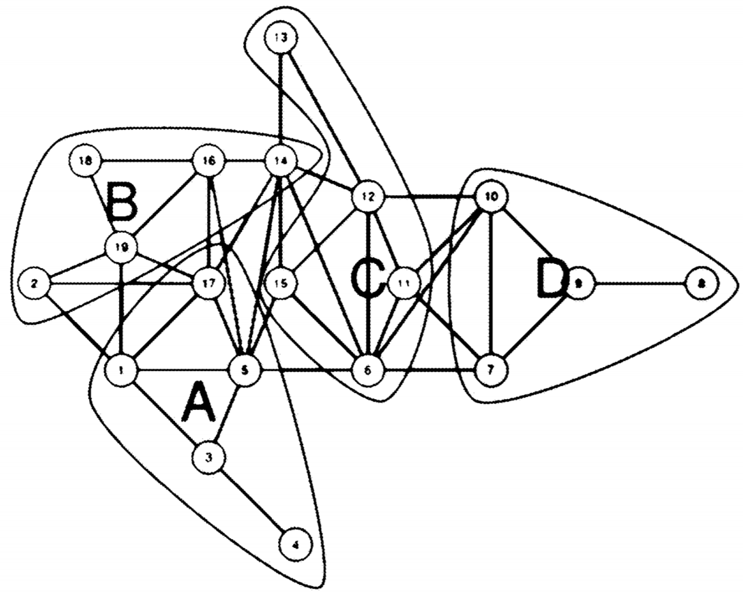


Fig. 1C

Creation of Multinodes

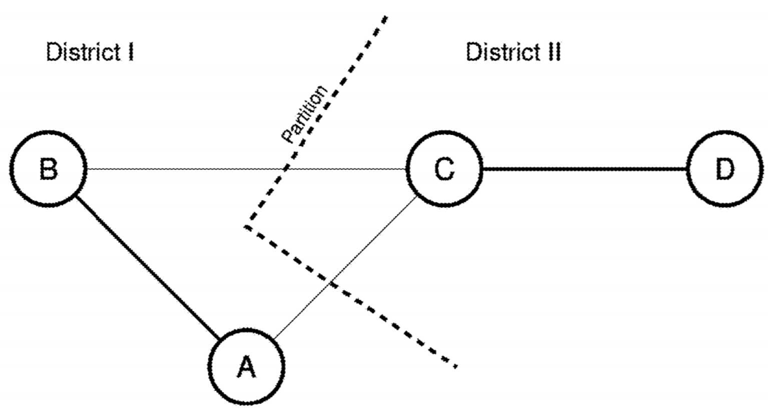


Fig. 1D

Partitioning the Simpler Graph

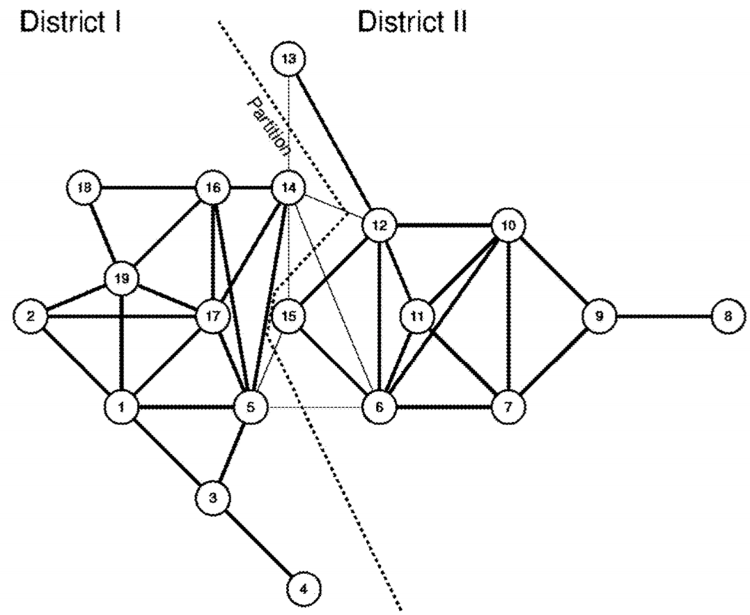


Fig. 1E

Initial Partition of the Graph

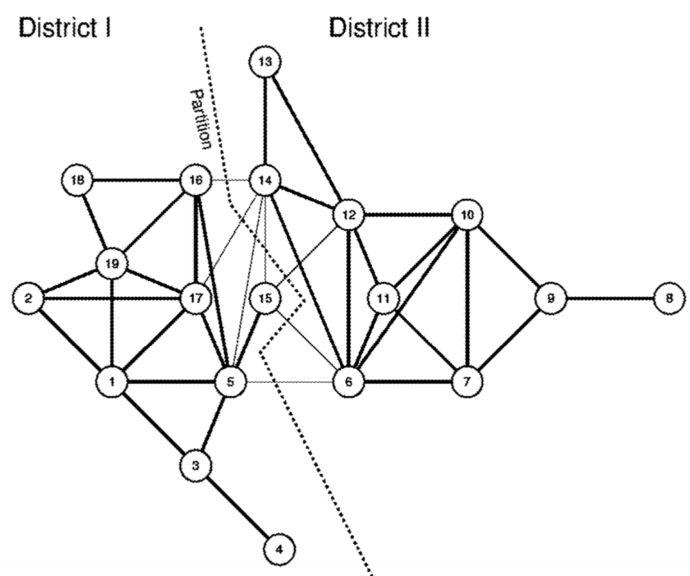


Fig. 1F

Optimal Partition of the Graph

Notes	More detailed notes on how the algorithm works follow: - The geographic data is made into a graph with weighted vertices depending on how populous each geographic region is. - Vertices represent geographic units and edges represent the borders between said units. - The graph is then partitioned into contiguous districts with (roughly) equal populations. - Begin with a graph G with vertex set A_0, edge set E_0, and a weight function w that assigns a weight to each vertex in A_0. - G is first split into a series of smaller graphs G_1, G_2, G_3, …, G_m such that vertices in G_i become contiguous multi-nodes in G_{i+1}. - To do this, the algorithm randomly selects an initial node and joins neighboring vertices with it to create multi-nodes. - This simplifies the graph while preserving information on connections (two multi nodes are connected if and only if at least one constituent vertex in each is connected) and weight. - The algorithm now partitions this coarsened graph of multi nodes. - A random multi node is selected and neighboring multi nodes are joined until a section of the graph with weight roughly 1/k is created, where k is the desired number of voting districts. - Discontiguous partitions created through this process will be discarded since continuity is a necessary criterion. - The resulting graph is then “un-coarsened” by reversing the steps used to coarsen it. - Partition assignments are preserved, so constituent elements of multi nodes in G_{t-1} preserve their district in partition P_{t-1}. - The last step is refinement, which is the main part of the algorithm. - The goal of this step is to make districts as equally balanced as possible. - During un-coarsening, the algorithm reassigns elements of one district to another if they would be more balanced. - This requires the implementation of a balancing algorithm, of which there exist several. - Finally, the algorithm is repeated until a sufficiently balanced redistricting has been created. - If this does not happen, the algorithm starts over with a new random seed. - This algorithm is superior to many currently used due to the fact that it avoids implicit bias. - The algorithm was shown not to produce districts of one variety over another.

	- It also was used to generate a distribution of possible voting maps, which was very similar to the known distribution, showing a lack of bias.
	- The proposed algorithm is also many orders of magnitude more efficient than any of its competitors.
	- To test for bias, a more symmetrical space (a square) was used, as opposed to the irregular populations of states.
	- Over 1 million trials of this symmetric space, little to no evidence of bias was found.
	- The asymptotic complexity of the algorithm can be formulated under two variables: n, the number of initial geographic units, and D, the number of desired voting districts.
	- The algorithm then scales under the function:
	    - f(n,D) = n^2(log(nD))(log(D)).
	- This is far more efficient than most similar algorithms.
	- A similar proposed algorithm by Cirincione, Darling, and O'Rourke (2000) scales according to:
	    - h(n,D) = n/(product of p(n,i) from i = 1 to D-1)
	- Here, p(n,i) is the probability that the next iteration will form a contiguous voting district.
	- As n and D become larger and larger, the difference in efficiency between these two algorithms becomes more and more apparent.
	- For example, the proposed algorithm is 10000 to 15000 times more efficient for something like New York.
	- The algorithm was used to draw neutral maps of Virginia, Texas, and Missouri.
	- All three states have significant minority populations, which adds another criterion, since minorities must be spread sufficiently by the Voting Rights Act.
	- The algorithm achieved expected results for all three states - i.e. the expected districting had mapmakers only considered geographic units and population.
	- Since the goal of the algorithm was to generate neutral districts, this was a favorable result.
	- Possible extensions on this algorithm include weighting the edges - higher-weighted edges mean that it is more imperative that two geographic units end up in the same district.
	- The algorithm can be altered to also minimize the weight of the edges it cuts, producing a favorable district with the above definition of edge weights.

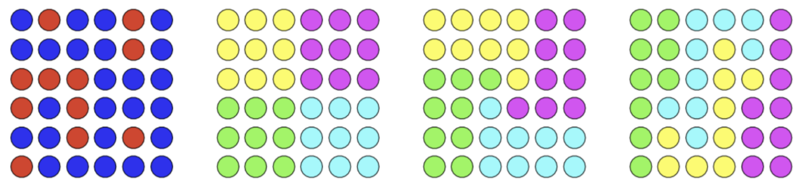
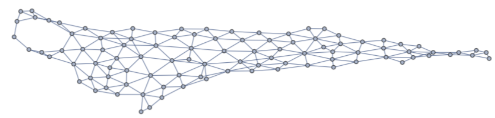
Cited references to follow up on	- Cirincione, Darling, and O'Rourke (2000): another gerrymandering algorithm that the proposed algorithm was compared to. - Hendrickson and Leland 1995, Kernighan and Lin 1970:

	sources on graph balancing algorithms, used in the refinement stage of the algorithm.
Follow up Questions	- What would implementing this algorithm in a programming language look like? Can it easily be implemented, and what programming language would be best for this purpose? - How can we use the author's suggestions to extend the algorithm to work with weighted edges? - Were the conducted tests sufficient to show that the algorithm is not biased? What other tests can be conducted? - During the coarsening of the graph, are they any criteria that should be taken into account when assigning vertices to multi nodes?

Article #2 Notes: Detecting Gerrymandering with Probability: A Markov Chain Monte Carlo Model

Article notes should be on separate sheets

Source Title	Detecting Gerrymandering with Probability: A Markov Chain Monte Carlo Model
Source citation (APA Format)	Barkstrom, J., Dalvi, R., & Wolfram, C. (2017). *Detecting Gerrymandering with Probability: A Markov Chain Monte Carlo Model*. Dam.Brown.Edu. https://www.dam.brown.edu/siam/2017/jrc.pdf
Original URL	https://www.dam.brown.edu/siam/2017/jrc.pdf
Source type	Paper
Keywords	Markov Chain, Monte Carlo, random generation, graph, adjacency graph, connected, efficiency gap, probability
Summary of key points (include methodology)	The authors tested three different algorithms of determining whether or not a given voting map was gerrymandered. They first tested their algorithms on a fictional state called “North Squarolina,” which was a 6 by 6 square grid with unanimous voting blocks. The algorithms were then tested on real states as well. The main idea was to pseudo randomly generate thousands of voting maps to compare the given one against. If the given voting map significantly differed from the randomly generated ones, then evidence of gerrymandering could be concluded. Three algorithms were tested: completely random districting, random districting so that the districts were of equal size, and pseudo random districting based on a Monte Carlo Markov Chain model. It was found that the third algorithm performed the best.
Research Question/Problem/ Need	What algorithms are the best for identifying partisan gerrymandering, while taking into account things like voter distribution and natural boundaries such as rivers.

Important Figures	 (V) (A) (B) (C) This figure shows the voting distribution of North Squarolina, as well as three potential districtings. Distrincting C was found to be intentionally gerrymandered.  Figure 8: Adjacency graph of North Carolina counties. This shows the adjacency graph of North Carolina, with vertices representing counties and edges representing adjacent counties.
Notes	- Efficiency gap is the most commonly used measurement of gerrymandering currently. - It basically counts how many votes a party “wastes”, either by going over the amount needed to win or by losing. - If this number is too big, the map is gerrymandered. - This approach is not great since it loses information like natural boundaries and natural voter distribution, which is not intentional gerrymandering. - Algorithms will be tested on North Squarolina, a 6 by 6 square grid with unanimous voting blocks, before being tested on the real state of North Carolina. - The first algorithm tested was completely random generation of districts. - The second algorithm tested was completely random generation of districts, but with the caveat that the districts had to be roughly equal in population. - The third algorithm started with a randomly generated districting of contiguous voting blocks, and at each step the smallest district was made bigger by flipping a block connected to it. - An immediate problem with the third algorithm is that it’s not completely random, and thus even after many steps the generated position will have some bias towards the original. - This can be improved by seeding the algorithm completely randomly. - Further improvement can be made by running algorithm 3 many times with different starting points, and averaging the

	results. - Algorithm 1 (completely random districting) was not very accurate when tested on North Squarolina, as it yielded districts of wildly different size, as well as non-contiguous districts, both of which are unfavorable. - As such, algorithm 1 gave basically no information about the potential gerrymandering of North Squarolina. - Algorithm 2 fared much better because the districts were of roughly equal size. However, non-contiguous districts were still an issue. - Algorithm 2 correctly identified a gerrymandered voting map of North Squarolina. - Algorithm 3 works the best because all districts are of roughly equal size and are contiguous at every iteration, meaning it truly generates hundreds of thousands of valid voting maps, as desired. - For a more complex state like North Carolina, we note that the above algorithms can be reduced to algorithms on graphs (North Squarolina was a grid graph). - A graph can be made by making each county a vertex and edges between connected counties. This is an adjacency graph. - Using algorithm 3, it was determined that North Carolina's 2012 election was not gerrymandered. - Although algorithm 3 is very good, there are some flaws, including the slight bias mentioned above. - Another flaw is the assumption of unanimous voting blocks, which may not be valid in many cases. However, the model should be easily generalizable, as it is independent of voting behavior. - For mathematical rigor, we can note a well-known theorem in the field of Markov chains, which states that the probability that the label of the initial seed is an epsilon-outlier (occurred epsilon times) is at most the square root of 2 times epsilon. - This gives much higher results than epsilon itself for small epsilon, which reduces the certainty of gerrymandering. - The efficiency gap, mentioned earlier, simply takes the difference between the number of wasted votes of party B and the number of wasted votes of party A and divides by the number of total votes. - However, this does not take into account natural voter distribution, which is very important: if democrats are naturally squashed together, it is impossible to avoid a somewhat gerrymandered arrangement.
Cited references to follow up on	Fifield, Benjamin, et al. A New Automated Redistricting Simulator Using Markov Chain Monte Carlo.,

	https://imai.fas.harvard.edu/research/files/redist.pdf
Follow up Questions	- Is there any way we can improve the mathematical bound of the square root of 2 times epsilon given in the paper? - Can we improve the efficiency gap to create a mathematical metric that takes into account more factors?

Article #3 Notes: A Computational Approach to Measuring Vote Elasticity and Competitiveness

Article notes should be on separate sheets

Source Title	A Computational Approach to Measuring Vote Elasticity and Competitiveness
Source citation (APA Format)	DeFord, D., Duchin, M. and Solomon, J. (2020, 26 May). A Computational Approach to Measuring Vote Elasticity and Competitiveness. 28 September 2020 tarihinde https://arxiv.org/abs/2005.12731 from retrieved.
Original URL	https://arxiv.org/pdf/2005.12731.pdf
Source type	Paper
Keywords	Redistricting, Gerrymandering, Markov Chains, Competitiveness
Summary of key points (include methodology)	This paper is a survey of many different measurements of competitiveness in voting maps, which is a way of measuring how uncertain the outcome of the election will be. The metrics measured include vote-band matrices, EG zeroing, and the Cook Partisan Voting Index (CPVI). In order to test the efficacy of these indices, the probabilistic approach of generating many districts was used. It was found that each competitiveness index had different problems.
Research Question/Problem/ Need	Are the current mathematical indices used to detect gerrymandering actually sufficient?

Important Figures

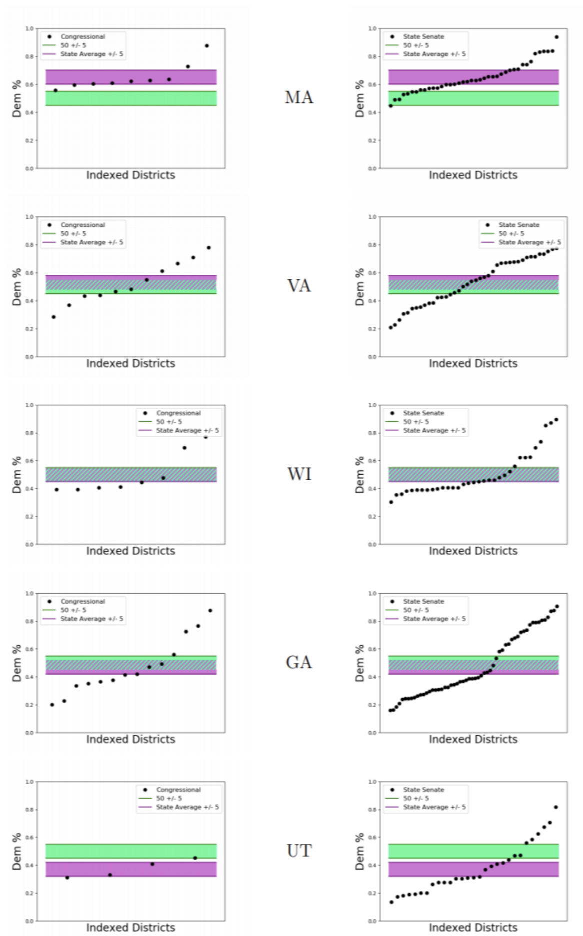


Figure 1: Comparison of the enacted Congressional and state Senate plans to $(5, 50)$ bands (green, competitive) and $(5, D_0)$ bands (purple, state-typical). Here and in most plots below, the districts are ordered from smallest D share to highest to facilitate comparisons across plans.

State, Type (#)	Enacted $(x, 5, 50)$	Ensemble $(x, 5, 50)$	D_0	Enacted $(x, 5, D_0)$	Ensemble $(x, 5, D_0)$
MA Cong (9)	0	0.24	64.7	5	3.85
VA Cong (11)	3	2.79	52.8	2	2.82
WI Cong (8)	1	1.76	49.6	1	1.81
GA Cong (14)	2	2.63	47.3	2	4.43
UT Cong (4)	1	0.62	37.6	2	1.05
MA Sen (40)	6	6.85	64.7	13	12.82
VA Sen (40)	6	8.60	52.8	6	8.00
WI Sen (33)	7	8.80	49.6	7	9.10
GA Sen (56)	2	8.57	47.3	4	8.78
UT Sen (29)	2	4.09	37.6	4	4.57

Table 2: Comparison of the number of $(5, 50)$ and $(5, D_0)$ districts—close to 50-50 and close to the state average, respectively—in enacted plans versus ensemble means. Most cases, the enacted plan has slightly fewer $(5, 50)$ districts than the ensemble average—the only examples where the enacted plan has more than the mean are Virginia's and Utah's Congressional plans. Georgia and Utah's Senate numbers stand out from the group in the other direction, with conspicuously fewer close seats than typical plans in the neutral ensemble.

Notes

- Competitiveness is the measure of how uncertain the result of an election in a particular district is.
- Many states have vague language on how districts should be drawn (New York just says "districts not be drawn to discourage competition")
- Missouri uses the efficiency gap in their law, but this has problems, as elaborated on later.
- New Jersey attempted to define competitiveness in terms of the previous census cycle, but this did not work since it is a majority democratic state.
- There are several mathematical indices for measuring competitiveness.
- We can measure evenness, which is the number of votes needed to change the outcome of a district. This is the intuitive idea of competitiveness.
- The Cook Partisan Voting Index (CPVI) is commonly used for this task.
- A district is described as competitive if its CPVI is between D+5 and R+5, i.e. within 5% of the nationwide average.
- However, this measures typicality instead of competitiveness by considering the case of a skewed average.
- In what follows, let D_0 be the Democratic vote share of a district.
- Define a district to be a (y,z)-district if its Democratic vote share is within y of z (a (5,50)-district has a Democratic vote share between 45% and 55%, for example)
- Then, we can define a districting to be (x,y,z)-compressed if it has at least x (y,z)-districts.
- Competitiveness can be targeted by setting z=50, and typicality can be targeted by setting z=D_0.

	- It is natural to ask how the number of (y,z)-districts varies with a fixed z and varying y: a y-value of 15-20% is needed to render about half the districts competitive -> this is a bigger skew than expected! - Missouri has a law in place to rigorously calculate how bad gerrymandering is: it calculates the efficiency gap for 11 different voter swings (-5,-4,-3,-2,-1,0,1,2,3,4,5), and the goal is to make these as small as possible. - It can be shown that this forces the number of (5,50) districts to be at most ⅕ the total -> similar results to other papers, on how the efficiency gap actually decreases competitiveness - Many neutral redistrictings will be developed for a variety of states, then evaluated through vote-band metrics. - We define the precinct dual graph, which has a node at each precinct and an edge between two precincts that are geographically adjacent. - In this way, a districting into k sections is just a subdivision of this graph into k subgraphs. - The 2016 presidential election was chosen for analysis. - GerryChain is used for neutral ensemble generation. - Districts are required to be connected and roughly equal in population (within 5% of each other). - Begin with a random seed, take 100,000 ReCom steps (to decrease similarity to the seed) and 1,000,000 Flip steps to generate a random districting. - A ReCom step involves taking two districts, merging them, creating the adjacency graph of their precincts, finding a spanning tree of that graph, and splitting it into two equally sized trees, if possible. - A flip step involves flipping one precinct to another district such that the districts remain contiguous. - The baseline range of voting maps generated by this method does not imply bounds on the possible districtings, but instead an image of what the general districting looks like. - All states (minus Virginia) had a mean-median score favoring Republicans, which shows that political geography must be taken into account when dealing with gerrymandering. - This adds to a “growing body of evidence” that you can’t apply one uniform metric to all states and expect to get a sensible result. You need to take into account local factors of each state. - One can also look at the effects of winnowing, which is picking a particular subset of the entire ensemble and comparing its properties to the whole. - In states with a strong skew towards one party, increasing the number of competitive districts ((5,50) districts) actually hurts the minority party since it’s easier to steal votes from them.

	- The paper concludes that Markov chains and ensemble analysis are particularly good ways to identify gerrymandering.
Cited references to follow up on	Jowei Chen and Jonathan Rodden. Unintentional gerrymandering: Political geography and electoral bias in legislatures. Quarterly Journal of Political Science, 8:239–269, 2013. Daryl DeFord, Moon Duchin, and Justin Solomon. Recombination: A family of Markov chains for redistricting. ArXiv:1911.05725, 2019.
Follow up Questions	- Is there some purely mathematical formula we can develop to evaluate the effectiveness of a certain voting map based on a variety of factors like population distribution and not just votes? - What happens if more Flip or less ReCom steps are used?

Article #4 Notes: Three Practical Tests for Gerrymandering: Application to Maryland and Wisconsin

Article notes should be on separate sheets

Source Title	Three Practical Tests for Gerrymandering: Application to Maryland and Wisconsin
Source citation (APA Format)	Wang, S. S.-H. (2016). Three Practical Tests for Gerrymandering: Application to Maryland and Wisconsin. *Election Law Journal: Rules, Politics, and Policy*, *15*(4), 367–384. https://doi.org/10.1089/elj.2016.0387
Original URL	https://web.math.princeton.edu/~sswang/wang16_ElectionLawJournal_gerrymandering-MD-WI_.pdf
Source type	Journal article
Keywords	Gerrymandering, lopsided, statistical test, court case, reliable win, intent, effect, partisan
Summary of key points (include methodology)	- In order to quickly and accurately identify partisan gerrymandering, statistical tests can be used. - These statistical tests allow for intent and effect to be identified, which are the key components to recognizing partisan gerrymandering. - The first test measures lopsided wins - if one party is winning in much higher margins than the other, then gerrymandering is likely to be at play. - The second test measures the reliability of wins, or if one party has gerrymandered the voting districts to win reliably. - These two tests test for intent. - The third test tests for effect by running computer simulations to compare the given voting map against the average one.
Research Question/Problem/ Need	Is there an easy and quick way to identify partisan gerrymandering that does not require intensive modeling, so that it can be used in courts?

<table>
<tr>
<td>Important Figures</td>
<td>

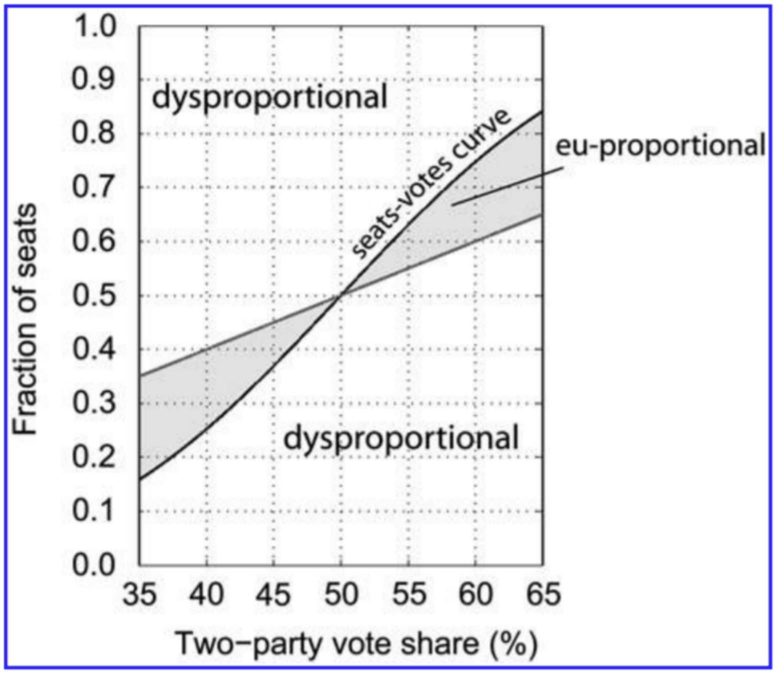
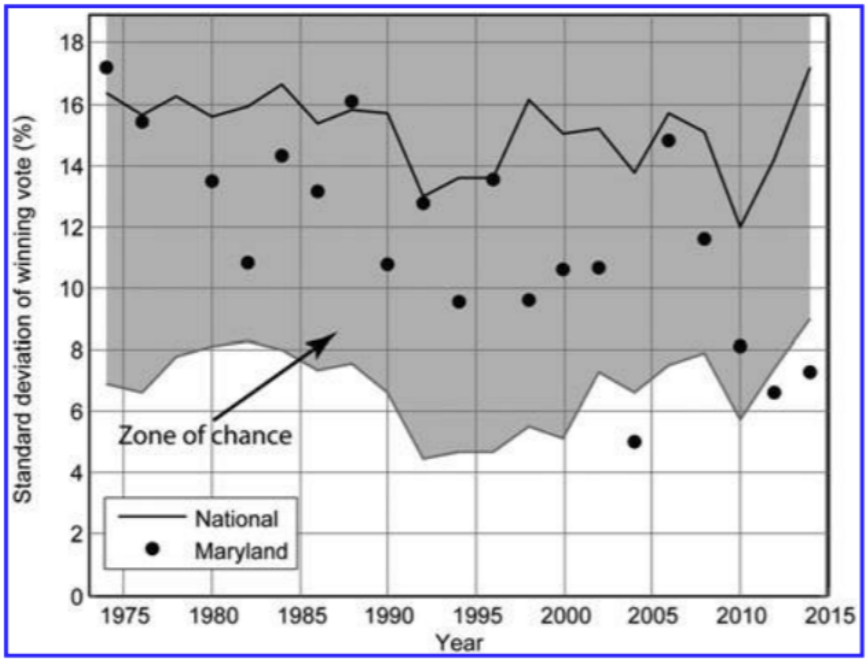


FIG. 1. A representation plot for classifying redistricting schemes. The seats/votes curve indicates the average relationship between seats won (*vertical* axis) and the popular vote share (*horizontal* axis), calculated by creating hypothetical delegations using 2012 House district election results. The *diagonal straight line* indicates proportional representation. Redistricting schemes that fall in the *shaded* zone between the *curve* and the *line* are termed eu-proportional; other outcomes are termed dysproportional. For clarity, the zone of chance (see text) is not shown.

The shaded zone is the zone of chance, with other zones being suspicious (either too much percentage for too few seats or the other way around). This forms a bowtie shape.

</td>
</tr>
</table>

	Standard deviation of winning vote (%) Year Zone of chance National Maryland 0 2 4 6 8 10 12 14 16 18 1975 1980 1985 1990 1995 2000 2005 2010 2015

<table>
<tr>
<td></td>
<td>

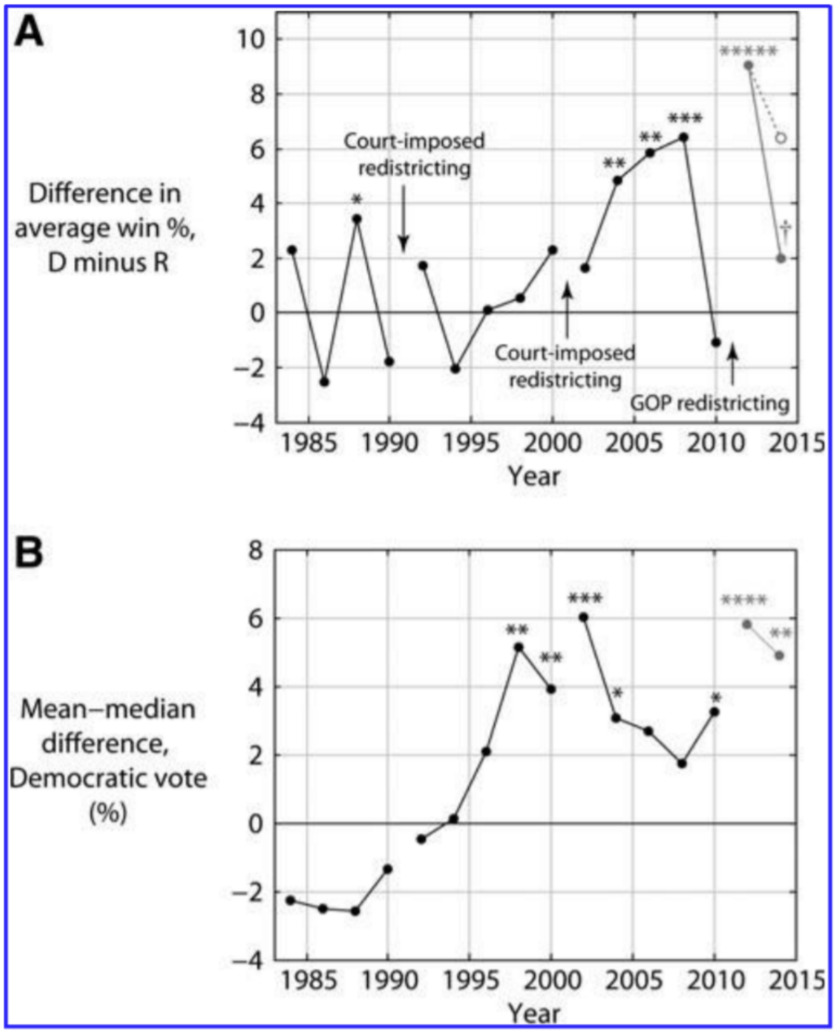


FIG. 5. Application of gerrymandering tests to the Wisconsin State Assembly, 1984–2014. **(A)** Application of Test 1, the difference between average Democratic win margin and average Republican win margins. Statistical significance was tested with a two-tailed unpaired *t*-test. The open symbol indicates the value calculated without imputing support. **(B)** Application of Test 3, the mean-median difference. Significance levels: $*p<0.05$; $**p<0.01$; $***p<0.001$; $****p<0.0001$; $*****p<10^{-7}$. †a majority of Assembly seats were uncontested, diminishing the numerical and statistical value of Test 1.

</td>
</tr>
<tr>
<td>Notes</td>
<td>

- Partisan gerrymandering, in short, is the act of drawing voting districts to give one side an unfair advantage.
- The Supreme Court has not yet decided on a good metric for measuring gerrymandering.

</td>
</tr>
</table>

	- One proposed approach to this was the absence of partisan symmetry, where a reversed popular vote leads to a reversed outcome. The absence of such symmetry yields partisan gerrymandering. - Diagnosing partisan gerrymandering requires examining the whole state at once. - The unconstitutionality of partisan gerrymandering is due to the Fourteenth Amendment's Equal Protection Clause and the First Amendment's protection of speech of association. - To legally identify partisan gerrymandering, both intent and effect have to be established. - The Supreme Court has developed a distinction between racial and partisan gerrymandering. - In gerrymandered districts, the only chance for voters of the gerrymandered party to express their interests is in the primary election. - In 2012 Pennsylvania, a well-known gerrymandered map, Pennsylvania had 5 out of 18 districts Democrat with an average of 76% voters in those 5 districts, as opposed to 59% Republican in the other 13. This is asymmetrical. - Getting more votes in a district than the popular vote is only seen as an offense if the gap is large enough - how do we quantify this? - State gerrymandering occurs by packing and cracking. First, opposing voters are packed into a few districts so the majority will be won by the preferred party. Voters can also be cracked so they are split among many districts which they lose instead of a few which they win. - The key component to either approach is asymmetry. - We can use a "lopsided-margins test" to detect when the targeted party wins with unusually large margins. - The difference between the mean and median is a good measure of skewness. - Calculating this tells you how many people were essentially disenfranchised in voting, telling you how badly the maps were skewed. - One exception to gerrymandering is in the case of ethnic minorities, who are encouraged to be placed together in a district to ensure representation. - Majority-minority districts should not comprise a fraction of all districts that is above the proportion of minority population in the state. - A standard for partisan gerrymandering should differentiate between partisan proportionality and disproportionality. - We must also define what exactly is chance and what isn't. - We do this by defining a zone of chance, a zone where voting maps in the zone could reasonably be randomly created.

	- Take the case where the two parties are equally represented in a state. - Then, party A's seat-share is on average N/2, and it wins a district with probability 0.5. - One standard deviation of this occurs about ⅔ of the time, and is therefore a rather unsurprising outcome. - Computer simulations can be used to answer the question of how different districtings affect the final election result. - This can then act as a baseline for spotting asymmetries. - Standard deviation will be used to define the zone of chance. - 1.6 standard deviations or more occurs by chance 5% of the time, the threshold for statistical significance. Thus, this can be used to define the zone of chance. - We now propose three quantitative tests on intens and effects in partisan gerrymandering. - The first test is the lopsided outcome test, which is the comparison of the difference between the share of Democratic votes in Democrat-won districts and the share of Republican votes in Republican-won districts. - Use a grouped t-test to compare the two and see if there is a major discrepancy. - The second test is the reliable-wins test. In a closely divided state, calculate the difference between a party's statewide average district vote share and its median vote share. A gerrymander occurs when a party's median share is far below its average vote share. - If the redistricting party is dominant, then calculate the standard deviation of the redistricting party's vote share in its districts and the standard deviation of the vote share nationwide. Compare these two with a chi-square test to define zones of chance. - The third test is the excess seats test. Calculate the expected number of won seats with simulations and compare this to the given to determine the size of the gerrymander. - The proposed tests do not require detailed maps. Also, they are derived from election results only, meaning they are independent of the districting itself. - The tests can also be used together for a more reliable result. - Test 1 is very simplistic. - Test 2 measures how reliable a win is for the redistricting party. It is also quite simple. - Test 3 quantifies effects, which makes it very powerful, but also harder to compute since it requires computer simulation. - These tests are applied to three real-world elections to show their effectiveness (details omitted in the notes). - By establishing a mathematically rigorous zone of chance, these tests are usable for judges.

	- An intents-and-effects based test is also unambiguous and can bypass the need to demonstrate intent in other ways. - Since the statistics involved in these tests is relatively simple, experts are not required in the courtroom. - These three tests can be calculated using information available before an election, as to prevent a gerrymandered election from even occurring.
Cited references to follow up on	Samuel S.-H. Wang, Three Tests for Practical Evaluation of Partisan Gerrymandering, 68 Stanford Law Review 1263 (2016).
Follow up Questions	- Are these tests really as accurate as the rigorous Markov chain modeling presented in other papers? - Is there any way we can test this? - Are there other methods that can be used to detect intent or measure effect?

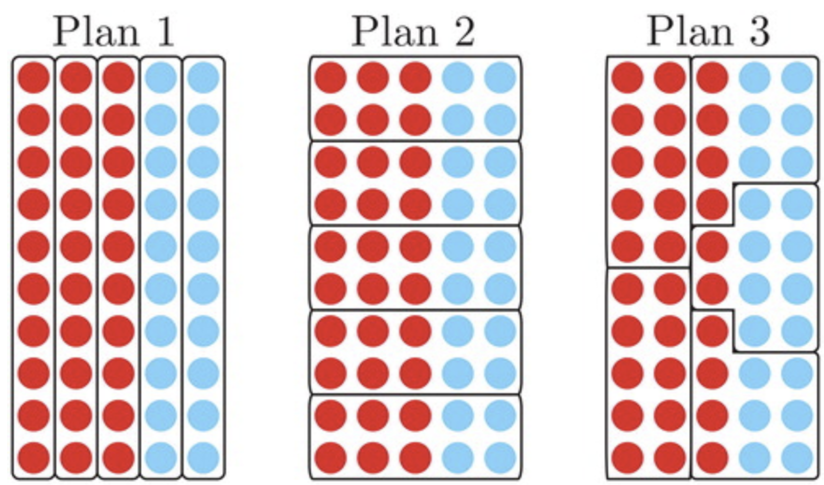
Article #5 Notes: Measuring Political Gerrymandering

Article notes should be on separate sheets

Source Title	Measuring Political Gerrymandering
Source citation (APA Format)	Tapp, K. (2019). Measuring Political Gerrymandering. *The American Mathematical Monthly*, *126*(7), 593–609. https://doi.org/10.1080/00029890.2019.1609324
Original URL	https://maa.tandfonline.com/doi/full/10.1080/00029890.2019.1609324
Source type	Journal article
Keywords	Gerrymandering, metric, efficiency gap, weighted efficiency gap, relative efficiency gap, symmetry
Summary of key points (include methodology)	This paper reviews the different mathematical metrics used to measure gerrymandering that are based solely on the district outcomes of the election. These are symmetry methods, the efficiency gap, weighted efficiency gap, and relative efficiency gap. In general, the relative efficiency gap with weight 2 was found to be the most useful and accurate metric, although all could be improved by adding consideration of other factors. The others had problems in various areas; symmetry methods assume too much and the weighted and unweighted efficiency gaps are not very useful in lopsided elections.
Research Question/Problem/ Need	What are the different mathematical metrics, only based on district outcome data, in use to determine the existence of partisan gerrymandering?

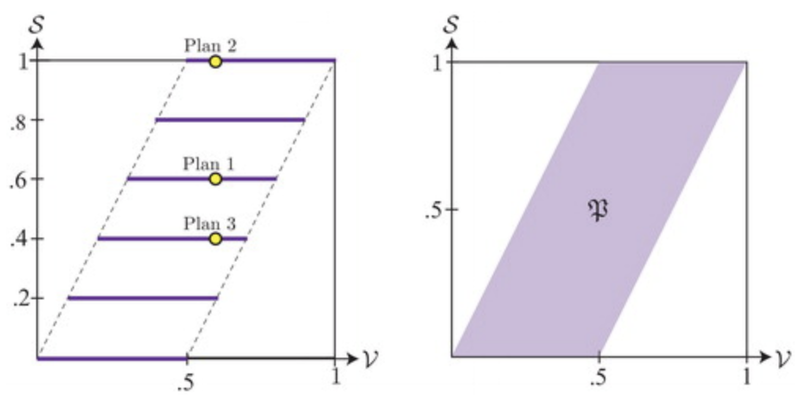
<table>
<tr>
<td>Important Figures</td>
<td>

Fig. 1 Three ways to divide 50 voters into 5 districts.



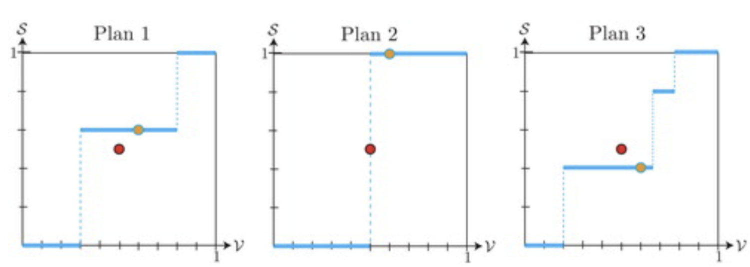
How redistricting can affect the outcome of elections.

Fig. 2 All possible $(\mathscr{V}, \mathscr{S})$-outcomes lie in $\mathfrak{P}$.



The space of possible outcomes for the election. V is the proportion of votes for a party and S is the proportion of districts won by that party.

Fig. 3 Seats–votes curves for the plans from Figure 1.

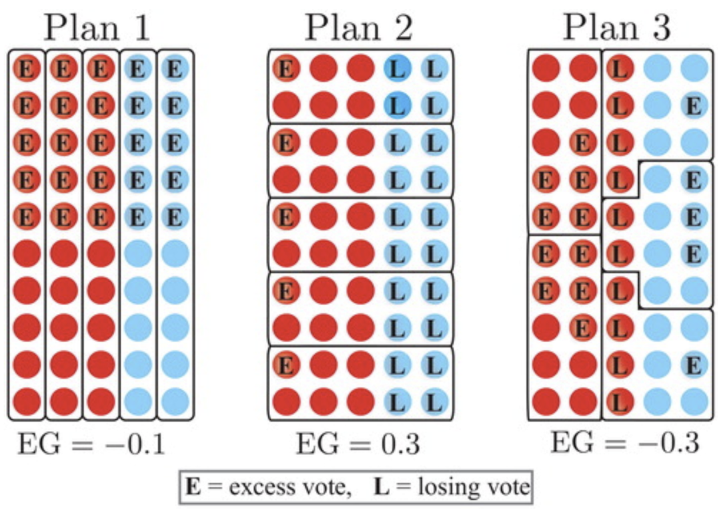


This maps the possible outcomes for voters switching in each of the

</td>
</tr>
</table>

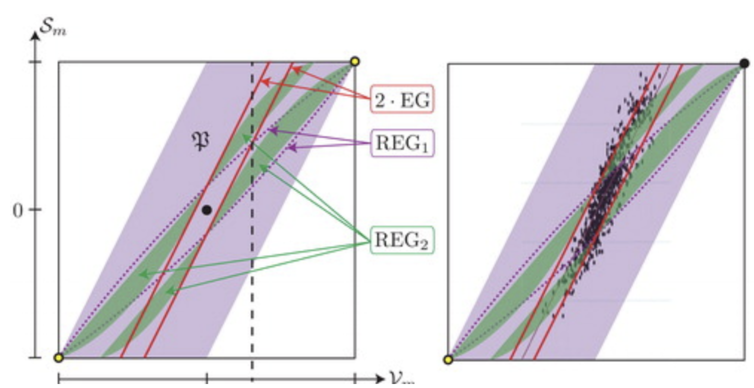
<table>
<tr>
<td></td>
<td>
plans in figure 1. Only plan 2 is fair (in this metric) since it is symmetric about the center of the square.

Fig. 5 The efficiency gap of the plans from Figure 1.



An efficiency gap on the three maps provided in figure 1. All of these pass the benchmark of 0.08, which conflicts the analysis provided by the vote-seat curves.

Fig. 8 Contours for the cutoffs $\pm.16$.



Comparing the allowable regions by 2 times EG, REG_1, and REG_2. REG_2 interpolates between regions allowed by EG and REG_1, making it a superior choice overall.
</td>
</tr>
<tr>
<td>Notes</td>
<td>

- The Supreme Court has not yet convicted any gerrymandering.
- The efficiency gap was proposed by Stephanopoulos and McGhee, and has been used in court, as a way to measure gerrymandering.
- Alternative versions of the efficiency gap have been proposed by others.
- Define Vi^P to be the number of votes for party P in district i,
</td>
</tr>
</table>

	S_i^P to be 1 if party P wins district i and 0 otherwise, and T^P to be the set of districts won by party P. - If there is no subscript or superscript, we assume things are being summed over. - Also let $v=V^A/V$ (the proportion of votes for party A) and $s=S^A/S$ (the proportion of seats for party A). We assume that no two districts are tied. - We also assume that each district has equal voter turnout - note that this does not directly follow from the fact that district must be equipopulous. - This allows us to rigorously show that the parallelogram in figure 2 is indeed the set of all possible elections. - For the remainder of the article, we only consider metrics that determine gerrymandering based solely on district outcome data (improve on this for my project by adding consideration of other metrics?) - Say we have some district outcome data, and exactly t voters switch parties in each district. - Ideally, if votes flip, then the outcome of an election should flip. If this happens in every case, then the districting is fair. - More sophisticated seats-votes curves have been created, but they haven't been used in court since a lot of arbitrary assumptions are made in their creation. - This prompted the development of the efficiency gap, which identifies gerrymandering based solely on votes. - If a party loses a district, then all its votes are in excess. If a party wins a district, then all votes that are over the required number to win (½) are excess. The difference in the number of wasted votes of the two parties, divided by the total number of votes, is the efficiency gap. - An efficiency gap far from 0 is evidence of gerrymandering, with the sign determining which party gains an advantage. - Efficiency gap alone is not enough to determine gerrymandering, since it can be chalked up to other factors like voter distribution. (MIT paper) - Thus, it often must be used in conjunction with computer simulations. - One can show that the efficiency gap is $S_m - 2V_m$, where $S_m = s-½$ and $V_m = v-½$. This replaces the symmetric principle from earlier, by suggesting that fairness stems from the seat margin should be twice the voter margin. - This shows that, in extremely lopsided elections, the efficiency gap is useless since it indicates the losing party manipulated the map, which is highly unlikely. - If one defines a packed district as one where the winning party receives more than 75% of the vote and a cracked district as one where the losing party loses with more than

	25% of the vote, then the efficiency gap can effectively prevent packing and cracking. - Thus, one can define competitiveness as the difference between the winning share and losing share of the votes, divided by the total. Thus, a competitiveness of .5 is neutral and contributes nothing to the efficiency gap. - One can prove that "The efficiency gap is the seat-share-weighted difference of the average of the amounts by which the competitiveness of districts won by parties A and B differs from ½". - A natural question about the efficiency gap is: if a district is won with 60% of the vote, then wasn't only 40% needed to win in the first place? - This leads to the development of the weighted efficiency gap, which is the same as the efficiency gap except it multiplies excess winning votes by some multiplier (weight) lambda. - The weighted version of the original efficiency gap lemma becomes S_m - (1+lambda)V_m, where lambda is the weight. - The efficiency gap occurs at lambda=1. - Choosing an absolute value of lambda for the whole election is always problematic, since elections can be made that mess up the formula. - This leads to the development of the relative efficiency gap, which is the difference of the proportion of votes the two parties wasted. - We can create a global formula for relative efficiency gap as well, although it depends on competitiveness as well as seat and voter margins. - Now, consider comparing the efficiency gap, the relative efficiency gap with lambda=1, and the relative efficiency gap with lambda=2. - REG_2 is the best choice here for a few reasons: it generalizes well to elections with more than two parties, and it is useful for lopsided elections, unlike the efficiency gap. - McGhee's efficiency principle states that a gerrymander detection measurement must increase when a party increases its seat share without increasing its vote share. - However, it can be shown axiomatically that this principle restricts the measurement to only depending on the district and vote share - relative efficiency gap does not satisfy this, since its global formula involves competitiveness as well. - The unweighted competitive differential (difference between the competitiveness of the two parties) also fails this principle. - As such, this intuitive principle is actually not a good thing to follow when examining a gerrymander detection metric.

Cited references to follow up on	● McGhee, E. (2017). Measuring efficiency in redistricting. *Elect. Law J.* 16(4): 417–442. DOI: 10.1089/elj.2017.0453. [Crossref], [Web of Science ®], [Google Scholar] ● https://maa.tandfonline.com/doi/full/10.1080/00029890.2018.1517571?src=recsys -> Shows rigorously that the efficiency gap is not a great measurement ● Stephanopoulos, N., McGhee, E. (2018). The measurement of a metric: The debate over quantifying patrisan gerrymandering. *Stanf. Law Rev*. 70(5): 1503–1568. [Google Scholar]
Follow up Questions	- Are there other commonly used metrics that were not considered in this paper? - In the paper, equal voter turnout was assumed. How do the efficacies of the metrics change when this assumption is removed? - Why are other choices of lambda in the weighted and relative efficiency gaps not realistic; could any information be gained from a choice of lambda not equal to 1 or 2?

Article #6 Notes: An impossibility theorem for gerrymandering

Article notes should be on separate sheets

Source Title	An impossibility theorem for gerrymandering
Source citation (APA Format)	Alexeev, B., & Mixon, D. G. (2018). An Impossibility Theorem for Gerrymandering. *The American Mathematical Monthly*, *125*(10), 878–884. https://doi.org/10.1080/00029890.2018.1517571
Original URL	https://arxiv.org/pdf/1710.04193.pdf
Source type	Journal article
Keywords	Efficiency gap, impossibility, Polsby-Popper, equal voter turnout
Summary of key points (include methodology)	First, three mathematical characteristics are defined in order to evaluate the efficacy of a given districting plan. These are equal voter populations in each district, Polsby-Popper compactness (measures how weird the shape of a district is, as weird district shapes are often a telltale sign of gerrymandering), and the efficiency gap. It is then shown that not all three characteristics can hold; that is, a low efficiency gap encourages weirdly shaped districts.
Research Question/Problem/ Need	Is the efficiency gap really a good way of measuring the fairness of a given voting map?

Important Figures

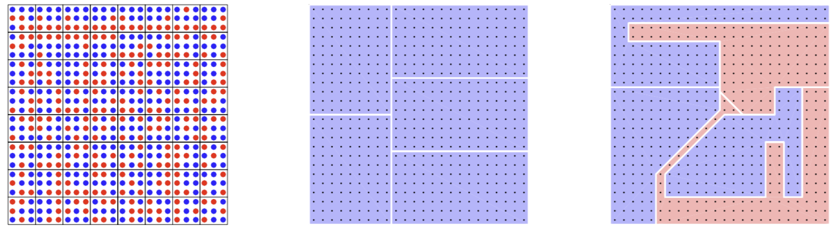


Figure 1: **(left)** Voter locations in $[0,1]^2$. The blue and red dots correspond to A and B, respectively. In this example, the unit square is partitioned into smaller squares of side length $\epsilon = 1/8$. Each ϵ-square contains 9 voters in an $\ell \times \ell$ subset of a lattice with $\ell = 3$. Of these voters, $a = 5$ belong to A and $b = 4$ belong to B. **(middle)** Five districts drawn according to the shortest splitline algorithm proposed by the Center for Range Voting in [6]. This algorithm is specifically designed to produce districts that satisfy desiderata (i) and (ii). In particular, this algorithm ignores voter preferences. In this case, the number of voters in each district is within $\delta = 0.07$ of the average and the smallest Polsby–Popper score $4\pi|D_i|/|\partial D_i|^2$ is over 0.70. (According to the isoperimetric inequality [12], the largest score possible is 1, which is achieved uniquely by the circle.) However, despite B making up 44% of the vote, A won every district. This is reflected in the efficiency gap being over 38% in favor of A. **(right)** One may attempt to decrease the efficiency gap by exploiting clusters in B. In this spirit, we hand-drew districts of similar size. The result is five districts within $\delta = 0.04$ of the average and an efficiency gap of about 2% in favor of A. In exchange for this partisan efficiency, the smallest Polsby–Popper score is now about 0.12. For reference, Case [4] suggests flagging scores below 0.20 as instances of possible gerrymandering. Our main result (Theorem 2) establishes that this tradeoff is unavoidable with any districting system.

The caption summarizes this quite rigorously. As a more high-level view, the left picture shows the voter distribution, where red and blue are the two different parties. The middle shows nicely shaped districts of equal size, but the efficiency gap is high. The right shows a decreased efficiency gap in exchange for weirdly shaped districts. All three characteristics cannot be satisfied at the same time.

Notes

- Fun fact: gerrymander comes from an 1812 political cartoon that compared the weird shape of Massachusetts districts to a salamander
- Sometimes, a small efficiency gap incentivizes the use of weirdly shaped districts
- This is counterintuitive, since geometry is one of the main indicators of a gerrymander
- We first define a districting system to be a function f(A,B,k) that outputs a state with voting population split into parties A and B into k districts
- We use a unit square to approximate a state - this does not affect the results of the proof
- There are three characteristics one might want to test for when evaluating the fairness of a districting system
    - We would like all districts to have a roughly equal number of voters. Assuming uniform voter turnout throughout the state, this corresponds to equal population throughout districts
    - Evaluate the geometry of the district with the Polsby-Popper score, which is the ratio of 4pi times

	the area over the perimeter squared - this follows from the isoperimetric inequality, meaning the score maximizes at 1 for a circle. The further a district is from a circle (the weirder its shape is), the lower its score is. A score of 0.2 should be cause for suspicion. - We also look at the efficiency gap. - We can show that no districting system satisfies all three characteristics - in other words, for some choice of voter distribution, not all three characteristics will be satisfied. - The idea is to consider when the A and B are roughly evenly matched, with A slightly ahead. If B was to win a district and stay equal size, it would have to use a weird shape. Thus, A will win most districts by a small margin, leading to a large efficiency gap. - The proof itself is quite technical - basically split the unit square into n^2 unit squares for some arbitrarily large n, and split A and B roughly equally among these squares. One can show that this yields A winning every district, which pushes the efficiency gap arbitrarily close to its minimum value as n gets large, as desired.
Cited references to follow up on	M. Bernstein, M. Duchin, A formula goes to court: Partisan gerrymandering and the efficiency gap, Available online: arXiv:1705.10812
Follow up Questions	- What other mathematical metric can be substituted for the efficiency gap to invalidate the proof of this theorem (weighted efficiency gap, relative efficiency gap, etc.)? - Is there a better number to bound the Polsby-Popper index by, other than the rather arbitrary 0.2?

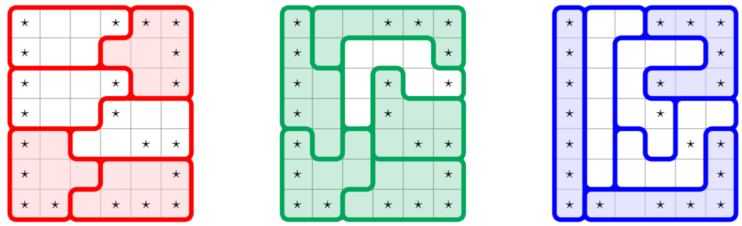
Article #7 Notes: A Formula Goes to Court: Partisan Gerrymandering and the Efficiency Gap

Article notes should be on separate sheets

Source Title	A Formula Goes to Court: Partisan Gerrymandering and the Efficiency Gap
Source citation (APA Format)	Bernstein, M., & Duchin, M. (2017). A Formula Goes to Court: Partisan Gerrymandering and the Efficiency Gap. *Notices of the American Mathematical Society*, *64*(09), 1020–1024. https://doi.org/10.1090/noti1573
Original URL	https://arxiv.org/pdf/1705.10812.pdf
Source type	Journal article
Keywords	Gerrymandered, packing, cracking, efficiency gap, vote lean, seat lean, non granularity
Summary of key points (include methodology)	The efficiency gap is first shown to not depend on local deviations in voting maps, which leads to a host of problems. As such, it is shown that it cannot be used as a standalone mathematical metric. However, it can be used effectively in court in the presence of other factors, which is how the authors predict the formula will be used legally in the coming years.
Research Question/Problem/ Need	How is the efficiency gap used in courts, and is it effective there as well as a standalone mathematical formula?

Important Figures

PARTISAN GERRYMANDERING AND THE EFFICIENCY GAP 3



Plan I: $EG = 0$ Plan II: $EG = -1/3$ Plan III: $EG = 0$

Of course, in real life, no districting plan will have an efficiency gap of exactly 0. How high is too high? In [3], the authors argue that $EG = .08$ corresponds to a historically robust threshold for unacceptable partisan gerrymandering.[4] In the Wisconsin case, for example, the plaintiffs demonstrated that the last three elections for the state assembly had efficiency gaps between 0.1 and 0.13 in favor of Republicans. Since it was a Republican legislature that had drawn the map, and there was plentiful evidence that they had intentionally done so to disfavor Democrats, the court ruled that the plan was an unconstitutional partisan gerrymander. QED.

Plan 1 is pretty fair overall, plan 2 is skewed towards the star party, and plan 3 has lots of packing and cracking but an efficiency gap of 0 since it is packed and cracked roughly symmetrically.

Notes

(Probably the last paper on the efficiency gap I will be reading - provides lots of a good information about how the efficiency gap is used in court as well as a good summary on why, on its own, it is not an effective mathematical metric)

- There are many types of gerrymandering, including racial gerrymandering (minority group's voting strength is reduced), partisan gerrymandering (one party tries to secure more seats for itself), and incumbent gerrymandering (officials gerrymander in an attempt to equalize the election).
- The Supreme Court has always disagreed on how to identify partisan gerrymandering.
- The efficiency gap was proposed as a metric to measure gerrymandering, and *can be useful* (something not really touched on in previous papers), but only with the help of other evidence.
- The next few pages are a definition of the efficiency gap as well as a derivation of the fact that it is just the vote lean minus half the seat lean (in other words, the efficiency gap does not measure local deviations at all). These notes are above, so I won't take them again here.
- Problems:
    - The efficiency gap penalizes proportionality (a map where party A gets 60% of the votes and seats would be measured as gerrymandered towards B)
    - In extremely competitive elections, a slight skew will generate a lot of wasted votes, yielding evidence of a gerrymander when there isn't one.
    - The only way to have a district with efficiency gap 0 is to have it 75-25 to one side, which is not good.

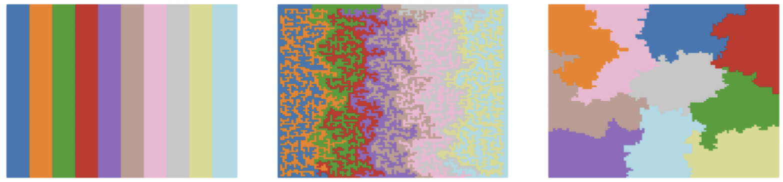
	- If the elections are extremely lopsided (79%+ in one direction), the efficiency gap will also be broken. - The efficiency gap does not depend on local deviations, making it extremely “nongranular” (in the words of the article) for states with a small number of districts. - The original authors address most of these concerns. They say the efficiency gap can only be used if it can be shown it persists under a statewide swing in either direction (to show durability over time). - They say lopsided elections don’t happen at the state level, and indirectly address non granularity by only considering elections with at least 8 districts. - They also justify the condemning of proportionality by saying the winning party deserves a seat bonus. - As used in court, a plan is not thrown out immediately due to high EG, but EG is simply used as a factor to determine gerrymandering. - In all, it can be argued that the efficiency gap can be a useful measure to determine gerrymandering in an array of circumstances, but not in all, and it certainly is not a be all and end all measurement. - EG could be coded into a simulation analysis by using the vote lean minus half the seat lean as a way to “score” the deviation from proportionality of a certain plan.
Cited references to follow up on	I don’t think it would be a good idea to follow up on the references here, since they all pertain to the efficiency gap and at this point most information I get about it will be repeated.
Follow up Questions	- How can the problems with the efficiency gap be addressed (somewhat answered in the review article with the relative efficiency gap that works well in lopsided elections)?

Article #8 Notes: Recombination: A Family of Markov Chains for Redistricting

Article notes should be on separate sheets

Source Title	Recombination: A Family of Markov Chains for Redistricting
Source citation (APA Format)	DeFord, D., Solomon, J., & Duchin, M. (2019, November 14). *Recombination: A family of Markov chains for redistricting*. ArXiv. https://arxiv.org/pdf/1911.05725.pdf
Original URL	https://arxiv.org/pdf/1911.05725.pdf
Source type	Paper
Keywords	Flip, ReCom, tree partitioning, Markov Chain, redistricting, gerrymandering
Summary of key points (include methodology)	Currently, Flip algorithms are the most commonly used Markov Chain-based algorithm to generate a neutral ensemble of voting districts for gerrymandering analysis. However, the proposed ReCom algorithm performs better in almost every category by being faster, taking less steps, and producing a more diverse ensemble of voting plans. The Flip algorithm takes random districts and flips them to their neighbors, while the ReCom algorithm merges two districts and partitions the dual graph of those two. The algorithms are tested on real elections as well as toy examples on simpler territories such as grids.
Research Question/Problem/ Need	How can the current computational algorithms used to identify gerrymandering be improved, specifically in the field of Markov Chain based algorithms?

Important Figures



(a) Initial Partition (b) 1,000,000 Flip steps (c) 100 ReCom steps

Figure 1: Comparison of the basic Flip proposal versus the spanning tree ReCom proposal to be described below. Each Markov chain was run from the initial partition of a 100×100 grid into 10 parts shown at left.

A comparison of Flip vs ReCom. The ReCom algorithm is much farther from the original than the Flip algorithm, even with far fewer steps.

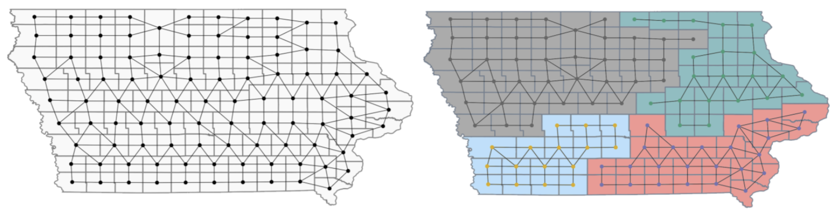
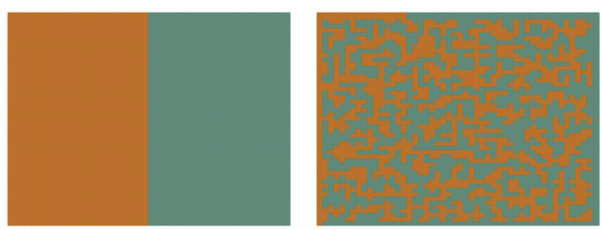


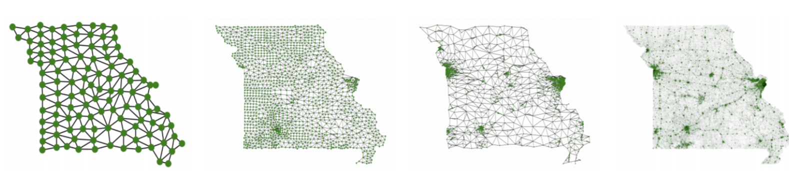
Figure 2: Iowa is currently the only state whose congressional districts are made of whole counties. The dual graph of Iowa's counties is shown here together with the current Iowa congressional districts.

(a) Sequence of four flip steps



(b) Outcome of 500,000 flip steps

Figure 3: At each flip step, a single node on the boundary changes assignment, preserving contiguity. This is illustrated schematically on a 5×4 grid and then the end state of a long run is depicted on a 50×50 grid.



(a) County (b) County Subunit (c) Census Tract (d) Census Block

Figure 6: Four dual graphs for Missouri at different levels of geography in the Census hierarchy. The graphs have 115, 1,395, 1,393, and 343,565 nodes respectively.

Notes

- There are far too many redistricting plans for a given state,

	even after applying criterion, so it becomes infeasible to enumerate all of them.
	- Thus, it suffices to consider a well-chosen sample.
	- Recombinations of Markov chains (ReCom algorithms) are a new class of proposed algorithms that are superior to Flip algorithms.
	- A Flip algorithm randomly flips neighboring squares of districts.
	- The first type of redistricting algorithm used in the literature is optimization algorithm, which focuses on maximizing some sort of objective function while adhering to a certain set of constraints.
	- It is, however, difficult to put these algorithms into practice.
	- Assembly algorithms create a basis of neutral plans to compare the given one against.
	- The most common way to do this is through a flood-fill approach by growing outwards from a set of random seeds. However, the resulting plans are most often not valid, so these algorithms are not feasible to use.
	- Random walk algorithms incrementally transform a given random seed in order to gain a basis.
	- Any Markov chain can be accurately modeled as a random walk on a directed graph.
	- Most MCMC (Markov Chain Monte Carlo) methods utilize Flip steps, but ReCom steps are described here.
	- We first create the dual graph for a state, with a vertex for each geographical unit and edges between adjacent geographical units.
	- A districting plan is then the assignment of each node to a district.
	- Seeds are needed for Markov chain generation; these can be generated with either flood fill or spanning tree methods.
	- There are a few criterion to keep in mind for a good districting plan:
	    - Roughly equal population between the districts.
	    - Continuous districts.
	    - Districts should be a reasonable shape - here, we can count the number of cut edges (edges whose endpoints are in different districts)
	    - Districting plans should respect larger areas like counties and municipalities
	    - Voting Rights Act: make sure minorities get fair representation in a district.
	    - Partisan data should not be included while generating the districting plan.
	- Given a graph G=(V,E), a districting is a decomposition of V into k subsets.

	- We also define the set of cut edges and the set of boundary nodes, nodes who are part of cut edges.
	- If we weight each node by population, we can impose a mathematically rigorous criterion for equal populations:
	$$(1-\varepsilon)\frac{\sum_V w(v)}{k} \leq \lvert V_i \rvert \leq (1+\varepsilon)\frac{\sum_V w(v)}{k}.$$
	- Simply flipping a node can lead to non contiguous districts - thus, we do simple random walks on the space to choose nodes to flip and maintain contiguity.
	- Flip methods, in general, don't work very well on their own since they struggle to get far away from the seed while maintaining criterion (see figure 1).
	- This encourages the use of ReCom, or recombination, methods.
	- These work by taking an induced subgraph of a number of vertices of the dual graph and partitioning the subgraph while retaining continuity.
	- The paper focuses on ReCom algorithms concerning spanning trees.
	    - First, pick a spanning tree from the merged region (we first randomly pick two districts to merge) at random.
	    - Find an edge to cut that maintains population balance: this may not be possible in the current tree, so sometimes multiple iterations will have to be done. However, this is still pretty fast.
	- ReCom algorithms tend to generate nicely shaped districts far from the initial seed.
	- Contrast this to Flip algorithms, which generate districts with weird boundaries and plans that are close to the initial state, even after a lot of steps.
	- Sample cases showed that Flip algorithms struggle with producing lots of effective and distinct plans under the duress of criterion.
	- Even on a simple grid graph, the Flip algorithm is unable to provide diverse plans, whereas the ReCom algorithm is.

Cited references to follow up on	Richard Barnes and Justin Solomon. Gerrymandering and compactness: Implementation flexibility and abuse. Political Analysis (to appear), 2019.
Follow up Questions	The article provides many good follow-up questions, a few of which are listed here: - What are other bipartitioning methods to replace spanning trees? - Can someone with knowledge of Markov chain sampling create a redistricting that fools these algorithms?

	- Explore the mathematical properties of spanning tree bipartitioning - what is the probability that this is possible? - How do perturbations of the underlying geographical data affect the running of the algorithm?

Article #9 Notes: A Comparison of Partisan-Gerrymandering Measures

Article notes should be on separate sheets

Source Title	A Comparison of Partisan-Gerrymandering Measures
Source citation (APA Format)	Warrington, G. S. (2019). A Comparison of Partisan-Gerrymandering Measures. *Election Law Journal: Rules, Politics, and Policy*, *18*(3), 262–281. https://doi.org/10.1089/elj.2018.0508
Original URL	https://arxiv.org/pdf/1805.12572.pdf
Source type	Journal article
Keywords	Gerrymandering, efficiency gap, declination, lopsided means, mean-median difference, partisan bias
Summary of key points (include methodology)	Fourteen different metrics for quantifying gerrymandering were compared. All metrics depended only on the final district voting data. The metrics compared included efficiency gap and its variants, mean-median difference, and declination. To compare these, they were all used on a slew of real elections as well as 12 hypothetical elections that were designed to catch extreme cases. It was concluded that declination was the most effective metric, as it had the least false positives and performed the best out of all other metrics.
Research Question/Problem/ Need	Among the many metrics (based solely on voting data) that are used to quantify gerrymandering, which are more effective than others?

Important Figures

TABLE 1. Summary of measures considered and their acronyms.

Acronym	Measure
EG^1	*Efficiency gap:* comparison of wasted votes.
EG^2	*Margin efficiency gap:* Variant of EG^1 in which wasted votes are relative to losing party vote.
EG^0	*Losing efficiency gap:* Variant of EG^1 in which only losing wasted votes are compared.
EG^1_v	*Vote-centric efficiency gap:* Variant of EG^1 in which each comparison is between the fraction of votes wasted by each party rather than between the absolute amounts.
EG^2_v	*Vote-centric losing efficiency gap:* Combination of EG^2 and EG^1_v.
EG^1_τ	*τ-Gap:* Variant of EG^1 in which votes are weighted according to a function parameterized by τ.
Dec	*Declination:* A measure of differential responsiveness.
Dec′	*Buffered declination:* Variant of Dec that is less sensitive in cases in which one party wins most of the votes.
MM	*Mean-median difference:* A comparison of the median and mean of distribution.
Bias	*Partisan bias:* A comparison of the seat share at 50% of the statewide vote.
Bias′	*Specific asymmetry:* Variant of partisan bias that focuses on symmetry relative to observed vote.
Lop	*Lopsided means:* A comparison of the average winning margins of parties.
EVW	*Equal vote weight:* Variant of MM that requires an anti-majoritarian outcome for a positive result.

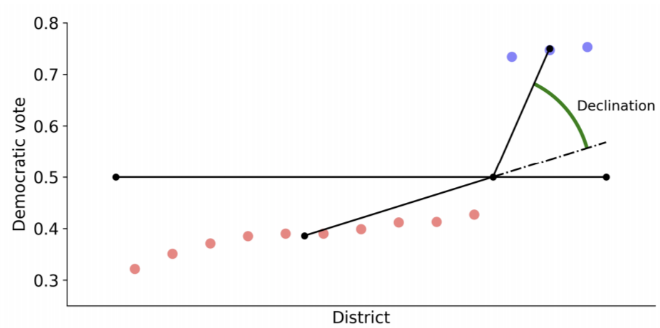
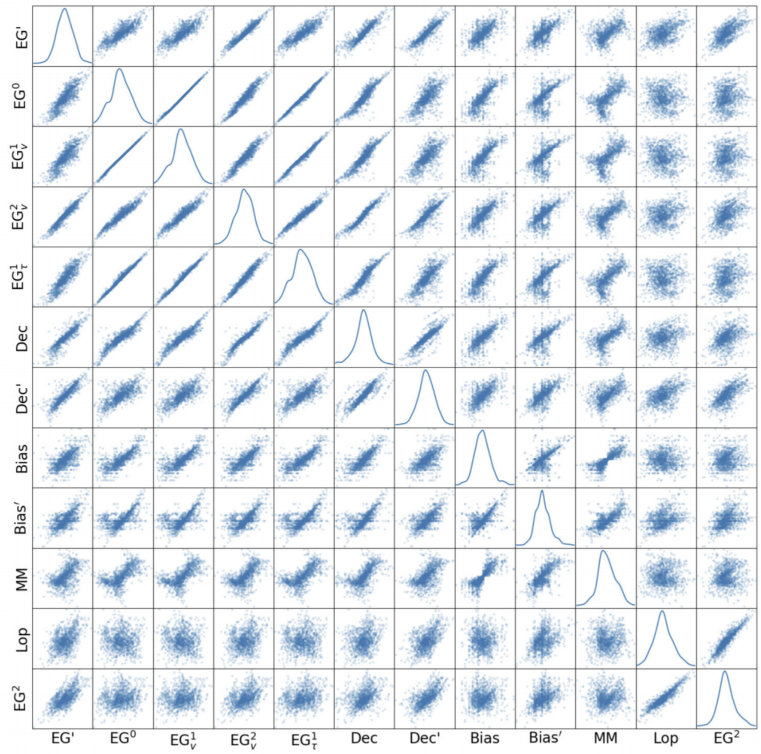


FIGURE 1. Definition of the declination measure.

TABLE 4. Values of scaled measures on hypothetical elections from Figure 4. Column N provides the number of districts in each election while the statewide Democratic vote share is indicated in the Mean column. Bold entries indicate false positives/negatives.

	N	Mean	EG^1	EG^2	EG^0	EG^1_v	EG^2_v	EG^1_τ	Dec	Dec′	MM	Bias	Bias′	Lop
1-PROP	23	0.60	1.2	**2.3**	-0.1	-0.1	0.6	0.5	0.1	0.7	0.0	0.2	0.0	**2.0**
2-PROP	25	0.60	0.3	1.5	-0.7	-0.7	-0.1	-0.6	0.1	0.8	0.0	0.2	0.0	1.9
3-PROP	25	0.60	-1.3	0.0	-1.8	-1.8	-1.5	-1.8	0.0	0.8	0.0	0.2	0.0	1.8
SWEEP	10	0.64	**-2.9**	-1.0	**-3.3**	**-3.4**	**-3.3**	**-3.3**			0.0	0.0	0.0	
COMP	12	0.52	**-3.9**	**-3.4**	**-2.8**	**-2.7**	**-3.5**	**-2.7**	-0.1	0.1	0.3	0.9	**-2.0**	0.3
COMP’	10	0.51	**-2.4**	**-2.1**	-1.7	-1.6	**-2.1**	-1.6	-1.0	-0.7	-0.1	**-2.2**	0.0	-0.5
UNCOMP	10	0.52	-0.7	-0.4	-0.7	-0.7	-0.7	-0.7	-0.8	-0.8	**-2.1**	-1.1	-1.6	-0.5
V-UNCOMP	10	0.52	-0.7	-0.4	-0.7	-0.7	-0.8	-0.9	-1.0	-1.2	**-4.4**	-1.1	-1.6	-1.4
CUBIC	10	0.57	-2.1	**-1.1**	-2.1	-2.0	-2.2	**-1.8**	-2.5	-2.0	**0.0**	**0.0**	**0.0**	**-1.5**
ANTI-M	10	0.44	-2.9	-3.3	**-1.4**	**-1.4**	-2.3	**-1.6**	-2.0	-2.7	-2.1	**-1.1**	**-1.6**	-3.4
CLASSIC	10	0.50	2.7	2.5	**1.8**	**1.7**	2.4	**1.8**	2.2	2.2	**1.4**	2.2	3.2	2.2
INVERTED	10	0.30	-2.7	-4.9	**0.0**	**0.0**	**-1.6**	**-0.3**	-2.0	-3.2	**1.4**	**2.2**	-2.4	-5.0

<table>
<tr>
<td></td>
<td>


This figure shows scatterplots of correlation between each pair of indices, based on the historical data. This is useful for seeing how similar two indices are to each other.
</td>
</tr>
<tr>
<td>Notes</td>
<td>

The authors compare fourteen different partisan gerrymandering measures based solely on district voting data.
The assumption is that voter turnout is equal in each district.
Identifying a gerrymander basically means showing that the drawing of districts contributes to inequality (i.e. geography is not the only reason for inequality between parties)
A linear vote distribution, where a distribution is linear if the vote fractions, arranged in increasing order, lie on a line.
Additionally, we also characterize a districting as unfair if the winning margins for one party are severely less than the winning margins for the other.
Note that there is no mention of competitiveness in either of the two above points. This is because lack of competitiveness is a symptom of bipartisan gerrymandering, not necessarily partisan gerrymandering (this is an interesting point that other articles did not bring up).
The winning efficiency is the difference between the total wasted votes of Democrats and Republicans, divided by 2

</td>
</tr>
</table>

times the number of districts.
- This reduces to the difference between the statewide support for each party, which is a useless metric.
- One can also compare the fraction of Republican votes wasted to the fraction of Democrat votes wasted, where wasted votes are defined as in the definition of the weighted efficiency gap.
- To calculate the declination, start by calculating the average winning margin for Republicans and Democrats.
- Then, calculate corresponding arctangents and the declination as shown below (y,z are the average winning margins for Republicans and Democrats respectively)

If $k = 0$ or $k' = 0$, the declination is undefined. Otherwise, we set $\theta_D = \arctan\left(\frac{2z}{k'/N}\right)$ and $\theta_R = \arctan\left(\frac{2y}{k/N}\right)$ and define the declination as $\mathrm{Dec} = 2(\theta_D - \theta_R)/\pi$ (see Figure 1). Note that the declination is not using the Democratic vote in the median district won by Democrats, but rather the mean of these values.

- We also consider the mean-median difference, as well as bias, which computes how different the fractions of districts each party wins would be if support was shifted uniformly to 50%:

$$\mathrm{Bias} = \frac{1}{2} - \frac{|\{i : p_i > \overline{p}\}|}{N}.$$

For the second share, we compare the seat share the Democrats hold at the level of $\overline{p}$ for statewide support with the seat share the Republicans would hold at the level of $1 - \overline{p}$ statewide support.

7

The partisan bias at the observed level of support is then:

$$\mathrm{Bias}' = \frac{|\{i : p_i + 1 - 2\overline{p} < 1/2\}| - |\{i : p_i > 1/2\}|}{2N}.$$

This variant is referred to as the *specific asymmetry* in (Baas & McAuliffe, 2017).

- The lopsided means measure is the difference between the average winning margins for each party.
- Equal vote weight standard is numerically equivalent to the mean-median measure, but it returns a gerrymander exactly when the party that wins the popular vote gets a minority of the seats.
- The measures were compared using real elections as well as hypothetical elections (results shown in third figure in “Important Figures”).
- The hypothetical figures were important to try to garner false positives/negatives from the metrics in more extreme cases (could be an important kind of testing to do in my project).
- Sensitivity to uniform vote swings was also tested, by applying 10 uniform vote swings from -5% to 5%.
- The weighted efficiency gaps don’t work very well.
- The relative efficiency gap works decently well, but not perfectly (take election 3-PROP).
- Mean-median vote and partisan bias are both immune to

	uniform voting swings, which makes them miss one of the primary goals of partisan gerrymandering. - Declination performed the best out of all measures tested here. - Disadvantages are still present; the measure is defined for party sweeps and quite sensitive to changes near the edge of a sweep. - The lopsided means test is very sensitive to small voter shifts, as it works best when the parties have equal support statewide. - The frame considered in the article was very narrow, and the author encourages broader study of the field, with consideration of more factors.
Cited references to follow up on	Warrington, G. S. (2018b). Introduction to the declination function for gerrymanders. ArXiv e-prints. http://arxiv.org/abs/1803.04799.
Follow up Questions	- Can the declination measure be modified in any way to keep its current properties but be defined for completely swept elections? - Are there any other metrics not assessed in this paper that are worth noting?

Article #10 Notes: Automated Voting District Generation Using Pre Existing Geopolitical Boundaries

Article notes should be on separate sheets

Source Title	Automated Voting District Generation Using Pre Existing Geopolitical Boundaries
Source citation (APA Format)	Peterson, D. W. and Osgood, C. E. (2008, 24 July). Automated Voting District Generation Using Preexisting Geopolitical Boundaries.
Original URL	https://www.freepatentsonline.com/20080177555.pdf
Source type	Patent
Keywords	Super district, library, voting maps, geopolitical boundaries
Summary of key points (include methodology)	The goal is to create a method to generate voting maps that maximizes use of existing geopolitical boundaries, as to reduce the chance of gerrymandering. To do this, first divide the state into super districts that have an integer multiple of the required population of each district, as well as follow all existing geopolitical boundaries. Then, there aren't many ways to divide super districts into districts, reducing the chance for gerrymandering.
Research Question/Problem/ Need	How can existing geopolitical boundaries be used to generate voting maps in such a way as to minimize opportunity for gerrymandering?

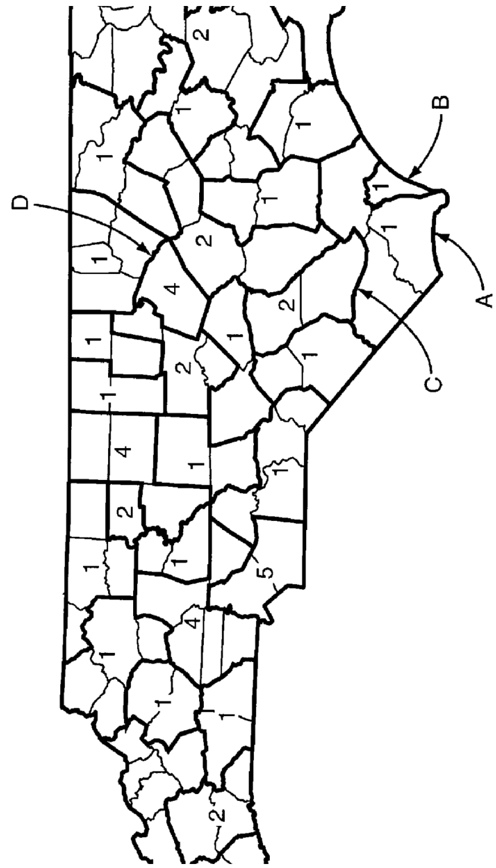
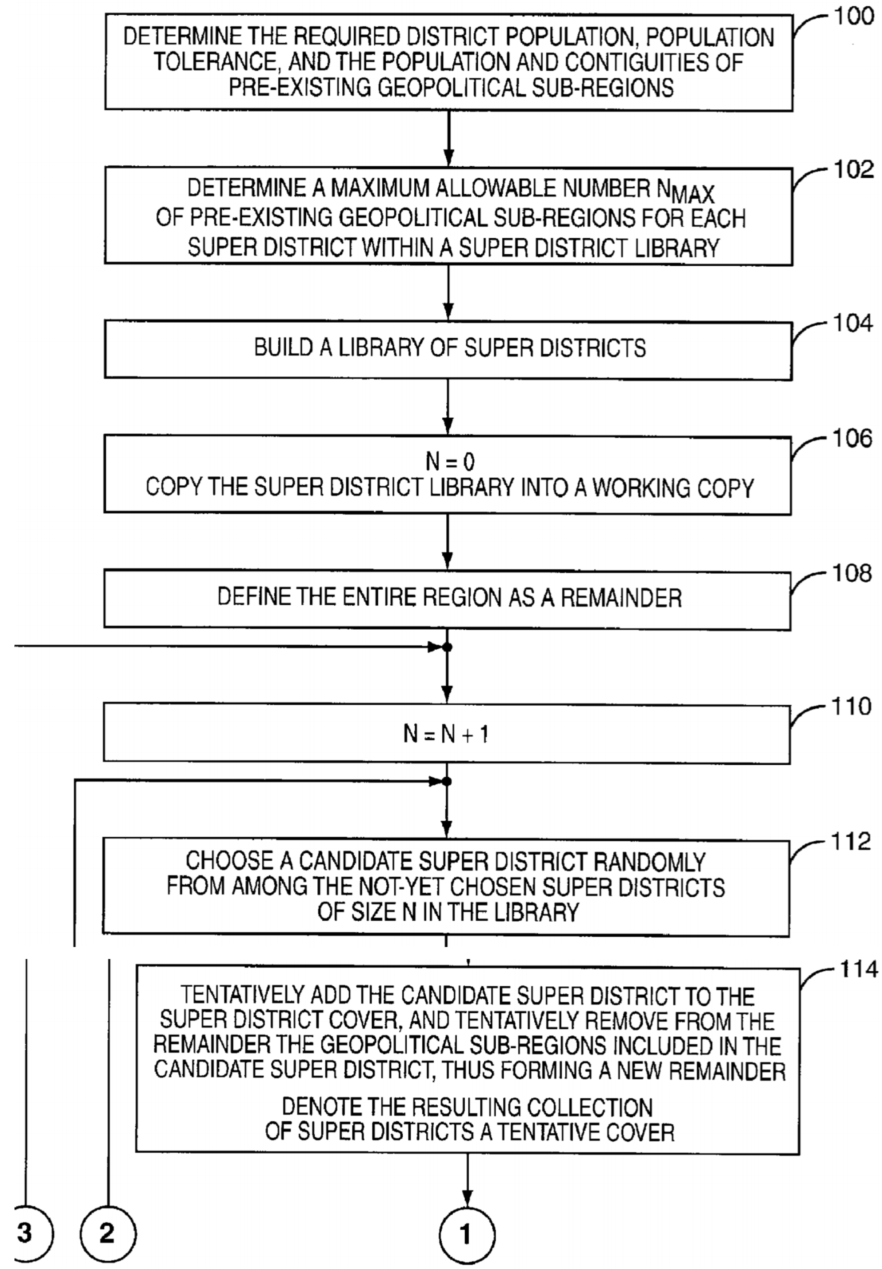
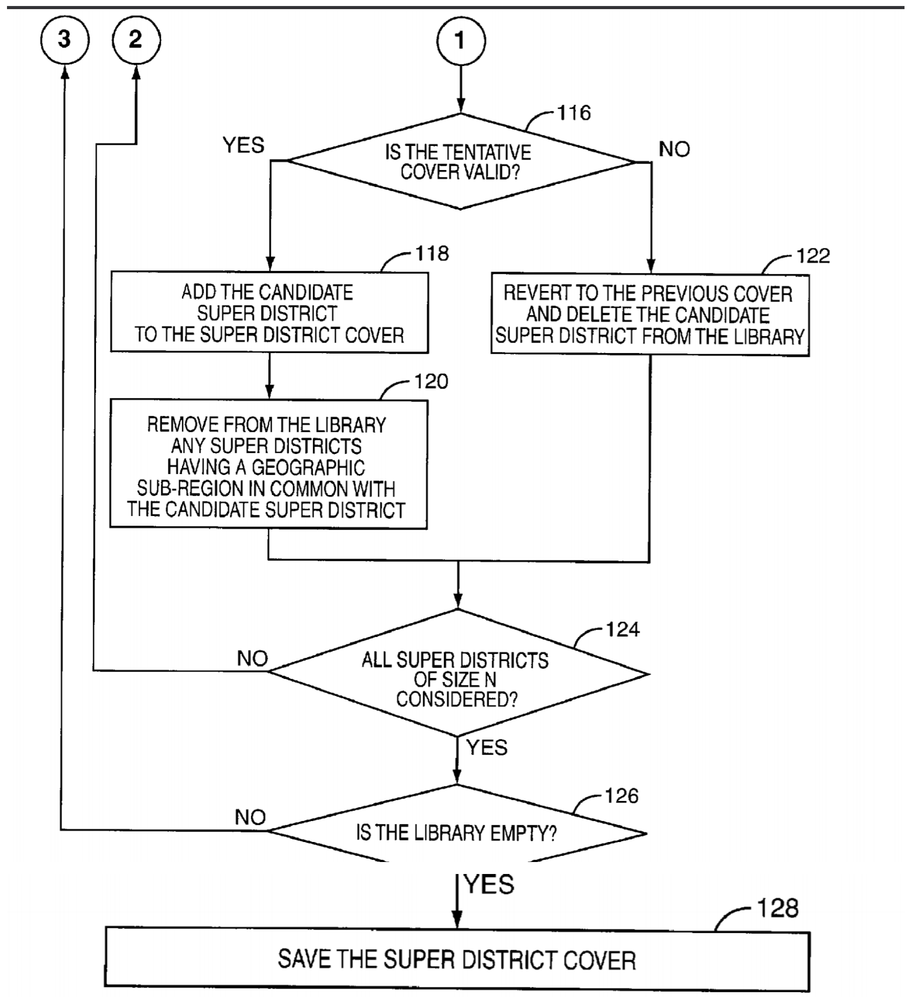
<table>
<tr>
<td>Important Figures</td>
<td>


FIG. 1

A sample super districting of North Carolina. The whole state did not fit in the screenshot, but this gives an idea.
</td>
</tr>
</table>


DETERMINE THE REQUIRED DISTRICT POPULATION, POPULATION TOLERANCE, AND THE POPULATION AND CONTIGUITIES OF PRE-EXISTING GEOPOLITICAL SUB-REGIONS
100
DETERMINE A MAXIMUM ALLOWABLE NUMBER N_{MAX} OF PRE-EXISTING GEOPOLITICAL SUB-REGIONS FOR EACH SUPER DISTRICT WITHIN A SUPER DISTRICT LIBRARY
102
BUILD A LIBRARY OF SUPER DISTRICTS
104
N = 0
COPY THE SUPER DISTRICT LIBRARY INTO A WORKING COPY
106
DEFINE THE ENTIRE REGION AS A REMAINDER
108
N = N + 1
110
CHOOSE A CANDIDATE SUPER DISTRICT RANDOMLY FROM AMONG THE NOT-YET CHOSEN SUPER DISTRICTS OF SIZE N IN THE LIBRARY
112
TENTATIVELY ADD THE CANDIDATE SUPER DISTRICT TO THE SUPER DISTRICT COVER, AND TENTATIVELY REMOVE FROM THE REMAINDER THE GEOPOLITICAL SUB-REGIONS INCLUDED IN THE CANDIDATE SUPER DISTRICT, THUS FORMING A NEW REMAINDER
DENOTE THE RESULTING COLLECTION OF SUPER DISTRICTS A TENTATIVE COVER
114
3
2
1

	 The above is a complete flow chart for the super districting algorithm.
Notes	- The patent proposes a method to divide states into districts based on existing geographical and political boundaries. - First, divide the state into a set of super districts that adhere to the set boundaries and have an integral multiple of the required population in each district (again, we require districts to be equipopulous). - A super district cover is a set of super districts covering the entirety of the state. - First, determine all constraints: geopolitical boundaries, population in each district, and so on. - We also define a maximum number of districts to put into one super district. - We then construct a library of all possible super districts, with the constraint that no super district is in the other. Save this somewhere. - To construct a valid cover,keep adding random super districts

	until either the covering is complete or the districting cannot be completed in a valid manner. If the latter holds, go back to the previous stage with a possible valid finish and take a different path. - Also, remove any overlapping super districts as the process goes on, as super districts in the final cover cannot overlap. - Repeat the above process to generate more covers. - Super district covers can be used as starting points for generating fair voting maps. - Given a super district cover, all that must be done is dividing the super districts into districts of equal population. This makes maximum use of existing geographical and political boundaries and minimizes the chance for gerrymandering.
Cited references to follow up on	No cited references in this patent (somewhat odd).
Follow up Questions	- Is it always possible to divide a state into a super district cover? - What error must we allow in the equipopulous condition (it is obviously infeasible to expect completely equal populations throughout all districts)? - Is the maximum number of districts in a super district meant to be chosen arbitrarily, or are there any criteria to consider here?

Article #11 Notes: Gerrymandering and Compactness: Implementation Flexibility and Abuse

Article notes should be on separate sheets

Source Title	Gerrymandering and Compactness: Implementation Flexibility and Abuse
Source citation (APA Format)	Barnes, R., & Solomon, J. (2018, March 7). *Gerrymandering and Compactness: Implementation Flexibility and Abuse*. ArXiv. https://arxiv.org/pdf/1803.02857.pdf
Original URL	https://arxiv.org/pdf/1803.02857.pdf
Source type	Paper
Keywords	Gerrymandering, compactness, Polsby-Popper, holes, Reock, convex hull, geographical boundaries
Summary of key points (include methodology)	The paper basically gives a list of criteria a good compactness score should satisfy, and also reviews three widely-used scores in this regard. The three scores evaluated were the Polsby-Popper score, Reock score, and convex hull score. All had strengths and weaknesses - particularly, the Polsby-Popper score could not deal with predetermined borders, which is a definite minus. Some criteria given for a good compactness score and good computation of compactness are high-resolution data, consideration of predetermined boundaries, and successfully dealing with holes in districts.
Research Question/Problem/ Need	What constitutes a good compactness score, and how well do the currently used compactness scores fare?

Important Figures

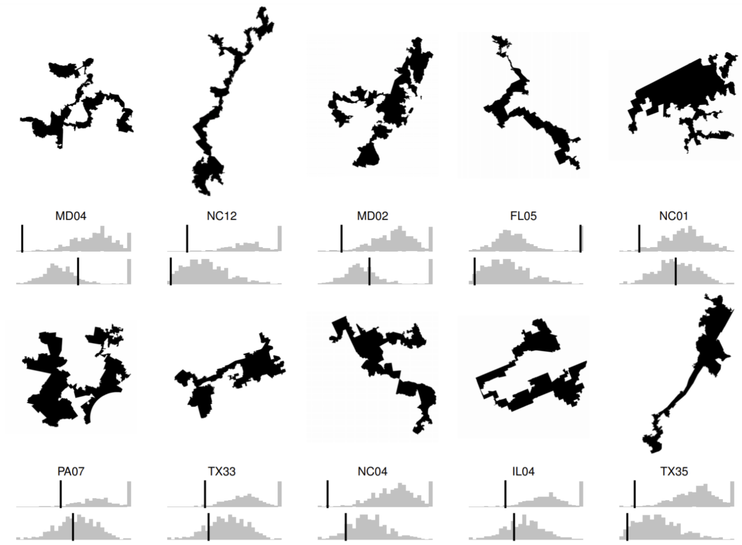


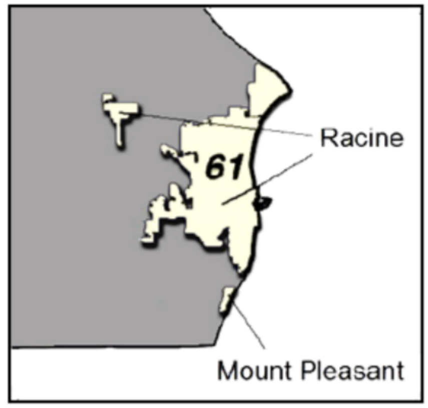
Fig. 1. Applied gerrymandering: abusing implementation flexibility. This figure shows several districts from the 114th Congress that appear incontrovertibly gerrymandered. The compactness scores of all the districts are shown in histograms below the districts' pictures with a black line indicating where the focal district falls on the distribution. Compactness ranges from 0 on the left-hand side of each histogram to 1 on the right-hand side. Scores for districts were generated by performing a grid search over a range of values for each implementation choice and choosing the minimum/maximum value across all choices. The grid search could also choose whether or not to include sole districts (§F) in the histogram. The result for each district is the pair of configurations in which the district appeared both the most gerrymandered as well as the most reasonable. Further details for the figure are in the Supplemental Information (see Table 1).

Ten widely known to be gerrymandered elections. Notice how weirdly the districts are shaped, and how noncompact they are.

sin
wit
ma
de
Re
inv
rec

Fig. 6. Misaligned borders. The border shown lies between Maryland 06 and West Virginia 01. The "true boundary" is drawn from data at 1:500,000 resolution and is shown by the transition between greys, while the black line represents the state boundary between the two districts and is drawn from 1:5,000,000 data. Note that differing data resolutions is only one way whereby such a mismatch might occur: shifts in data (as from projections), differing collection procedures, or deliberate manipulation are all possible.

The importance of using high-resolution data is clear here.

	 **Fig. 5.** Wisconsin's 61st Assembly District showing non-contiguous regions. See text for discussion. Figure drawn from (30). An example of a non-contiguous district.

Notes	- Many compactness scores exist since non-compact districts are a telltale sign of gerrymandering.
	- Choices that must be made when computing a compactness score:
	    - definition of compactness
	    - how to handle non-contiguous districts
	    - districts with holes
	    - existing geopolitical boundaries
	    - map boundaries
	- When creating a compactness score, be explicit about what area entails - are holes included, for example?
	- Take into account existing boundaries, so you don't penalize a district for having an odd shape when no other shape was possible.
	- Eliminate districts that comprise a whole geographical unit, as no other choice of district was possible in that case.
	- Although the Supreme Court has ruled partisan gerrymandering unconstitutional, they have not decided on a way to identify and measure it - motivation for this whole field of research.
	- The authors consider three widely used compactness scores:
	- Polsby-Popper score: 4pi times the area over the perimeter squared (contrast with the isoperimetric inequality)
	- Reock score: ratio of area of district to the area of its minimum bounding circle.
	- Convex hull: Ratio of area of district to the area of its convex hull (convex polygon of minimum size that contains it)
	- There is no federal requirement for contiguous districts, but it's preferable in many cases.
	- If districts are non-contiguous, then treating it as one unit

	decreases compactness while treating it as several units increases compactness. - One must also consider borders mandated by geography or existing political boundaries. The Reock and convex hull scores can be modified for this - simply allow the bounding circle or bounding convex polygon to intersect that section of the perimeter. However the Polsby-Popper score cannot be modified in this way. - If only one choice exists for a district, then it cannot be gerrymandered by definition. - Projection can also be considered, since technically districts don’t lie on a plane, but this seems too technical and specific for the purposes of my project. - Use data of high resolution in order to get an accurate model of what borders actually look like.
Cited references to follow up on	Chambers C, Miller A (2010) A Measure of Bizarreness. Quarterly Journal of Political Science 5(1):27–44 Altman M (1998) Chapter 2: The Consistency and Effectiveness of Mandatory District Compactness Rules in PhD Thesis: Districting principles and democratic representation. (California Institute of Technology). Niemi RG, Grofman B, Carlucci C, Hofeller T (1990) Measuring Compactness and the Role of a Compactness Standard in a Test for Partisan and Racial Gerrymandering. The Journal of Politics 52(4):1155–1181.
Follow up Questions	- What compactness scores were not mentioned here? - What other criterion could be factored into a compactness score?

Article #12 Notes: Introduction to the Declination Function for Gerrymanders

Article notes should be on separate sheets

Source Title	Introduction to the Declination Function for Gerrymanders (reading this for a better introduction to declinations, which were introduced in article #9 but not very in-depth)
Source citation (APA Format)	Warrington, G. (2018, March 13). *Introduction to the Declination Function for Gerrymanders*. ArXiv. https://arxiv.org/pdf/1803.04799.pdf
Original URL	https://arxiv.org/pdf/1803.04799.pdf
Source type	Journal article
Keywords	Declination, gerrymanders, lopsided means
Summary of key points (include methodology)	The authors introduce the declination function to identify partisan gerrymandering. The declination function works roughly by comparing the fractions of winning margins to the fraction of votes in the state in each party. The authors compare this to the lopsided means test and show how it addresses the existing problems in the lopsided means test.
Research Question/Problem/ Need	What is a metric that can be effectively used to identify gerrymandering, and how does it pick up on some of the problems with other existing metrics?

<table>
<tr>
<td>Important Figures</td>
<td>

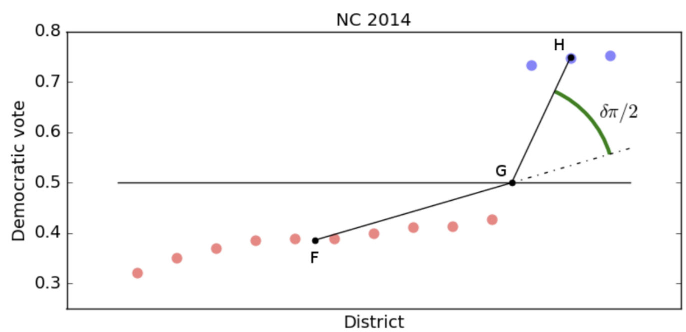


FIGURE 1. Example illustrating the three points F, G and H arising in the definition of the declination. Data is from the 2014 North Carolina election for the US House.

Example of declination.

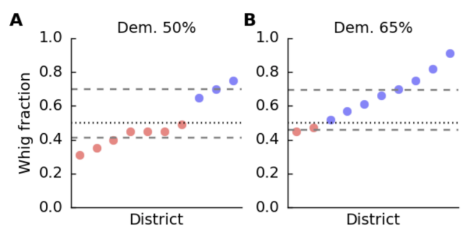


FIGURE 5. Distributions illustrating the limitations of comparing average margins of victory between the two parties.

Why just computing average margins doesn't work: democrats have a high average margin but still win most of the districts in the second diagram.

</td>
</tr>
<tr>
<td>Notes</td>
<td>

- First, place the democratic vote fractions of each district in increasing order.
- Find the center of mass F of the republican districts and the center of mass H of the democratic districts.
- Also plot a point G at y-value ½ and x-value between the transition from republican to democratic districts.
- Find the angle between FG and GH and multiply by 2/pi. This is known as the declination of the voting data.
- Positive values favor Republicans while negative values favor Democrats.
- The declination has many strengths:
    - Does not assume any proportionality between votes and seats.
    - Used well in conjunction with computer simulations to identify gerrymandering.
    - Increases in magnitude when packing or cracking happens, which are two telltale signs of

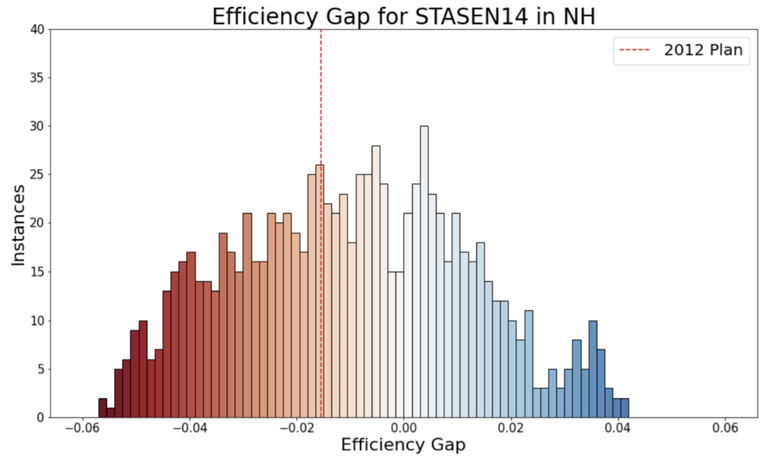
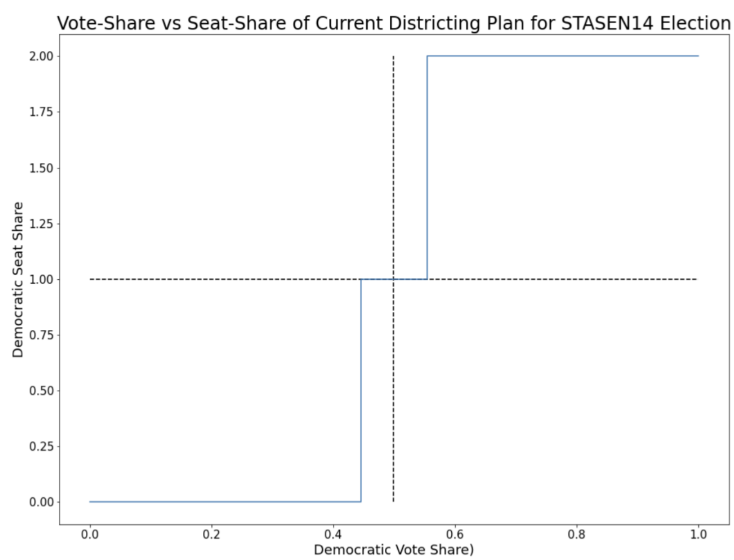
</td>
</tr>
</table>

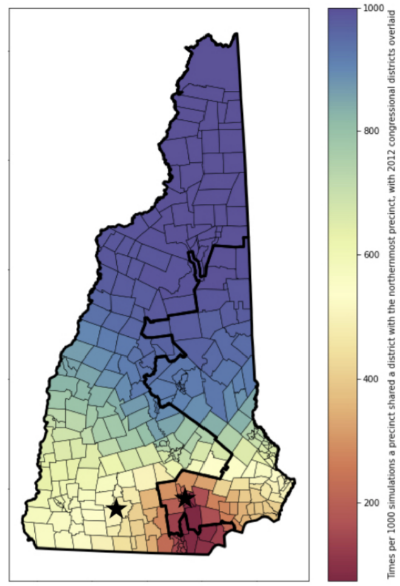
	gerrymandering. - Insensitive to incumbent gerrymandering (done by both parties), as well as to competitiveness. - However, it is undefined when one party sweeps the election and very noisy when one party is close to sweeping the election. - Intuition for the motivation of the declination (possibly useful for when I create my own metric): - The lopsided means test doesn't work well, as illustrated by the second figure above. - This is combated by the declination computing the ratio of average winning margin to fraction of seats won, as this is a better measure of the fairness of a plan. - These ratios should be comparable between the parties. - Basically, just try and find problems with existing metrics and figure out ways to combat it - this seems to be the way to go when developing a new metric.
Cited references to follow up on	J. S. Buzas and G. S. Warrington. Gerrymandering and the net number of US House seats won due to vote-distribution asymmetries. ArXiv e-prints, July 2017. https://arxiv.org/abs/1707.08681. - A more rigorous treatment of the declination
Follow up Questions	- Are there any other problems with other metrics that the declination effectively or does not effectively address? - How can we improve the existing problems with the declination?

Article #13 Notes: Assessing Congressional Districting in ME and NH

Article notes should be on separate sheets

Source Title	Assessing Congressional Districting in ME and NH
Source citation (APA Format)	Asgari, S., Basewitz, Q., Bergmann, E., Brogsol, J., Cox, N., Davis, D., . . . Torrance, D. (2020, November 12). Assessing congressional districting in Maine and New Hampshire. Retrieved November 23, 2020, from https://arxiv.org/abs/2011.06555
Original URL	https://arxiv.org/abs/2011.06555
Source type	Paper
Keywords	Incumbent gerrymandering, ReCom, dual graph, Maine, New Hampshire
Summary of key points (include methodology)	The authors found that Maine and New Hampshire were most likely gerrymandered in 2011 by comparing the given voting maps of that time to a background distribution of thousands of neutral maps, and noticing large amounts of discrepancies. Attention is brought to the possibility of creating a metric that is useful in states with only two congressional districts.
Research Question/Problem/ Need	Were the 2012 districting plans in Maine and New Hampshire gerrymandered, partisanly or otherwise?

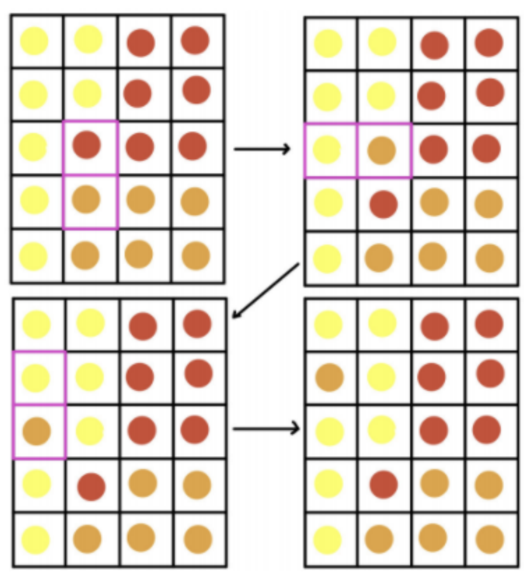
Important Figures	 FIGURE 3. Histogram of the efficiency gap among an ensemble of 1000 plans for the 2014 State Senate Election in New Hampshire. The score for the 2012 plan is marked with a vertical dashed line. 

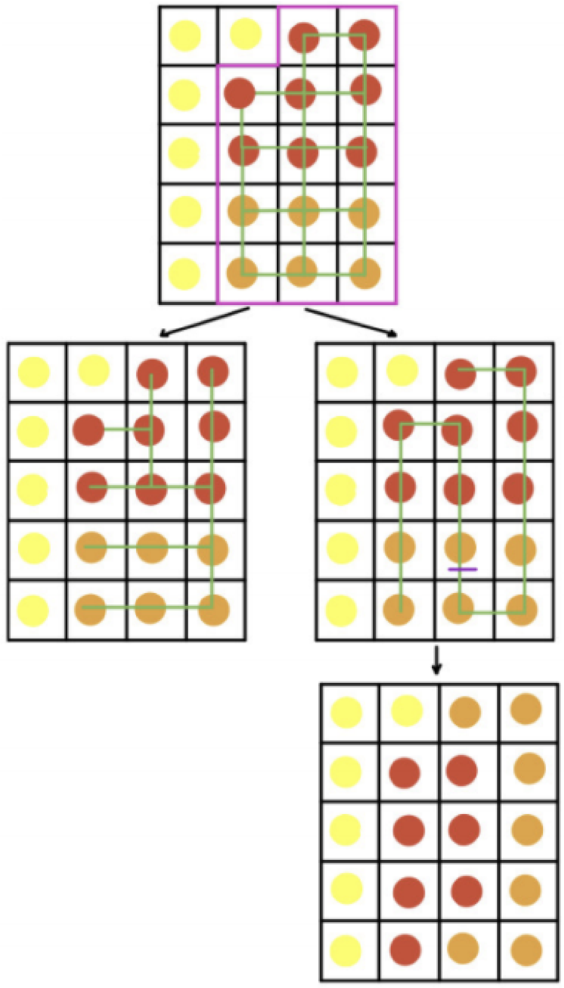
	 FIGURE 8. A heat map showing the proportion of plans in which a precinct shared a district with the northernmost precinct (Pittsburg). The 2012 enacted division boundary is outlined, and the hometowns of the incumbent representatives are indicated with stars.
Notes	- Both Maine and New Hampshire are swing states, making them important for analysis. - The second district of Maine usually leans conservative while the first leans Democrat. However, a large portion of the state is registered as independent. - New Hampshire voters are actually mostly independent (42%), and elections can go either way in this state. - Many mathematical metrics fall flat in states with two districts such as New Hampshire and Maine. - For example, the mean-median score and partisan bias score, two of the most popular metrics, are both zero if the state has two districts, rendering them unhelpful. - The efficiency gap is unhelpful in the case that one party wins both districts, but if each party wins one district the metric proves to be somewhat useful. - Good place for future work! - New Hampshire and Maine shapefiles available online at Secretary of State websites - perhaps other shape files could be gathered for other states at their respective Secretary websites. - Analysis in New Hampshire found Independent voters to be very influential in the election. - The 2012 districting did not seem to favor either party (this was the same result I found and presented in the December Fair).

	- However, it was found that the expected dividing line between districts was perpendicular to the enacted one, leading to cause for concern. - This could be evidence of incumbent gerrymandering, gerrymandering done to keep the current representatives in power. In this case, the expected dividing line would have made the two incumbent representatives compete with each other, so a perpendicular line was instead enacted. - A similar phenomenon occurred in Maine.
Cited references to follow up on	DeFord, Dhamankar, Duchin, Gupta, McPike, Schoenbach, and Sim, Implementing partisan symmetry: Problems and paradoxes, preprint (2019), available at https://mggg.org/uploads/PSymm. pdf
Follow up Questions	- How can a metric be developed that is effective in the case of states with two districts, such as New Hampshire and Maine? - How does political geography affect districting plans in states (for example, Maine's complicated political geography played a role in the enacted plan).

Article #14 Notes: Using Gerrymandering for School Districting

Article notes should be on separate sheets

Source Title	Using Gerrymandering for School Districting
Source citation (APA Format)	Park, J., Suo, A., Sy, J., & Xie, H. (2020). Using Gerrymandering for School Districting. Retrieved December 12, 2020.
Original URL	N/A
Source type	Paper
Keywords	Racial gerrymandering, Flip, ReCom, Gerrychain, splittable edge, probability
Summary of key points (include methodology)	- Scores are assigned to plans generated by the Flip and ReCom algorithms in order to determine how racially gerrymandered they are. - Due to a variety of factors such as overspecificity of data, the analysis was overall unsuccessful.
Research Question/Problem/ Need	How can racial gerrymandering in Massachusetts be analyzed using algorithms commonly utilized to analyze partisan gerrymandering, such as the Flip and ReCom algorithms.
Important Figures	 An informative diagram on the Flip algorithm, possibly could be used in presentation.

<table>
<tr><td></td><td>
An informative diagram on the ReCom algorithm, possibly could be used in the presentation.</td></tr>
<tr><td>Notes</td><td>Gerrymandering is mainly partisan, but can also apply to race.
In 2019, ⅔ of the students of color in Massachusetts attend intensely segregated schools, with over 90 percent of students being of color.
Flip algorithms make it hard to generate a proper neutral ensemble of plans, as not many changes happen each step of the algorithm.
In contrast, it is hard to identify the initial partition after even one step of a ReCom algorithm.
Interesting conjectures: The probability of an unlabeled tree on 2n vertices being splittable is less than ½ and approaches 0 as n approaches infinity.</td></tr>
</table>

	- Data on race, income, etc. can be found on NHGIS website, as well as various shape files. - A more efficient algorithm to find an edge to split a tree is proposed, but runtime of the ReCom algorithm does not seem to be a major issue at the moment. - We can assign scores to plans to check how gerrymandered they are based on race, religion, or other factors. - The analysis in the paper was unsuccessful due to overspecificity of data as well the inability to set proper bounds in the case of Suffolk County, which has over a hundred schools.
Cited references to follow up on	Mira Bernstein, Gerrymandering: Why It's More Complicated Than You Might Think, talk at Bryn Mawr College, April 15, 2019.
Follow up Questions	- Can any probabilities of splitting into equal parts be calculated for weighted trees (this is a very general problem, so perhaps impose some restrictions on the weights)? - How can scores be added to GerryChain analysis in order to test for gerrymandering in different areas such as race? Here, it was done by assigning scores, but are there other ways to do it?

Article #15 Notes: Gerrymandering

Article notes should be on separate sheets

Source Title	Gerrymandering
Source citation (APA Format)	Du, C., Kong, T., Shleifer, Z., & Tang, S. (2020). Gerrymandering. Retrieved December 12, 2020.
Original URL	N/A
Source type	Paper
Keywords	Gerrymandering, school segregation, minority, population, induced subgraph, ReCom, Markov chain
Summary of key points (include methodology)	- Current gerrymandering algorithms and metrics were combined with scores to evaluate the school segregation in Pennsylvania. - This was done by modifying the ReCom algorithm by scoring the generated plans a different way. - It was found that schools in Pennsylvania were intentionally segregated on the basis of race, and a better plan was introduced using the algorithm described in the paper.
Research Question/Problem/ Need	How can current gerrymandering algorithms be used to evaluate school segregation in Pennsylvania?
Important Figures	• Let $C(s_i)$ denote the enrollment capacity of a the school s_i. • Let $P(s_i)$ denote the population of the school district given by s_i. • Let $M(s_i)$ denote the minority population of the school district given by s_i. • Let $R(s_i)$ be the ratio $M(s_i)/P(s_i)$. • Let P_{tot} denote the total population of the region. • Let R_{ave} be the percentage of minority population of the entire region. We define the Overall Minority Distribution Score to be: $M = \sum_{si \in S} C(s_i) * (R(s_i) - R_{ave})^2 / P_{tot}$

<table>
<tr>
<td></td>
<td>

FIGURE 6. Planning With Improve Of All Scores

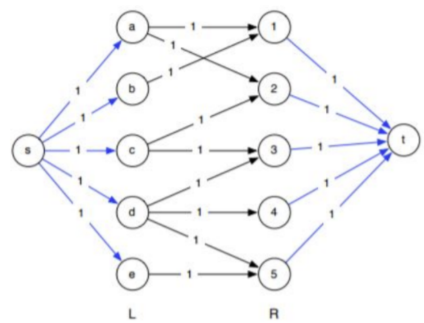


FIGURE 7. Bipartite Matching with Flow Networks

</td>
</tr>
<tr>
<td>Notes</td>
<td>

- The goal is to apply gerrymandering algorithms to school segregation in Pennsylvania.
- The ReCom algorithm is modified in order to account for several factors.
- These factors include travel time, school enrollment, and minority population.
- A “minority distribution” score of a plan is created by combining all of these factors (this is good and something I could maybe implement in my own project - the score is in the figures section)
- Travel time and compactness scores are also defined.
- The modification to the ReCom algorithm is: when a spanning tree of the adjacency graph of two neighboring districts is created, cut every edge and take the plan with the best improvements in all of the three scores listed earlier.
- A plan was created with an improvement in all scores.
- This paper is a good example of combining several different metrics in a computer simulation, something I want to do in my project -> it was a very beneficial read

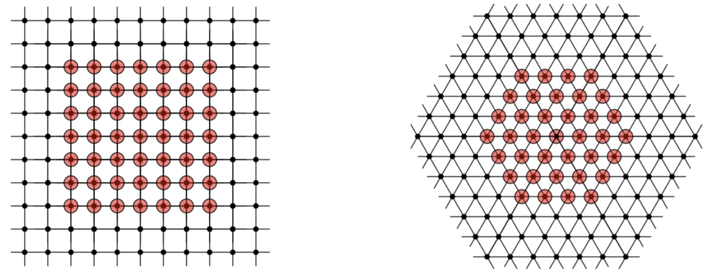
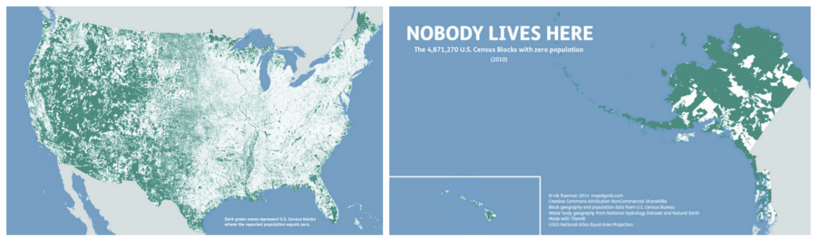
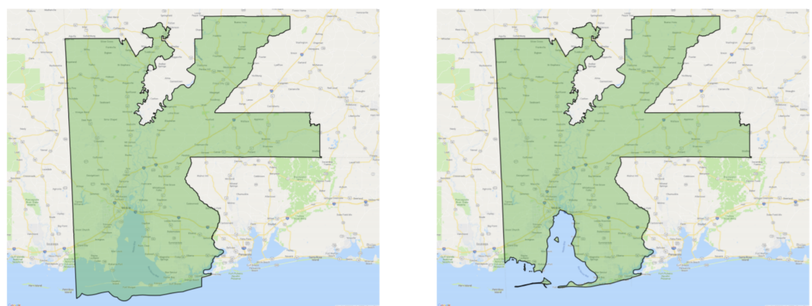
</td>
</tr>
</table>

	- Pennsylvanian schools were intentionally segregated on the basis of race
Cited references to follow up on	Ahmed, Faez, John P. Dickerson, and Mark Fuge. “Diverse Weighted Bipartite b-Matching.” Proceedings of the Twenty-Sixth International Joint Conference on Artificial Intelligence (2017): n. pag. Crossref. Web.
Follow up Questions	How can redistricting laws in the state be taken into account when creating a plan? How can the different types of schools, such as magnet schools, public schools, and private schools be taken into account when creating a plan? (possibly with a bipartite flow network, the third feature in important figures) How can the given algorithm be sped up, as checking every edge of a tree can take a while, especially for larger ones?

Article #16 Notes: Discrete Geometry For Electoral Geography

Article notes should be on separate sheets

Source Title	Discrete Geometry For Electoral Geography
Source citation (APA Format)	Duchin, M., & Tenner, B. E. (2018, August 15). Discrete geometry for electoral geography. Retrieved December 13, 2020, from https://arxiv.org/abs/1808.05860
Original URL	https://arxiv.org/abs/1808.05860
Source type	Paper
Keywords	Discrete compactness, boundary, induced subgraph, Polsby-popper, Census, block, block group, district, Reock, isoperimetric
Summary of key points (include methodology)	- The issues of contour-based compactness are shown in detail, which include coastline, projection, and empty space effects. - A new measure of discrete compactness is introduced based on discrete blocks of population rather than the geographical shape of the district. - This measure is shown to immediately get rid of the problem of empty space, but research remains to show how it addresses the other relevant issues.
Research Question/Problem/ Need	How can the issues of contour-based compactness measures be resolved through a different measure?

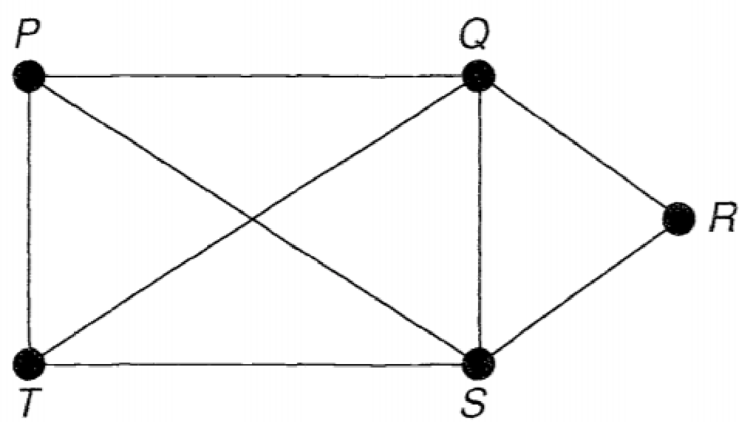
Important Figures	 FIGURE 9. Square and triangular lattices with square and hexagonal "districts" Ω_7 and Ω'_4, respectively.  FIGURE 4. Unpopulated census blocks, excluding the Great Lakes, are depicted in dark green in these maps by Nik Freeman [7].  FIGURE 2. Alabama's 1st district has boundary partly defined by the Gulf of Mexico and the Tombigbee and Alabama rivers. The TIGER/Line (left) and Cartographic 500K (right) maps are shown here, illustrating that the Polsby-Popper quantification of compactness leaves the modeler caught in an unpleasant choice between a map subject to coastline effects (Issue A) or to empty space effects (Issue D).
Notes	- Many of the compactness scores currently commonly used suffer from the same issues because they are all based on the contours of the state. - Here, compactness metrics based on a discrete, graphical version of the state are proposed, which will hopefully alleviate some of the issues commonly faced currently. - There are many reasons to desire compact districts - justice as well as convenience for those traveling to vote. - Census geography splits states into blocks, then block groups, then tracts, then VTDs, which are the closest unit of measurement to what is reported out as voting data. - Congressional districts form another partition of the

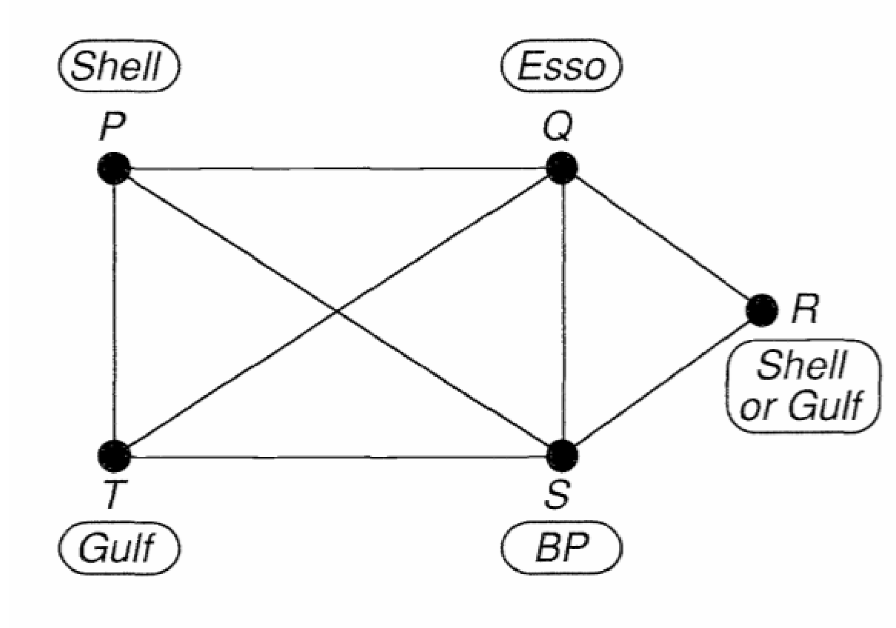
	state,often cutting through tracts and block groups. - Polsby-Popper is an isoperimetric score that compares the perimeter and area of the district. - The Polsby-Popper score rewards districts closer to shape to circles with higher scores - The Reock score differs since it is not isoperimetric, even though it also rewards a circle with the highest score. - There are four main issues with contour-based scores. - Districts with noncompact physical features such as coastlines are automatically penalized, even though there is no plan that can avoid this. - The resolution at which the map is viewed can have a great effect on the compactness score, as the contour becomes more or less visible. - Choices of map projection can lead to vastly different shapes - Unpopulated regions, while having no effect on the outcome of the election, have a large effect on contour-based compactness scores, which doesn't make sense. - Given a graph G and an induced subgraph of a subset of its vertices H, the internal boundary of H is the subset of vertices of H that have neighbors in G outside of H. - A graph with boundary is a graph G together with some vertices designated as boundary. - An induced subgraph with boundary is the induced subgraph, its internal boundary, and any boundary vertices of the whole graph contained in the subgraph. - This has foundations in topology (manifolds with or without boundary) - Create the dual graph of a state with vertices weighted based on the population of the corresponding block. - One can mark boundary vertices in the best way possible, for example if they are on the geographical boundary of the state, on a coastline or something similar. - We can define the discrete area of a graph to be the number of vertices in the graph, and the discrete perimeter to be the number of vertices on its boundary. - This suggests analogs of the Polsby-Popper score. - This could be a very good area to study and survey in my studies of compactness, to provide a starting point for this type of research.
Cited references to follow up on	M. Altman, Modeling the effect of mandatory district compactness on partisan gerrymanders, Political Geography 17 (1998), 989–1012. R. G. Niemi, B. Grofman, C. Carlucci, T. Hofeller, Measuring compactness and the role of a compactness standard in a test for partisan and racial gerrymandering. The Journal of Politics 52 (1990), 1155–1181.

Follow up Questions	- How good is discrete compactness at reducing the effects of coastlines? - Does the choice of unit of census geography have an effect on discrete compactness (for example, blocks vs block groups)? - What features of districts are encouraged or discouraged by the discrete compactness score?

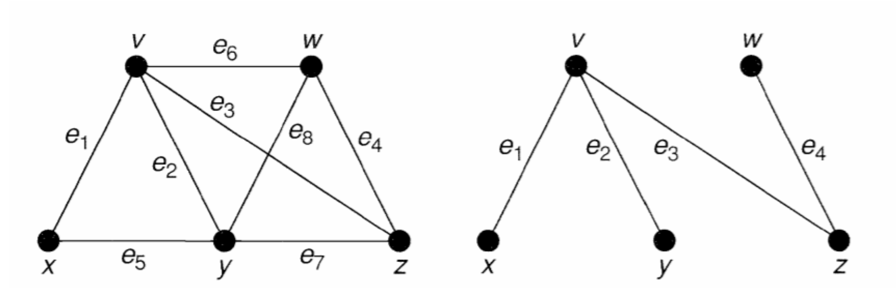
Article #17 Notes: Introduction to Graph Theory

Article notes should be on separate sheets

Source Title	Introduction to Graph Theory
Source citation (APA Format)	Wilson, R. J. (2015). *Introduction to graph theory*. Harlow, United Kingdom: Prentice Hall.
Original URL	https://www.maths.ed.ac.uk/~v1ranick/papers/wilsongraph.pdf
Source type	Book
Keywords	Tree, spanning tree, connected, vertices, edges, graph
Summary of key points (include methodology)	This book just provides background on basic graph theoretical concepts. The following notes only pertain to the concepts important for the project, which are spanning trees and the basic definition of a graph.
Research Question/Problem/ Need	What are the graph theoretical concepts required to analyze gerrymandering?
Important Figures	An example of a graph:  This graph being used to represent a real-world situation:



A graph and its spanning tree:



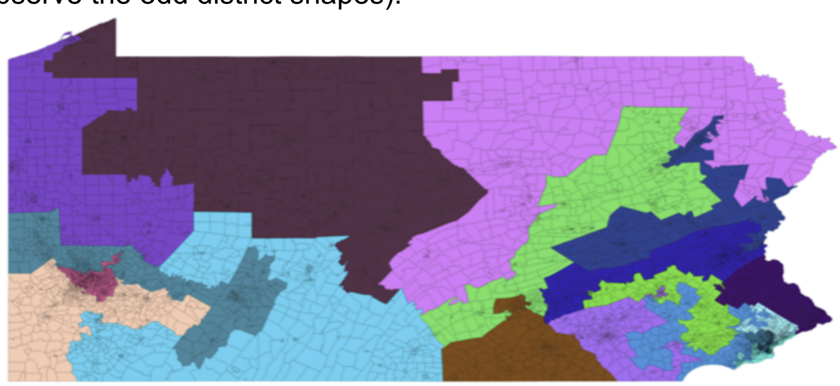
Notes

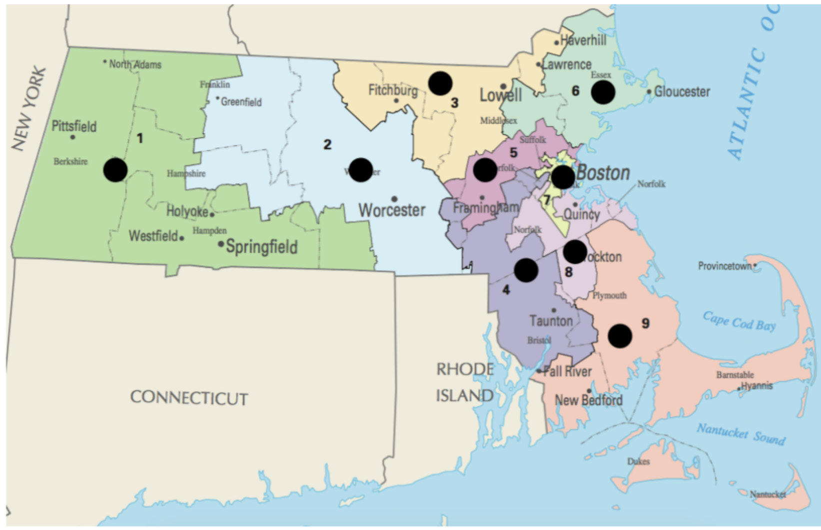
- A graph is simply a collection of nodes and edges between those nodes.
- The nodes of a graph can be used to represent anything.
- Graphs are usually used to simplify complex situations (as in the case of gerrymandering, where adjacency graphs simplify the complex structure of cities).
- A path in a graph is a walk between two vertices along edges such that no vertex is repeated.
- A graph is connected if there exists a path between any two vertices.
- A cycle in a graph is a walk around non-repeated vertices that starts and ends at the same vertex.
- A graph is acyclic if it has no cycles, and a connected acyclic graph is known as a tree.
- Trees have many special properties - they always have one less edge than vertices, there is a unique path between any two vertices, and so on.
- Trees are especially important because of the spanning tree,

	which is a tree embedded in any graph (spanning trees are important to the ReCom algorithm). - A spanning tree is a subgraph of a connected graph that contains all the vertices and a subset of the edges such that it is connected. - Any connected graph contains a spanning tree. - Adjacency graphs are almost always connected, which allows the ReCom algorithm to be used since a spanning tree can almost always be found.
Cited references to follow up on	C. Berge, Graphs, North-Holland. 1985.
Follow up Questions	- How do we know when partitioning a tree into two equal parts is possible (this question was not addressed in this book)?

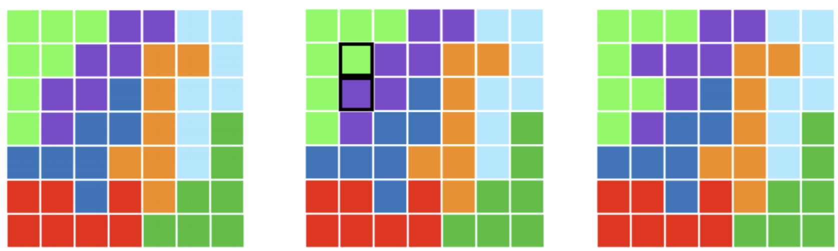
Article #18 Notes: Gerrymandering project: Cutting trees

Article notes should be on separate sheets

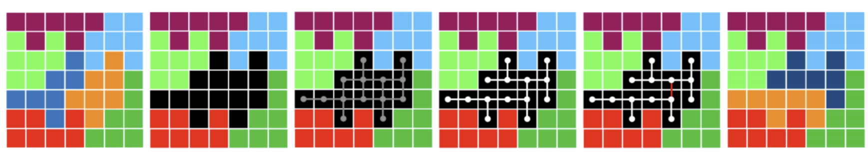
Source Title	Gerrymandering project: Cutting trees
Source citation (APA Format)	Davis, D. (2020). Gerrymandering project: Cutting trees. Retrieved December 21, 2020.
Original URL	N/A (private resource)
Source type	Anthology
Keywords	Gerrymandering, flip, ReCom, simulation, ensemble, adjacency graph, spanning tree, algorithm
Summary of key points (include methodology)	This document gives background on the basics of creating metrics to detect gerrymandering as well as the computer algorithms currently used to identify it. It also contains many research questions good for further pursuit.
Research Question/Problem/ Need	What are some researchable questions in the field of modeling and identifying gerrymandering?
Important Figures	The 2011 map of Pennsylvania, well-known to be gerrymandered (observe the odd district shapes):  The (vertices of the) adjacency graph of Massachusetts:



The Flip algorithm:



The ReCom algorithm:



Notes

- It is unrealistic to have a gerrymandering measure that is able to accurately assign a score to every map.
- This is because it is very hard to accurately capture what is going in a plan with just one number.
- The score of a plan should be -1 if it completely favors democrats, 0 if it is completely neutral, and 1 is it completely favors Republicans.
- Thus, negative scores favor Democrats and positive scores favor Republicans.
- Some plans and states have built-in advantages for certain parties by virtue of the geography or something similar.
- Because of this, we need to take geography into account when attempting to identify gerrymandering.
- The flip algorithm is one way to create a large ensemble of plans from an initial seed, done by continually flipping two

	adjacent voting blocs of different districts. - The Flip algorithm goes very slowly through the space of all possible districting plans, making it a suboptimal algorithm. - The ReCom algorithm goes faster through this space by merging two districts and splitting the spanning tree of the resulting adjacency graph into two equally sized trees by cutting an edge. - These algorithms can be used in conjunction for the best results. - There are several questions in theoretical mathematics that can be pondered (these are outlined in the future questions section). - The minimal spanning tree of an adjacency graph can be found using Dijkstra's Algorithm, which finds the minimal path between any two vertices in a graph. - These same algorithms can be used in other areas such as school redistricting.
Cited references to follow up on	Mira Bernstein, Gerrymandering: Why It's More Complicated Than You Might Think, talk at Bryn Mawr College, April 15, 2019. Jowei Chen and Jonathan Rodden, Unintentional gerrymandering: political geography and electoral bias in legislatures. Quarterly Journal of Political Science, 2013, 8: 239–269.
Follow up Questions	What is the probability a tree on 2n unlabeled vertices can be evenly split into two trees on n vertices by cutting an edge? This question provides important insight into the efficiency of the ReCom algorithm. Is there a better method to find an edge to cut during the ReCom algorithm than trying every edge? Given a weighted tree on 2n vertices with total weight 2, what is the probability an edge can be cut so that the two resulting trees each have total weight between 0.99 and 1.01? This also ties back to the efficiency of the ReCom algorithm.

Article #19 Notes: A Two Hundred-Year Statistical History of the Gerrymander

Article notes should be on separate sheets

Source Title	A Two Hundred-Year Statistical History of the Gerrymander
Source citation (APA Format)	Ansolabehere, S., & Palmer, M. (2016). A Two Hundred-Year Statistical History of the Gerrymander. *Ohio Law Journal, 77*(4), 741-762.
Original URL	https://maxwellpalmer.com/research/Ansolabehere_Palmer_Gerrymander_Compactness.pdf
Source type	Journal article
Keywords	Gerrymandering, Reock, convex hull, Polsby-Popper, Schwartzberg, compact, district
Summary of key points (include methodology)	The original gerrymander was used as a benchmark to determine whether other districts were gerrymandered or not, through the measure of compactness. It was found that districts have become significantly less compact over time, with 28% scoring worse than the original gerrymander on one test and 1% being definitively less compact than the original gerrymander. This trend of compactness is not necessarily due to gerrymandering, but may be due to other things such as the Voting Rights Act.
Research Question/Problem/ Need	How can measures of compactness be made more objective, and have districts become more or less compact over time?

Important Figures

Figure 2: *Illustrations of Compactness Measures Using the Original Gerrymander*

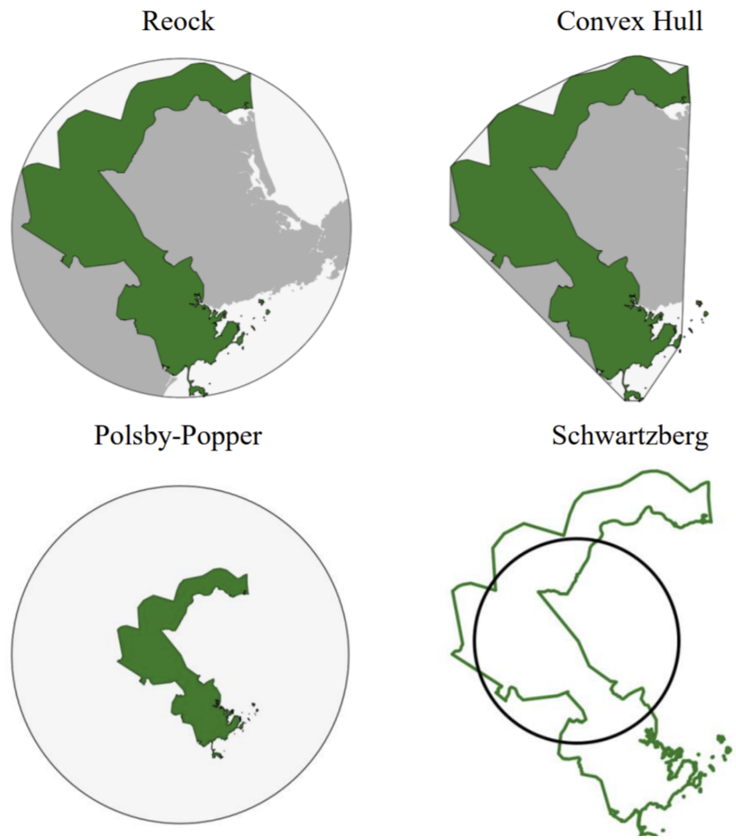


Figure 3: *Historical Trends in District Compactness*

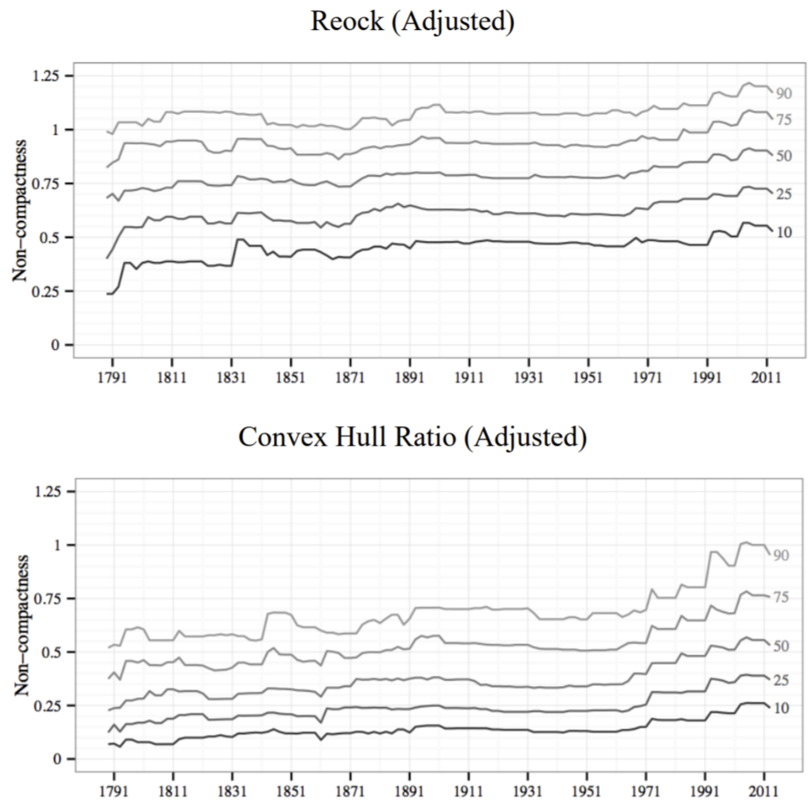
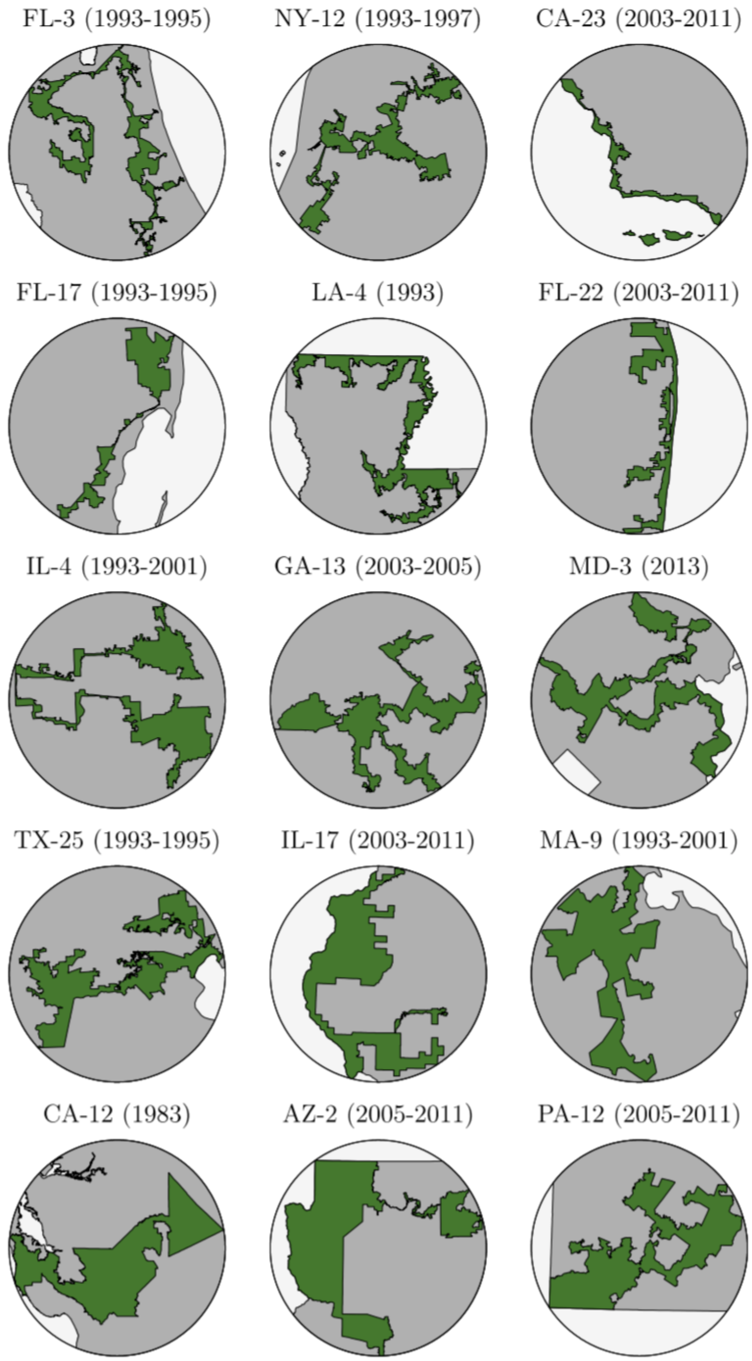


Figure 5: *Examples of Highly-Gerrymandered Districts*



Notes	- The article analyzes the compactness of districts in many previous elections to try and notice any statistical patterns. - Specifically, the article aims to set a threshold for how bad a

	non compact district has to be before being suspicious, and how to make this measure less subjective. - Thus, the shape of each district is compared against the original gerrymander. - Interestingly, one in five districts in the present day are less compact than the original gerrymander. - This could be due to a variety of reasons, including legal reasons such as the Voting Rights Act, which encourages oddly shaped districts if they favor minorities. - The original gerrymander should be used as the benchmark to measure other districts against -> this also holds when using compactness scores. - By standardizing the approach, we can say that if a district scores worse than the original gerrymander, it is also gerrymandered. - In some cases, non-compactness is an indicator of possible gerrymandering, as compactness is usually required either legally or geographically. - Using this method, one can also find out things such as which party compactness benefits more, and so on. - The Reock score finds the ratio of the area of a district to the area of its minimum bounding circle. - The convex hull score finds the ratio of the area of a district to the area of its convex hull. - Statistical analysis shows that districts are becoming less and less compact as time goes on, due to the legal and geographical criteria described earlier. - This trend is especially clear in the Reock and convex hull scores, but not in the Polsby-Popper score. - 28% of current districts are worse than the original gerrymander on at least one measure, but only 1% are worse on all three. - This shows the importance of using multiple criteria to test your hypotheses. - It can be safely concluded that these 1% of districts are gerrymandered. - The convex hull score scores the original gerrymander extremely low due to the large protruding neck, which means only 2% of current districts scored lower than it on the convex hull test, as opposed to 19% on the Reock test. - A figure of the most non compact districts in history found by the article is shown in the Important Figures section. - It was found that Democratic vote share was heavily tied to non compactness. - This could have to do with the fact the Democrats tend to pack and crack themselves geographically. - This could also be due to Republican gerrymandering against

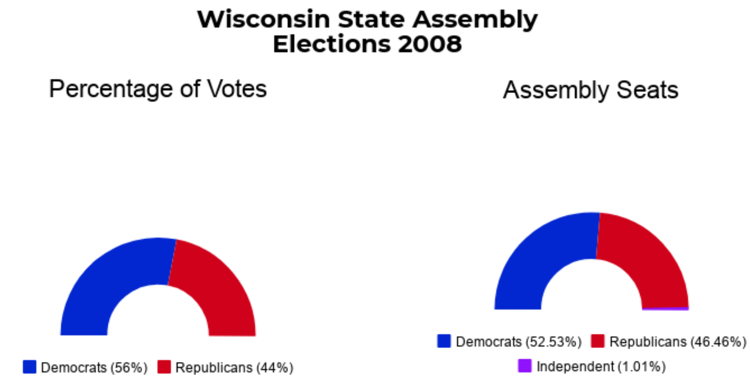
	Democrats.
Cited references to follow up on	Richard G. Niemi et al., Measuring Compactness and the Role of a Compactness Standard in a Test for Partisan and Racial Gerrymandering, 52 J. POL. 1155, 1157 (1990)
Follow up Questions	- Are there other gerrymanders that can be used as benchmarks? (For example, the few districts that were consistently worse than the original gerrymander in this paper). - Would other compactness scores return similar trends over time as the three tested in this paper?

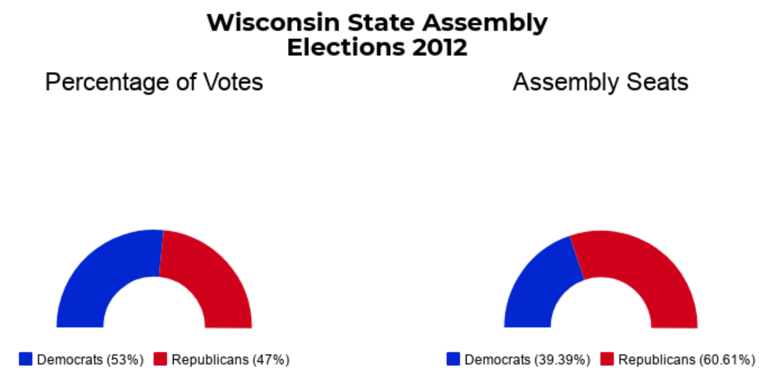
Article #20 Notes: Gerrymandering - The Salamander That Could… And Did

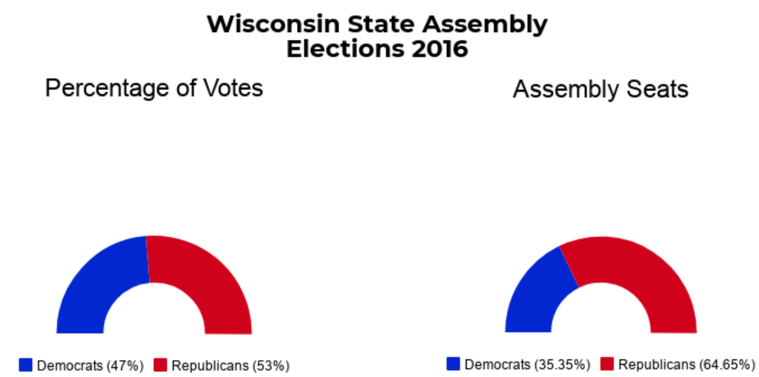
Article notes should be on separate sheets

Source Title	Gerrymandering - The Salamander That Could… And Did
Source citation (APA Format)	Anusauskas, L. (2019, March 22). Gerrymandering - The Salamander That Could ... And Did. Retrieved November 24, 2020, from http://publicpolicycenter.org/gerrymandering-the-salamander-that-could-and-did/
Original URL	http://publicpolicycenter.org/gerrymandering-the-salamander-that-could-and-did/
Source type	Article
Keywords	Gerrymandering, split lining, bipartisan
Summary of key points (include methodology)	The article gives background on gerrymandering in history as well as in the present day, with examples. Basic redistricting techniques are also shown.
Research Question/Problem/ Need	What is gerrymandering and why is such a hard problem to deal with?

Important Figures



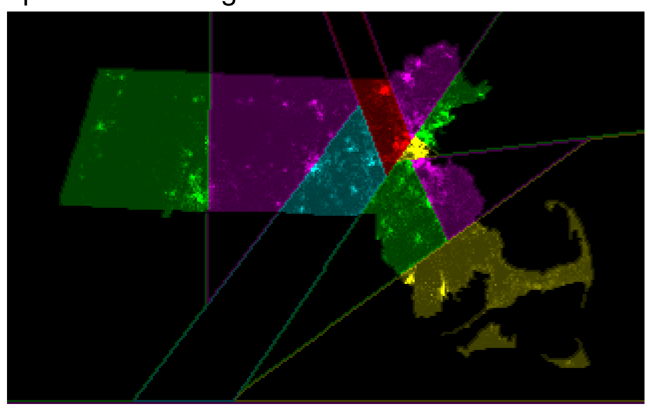




The original gerrymander:

<table>
<tr>
<td></td>
<td>


Splitline districting of Massachusetts:


</td>
</tr>
<tr>
<td>Notes</td>
<td>

- In 1812, the original gerrymander was conceived by Massachusetts senator Elbridge Gerry, who drew a district so non compact it resembled a salamander (hence the name)
- District lines are redrawn every ten years
- There are several requirements to keep in mind during redrawing, such as the Voting Rights Act and equal populations
- Recent midterm elections have shown that people want to stop gerrymandering
- Some states are assigning independent committees to draw districts, as to limit partisan gerrymandering
- In a perfect world, the percentage of seats each party receives would be equal to the percentage of votes
- Check the Wisconsin figure for a case when this is not true
- It is very difficult for courts to make decisions on gerrymandering, as intent must be established, and this is oftentimes unclear.
- Metrics and simulations help in this regard.
- Splitline districting is splitting the state into equal population

</td>
</tr>
</table>

	districts, ignoring politics - This works in theory, but it could split geographical boundaries and property, making it hard to put directly into practice
Cited references to follow up on	Knudson, Kevin. (2015, August 3). The Conversation. *Can math solve the congressional districting problem?* Retrieved 2019, February. https://theconversation.com/can-math-solve-the-congressional-districting-problem-44963
Follow up Questions	How can split lining be modified so it doesn't cut through geographical boundaries or houses?