CSE6740: Computational Data Analysis
Homework 2

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2.1

Denote λ_i, v_i as eigenvalue and eigenvector of block A_i, then $L_i v_i = \lambda_i v_i$. Pick $v_i = \mathbf{1}$, $L_i v_i = (D_i - A_i)\mathbf{1} = 0 = \lambda_i \mathbf{1}$. Hence, $\lambda_i = 0$. WLOG, we can show that there are m eigenvectors of L corresponding to eigenvalue 0.
Let's consider eigenvectors to eigenvalue 0 for each block A_i. If v is the eigenvector to eigenvalue zero, then

$$v^T L_i v = v^T (D_i - A_i) v = \sum_{i=1}^{n} d_i v_i^2 - \sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} v_i v_j = \sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} (v_i - v_j)^2 = 0$$

The only solution is $v_1 = v_2 = ... = v_n$. Thus, the only eigenvector to eigenvalue zero is scaled identity vector.
Since $L = \text{diag}(L_1, ..., L_m)$, the eigenvectors to eigenvalue zero are linear combinations of $I_{A_1}, ..., I_{A_m}$