

$$p(x^{(i)} | y=c) = \prod_{k=1}^n \theta_{c_k}^{x_k^{(i)}}$$

$$p(X|\theta, y=c) = \prod_{i=1}^m \prod_{k=1}^n \theta_{c_k}^{x_k^{(i)}}$$

$$p(X|\theta, y=c) = \prod_{k=1}^n \theta_{c_k}^{\sum_{i=1}^m x_k^{(i)}}$$

$$\log P = \sum_{k=1}^n \log \theta_{c_k}^{\sum_{i=1}^m x_k^{(i)}} - \lambda \sum_{k=1}^n \theta_{c_k}$$

$$\frac{\log P}{\partial \theta_{c_k}} = \frac{\sum_{i=1}^m x_k^{(i)}}{\theta_{c_k}} - \lambda = 0$$

$$\Leftrightarrow \theta_{c_k} = \frac{\sum_{i=1}^m x_k^{(i)}}{\lambda}$$

$$\log P(\lambda) = \sum_{k=1}^n \log \left(\frac{\sum_{i=1}^m x_k^{(i)}}{\lambda} \right) - \lambda \sum_{k=1}^n \frac{\sum_{i=1}^m x_k^{(i)}}{\lambda}$$

$$= \sum_{k=1}^n \left(\log \frac{1}{\lambda} + \log \sum_{i=1}^m x_k^{(i)} \right) - \lambda \sum_{k=1}^n \sum_{i=1}^m x_k^{(i)}$$

$$\frac{\partial \log P(\lambda)}{\partial \lambda}$$

$$= - \sum_{k=1}^n \frac{1}{\lambda} - 1 = 0 \Leftrightarrow \lambda = m$$