Title: "Analysis of Numerical Methods for Planetary Orbit Prediction"

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Project Statement:

This project describes an investigation into developing a polynomial model to predict Venus's orbital position using different numerical computation techniques. We will implement and evaluate three approaches: Kepler's equations using root-finding algorithms, regression analysis, and interpolation techniques. We aim to provide insights into the relative strengths and weaknesses of these methods for planetary orbit prediction and find answers to questions like, can the data be fitted using polynomial models?, and how effectively can numerical methods be applied to analyze this data?

Algorithm:

We need to implement multiple data analysis techniques and numerical methods to come up with a good solution for this problem statement. Algorithms like Root-finding algorithm (e.g., Newton-Raphson method) to solve Kepler's equation, Polynomial regression (e.g., least squares method), and Interpolation technique (e.g., cubic spline interpolation) are to be used.

Background:

Venus, the second planet from the Sun, orbits at an average distance of 0.72 AU(Astronomical Units) with a sidereal period of 224.7 Earth days. Accurate prediction of planetary positions is crucial for space mission planning, including flyby trajectories and long-term mission scheduling. Kepler's equations provide a theoretical basis for orbital mechanics, while we hypothesize that regression and interpolation offer better data-driven approaches to position prediction.

Initial Findings:

Orbital positions of a planet can be determined by a few laws and combination of trigonometric and algebraic equations. Johannes Kepler developed these laws of planetary motion, which describe the elliptical orbits of planets around the Sun. Preliminary analysis of Venus's orbital data shows periodic patterns consistent with its known orbital characteristics. In this scenario, input data's resolution and time span will be sufficient for implementing and comparing the proposed numerical methods. We are considering 30 years of spatial position and velocity data, from 1964 to 1994 at the resolution of a day.

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We would perform exploratory Data Analysis to detect any outliers using statistical methods (Z-score, IQR) and handle missing data if any to smoothen the data, by applying **spline interpolation** techniques to fill in gaps.

Schedule:

Rough estimate of deadline is:

- 1) Data preprocessing and analysis: 10/17 10/22
- 2) Implementation of Kepler's equation solver using root-finding: 10/23 10/29
- 3) Development of regression model for position prediction: 10/30 11/05
- 4) Implementation of interpolation technique: 11/06 11/12
- 5) Comparative analysis of all three methods: 11/13 11/19
- 6) Error analysis and performance evaluation: 11/20 12/03

Group Member Contributions:

Yet to be discussed. Points 2, 3 and 4 from above can be done in parallel which have to be incorporated at the end for analysis. Work will be split and almost equal efforts would be carried out by each member.

Potential Difficulties:

Few of the difficulties that can be foreseen are handling numerical instabilities in root-finding algorithms, selecting appropriate regression models for multi-dimensional orbital data, and ensuring fair comparison between theoretical and data-driven methods.

Bibliography:

- A) https://oer.pressbooks.pub/lynnanegeorge/chapter/chapter-6-keplers-prediction-problem/
 Discusses about orbital elements and how they are used.
- B) https://nssdc.gsfc.nasa.gov/planetary/factsheet/venusfact.html
 Provides basic orbital parameters and characteristics of Venus.
- C) https://ssd.jpl.nasa.gov/horizons/tutorial.html
 Provides the true data and documentation on how the data was collected.
- D) ChatGPT for abstract domain specific clarifications.