Kien Vu Homework 2 ISYE6400

Problem 1: k-out-of-n system

Probability of a component working when checking at time t = 3 months, assuming n =8, k=4, r=3/2 and **λ = 1/10=>** **p= exp{-0.1t$^{3/2}$}**

a) Probability that a k-out-of-n system is still operation when checked at time t =3:

P(X>=4) = $\sum_{k=4}^{8}\binom{8}{k}p^k\,(1-p)^{8-k}$ = 0.5351

b) Probability that a 5-out-of-n system is still operation when checked at time t =3:

P(X==5) = $\binom{8}{5}p^5(1-p)^3$ = 0.2773

Probability that exact 5 components working while the system working when checked at time t

P(x==5|X>=4) = P(X==5)/P(X>=4) = 0.2773/0.5351 = 0.5182

Problem 2:

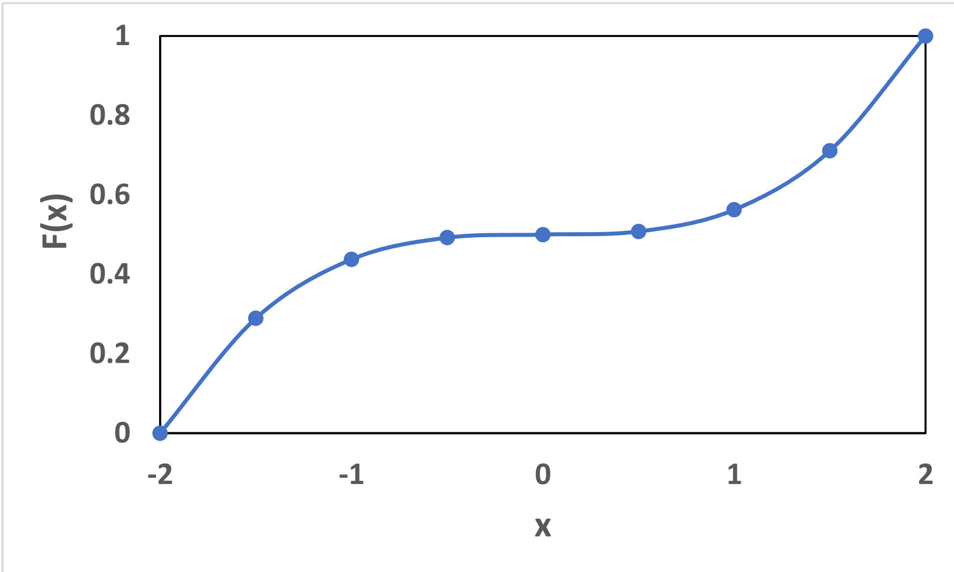
$$f(x) = \begin{cases} \frac{3x^2}{16}, & -2 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

a) Probability that a randomly chosen measurement can be classified as accurate (abs(X) < 0.5)

P(abs(X)<0.5) = $\int_{-0.5}^{0.5} f(x)\,dx = \frac{3}{16}\int_{-0.5}^{0.5} x^2\,dx = \frac{3}{16}\,{}_{-0.5}^{0.5}(\frac{x^3}{3})$ =

0.015625

b) Cumulative distribution F(x) = $\int_{-2}^{2} f(x)\,dx = \frac{3}{16}\int_{-2}^{2} x^2\,dx = \frac{x^3}{16}$



c) Expected loss (mean of Y) :

$$\mathbf{E(Y) = E(}\boldsymbol{X^2}\mathbf{) =} \int_{-2}^{2} \boldsymbol{x^2 f(x)\, dx} = \int_{-2}^{2} \boldsymbol{x^2 (\frac{3}{16} x^2)\, dx}$$

$$\mathbf{E(Y) =} \frac{3}{16} \int_{-2}^{2} \boldsymbol{x^4\, dx} = \frac{3}{16*5} \left(\boldsymbol{x^5}\right)\Big|_{-2}^{2} = \mathbf{2.4}$$

d) Probability of Y is less than \$3:

$$\text{Y} < \$3 \xrightarrow{yields} X^2 < \frac{3}{1000} \xrightarrow{yields} -\sqrt{\frac{3}{1000}} < \text{X} < \sqrt{\frac{3}{1000}}$$

$$\mathbf{P(abs(X)<} \sqrt{\frac{3}{1000}}\mathbf{) =} \int_{-\sqrt{\frac{3}{1000}}}^{\sqrt{\frac{3}{1000}}} \boldsymbol{f(x)\, dx} = \frac{3}{16} \int_{-\sqrt{\frac{3}{1000}}}^{\sqrt{\frac{3}{1000}}} \boldsymbol{x^2\, dx} = \frac{3}{16} {}_{-\sqrt{\frac{3}{1000}}}^{\sqrt{\frac{3}{1000}}} \boldsymbol{(\frac{x^3}{3})} =$$

2.054e-5

<u>Problem 3:</u>

$$f(x,y) = \begin{cases} x+y, & 0 \le x \le 1;\ 0 \le y \le 1 \\ 0, & \text{else} \end{cases}$$

a) Marginal distribution of $f_X(x)$

$$f_X(x) = \int_{-inf}^{inf} f(x,y)\,dy$$

$$= \int_{-inf}^{0} f(x,y)\,dy + \int_{0}^{1} f(x,y)\,dy + \int_{1}^{0} f(x,y)\,dy$$

$$= \int_{0}^{1} f(x,y)\,dy = \int_{0}^{1} (x+y)\,dy = \left(xy + \frac{y^2}{2}\right)\Big|_0^1$$

$$= \left(x + \frac{1}{2}\right)$$

When 0<=x<=1, else 0

b) Conditional distribution **f(y|x)** $= \frac{f(x,y)}{f_X(x)} = \frac{x+y}{x+\frac{1}{2}}$ **when x, y in [0,1]**

Else f(y|x) is undefined

Problem 4:

$$\pi(\theta|x) = f(u1, u2, \ldots, u34|\theta)\pi(\theta) = (\theta^{-34}1)\frac{1}{\theta}\ (\theta > M)$$

Pareto family: $\pi(\theta|x) = (\theta^{-34}1)\frac{1}{\theta} = \theta^{-35} = \frac{\alpha\, c^{\alpha}}{\theta^{\alpha+1}}1\ (\theta >$

$M)$

Therefore **c = M = 0.54876 and α = 34**

<u>c)</u> $\widetilde{\boldsymbol{\theta}} = \frac{c\alpha}{\alpha-1} = \frac{(0.54876)*34}{34-1}$ **= 0.5654**

$$F(\theta) \geq 0.95 \xrightarrow{yields} \left[1-\left(\frac{c}{\theta}\right)^{\alpha}\right] 1(\theta > 0.54876) \geq 0.95$$

$$Yield\ 1-\left(\frac{0.54876}{\theta}\right)^{34} \geq 0.95$$

$$Yield\ \left(\frac{0.54876}{\theta}\right)^{34} \leq 0.05$$

$$\theta \geq 0.5993$$

Therefore, the true value of parameter (0.6) in the credible set.