MIDTERM EXAM

ISyE6420
Spring 2020

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Name ______________________________

Problem	Six Neurons	Ball Bearings	Mating Calls	Total
Score	/33	/33	/34	/100

1. Six Neurons. Six neurons $N_1, N_2, ..., N_6$ are connected as in the right panel in Fig. 1. If a stimulus is present, the neuron will fire with probability 0.9. When the stimulus is not present, the neuron may still fire but with a small probability of 0.05. Firing of a neuron serves as a stimulus for the subsequent neuron. N_1 is given a stimulus.

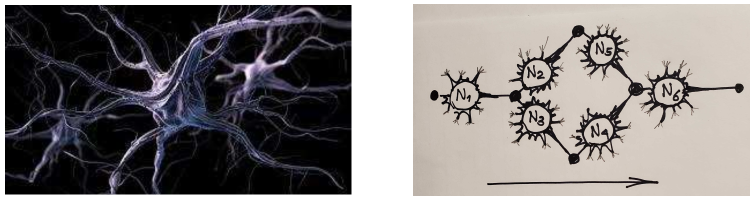

Figure 1: Neuron Fires!

(a) What is the probability that N_6 will fire?
(b) What is the probability that N_6 will fire if N_4 did not fire?
(c) If N_6 did not fire, what is the probability that N_5 received stimulus.

Hint: You can solve this problem by any of the 3 ways: (i) use of WinBUGS or OpenBUGS, (ii) direct simulation using Octave/MATLAB, R, or Python, and (iii) exact calculation. Use just one of the three ways to solve it.

2. Endurance of Deep Groove Ball Bearings. The data analyzed by Lawless (1986; page 228)[1] arose in tests on endurance of deep groove ball bearings (Fig. 2). The data, in units of 10^7 revolutions before failure for each of the 23 ball bearings in the life test, are: 1.788, 2.892, 3.300, 4.152, 4.212, 4.560, 4.880, 5.184, 5.196, 5.412, 5.556, 6.780, 6.864, 6.864, 6.888, 8.412, 9.312, 9.864, 10.512, 10.584, 12.792, 12.804, and 17.340.

Figure 2: Deep groove ball bearing

Assume that observations are coming from gamma $\mathcal{G}a(r, \lambda)$ distribution, where shape parameter is known, $r = 4$, and rate parameter λ is to be estimated in a Bayesian fashion.

[1]Lawless, J. F., 1982. *Statistical Models and Methods for Lifetime Data*, Wiley, New York.

An expert elicits a gamma prior on λ,

$$\pi(\lambda) = \frac{\lambda^{\alpha-1}\beta^{\alpha}}{\Gamma(\alpha)} e^{-\beta\lambda},$$

with hyperparameters $\alpha = 3$ and $\beta = 5$.

(a) Find Bayes estimator of λ (posterior mean), the 95% equitailed credible set for λ, and the posterior probability of hypothesis $H : \lambda \leq 0.5$.

Hint: No WinBUGS should be used, the problem is conjugate. You will need Octave or R or Python, to calculate gamma cdf and quantiles.

Mating Calls. In a study of mating calls in the gray tree frogs *Hyla hrysoscelis* and *Hyla versicolor*, Gerhart (1994)[2] reports that in a location in Lousiana the following data on the length of male advertisement calls have been collected:

	Sample size	Average duration	SD of duration
Hyla chrysoscelis	43	0.65	0.18
Hyla versicolor	12	0.54	0.14

The two species cannot be distinguished by external morphology, but *H. chrysoscelis* (Fig. 3) are diploids while *H. versicolor* are tetraploids. The triploid crosses exhibit high mortality in larval stages, and if they attain sexual maturity, they are sterile. Females responding to the mating calls try to avoid mismatches.

Figure 3: Hyla chrysoscelis

Assume that duration observations are normally distributed with means μ_1 and μ_2, and precisions τ_1 and τ_2, for the two species respectively. For $i = 1, 2$, assume normal priors on μ_i's as $\mathcal{N}(0.6, 1)$ and gamma priors on τ_i's as $\mathcal{G}a(20, 0.5)$, where 0.5 is a rate hyperparameter.

Do the following simulations in Octave (MATLAB), or Python, or R. Based on observations and given priors, in the same loop construct two Gibbs samplers, one for (μ_1, τ_1) and the other for (μ_2, τ_2).

[2]Gerhardt, H. C. (1994). Reproductive character displacement of female mate choice in the grey treefrog, *Hyla chrysoscelis*. *Anim. Behav.*, **47**, 959–969.

Form a sequence of differences $\mu_{1,j} - \mu_{2,j}$, $j = 1, \ldots, 11000$, and after rejecting the initial 1000 differences, from the remaining simulations estimate 95% credible set for $\mu_1 - \mu_2$.

Does this set contain zero? What can you say about the hypothesis $H_0 : \mu_1 = \mu_2$ based on this credible set? Based on this analysis elaborate whether the length of call is a discriminatory characteristic?

Hint: When no raw data are given, that is, when data are summarized via sample size, sample mean, and sample standard deviation, the following identity may be useful:

$$\sum_{i=1}^{n}(y_i - \mu)^2 = (n-1)s^2 + n(\overline{y} - \mu)^2,$$

where $n, \overline{y}$, and s are sample size, sample mean, and sample standard deviation, respectively.