problem 3: Solving RPCA - Algorithm

$$\underset{L,S}{\arg\min} \|M-(L+S)\|_F^2 + \gamma\|L\|_* + \lambda\|S\|_1 \quad (*)$$

① Block coordinate Gradient descent:

$$(*) \Leftrightarrow \underset{L,S}{\arg\min} \frac{1}{\gamma}\|M-(L+S)\|_F^2 + \|L\|_* + \frac{\lambda}{\gamma}\|S\|_1$$

Let $\frac{1}{\gamma} = \frac{\mu}{2}$; $\frac{\lambda}{\gamma} = \lambda^*$

We have: $(*) \Leftrightarrow \underset{L,S}{\arg\min} \frac{\mu}{2}\|M-(L+S)\|_F^2 + \|L\|_* + \lambda\|S\|_1$

Block Coordinate descent:

Initializing: $S_0 ; Y_0 = 0$; $\mu \geq 0$

① Give S, Y, update L

$$\underset{L}{\arg\min} \|L\|_* + \frac{\mu}{2}\|M-L-S+\frac{Y}{\mu}\|_F^2$$

$$X = M - S + \frac{Y}{\mu} \Rightarrow L = D_{1/\mu}(X)$$

② Give L, Y update S

$$\underset{S}{\arg\min} \lambda\|S\|_1 + \frac{\mu}{2}\|M-L-S+\frac{Y}{\mu}\|_F^2$$

$$S_{ij} = S_{\frac{\lambda}{\mu}}(X) = \text{sign}(X)\max($$

③ Give L and S, update Y