ISyE 6810 - Homework 7

Question 1 - Compressive Sensing: Consider a noiseless signal given in "full.mat". Although the signal is not sparse in the time domain, its wavelet representation is sparse. A Haar wavelet basis matrix is also given in "haar.mat". The goal is to first compress the signal and then retrieve the compressed signal using compressive sensing approach.

1. Create an $n \times 1024; n = \{600, 700, 800, 900\}$ noiselet sensing matrix randomly using the function "noiselet.m". Use this matrix to compress the signal.
2. Use the haar matrix as the basis matrix (Ψ) to recover the signal. Plot the reconstruction error against n.
3. Add standard normal noise to the original signal an repeat part 1 and 2 for the noisy signal.
4. compare and comment on the reconstruction errors for noisy and noiseless cases.

Question 2 - Matrix completion: A noisy image is given in "image.mat". Load the image and randomly remove 15% of pixels (replace them with 0).

1. Code the PFBS algorithm given in Slide 42 to complete the matrix.
2. Using imagesc plot both original and recovered image.
3. Compute the recovery error for missing pixels as well as the whole image.

Question 3 - Robust PCA: Consider the Robust PCA problem for noisy data. If we move constraint to the objective function we have

$$\underset{L,S}{argmin} \, \|M - (L + S)\|_F^2 + \gamma \|L\|_* + \lambda \|S\|_1$$

1. Use the block coordinate descent method to develop an optimization algorithm for this problem. Write the pseudo code.
2. Apply your algorithm on the noisy image data given in "image_anomaly.mat".

Question 4 - RKHS Ridge Regression: A noisy smooth image is given in "peaks.mat". The goal is to denoise the image using the RKHS.

1. Create a 2D Gaussian kernel basis (Gram matrix) by finding the Kronecker product of two 1D Gaussian kernels with bandwidth 1.
2. Use these Gram matrices to estimate the value of each pixels using the information of other pixels following the RKHS Ridge Regression procedure.
3. Compute and plot the smooth image and estimate the noise standard deviation.