rark:-

## M-3

Gradient :- Let $\phi(x,y,z)$ be as scalar Point

$$\nabla\phi = \bar{i}\frac{\partial\phi}{\partial x} + \bar{j}\frac{\partial\phi}{\partial y} + \bar{z}\frac{\partial\phi}{\partial z} \quad (1,1,-2).$$

ex :- $\phi = x^3+y^3+3xyz$ at $(1,1,-2)$.

$$\text{grad}\,\phi = \nabla\phi = \left(\bar{i}\frac{\partial}{\partial x} + \bar{j}\frac{\partial}{\partial y} + \bar{z}\frac{\partial}{\partial z}\right).$$

$$(x^3+y^3+3xyz).$$

$$= \bar{i}\frac{\partial}{\partial x}(x^3+y^3+3xyz) + \bar{j}\frac{\partial}{\partial y}(x^3+y^3+3xyz) + \bar{K}\frac{\partial}{\partial z}(x^3+y^3+3xyz).$$

$$= \bar{i}\,(3\cdot1^2+3\cdot1\cdot(-2)) + \bar{j}\,(3\cdot1^2+3\cdot1\cdot(-2)) + \bar{K}(3\cdot1\cdot1).$$

$$= -3\bar{i} - 3\bar{j} + 3\bar{K}.$$

$x=1, y=1, z=-2$.

derectional ~~di~~ derivative

Given $\phi = x^2-2y^2+4z^2$

$$\nabla\phi = \left(\bar{i}\frac{\partial}{\partial x} + \bar{j}\frac{\partial}{\partial y} + \bar{K}\frac{\partial}{\partial z}\right)(x^2-2y^2+4z^2).$$

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$= \bar{i}(2x) + \bar{j}(-4y) + \bar{k}(8z)$ At $(1, 1, -1)$

$\nabla\phi = 2\bar{i} + \bar{j} - \bar{k}$

Let $\bar{e}$ be the unit vector in the direction.

$$\bar{e} = \frac{2\bar{i} + \bar{j} - \bar{k}}{|2\bar{i} + \bar{j} - \bar{k}|}$$

$$= \frac{2\bar{i} + \bar{j} - \bar{k}}{\sqrt{2^2 + 1^2 + (-1)^2}}$$

$$= \frac{2\bar{i} + \bar{j} - \bar{k}}{\sqrt{6}}.$$

The directional derivative of $\phi$ in the direction of unit even $\bar{e}$ is.

$$\bar{e} \cdot \nabla\phi.$$

$$= \left(\frac{2\bar{i} + \bar{j} - \bar{k}}{\sqrt{6}}\right) \cdot (2\bar{i} - 4\bar{j} - 8\bar{k}).$$

$$= \left(\frac{2}{\sqrt{6}}\right) 2 + \left(\frac{1}{\sqrt{6}}\right)(-4) + \left(\frac{-1}{\sqrt{6}}\right)(-8)$$

$$= \frac{4}{\sqrt{6}} - \frac{4}{\sqrt{6}} + \frac{8}{\sqrt{6}}.$$

$$= \frac{4 - 4 + 8}{\sqrt{6}} = \frac{8}{\sqrt{6}}.$$

$\therefore$ Normal of surface $3yx^2 + y + z = 0$.

$\nabla\phi$

$f = \text{grad}\,(x + y + z - 3xyz).$

$= \bar{i}\,(3x^2 - 3yz) + \bar{j}\,(3y^2 - 3xy) + \bar{k}\,(3z^2 - 3xy).$

$= f_1\bar{i} + f_2\bar{j} + f_3\bar{k}$ (say).

$\text{div}\,\bar{f} = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}.$

$= \frac{\partial}{\partial x}(3x^2 - 3yz) + \frac{\partial}{\partial y}(3y^2 - 3xz) + \frac{\partial}{\partial z}(3z^2 - 3xy).$

$= 6x + 6y + 6z$

* unit vector :- $f(x, y, z) = x^2 + y^2 + 2z^2 - 26 = 0.$

$\frac{\partial f}{\partial x} = 2x, \frac{\partial f}{\partial y} = 2y, \frac{\partial f}{\partial z} = 4z.$

$\text{grad}\,f = \Sigma i \frac{\partial f}{\partial x} = 2x\bar{i} + 2y\bar{j} + 4z\bar{k}.$

Normal vector $= [\nabla f](2, 2, 3)$

$= 4i + 4j + 12k$

unit $= \frac{\nabla f}{|\nabla f|}.$

* angle between 2 vectors.

$\cos\theta = \frac{\bar{n}_1 \cdot \bar{n}_2}{|\bar{n}_1|\,|\bar{n}_2|}.$

* curl $f = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy^2 & 2x^2yz & -3yz^2 \end{vmatrix}$

$= -(3z^2 + 2x^2 y)\bar{i} + (4xyz - 2xy)\bar{j}$.

curl f at (1,1,1)

$= -\bar{i} - 2\bar{k}$

$\therefore \bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ and $r = |\bar{r}|$ then

$r^2 = x^2 + y^2 + z^2$

* curl $= \nabla = \sum \bar{i} \frac{\partial}{\partial x}$

* Solenoid $r = x\bar{i} + y\bar{j} + z\bar{k}$.

multiple integral

* $\iint y \, dx \, dy$

$y^2 = 4x \quad x^2 = 4y$

$\Rightarrow y = \frac{x^2}{4}$

$\cancel{x^4}$

$\frac{x^4}{64} = 4x$

$x = 4$ & 0.

$x = 4, y = 2\sqrt{x}$

$$\int_{x=0} \int_{y=x^2/4} y \, dy \, dx.$$

$$= \int_0^4 \left[ \frac{y^2}{2} \right]_{\frac{x^2}{4}}^{2\sqrt{x}} dx.$$

$= \frac{48}{5}$

* Limits

$x=0, y=0, z=0$ $\qquad$ $2x+3y+4z=12$.

$y=\frac{1}{3}(12-2x)$. $\qquad$ $\Rightarrow 4z=12-2x-3y$.

$x=6$. $\qquad$ $\Rightarrow z=\frac{1}{4}(12-2x-3y)$.

$$A=\begin{bmatrix} -7 & -6 & 2 \\ -6 & -8 & -4 \\ 2 & -4 & -12 \end{bmatrix}$$

$A-\lambda I=0$.

$=-\lambda^3-27\lambda^2-184\lambda-14$

$\lambda=$ answer like

$(2-\lambda)(2-\lambda)(8-\lambda)$.

$\lambda=2,2,8$.

Let $\lambda=2$ ...

$(A-\lambda I)\cancel{X}=0$

$2x-4y+z=0$.

$-5y+5z=0$.

$-3z=0$.

$$\left.\begin{aligned} x&= \\ y&= \\ z&= \end{aligned}\right\}$$

Cayley hamilton

first same process $|A-\lambda I|=0$.

upto ~~e~~ equation form like

$\lambda^3 - 5\lambda^2 + 7\lambda - 3 = 0$.

$A^3 - 5A^2 + 7A - 3I = 0$.

$A^2 = ?$ , $A \cdot A^2 = ?$

* inverse of cayley hamilton.

multiply with $A^{-1}$ with $A$ no.

$A^3 - 5A^2 + 7A - 3I = 0$.

$A^{-1}[A^3 - 5A^2 + 7A - 3I] = 0$.

$A^2 - 5A + 7 - 3A^{-1} = 0$.

$A^{-1} =$ .

* Quadratic forms :-

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 5 & -1 \\ 5 & 1 & 6 \\ -1 & 6 & 2 \end{bmatrix}$$

$$X^T A X = [x \ y \ z] \begin{bmatrix} 0 & 5 & -1 \\ 5 & 1 & 6 \\ 1 & 6 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= [x \ y \ z] \begin{bmatrix} 5y - z \\ 5x + 1y + 6z \\ -1x + 6y + 2z \end{bmatrix}$$

$$= x(5y - z) + y(5x + 1y + 6z) + z(-1x + 6y + 2z)$$

Symmetric :-

the given eq can be write as

$$\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} x^2 & xy & xz \\ yx & y^2 & yz \\ zx & zy & z^2 \end{bmatrix} \end{array}$$

Nature λ values.

* Reduce quadratic to conical.

$$7x^2+6y^2+5z^2-4xy-4yz.$$

$$A = \begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 6 \end{bmatrix}$$

* rank means No of Non-zero terms
* index of the quadratic term No. of Positive terms = 1
* Signature $2s-r = 2(1)-3$

$$= -1.$$

* the conical form is

$$A^T D A.$$

# Matrices

* Rank = No. of Non-zero Rows.
* Gauss elimination.

$$AX = B.$$

Augmented Matrix $\left[\frac{A}{B}\right] = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 3 & 8 & 2 \\ 1 & 1 & 1 & 3 \end{bmatrix}$

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & -4 & 2 \end{bmatrix}.$$

it is in echelon form.

$x + 2y + 3z = 1.$

$-y + 2z = 0.$

$-4z = 2.$

$z = -\frac{1}{2}, y = -1, x = \frac{9}{2}$

* Solution of system of eq.

1.) $P(A) \neq P(A/B)$. No Solution.

2.) $P(A) = P(A/B)$ Number of variables the system has unique Sol.

3.) $P(A) = P(A/B) <$ num of variables system has infinite.

* DAQ = A = I A I

* Gauss - Seidal

$x_1 =$

$x_2 =$

$x_3 =$

Iteration I II III

$x_1 =$ Put $x_2 = 0$

$x_2 = 0$

$x_1 = 0.6$

$x_2 =$ $x_1 = 0.6$

$x_2 = 0$

$x_3 =$ $x_1 = 0.6$

$x_2 = 0.54$

Similarly :-

Gauss jacobian

Iteration I II III

$x_1 =$ Put $x_3 = 0$.

$x_2 = 0$,

$x_2 =$ Put $x_1 = 0$.

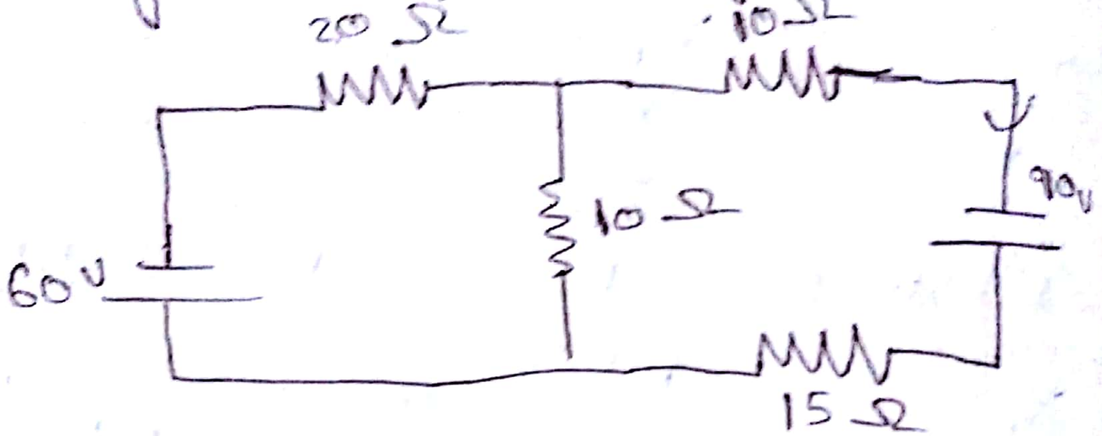
$x_3 = 0$,

$x_3 =$ Put $x_1 = 0$.

$x_2 = 0$.

* Circuits :-

by Kurcoff voltage Law.




for ABCD loop.

$80 - 20I_1 - 10I_2 = 0 \longrightarrow ①$.

$2I_1 + I_2 = 8$.

for CEFD loop

$2I_2 + 5I_3 = 10 \longrightarrow ②$

by Kurfcoff voltage Law.

$I_2 = I_1 + I_3$

$I_1 - I_2 + I_3 = 0 \longrightarrow ③$

# STANDARD RESULTS

1. $\frac{d}{dx}(x^n) = nx^{n-1}$

2. $\frac{d}{dx}(a^x) = a^x \log_e a$

3. $\frac{d}{dx}(e^x) = e^x$

4. $\frac{d}{dx}(\log_e x) = \frac{1}{x}$

5. $\frac{d}{dx}(\log_{10} x) = \frac{1}{x}\log_{10} e$

6. $\frac{d}{dx}(\sin x) = \cos x$

7. $\frac{d}{dx}(\cos x) = -\sin x$

8. $\frac{d}{dx}(\tan x) = \sec^2 x$

9. $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

10. $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

11. $\frac{d}{dx}(\sec x) = \sec x \tan x$

12. $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

13. $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$

14. $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

15. $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$

16. $\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$

17. $\frac{d}{dx}(\operatorname{cosec}^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$

18. $\sinh x = \frac{e^x - e^{-x}}{2}$

19. $\cosh x = \frac{e^x + e^{-x}}{2}$

20. $\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

21. $\cosh^2 x - \sinh^2 x = 1,\ \operatorname{sech}^2 x + \tanh^2 x = 1,\ \coth^2 x = 1 + \operatorname{cosech}^2 x$

22. $\cosh^2 x + \sinh^2 x = \cosh 2x$

23. $\sinh^{-1} x = \log(x + \sqrt{x^2+1})$

    $\cosh^{-1} x = \log(x + \sqrt{x^2-1})$

24. $\frac{d}{dx}(\sinh x) = \cosh x$

25. $\frac{d}{dx}(\cosh x) = \sinh x$

26. $\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$

27. $\frac{d}{dx}(\coth x) = -\operatorname{cosech}^2 x$

28. $\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$

29. $\frac{d}{dx}(\operatorname{cosech} x) = -\operatorname{cosech} x \coth x$

30. Product rule : $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

31. Quotient rule : $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

**32.** $\dfrac{dy}{dx} = \dfrac{dy}{dt} \cdot \dfrac{dt}{dx}$ if $y = f_1(t)$ and $x = f_2(t)$

**33.** $\sin^{-1} x + \cos^{-1} x = \dfrac{\pi}{2}$, $\tan^{-1} x + \cot^{-1} x = \dfrac{\pi}{2}$, $\sec^{-1} x + \operatorname{cosec}^{-1} x = \dfrac{\pi}{2}$

**34.** $\tan^{-1}\left(\dfrac{a-b}{1+ab}\right) = \tan^{-1} a - \tan^{-1} b$, $\tan^{-1}\left(\dfrac{a+b}{1-ab}\right) = \tan^{-1} a + \tan^{-1} b$

**35.** $\tan^{-1}\left(\dfrac{2x}{1-x^2}\right) = \sin^{-1}\left(\dfrac{2x}{1+x^2}\right) = 2\tan^{-1} x$

**36.** $\sin 3x = 3\sin x - 4\sin^3 x$, $\cos 3x = 4\cos^3 x - 3\cos x$, $\tan 3x = \dfrac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$

$\sin 2x = 2\sin x \cos x$ ; $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$,

$\cos 2x = 2\cos^2 x - 1 = 1 - 2\sin^2 x = \cos^2 x - \sin^2 x = \dfrac{1-\tan^2 x}{1+\tan^2 x}$

**37.** $\sin x = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dfrac{x^7}{7!} + \ldots$

$\cos x = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \dfrac{x^6}{6!} + \ldots$

$e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \ldots$

$(1-x)^{-1} = 1 + x + x^2 + x^3 + \ldots$ ; $|x| < 1$

$(1+x)^{-1} = 1 - x + x^2 - x^3 + \ldots$

$(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \ldots$

$(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \ldots$

**38.** $\sin C + \sin D = 2\sin\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$, $\sin C - \sin D = 2\cos\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

$\cos C + \cos D = 2\cos\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$, $\cos C - \cos D = 2\sin\dfrac{C+D}{2}\sin\dfrac{D-C}{2}$

**39.** $2\cos A\cos B = \cos(A+B) + \cos(A-B)$, $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

$2\sin A\cos B = \sin(A+B) + \sin(A-B)$, $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

**40.** $\sin(A+B) = \sin A\cos B + \cos A\sin B$, $\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$, $\cos(A-B) = \cos A\cos B + \sin A\sin B$

**41.** $\dfrac{d}{dx}(\sinh^{-1} x) = \dfrac{1}{\sqrt{1+x^2}}$, $\dfrac{d}{dx}(\cosh^{-1} x) = \dfrac{1}{\sqrt{x^2-1}}$

$\dfrac{d}{dx}(\tanh^{-1} x) = \dfrac{1}{1-x^2}$ where $|x| < 1$, $\dfrac{d}{dx}(\coth^{-1} x) = \dfrac{1}{x^2-1}$ where $|x| > 1$

$\dfrac{d}{dx}(\operatorname{sech}^{-1} x) = -\dfrac{1}{x\sqrt{1-x^2}}$, $\dfrac{d}{dx}(\operatorname{cosech}^{-1} x) = -\dfrac{1}{x\sqrt{x^2+1}}$

**42.** $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$, $(\cos\theta + i\sin\theta)^{-n} = \cos n\theta - i\sin n\theta$

**43.** $\sin^2\theta + \cos^2\theta = 1$, $\sec^2\theta - \tan^2\theta = 1$, $1 + \cot^2\theta = \text{cosec}^2\theta$

**44.**

| $\theta$ | $0°$ | $30°$ | $45°$ | $60°$ | $90°$ | $180°$ | $270°$ |
|---|---|---|---|---|---|---|---|
| $\sin\theta$ | 0 | $1/2$ | $1/\sqrt{2}$ | $\sqrt{3}/2$ | 1 | 0 | $-1$ |
| $\cos\theta$ | 1 | $\sqrt{3}/2$ | $1/\sqrt{2}$ | $1/2$ | 0 | $-1$ | 0 |
| $\tan\theta$ | 0 | $1/\sqrt{3}$ | 1 | $\sqrt{3}$ | $\infty$ | 0 | $\infty$ |

**45.**

| $\theta$ | $90° - \theta$ | $90° + \theta$ | $\pi - \theta$ | $\pi + \theta$ |
|---|---|---|---|---|
| $\sin\theta$ | $\cos\theta$ | $\cos\theta$ | $\sin\theta$ | $-\sin\theta$ |
| $\cos\theta$ | $\sin\theta$ | $-\sin\theta$ | $-\cos\theta$ | $-\cos\theta$ |
| $\tan\theta$ | $\cot\theta$ | $-\cot\theta$ | $-\tan\theta$ | $\tan\theta$ |

**46.** sine formula : $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine formula : $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

**47.** Area of triangle $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{a+b+c}{2}$

**48.** ${}^nC_r = \dfrac{n!}{r!\,n-r!}$

**49.** $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + c\ ;\ n \neq -1$

$\int \frac{1}{x}\,dx = \log_e x + c\ ;\ \int e^x\,dx = e^x + c\ ;\ \int a^x\,dx = \dfrac{a^x}{\log_e a} + c$

$\int \sin x\,dx = -\cos x + c\ ;\ \int \cos x\,dx = \sin x + c$

$\int \tan x\,dx = \log \sec x + c\ ;\ \int \cot x\,dx = \log \sin x + c$

$\int \sec x\,dx = \log(\sec x + \tan x) + c = \log\tan\left(\dfrac{\pi}{4} + \dfrac{x}{2}\right) + c$

$\int \text{cosec}\,x\,dx = \log(\text{cosec}\,x - \cot x) + c = \log\tan\dfrac{x}{2} + c$

$\int \sec x\tan x\,dx = \sec x + c\ ;\ \int \text{cosec}\,x\cot x\,dx = -\text{cosec}\,x + c$

**50.** $\int \dfrac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\dfrac{x}{a}\right) + c\ ;\ \int \dfrac{-dx}{\sqrt{a^2 - x^2}} = \cos^{-1}\left(\dfrac{x}{a}\right) + c$

$\int \dfrac{dx}{a^2 + x^2} = \dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right) + c\ ;\ \int \dfrac{-dx}{a^2 + x^2} = \dfrac{1}{a}\cot^{-1}\left(\dfrac{x}{a}\right) + c$

$\int \dfrac{dx}{a^2 - x^2} = \dfrac{1}{2a}\log\left(\dfrac{a+x}{a-x}\right) + c\ ;\ \int \dfrac{dx}{x^2 - a^2} = \dfrac{1}{2a}\log\left(\dfrac{x-a}{x+a}\right) + c$

$\int \dfrac{dx}{x\sqrt{x^2 - a^2}} = \dfrac{1}{a}\sec^{-1}\left(\dfrac{x}{a}\right) + c\ ;\ \int \dfrac{-dx}{x\sqrt{x^2 - a^2}} = \dfrac{1}{a}\text{cosec}^{-1}\left(\dfrac{x}{a}\right) + c$

**51.** $\int \operatorname{sech}^2 x\, dx = \tanh x + c$, $\int \operatorname{cosech}^2 x\, dx = -\coth x + c$

$\int \sinh x\, dx = \cosh x + c$, $\int \cosh x\, dx = \sinh x + c$

$\int \operatorname{sech} x \tanh x\, dx = -\operatorname{sech} x + c$, $\int \operatorname{cosech} x \coth x\, dx = -\operatorname{cosech} x + c$

**52.** $\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2} x \sqrt{a^2 - x^2} + \frac{1}{2} a^2 \sin^{-1} \frac{x}{a} + c$

$\int \sqrt{a^2 + x^2}\, dx = \frac{1}{2} x \sqrt{a^2 + x^2} + \frac{1}{2} a^2 \log (x + \sqrt{a^2 + x^2}) + c$

$\int \sqrt{x^2 - a^2}\, dx = \frac{1}{2} x \sqrt{x^2 - a^2} - \frac{1}{2} a^2 \log (x + \sqrt{x^2 - a^2}) + c$

$\int \frac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + c$ ; $\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + c$

**53.** $\int_a^b f(x)\, dx = \int_a^b f(y)\, dy$ ; $\int_a^b f(x)\, dx = -\int_b^a f(x)\, dx$, $\int_0^a f(x)\, dx = \int_0^a f(a - x)\, dx$

$$\int_{-a}^{a} f(x)\, dx = \begin{cases} 2\int_0^a f(x)\, dx & , \text{ if } f(x) \text{ is even function} \\ 0 & , \text{ if } f(x) \text{ is odd function} \end{cases}$$

$$\int_0^{2a} f(x)\, dx = \begin{cases} 2\int_0^a f(x)\, dx & , \text{ if } f(2a - x) = f(x) \\ 0 & , \text{ if } f(2a - x) = -f(x) \end{cases}$$

**54.** Leibnitz rule for differentiation under the integral sign

$$\frac{d}{dx}\int_{\phi(\alpha)}^{\psi(\alpha)} f(x, \alpha)\, dx = \int_{\phi(\alpha)}^{\psi(\alpha)} \frac{\partial}{\partial \alpha}\{f(x, \alpha)\}\, dx + f\{\psi(\alpha), \alpha\} \frac{d\psi(\alpha)}{d\alpha} - f\{\phi(\alpha), \alpha\} \frac{d\phi(\alpha)}{d\alpha}$$

**55.** If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$ and $\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2 + y^2 + z^2}}$

**56.** $\overrightarrow{AB}$ = position vector of B-position vector of A $= \overrightarrow{OB} - \overrightarrow{OA}$

**57.** $\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta$ ; work done $= \int_c \vec{F} \cdot d\vec{r}$

**58.** $\vec{a} \times \vec{b} = |\vec{a}|\,|\vec{b}| \sin\theta\, \hat{n}$

**59.** Area of parallelogram $= \vec{a} \times \vec{b}$, Moment of force $= \vec{r} \times \vec{F}$

**60.** $\vec{a} \cdot (\vec{b} \times \vec{c}) = [\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = (\vec{a} \times \vec{b}) \cdot \vec{c}$

where $\vec{a} = \Sigma a_1 \hat{i}$, $\vec{b} = \Sigma b_1 \hat{i}$ and $\vec{c} = \Sigma c_1 \hat{i}$

If $\vec{a} \cdot (\vec{b} \times \vec{c}) = 0$ then $\vec{a}, \vec{b}, \vec{c}$ are coplanar.

**61.** $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\,\vec{b} - (\vec{a} \cdot \vec{b})\,\vec{c}$

**62.** $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = \begin{vmatrix} \vec{a} \cdot \vec{c} & \vec{a} \cdot \vec{d} \\ \vec{b} \cdot \vec{c} & \vec{b} \cdot \vec{d} \end{vmatrix}$

**63.** $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}\ \vec{b}\ \vec{d}]\ \vec{c} - [\vec{a}\ \vec{b}\ \vec{c}]\ \vec{d}$

**64.** $A\,(\text{Adj. } A) = |\,A\,|\, I$

**65.** $AA^{-1} = I = A^{-1} A$

**66.** $AI = A = IA$

**67.** $(ABC)' = C'B'A'$

**68.** $(AB)\,C = A(BC)$ ; $A(B + C) = AB + AC$

**69.** $A + B = B + A$ ; $A + (B + C) = (A + B) + C$

**70.** $(AB)^{-1} = B^{-1}A^{-1}$

**71.** **Walli's formula**

$$\int_0^{\pi/2} \sin^n \theta \, d\theta = \int_0^{\pi/2} \cos^n \theta \, d\theta = \begin{cases} \dfrac{n-1}{n} \cdot \dfrac{n-3}{n-2} \cdot \dfrac{n-5}{n-4} \cdots\cdots \dfrac{3}{4} \cdot \dfrac{1}{2} \cdot \dfrac{\pi}{2} & \text{if } n \text{ is eve} \\ \dfrac{n-1}{n} \cdot \dfrac{n-3}{n-2} \cdot \dfrac{n-5}{n-4} \cdots\cdots \dfrac{4}{5} \cdot \dfrac{2}{3} & \text{if } n \text{ is od} \end{cases}$$

**72.** $$\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx) + c$$

$$\int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx) + c$$

**73.** $\Gamma(1/2) = \sqrt{\pi}$, $\Gamma(-1/2) = -2\sqrt{\pi}$

**74.** $$\log(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} + \cdots\cdots$$

$$\log(1 - x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \frac{x^6}{6} - \cdots\cdots$$

**75.** $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

1. $\frac{d}{dx}(x^n) = nx^{n-1}$

2. $\frac{d}{dx}(a^x) = a^x \log_e a$

3. $\frac{d}{dx}(e^x) = e^x$

4. $\frac{d}{dx}(\log_e x) = \frac{1}{x}$

5. $\frac{d}{dx}(\log_{10} x) = \frac{1}{x}\log_{10} e$

6. $\frac{d}{dx}(\sin x) = \cos x$

7. $\frac{d}{dx}(\cos x) = -\sin x$

8. $\frac{d}{dx}(\tan x) = \sec^2 x$

9. $\frac{d}{dx}(\text{cosec } x) = -\text{cosec } x \cot x$

10. $\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$

11. $\frac{d}{dx}(\sec x) = \sec x \tan x$

12. $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

13. $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$

14. $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

15. $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$

16. $\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$

17. $\frac{d}{dx}(\text{cosec}^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$

18. $\sinh x = \frac{e^x - e^{-x}}{2}$

19. $\cosh x = \frac{e^x + e^{-x}}{2}$

20. $\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

21. $\cosh^2 x - \sinh^2 x = 1$, $\text{sech}^2 x + \tanh^2 x = 1$, $\coth^2 x = 1 + \text{cosech}^2 x$

22. $\cosh^2 x + \sinh^2 x = \cosh 2x$

23. $\sinh^{-1} x = \log(x + \sqrt{x^2+1})$

$\cosh^{-1} x = \log(x + \sqrt{x^2-1})$

24. $\frac{d}{dx}(\sinh x) = \cosh x$

25. $\frac{d}{dx}(\cosh x) = \sinh x$

26. $\frac{d}{dx}(\tanh x) = \text{sech}^2 x$

27. $\frac{d}{dx}(\coth x) = -\text{cosech}^2 x$

28. $\frac{d}{dx}(\text{sech } x) = -\text{sech } x \tanh x$

29. $\frac{d}{dx}(\text{cosech } x) = -\text{cosech } x \coth x$

30. Product rule : $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

31.