

## Homework 1

**Problem 1:**

Test Sensitivity (SS) of Tinel's sign, Phalen's test, and nerve conduction velocity test are"

$$SSa = 0.97, SSb = 0.92, SSc = 0.93$$

Test Specificity (SP) of Tinel's sign, Phalen's test, and nerve conduction velocity test are:

$$SPa = 0.91, SPb = 0.88, SPc = 0.87$$

a) In serial manner:  **$SSs = 0.97 * 0.92 * 0.93 = 0.83$**

$$SPs = 0.91 * 0.88 * 0.87 = 0.697$$

b) In parallel manner (at least 1 test having positive result given patient has carpal tunnel syndrome):

$$SSp = 0.97 * 0.92 * 0.93 + 0.97 * (1 - 0.92) * (1 - 0.93) + (1 - 0.97) * 0.92 * (1 - 0.93) + (1 - 0.97) * (1 - 0.92) * 0.93 + 0.97 * 0.92 * (1 - 0.93) + 0.97 * (1 - 0.92) * 0.93 + (1 - 0.97) * 0.92 * 0.93$$

$$SSp = 0.99983$$

Similarly,  **$SPp = 0.9906$**

c) Assuming 1000 people are tested with 3 combined test with 50 patient carrying carpal tunnel syndrome (prevalence = 50/1000)

$$\text{Sensitivity} = TP / (TP + FN)$$

$$\text{Specificity} = TN / (TN + FP)$$

-In serial manner:  $TP + FN = 50$  and  $\text{Sensitivity} = 0.83 \Rightarrow TP = 41.5$  and  $FN = 8.5$

$$TN + FP = 950 \text{ and } \text{specificity} = 0.697 \Rightarrow TN = 662.15 \text{ and } FP = 287.85$$

PPV in series manner =  $TP / (TP + FP) = 0.126$

Similarly, PPV in parallel manner = 0.8484

**Problem 2:**

$\Pr(\text{Went Beach} = 0) = 0.6$ ,  $\Pr(\text{Midterm} = 0 \mid \text{Went Beach} = 0) = 0.55$

$\Pr(\text{Finances} = 0 \mid \text{Went Beach} = 0) = 0.583$

$\Pr(\text{Friends Go} = 0 \mid \text{Went Beach} = 0) = 0.883$

$\Pr(\text{Forecast} = 0 \mid \text{Went Beach} = 0) = 0.283$

$\Pr(\text{Gender} = 0 \mid \text{Went Beach} = 0) = 0.616$

a) Probability that Michael will not go to the beach with given conditions:

$$\Pr_n = 0.55 * 0.583 * (1 - 0.883) * (1 - 0.283) * (1 - 0.616) * 0.6 = 0.619\%$$

$$\Pr = 0.495\%$$

Probability that Michael will go  $P = 0.495 / (0.495 + 0.619) = 44.4\% \Rightarrow$  He wont go

b) Meliisa

$$\Pr_n = (1 - 0.55) * (1 - 0.583) * (1 - 0.283) * 0.883 * 0.616 * 0.6 = 4.39\%$$

$$\Pr = 9.126\%$$

Probablity that Melissa will go

$$P = 9.126 / (9.126 + 4.39) = 67.5\% \Rightarrow \text{Melissa will go to the beach}$$

**Problem 3:**

a) Probability that student knows both of the answers is  $0.8 * 0.8 = 0.64$

Probability that students knows the answer of 1 question and correctly guess the other

$$\text{is } 2 * 0.8 * 0.2 * 0.25 = 0.08$$

Probability that students guesses correctly both the answers is  $(0.2 * 0.25)^2 = 0.0025$

Probability that student answers correctly both questions is

$$\text{Pr} = 0.64 + 0.08 + 0.0025 = 0.7225$$

b) If answering correctly, probability that student knows both of the answer is

$$\text{Pr} = 0.64 / 0.7225 = 0.8858$$

c) Generalized formula in case of n questions

Probability that student answers correctly n questions

$$\text{Pr} = (0.8 + 0.2 * 0.25)^n = 0.85^n$$

Probability that student knows the answers of n questions given answering correctly n questions

$$\text{Pr} = \left( \frac{0.8}{0.85} \right)^n$$

### **Proof: Induction Method**

Assuming that the equation holds true for n: Proving that the equation is true in case of (n+1)

Probability that student answer correctly n questions and knowing the answer of (n+1)th question

$$\text{Pr} = 0.85^n * 0.8$$

Probability that student answer correctly n questions and guess correctly the answer of (n+1)question :

$$\text{Pr} = 0.85^n * 0.2 * 0.25$$

Therefore, probability that student answers correctly  $n+1$  questions:

$$\Pr = 0.85^n \cdot 0.8 + 0.85^n \cdot 0.2 \cdot 0.25 = 0.85^{n+1}$$

Probabilities in (a) and (b) approach 0 as  $n$  approaches infinity.