Matrices

1. Show that the matrix $A = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & -5 & 5 \end{bmatrix}$ is idempotent.

Given, $A = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & -5 & 5 \end{bmatrix}$

$$A^2 = AA = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & -5 & 5 \end{bmatrix}\begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & -5 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1+3-5 & -1-3+5 & 1+3-5 \\ -3-9+15 & 3+9-15 & -3-9+15 \\ -5-15+25 & 5+15-25 & -5-15+25 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & -5 & 5 \end{bmatrix} = A$$

∴ The given matrix A is idempotent.

2. Show that $\begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$ is nilpotent matrix. Find its index.

let $A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+5-6 & 1+2-3 & 3+6-9 \\ 5+10-12 & 5+4-6 & 15+12-18 \\ -2-5+6 & -2-2+3 & -6-6+9 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 3 & 3 & 9 \\ -1 & -1 & -3 \end{bmatrix}$$

$$A^3 = A^2 A = \begin{bmatrix} 0 & 0 & 0 \\ 3 & 3 & 9 \\ -1 & -1 & -3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Q. show that $A = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$ is orthogonal.

$$A = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix} ; \quad A^T = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$$

$$AA^T = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix} \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 1+4+4 & -2-2+4 & -2+4-2 \\ -2-2+4 & 4+1+4 & 4-2-2 \\ -2+4-2 & 4-2-2 & 4+4+1 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$$AA^T = A^TA = I$$

$\therefore$ given A matrix is orthogonal.

4. Express matrix $\begin{bmatrix} 3 & 7 \\ 4 & 5 \end{bmatrix}$ as a sum of symmetric & skew symmetric matrices.

$\rightarrow$ let $A = \begin{bmatrix} 3 & 7 \\ 4 & 5 \end{bmatrix}$

$$A^T = \begin{bmatrix} 3 & 4 \\ 7 & 5 \end{bmatrix}$$

Symmetric matrix $= \dfrac{A+A^T}{2}$

$$= \frac{1}{2}\left[\begin{bmatrix} 3 & 7 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 7 & 5 \end{bmatrix}\right]$$

$$= \frac{1}{2}\begin{bmatrix} 6 & 11 \\ 11 & 10 \end{bmatrix} = \begin{bmatrix} 3 & 11/2 \\ 11/2 & 5 \end{bmatrix}$$

Skew symmetric matrix $= \dfrac{A-A^T}{2}$

$$= \frac{1}{2}\left[\begin{bmatrix} 3 & 7 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 4 \\ 7 & 5 \end{bmatrix}\right]$$

$$= \frac{1}{2}\begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 3/2 \\ -3/2 & 0 \end{bmatrix}$$

$$A = \frac{A+A'}{2} + \frac{A-A'}{2}$$

$$= \begin{bmatrix} 3 & 11/2 \\ 11/2 & 5 \end{bmatrix} + \begin{bmatrix} 0 & 3/2 \\ -3/2 & 0 \end{bmatrix}.$$

5. Write the properties of an orthogonal matrix.

1. If A is the orthogonal matrix then A^T and A^{-1} are also orthogonal.

$AA^{-1} = I$

$|AA^{-1}| = |I|$

$|A||A^{-1}| = |I|$

$|A||A| = 1$

$|A|^2 = 1$

$|A| = \pm 1$

2. If A is orthogonal then $A^{-1} = A^T$

3. If A is orthogonal then $|A| = \pm 1$.

4. If A and B are orthogonal matrices of same order then AB and BA are also orthogonal

Sub matrix:- The matrix obtained by eliminating a rows or columns or both is called a sub matrix of the given matrix.

Ex:- For the matrix $\begin{bmatrix} 1 & -1 & 0 & 3 \\ 2 & 1 & 4 & 6 \\ 0 & 5 & -2 & 3 \end{bmatrix}_{3\times4}$ some sub matrices are

$$\begin{bmatrix} 1 & 0 & 3 \\ 2 & 4 & 6 \\ 0 & -2 & 3 \end{bmatrix}_{3\times3} \begin{bmatrix} 1 & -1 \\ 2 & 1 \\ 0 & 5 \end{bmatrix}_{3\times2} \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}_{2\times2}$$

Minor of a matrix:- The determinant of a square sub matrix of the given matrix is called a minor: If the order of the sub matrix is t then its determinant is a minor of order t

Elimentary transformations (or) operations:-

1. Inter changing of two rows

If the rows R_i and R_j are interchanged then

It is denoted by $R_i \leftrightarrow R_j$

2. All the elements of a row are multiplied with a non-zero scalar.

If all the elements of R_i are multiplied with a non zero scalar k, then it is denoted by $R_i \to kR_i$

3. All the elements of a row are multiplied with a non-zero scalar and added to the corresponding elements of any other row

If all the elements of R_j are multiplied with $k \neq 0$ and added to the corresponding elements of R_i then it is denoted by $R_i \to R_i + kR_j$.

These three are called elementary row transformations. Similarly we define column transformation.

Rank of a matrix :- Let A is a mxn matrix.

If A is a null matrix then rank of A is zero

let A is non-zero matrix. r is said to be rank of the matrix if

1. Every minor of order r+1 is zero
2. There exists atleast one minor of order r is non-zero

The rank of A is denoted by $\rho(A)$

properties:-

1. The rank of every matrix is exist and it is unique.

2. The rank of identity matrix of order n is n

3. If A is non-singular matrix of order n then the rank of A is 1.

4. If A is an $m \times n$ matrix then rank of A is less than or equal minimum of m, n

$$\rho(A) \leq \min\{m, n\}$$

5. If A and B are equivalent matrices then rank of A = rank of B.

Equivalent matrices:- Two matrices A and B are said to be equivalent if one is obtained from the other by a finite no. of transformations and it is denoted by $A \sim B$

Zero row, non-zero row:- If all the elements of a row are equal to zero then it is called a zero row.

If atleast one element of a row is non-zero then it is called a non-zero row.

Rank - Echelon form:- let A is the given matrix the Echelon form of A has the following properties

1. zero rows if exists they lie below the non-zero rows.

2. The first non-zero element in any any non-zero row is '1'.

3. The no. of zeros before the 1st non-zero digit in a row is less than the no. of such zeroes in the next row

The no.of non-zero rows in the echlon form of A is called rank of A

Note:-

1. To get echelon form of a matrix reduce all the elements below the principal diagonal to "0".

problems:-

1. Find the rank of the matrix by reducing in to Echelon form.

i. $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 2 \\ 3 & 1 & 4 \end{bmatrix}$

let $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 2 \\ 3 & 1 & 4 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$\sim \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -2 \\ 0 & 4 & -2 \end{bmatrix}$

$R_3 \to 3R_3 - 4R_2$

$\sim \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -2 \\ 0 & 0 & 2 \end{bmatrix}$

$\therefore$ The rank of the matrix is "3".

2. $\begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ -1 & -2 & 6 & -7 \end{bmatrix}$

let $A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ -1 & -2 & 6 & -7 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 2 & -1 & 4 \\ 0 & 0 & 5 & -3 \\ 0 & 0 & 5 & -3 \end{bmatrix}$ $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 + R_1$

$$\sim \begin{bmatrix} 1 & 2 & -1 & 4 \\ 0 & 0 & 5 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_3 \to R_3 - R_2$$

$\therefore$ This is in echelon form

$\therefore$ The rank of A is "2"

3. $\begin{bmatrix} 3 & 1 & 4 & 6 \\ 2 & 1 & 2 & 4 \\ 4 & 2 & 5 & 8 \\ 1 & 1 & 2 & 2 \end{bmatrix}$

let $A = \begin{bmatrix} 3 & 1 & 4 & 6 \\ 2 & 1 & 2 & 4 \\ 4 & 2 & 5 & 8 \\ 1 & 1 & 2 & 2 \end{bmatrix}$

$$\sim \begin{bmatrix} 3 & 1 & 4 & 6 \\ 0 & 1 & -2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 2 & 2 & 0 \end{bmatrix} \quad \begin{matrix} R_2 \to 3R_2 - 2R_1 \\ R_3 \to 3R_3 - 4R_1 \\ R_4 \to 3R_4 - R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 3 & 1 & 4 & 6 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 6 & 0 \end{bmatrix} \quad \begin{matrix} R_3 \to R_3 - 2R_2 \\ R_4 \to R_4 - 2R_2 \end{matrix}$$

$$R_4 \to R_4 - 2R_3$$

$$\sim \begin{bmatrix} 3 & 1 & 4 & 6 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ This is in the echelon form

$\therefore$ The rank of A is "3"

4. $\begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$

let $A = \begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$ $\quad R_2 \to R_2 + R_1$

$R_3 \to R_3 + 2R_1$

$R_4 \to R_4 - R_1$

$$\sim \begin{bmatrix} -1 & -3 & 3 & -1 \\ 0 & -2 & 2 & -1 \\ 0 & -11 & 8 & -5 \\ 0 & 4 & -3 & 2 \end{bmatrix}$$

$R_3 \to 2R_3 - 11R_2$

$R_4 \to R_4 + 2R_2$

$$\sim \begin{bmatrix} -1 & -3 & 3 & -1 \\ 0 & -2 & 2 & -1 \\ 0 & 0 & -6 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$R_4 \to 6R_4 + R_3$

$$\sim \begin{bmatrix} -1 & -3 & 3 & -1 \\ 0 & -2 & 2 & -1 \\ 0 & 0 & -6 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This is in echelon form

$\therefore$ rank of $A = 4$.

5. $$\begin{bmatrix} 2 & -4 & 3 & -1 & 0 \\ 1 & 2 & -1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 5 \end{bmatrix}$$

let $$A = \begin{bmatrix} 2 & -4 & 3 & -1 & 0 \\ 1 & 2 & -1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 5 \end{bmatrix}$$

$R_2 \to 2R_2 - R_1$

$R_4 \to R_4 - 2R_1$

$$\sim \begin{bmatrix} 2 & -4 & 3 & -1 & 0 \\ 0 & 8 & -5 & -7 & 4 \\ 0 & 1 & -1 & 3 & 1 \\ 0 & 1 & -2 & -2 & 5 \end{bmatrix}$$

$R_3 \to 8R_3 - R_2$

$R_4 \to 8R_4 - R_2$

$$\sim \begin{bmatrix} 2 & -4 & 3 & -1 & 0 \\ 0 & 8 & -5 & -7 & 4 \\ 0 & 0 & -3 & 31 & 4 \\ 0 & 0 & -11 & -9 & 36 \end{bmatrix}$$

$R_4 \to 3R_4 - 11R_3$

$$\sim \begin{bmatrix} 2 & -4 & 3 & -1 & 0 \\ 0 & 8 & -5 & -7 & 4 \\ 0 & 0 & -3 & 31 & 4 \\ 0 & 0 & 0 & -363 & 64 \end{bmatrix}$$

This is in the echelon form

The rank of A is 4

6. For what value of k the matrix $\begin{bmatrix} 4 & 4 & -3 & 1 \\ 1 & 1 & -1 & 0 \\ k & 2 & 2 & 2 \\ 9 & 9 & k & 3 \end{bmatrix}$ rank is "3"

let $A = \begin{bmatrix} 4 & 4 & -3 & 1 \\ 1 & 1 & -1 & 0 \\ k & 2 & 2 & 2 \\ 9 & 9 & k & 3 \end{bmatrix}$

$R_1 \leftrightarrow R_2$

$\sim \begin{bmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ k & 2 & 2 & 2 \\ 9 & 9 & k & 3 \end{bmatrix}$

$R_2 \to R_2 - 4R_1$

$R_3 \to R_3 - kR_1$, $R_4 \to R_4 - 9R_1$

$\sim \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 2-k & 2+k & 2 \\ 0 & 0 & k+9 & 3 \end{bmatrix}$

given, $\rho(A) = 3$

$|A| = 0$.

expand along C_1

$$1 \begin{vmatrix} 0 & 1 & 1 \\ 2-k & 2+k & 2 \\ 0 & k+9 & 3 \end{vmatrix} = 0$$

$$(2-k)(-1) \begin{vmatrix} 1 & 1 \\ k+9 & 3 \end{vmatrix} = 0$$

$$(2-k)(3-k-9) = 0$$

$2-k=0$; $-k-6=0$

$k=2$; $k=-6$

7. $\begin{bmatrix} 10 & -2 & 3 & 0 \\ 2 & 10 & 2 & 4 \\ -1 & -2 & 10 & 1 \\ 2 & 3 & 4 & 9 \end{bmatrix}$

→ let the given matrix

$$A = \begin{bmatrix} 10 & -2 & 3 & 0 \\ 2 & 10 & 2 & 4 \\ -1 & -2 & 10 & 1 \\ 2 & 3 & 4 & 9 \end{bmatrix} \quad \begin{array}{l} R_2 \to 5R_2 - R_1 \\ R_3 \to 10R_3 + R_1 \\ R_4 \to 5R_4 - R_1 \end{array}$$

$$\sim \begin{bmatrix} 10 & -2 & 3 & 0 \\ 0 & 52 & 7 & 20 \\ 0 & -22 & 33 & 10 \\ 0 & 17 & 17 & 45 \end{bmatrix} \quad \begin{array}{l} R_3 \to -52R_3 + 22R_2 \\ R_4 \to 52R_4 - 17R_2 \end{array}$$

$$\sim \begin{bmatrix} 10 & -2 & 3 & 0 \\ 0 & 52 & 7 & 20 \\ 0 & 0 & -1563 & -80 \\ 0 & 0 & 765 & 2000 \end{bmatrix} \quad R_4 \to 7R_4 - 765R_2$$

$$\sim \begin{bmatrix} 10 & -2 & 3 & 0 \\ 0 & 52 & 7 & 20 \\ 0 & 0 & -1563 & -80 \\ 0 & 0 & 0 & 1300 \end{bmatrix}$$

This is in Echelon form

$\therefore$ rank of $A = 4$.

8/ $\begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ -2 & -3 & -1 & 4 & 3 \\ -1 & 6 & 7 & 2 & 9 \\ -3 & 3 & 6 & 6 & 12 \end{bmatrix}$

let $A = \begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ -2 & -3 & -1 & 4 & 3 \\ -1 & 6 & 7 & 2 & 9 \\ -3 & 3 & 6 & 6 & 12 \end{bmatrix} \quad \begin{array}{l} R_2 \to R_2 + 2R_1 \\ R_3 \to R_3 + R_1 \\ R_4 \to R_4 + 3R_1 \end{array}$

$$\sim \begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ 0 & 5 & 5 & 0 & 5 \\ 0 & 10 & 10 & 0 & 10 \\ 0 & 15 & 15 & 0 & 15 \end{bmatrix} \quad \begin{array}{l} R_3 \to R_3 - 2R_2 \\ R_4 \to R_4 - 3R_2 \end{array}$$

$$\sim \begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ 0 & 5 & 5 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

This is in Echelon form

rank of $A = 2$

Rank - Normal Form :- let r is the rank of an $m\times n$ matrix A then A can be reduced into one of the forms I_r (or) $[I_r \;\; 0]$ (or) $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$ (or) $\begin{bmatrix} I_r \\ 0 \end{bmatrix}$

where I_r is the unit matrix of order r the above forms are normal forms of A.

problems:-

1. Find the rank of the following matrices by redcuing into normal form.

1. $\begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$

let $A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$ $R_2 \to R_2 - R_1$, $R_3 \to R_3 - 3R_1$

$\sim \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & -2 & 3 & -2 \\ 0 & -2 & 3 & -2 \end{bmatrix}$ $R_3 \to R_3 - R_2$

$\sim \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & -2 & 3 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $C_2 \to C_2 - C_1$
$C_3 \to C_3 + C_1$
$C_4 \to C_4 - C_1$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -2 & 3 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $C_3 \to 2C_3 + 3C_2$
$C_4 \to C_4 - C_2$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $C_2 \to \frac{C_2}{-2}$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$

$\therefore$ The rank of $A = 2$.

2. $\begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & 4 & 5 \\ 1 & -3 & 2 & 0 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

let $A = \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & 4 & 5 \\ 1 & +3 & 2 & 0 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ $R_3 \to R_3 - R_1$, $R_4 \to R_4 - R_1$

$\sim \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & 4 & 5 \\ 0 & +3 & 5 & -2 \\ 0 & 1 & 1 & -2 \end{bmatrix}$ $R_3 \to R_3 - 3R_2$, $R_4 \to R_4 - R_2$

$\sim \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & 4 & 5 \\ 0 & 0 & -7 & -17 \\ 0 & 0 & -3 & -7 \end{bmatrix}$ $R_4 \to 7R_4 - 3R_3$

$\sim \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & 4 & 5 \\ 0 & 0 & -7 & -17 \\ 0 & 0 & 0 & 2 \end{bmatrix}$ $C_3 \to C_3 + 3C_1$, $C_4 \to C_4 - 2C_1$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 4 & 5 \\ 0 & 0 & -7 & -17 \\ 0 & 0 & 0 & 2 \end{bmatrix}$ $C_3 \to C_3 - 4C_2$, $C_4 \to C_4 - 5C_2$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -7 & -17 \\ 0 & 0 & 0 & 2 \end{bmatrix}$ $C_3 \to \frac{C_3}{-7}$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -17 \\ 0 & 0 & 0 & 2 \end{bmatrix}$ $C_4 \to C_4 + 17C_3$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$ $C_4 \to \frac{C_4}{2}$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} = I_4$

The rank of A is 4

3.
$$\begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & +1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

let
$$A = \begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & +1 \\ 0 & 1 & 1 & -1 \end{bmatrix} \qquad \begin{matrix} R_2 \to 2R_2 + R_1 \\ R_3 \to 2R_3 + R_1 \end{matrix}$$

$$\sim \begin{bmatrix} -2 & -1 & -3 & -1 \\ 0 & 3 & 3 & -3 \\ 0 & -1 & -1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix} \qquad R_2 \to \frac{R_2}{3}$$

$$\sim \begin{bmatrix} -2 & -1 & -3 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & -1 & -1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix} \qquad \begin{matrix} R_3 \to R_3 + R_2 \\ R_4 \to R_4 - R_2 \end{matrix}$$

$$\sim \begin{bmatrix} -2 & -1 & -3 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{matrix} C_4 \to C_4 - C_2 \\ C_3 \to C_3 - 3C_2 \end{matrix}$$

$$\sim \begin{bmatrix} -2 & -1 & 0 & 0 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad R_2 \to \frac{R_2}{-} \qquad \begin{matrix} C_3 \to \frac{C_3}{-2} \\ C_4 \to \frac{C_4}{-2} \end{matrix}$$

$$\sim \begin{bmatrix} -2 & -1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad C_4 \to C_4 - C_3$$

$$\sim \begin{bmatrix} -2 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad C_1 \to C_1/-2$$

$$\sim \begin{bmatrix} +1 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad C_2 \to C_2 + C_1$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad C_3 \to C_3 - C_2$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cong \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$$

The rank of A is 2.

5) $\begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ +1 & -2 & 1 & 2 \end{bmatrix}$

let $A = \begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ +1 & -2 & 1 & 2 \end{bmatrix}$ $\quad R_1 \to \frac{R_1}{2}$, $R_2 \to \frac{R}{2}$

$\sim \begin{bmatrix} 1 & -1 & 0 & 3 \\ 2 & 1 & 0 & 1 \\ 1 & -1 & 0 & 3 \\ +1 & -2 & 1 & 2 \end{bmatrix}$ $\quad R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - R_1$, $R_4 \to R_4 - R_1$

$\sim \begin{bmatrix} 1 & -1 & 0 & 3 \\ 0 & 3 & 0 & -5 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & -1 \end{bmatrix}$ $\quad R_3 \leftrightarrow R_4$

$\sim \begin{bmatrix} 1 & -1 & 0 & 3 \\ 0 & 3 & 0 & -5 \\ 0 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\quad R_3 \to 3R_3 + R_2$

$\sim \begin{bmatrix} 1 & -1 & 0 & 3 \\ 0 & 3 & 0 & -5 \\ 0 & 0 & 3 & -8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\quad C_4 \to C_4 + 3C_1$, $C_2 \to C_2 + C_1$, $C_3 \to C_3/3$

$\sim \begin{bmatrix} 1 & \cancel{-1}\ 0 & 0 & 0 \\ 0 & 3 & 0 & 4 \\ 0 & 0 & 1 & -8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\quad C_2 \to C_2 + C_1$, $C_4 \to C_4/4$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\quad C_4 \to 3C_4 - C_2$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\quad C_2 \to \frac{C_2}{3}$, $C_4 \to \frac{C_4}{-6}$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\quad C_4 \to C_4 - C_3$.

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix}$$

The rank of A is 3.

5. $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

let, $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ $R_2 \leftrightarrow R_1$

$$\sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix} \quad \begin{matrix} R_3 \to R_3 - 3R_1 \\ R_4 \to R_4 - R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \end{bmatrix} \quad \begin{matrix} R_3 \to R_3 - R_2 \\ R_4 \to R_4 - R_2 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad C_4 \to C_4 - C_3$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad C_4 \to C_4 - 2C_2$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad C_3 \to C_3 - C_1$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad C_3 \to C_3 + 3C_2$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$$

$\therefore$ The rank of A is 2

6. $\begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$

let, $A = \begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 4R_1$
$R_4 \to R_4 - 4R_1$

$\sim \begin{bmatrix} 2 & 1 & 3 & 5 \\ 0 & 0 & -5 & -7 \\ 0 & 0 & -5 & -7 \\ 0 & 0 & -15 & -21 \end{bmatrix}$

$R_3 \to R_3 - R_2$
$R_4 \to R_4 - 3R_2$

$\sim \begin{bmatrix} 2 & 1 & 3 & 5 \\ 0 & 0 & -5 & -7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$C_3 \to C_3 - 3C_2$
$C_4 \to C_4 - 5C_2$

$\sim \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 0 & -5 & -7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$C_1 \to C_1/2$
$C_3 \to C_3/-5$
$C_4 \to C_4/-7$

$\sim \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$C_4 \to C_4 - C_3$
$C_2 \to C_2 - C_1$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $C_3 \leftrightarrow C_2$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$

The rank of $A = 2$

7. $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$

let $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$ $\quad R_2 \to R_2 - 2R_1$

$\sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$ $\quad R_3 \to R_3 - 3R_1$, $R_4 \to R_4 - 6R_1$

$R_4 \to R_4 - R_3$

$\sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$ $\quad R_3 \leftrightarrow R_2$

$\sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & -3 & 2 \end{bmatrix}$

$R_4 \to R_4 - R_3$

$\sim \begin{bmatrix} 1 & 2 & +3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$C_3 \to 2C_3 - 3C_2$

$C_4 \to$

$\sim \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & -4 & 4 & 3 \\ 0 & 0 & -6 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\quad C_4 \to 4C_4 - 3C_3$

$\sim \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & -4 & 4 & 0 \\ 0 & 0 & -6 & 26 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$C_4 \to C_4 -$

$C_2 \to C_2 - 2C_1$, $\dfrac{C_4}{26}$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -4 & 4 & 0 \\ 0 & 0 & -6 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\quad C_3 \to C_3 + C_2$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 \\ 0 & 0 & -6 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\quad C_4 \to 6C_4 + C_3$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 \\ 0 & 0 & -6 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\quad C_2 \to C_2/-4$, $C_3 \to C_3/-6$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix}$ $\therefore$ the rank of $A = 3$.

8. $\begin{bmatrix} 1 & 2 & 2 & 4 \\ 2 & 3 & 4 & 6 \\ 3 & 5 & 6 & 10 \\ -1 & 1 & -2 & -2 \end{bmatrix}$

let, $A = \begin{bmatrix} 1 & 2 & 2 & 4 \\ 2 & 3 & 4 & 6 \\ 3 & 5 & 6 & 10 \\ -1 & 1 & -2 & -2 \end{bmatrix}$

$R_2 \to R_2 - 2R_1, \quad R_3 \to R_3 - 3R_1, \quad R_4 \to R_4 + R_1$

$$\sim \begin{bmatrix} 1 & 2 & 2 & 4 \\ 0 & -1 & 0 & -2 \\ 0 & -1 & 0 & -2 \\ 0 & 3 & 4 & 2 \end{bmatrix}$$

$R_3 \to R_3 - R_2, \quad R_4 \to R_4 + 3R_2$

$$\sim \begin{bmatrix} 1 & 2 & 2 & 4 \\ 0 & -1 & 0 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & -4 \end{bmatrix}$$

$R_4 \leftrightarrow R_3$

$$\sim \begin{bmatrix} 1 & 2 & 2 & 4 \\ 0 & -1 & 0 & -2 \\ 0 & 0 & 4 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_3 \to C_3 - C_2, \quad C_4 \to C_4 - 2C_2$

$$\sim \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 4 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_2 \to C_2 - 2C_1$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 4 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_3 \to C_3 + C_2$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 4 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_3 \to \frac{C_3}{-4}$, $C_4 \to \frac{C_4}{-4}$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_2 \to \frac{C_2}{-1}$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_4 \to C_4 - C_3$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix}$$

rank of $A = 3$.

Rank - PAQ form:- let A is an $m \times n$ matrix having rank r then their exist two non-singular matrix P and Q such that $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} = PAQ$

The given matrix "A" can be expressed as $A = I_m A I_n$ using row and column transformation reduces this matrix equation into the form $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} = PAQ$

Every row matrix transformation is applied to LHS and pre-matrix of RHS and every column transformation is applied to LHS and post matrix of RHS.

problems:-

1. Find two non-singular matrix P and Q such that PAQ is in normal form for the following matrix. Hence find the rank is

$$\begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$$

let, $A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$

$$A = I_3 A I_3$$

$$\begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & 1 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ 0 & -1 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_3 \to 3R_3 - 2R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & 1 \\ 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ -2 & 1 & 3 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ -2 & 1 & 3 \end{bmatrix} A \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_3 \to 3C_3 + C_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -6 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ -2 & 1 & 3 \end{bmatrix} A \begin{bmatrix} 1 & -1 & -4 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

$C_2 \to \dfrac{C_2}{-3}$, $C_3 \to \dfrac{C_3}{-6}$.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ -2 & 1 & 3 \end{bmatrix} A \begin{bmatrix} 1 & 1/3 & 2/3 \\ 0 & -1/3 & -1/6 \\ 0 & 0 & -1/2 \end{bmatrix}$$

$$I_3 = PAQ$$

where, $P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ -2 & 1 & 3 \end{bmatrix}$, $Q = \begin{bmatrix} 1 & 1/3 & 2/3 \\ 0 & -1/3 & -1/6 \\ 0 & 0 & -1/6 \end{bmatrix}$ are non-singular matrix.

rank of $A = 3$.

2. $\begin{bmatrix} 1 & -1 & -1 & 2 \\ 4 & 2 & 2 & -1 \\ 2 & 2 & 0 & -2 \end{bmatrix}$

let $A = \begin{bmatrix} 1 & -1 & -1 & 2 \\ 4 & 2 & 2 & -1 \\ 2 & 2 & 0 & -2 \end{bmatrix}$

$$A = I_3 A I_4$$

$$\begin{bmatrix} 1 & -1 & -1 & 2 \\ 4 & 2 & 2 & -1 \\ 2 & 2 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to R_2 - 4R_1$, $R_3 \to R_3 - 2R_1$

$$\begin{bmatrix} 1 & -1 & -1 & 2 \\ 0 & 6 & 6 & -9 \\ 0 & 4 & 2 & -6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_3 \to 3R_3 - 2R_2$

$$\begin{bmatrix} 1 & -1 & -1 & 2 \\ 0 & 6 & 6 & -9 \\ 0 & 0 & -6 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 2 & -2 & 3 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$C_2 \to C_2 + C_1$, $C_3 \to C_3 + C_1$, $C_4 \to C_4 - 2C_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 6 & 6 & -9 \\ 0 & 0 & -6 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 2 & -2 & 3 \end{bmatrix} A \begin{bmatrix} 1 & 1 & 1 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$C_3 \to C_3 - C_2$, $C_4 \to 2C_4 + 3C_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & -6 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 2 & -2 & 3 \end{bmatrix} A \begin{bmatrix} 1 & 1 & 0 & -1 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

$C_2 \to \frac{C_2}{6}$, $C_3 \to \frac{C_3}{-6}$.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 2 & -2 & 3 \end{bmatrix} A \begin{bmatrix} 1 & 1/6 & 0 & -1 \\ 0 & 1/6 & 1/6 & 3 \\ 0 & 0 & -1/6 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} I_3 & 0 \end{bmatrix} = PAQ$$

where, $P = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 2 & -2 & 3 \end{bmatrix}$, $Q = \begin{bmatrix} 1 & 1/6 & 0 & -1 \\ 0 & 1/6 & 1/6 & 3 \\ 0 & 0 & -1/6 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$.

rank of $A = 3$.

3) $\begin{bmatrix} 1 & 3 & 6 & -1 \\ 1 & 4 & 5 & 1 \\ 1 & 5 & 4 & 3 \end{bmatrix}$

let, $A = \begin{bmatrix} 1 & 3 & 6 & -1 \\ 1 & 4 & 5 & 1 \\ 1 & 5 & 4 & 3 \end{bmatrix}$

$A = I_3 A I_4$

$$\begin{bmatrix} 1 & 3 & 6 & -1 \\ 1 & 4 & 5 & 1 \\ 1 & 5 & 4 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 3 & 6 & -1 \\ 0 & 1 & -1 & 2 \\ 0 & 2 & -2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$$\begin{bmatrix} 1 & 3 & 6 & -1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$C_2 \to C_2 - 3C_1$, $C_3 \to C_3 - 6C_1$, $C_4 \to C_4 + C_1$

$$\begin{bmatrix}1&0&0&0\\0&1&-10&9\\0&0&0&0\end{bmatrix}=\begin{bmatrix}1&0&0\\-1&1&0\\1&-2&1\end{bmatrix}A\begin{bmatrix}1&-3&-6&0\\0&1&0&0\\0&0&1&0\\0&0&0&0\end{bmatrix}.$$

$C_3 \to C_3+C_1$, $C_4 \to C_4-2C_2$

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\end{bmatrix}=\begin{bmatrix}1&0&0\\-1&1&0\\1&-2&1\end{bmatrix}A\begin{bmatrix}1&-3&-9&-7\\0&1&1&-2\\0&0&-1&0\\0&0&0&1\end{bmatrix}.$$

$$\begin{bmatrix}I_2&0\\0&0\end{bmatrix}=PAQ$$

where,

$$P=\begin{bmatrix}1&0&0\\-1&1&0\\1&-2&1\end{bmatrix},\ Q=\begin{bmatrix}1&-3&-9&-7\\0&1&1&-2\\0&0&-1&0\\0&0&0&1\end{bmatrix}$$

$\therefore$ rank of $A=2$.

4. $\begin{bmatrix}3&2&-1&5\\5&1&4&-2\\1&-4&11&-19\end{bmatrix}$.

A can be written as $A=I_3AI_4$

$$\begin{bmatrix}3&2&-1&5\\5&1&4&-2\\1&-4&11&-19\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}A\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

$R_1 \leftrightarrow R_3$

$$\begin{bmatrix}1&-4&11&-19\\5&1&4&-2\\3&2&-1&5\end{bmatrix}=\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}A\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

$R_2 \to R_2-5R_1$, $R_3 \to R_3-3R_1$

$$\begin{bmatrix}1&-4&11&-19\\0&21&-51&93\\0&14&-34&62\end{bmatrix}=\begin{bmatrix}0&0&1\\0&1&-5\\1&0&-3\end{bmatrix}A\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

$R_3 \to 3R_3-2R_2$

$$\begin{bmatrix}1&-4&11&-19\\0&21&-51&93\\0&0&0&0\end{bmatrix}=\begin{bmatrix}0&0&1\\0&1&-5\\3&-2&1\end{bmatrix}A\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

$C_2 \to C_2+4C_1$, $C_3 \to C_3-11C_1$, $C_4 \to C_4+19C_1$

$$\begin{bmatrix}1&0&0&0\\0&21&-51&93\\0&0&0&0\end{bmatrix}=\begin{bmatrix}0&0&1\\0&1&-5\\3&-2&1\end{bmatrix}A\begin{bmatrix}1&4&-11&19\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

$C_2 \to \frac{C_2}{21}, \quad C_3 \to \frac{C_3}{51}, \quad C_4 \to \frac{C_4}{93}.$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -5 \\ 3 & -2 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 4/21 & 11/51 & 19/93 \\ 0 & 1/21 & 0 & 0 \\ 0 & 0 & -1/51 & 0 \\ 0 & 0 & 0 & 1/93 \end{bmatrix}$$

$C_3 \to C_3 - C_2, \quad C_4 \to C_4 - C_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -5 \\ 3 & -2 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 4/21 & 3/119 & 3/217 \\ 0 & 1/21 & -1/21 & -1/21 \\ 0 & 0 & -1/51 & 0 \\ 0 & 0 & 0 & 1/93 \end{bmatrix}$$

$$\begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix} = PAQ$$

Here, $P = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -5 \\ 3 & -2 & 1 \end{bmatrix}, \quad Q = \begin{bmatrix} 1 & 4/21 & 3/119 & 3/217 \\ 0 & 1/21 & -1/21 & -1/21 \\ 0 & 0 & -1/51 & 0 \\ 0 & 0 & 0 & 1/93 \end{bmatrix}$

Rank of $A = 2$.

Inverse of a matrix using row transformation:-

Let A is a non-singular matrix of order n. A can be written as $A = I_n A$ using row transformation and reduce this matrix eqn into the form $I_n = BA$ then by the definition of inverse of matrix we have $B = A^{-1}$

1. $\begin{bmatrix} 1 & 0 & 1 \\ -2 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$

$\to$ A can be written as $A = I_3 A$

$$\begin{bmatrix} 1 & 0 & 1 \\ -2 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_2 \to R_2 + 2R_1$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix} A$$

$R_1 \to 3R_1 - R_3$, $R_2 \to 3R_2 - 2R_3$

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -1 \\ 2 & 1 & -2 \\ 2 & 1 & 1 \end{bmatrix} A$$

$R_1 \to \frac{R_1}{3}$, $R_2 \to \frac{R_2}{3}$, $R_3 \to \frac{R_3}{3}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/3 & -1/3 & -1/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & 1/3 & 1/3 \end{bmatrix} A$$

$I_3 = BA$

$$A^{-1} = B = \frac{1}{3}\begin{bmatrix} 1 & -1 & -1 \\ 2 & 1 & -2 \\ 2 & 1 & 1 \end{bmatrix}$$

2. $$\begin{bmatrix} 0 & 1 & 2 & 2 \\ 1 & 1 & 2 & 3 \\ 2 & 2 & 2 & 3 \\ 2 & 3 & 3 & 3 \end{bmatrix}$$

let $A = \begin{bmatrix} 0 & 1 & 2 & 2 \\ 1 & 1 & 2 & 3 \\ 2 & 2 & 2 & 3 \\ 2 & 3 & 3 & 3 \end{bmatrix}$

A can be written as $A = I_4 A$

$$\begin{bmatrix} 0 & 1 & 2 & 2 \\ 1 & 1 & 2 & 3 \\ 2 & 2 & 2 & 3 \\ 2 & 3 & 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A$$

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 1 & 2 & 2 \\ 2 & 2 & 2 & 3 \\ 2 & 3 & 3 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A$$

$R_3 \to R_3 - 2R_1$, $R_4 \to R_4 - 2R_1$

$$\begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & -2 & -3 \\ 0 & 1 & -1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{bmatrix} A$$

$R_4 \to R_4 - R_2$

$$\begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & -2 & -3 \\ 0 & 0 & -3 & -5 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ -1 & -2 & 0 & 1 \end{bmatrix} A$$

$R_4 \to 2R_4 - 3R_3 \,,\, R_2 \to R_2 + R_3$

$$\begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -2 & -3 \\ 0 & 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & -2 & 1 & 0 \\ -2 & 2 & -3 & 2 \end{bmatrix} A$$

$R_1 \to R_1 - R_2$

$$\begin{bmatrix} 1 & 0 & 2 & 4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -2 & -3 \\ 0 & 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 3 & -1 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & -2 & 1 & 0 \\ -2 & 2 & -3 & 2 \end{bmatrix} A$$

$R_1 \to R_1 + R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -2 & -3 \\ 0 & 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & -2 & 1 & 0 \\ -2 & 2 & -3 & 2 \end{bmatrix} A$$

$R_1 \to R_1 + R_4 \,,\, R_2 \to R_2 - R_4 \,,\, R_3 \to R_3 - 3R_4$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -3 & 3 & -3 & 2 \\ 3 & -4 & 4 & -2 \\ 6 & -8 & 10 & -6 \\ -2 & 2 & -3 & 2 \end{bmatrix} A$$

$R_3 \to \frac{R_3}{-2} \,,\, R_4 \to \frac{R_4}{-1}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 3 & -3 & 2 \\ 3 & -4 & 4 & -2 \\ -3 & 4 & -5 & 3 \\ 2 & -2 & 3 & -2 \end{bmatrix} A$$

$I_4 = BA$

Inverse of A is

$$B = \begin{bmatrix} -3 & 3 & -3 & 2 \\ 3 & -4 & 4 & -2 \\ -3 & 4 & -5 & 3 \\ 2 & -2 & 3 & -2 \end{bmatrix}$$

System of Linear Equations :- Condition ...

'm' equations in 'n' variables.

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m$$

These equations can be written as the matrix equation, $Ax = B$

where, $A = [a_{ij}]$ $x = (x_1 x_2 \cdots x_n)^T$ $B = (b_1 b_2 \cdots b_m)^T$

If $B \neq 0$ then it is called Non-Homogeneous System of linear equations.

If $B = 0$ then it is called Homogeneous system of equations.

An ordered n-tuple $(x_1, x_2, \cdots x_n)$ is called a solution of the system of equations. If it's satisfies all the equations.

Non-Homogeneous system of Equations:-

A Non-Homogenous system of equations may have

1. have unique solution may have
2. No solution.
3. Infinite no of solutions.

Gauss elimination method:- For the given system of linear equation $Ax = B$ then augmented matrix is $[A \quad B]$. Reduce the augmented matrix into Echelon form by backward substitution, we can find the solution.

Solve the following system of equations using Gauss elimination method.

1/ $x+2y+3z=1$, $2x+3y+8z=2$, $x+y+z=3$.

→ Augmented matrix of the system of equations is

$$[A \quad B] = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 3 & 8 & 2 \\ 1 & 1 & 1 & 3 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - R_1$

$$= \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & -1 & 2 & 0 \\ 0 & -1 & -2 & 2 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$= \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & -4 & 2 \end{bmatrix}$$

This is in Echelon form. By Backward substitution

$-4z = 2$, $-y+2z=0$, $x+2y+3z=1$.

$z = -\frac{1}{2}$; $-y+2(\frac{-1}{2})=0$; $x+2(-1)+3(\frac{-1}{2})=1$.

$-y-1=0$; $x-2-\frac{3}{2}=1$.

$-y=1$; $x=1+2+\frac{3}{2}$

$y=-1$; $x = 3+\frac{3}{2} = \frac{6+3}{2}$

$\therefore x = \frac{9}{2}$

$\therefore x = \frac{9}{2}$, $y=-1$, $z=-\frac{1}{2}$.

2/ $3x+y+2z=3$, $2x-3y-z=-3$, $x+2y+z=4$

→ Augmented matrix.

$$[A \quad B] = \begin{bmatrix} 3 & 1 & 2 & 3 \\ 2 & -3 & -1 & -3 \\ 1 & 2 & 1 & 4 \end{bmatrix}$$

$R_2 \to 3R_2 - R_1$

$$\sim \begin{bmatrix} 3 & 1 & 2 & 3 \\ 0 & -11 & -7 & -15 \\ 0 & 5 & 1 & 9 \end{bmatrix} \quad R_3 \to 11R_3 + 5R_2$$

$$\sim \begin{bmatrix} 3 & 1 & 2 & 3 \\ 0 & -11 & -7 & -15 \\ 0 & 0 & -24 & 24 \end{bmatrix}$$

This is in the form of Echelon.

By Backward sub.

$$3x + y + 2z = 3$$

$$-11y - 7z = -15$$

$$-24z = 24$$

$z = -1$; $-11y + 7 = -15$; $3x + \cancel{2} - \cancel{2} = 3$

$-11y = -22$; $x = 1$.

$y = 2$

$\therefore x = 1,\ y = 2,\ z = -1$.

3. $2x_1 + x_2 + 2x_3 + x_4 = 6$, $6x_1 - 6x_2 + 6x_3 + 12x_4 = 36$

$4x_1 + 3x_2 + 3x_3 - 3x_4 = -1$, $2x_1 + 2x_2 - x_3 + x_4 = 10$.

Augmented matrix,

$$[A \quad B] = \begin{bmatrix} 2 & 1 & 2 & 1 & 6 \\ 6 & -6 & 6 & 12 & 36 \\ 4 & 3 & 3 & -3 & -1 \\ 2 & 2 & -1 & 1 & 10 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - 2R_1$

$R_4 \to R_4 - R_1$

$$\sim \begin{bmatrix} 2 & 1 & 2 & 1 & 6 \\ 0 & -9 & 0 & 9 & 18 \\ 0 & 1 & -1 & -5 & -13 \\ 0 & 1 & -3 & 0 & 4 \end{bmatrix}$$

$R_3 \to 9R_3 + R_2$, [illegible]

$$\sim \left[\begin{array}{ccccc} 2 & 1 & 2 & 1 & 6 \\ 0 & -9 & 0 & 9 & 18 \\ 0 & 0 & -9 & -36 & -99 \\ 0 & 0 & -27 & 9 & 54 \end{array}\right]$$

$R_4 \to R_4 - 3R_3$

$$\sim \left[\begin{array}{ccccc} 2 & 1 & 2 & 1 & 6 \\ 0 & -9 & 0 & 9 & 18 \\ 0 & 0 & -9 & -36 & -99 \\ 0 & 0 & 0 & 117 & 351 \end{array}\right]$$

This is in echelon form, by backward sub

$117x_4 = 351 \Rightarrow x_4 = \frac{351}{117}$

$\boxed{x_4 = 3}$

$-9x_3 - 36x_4 = -99$

$-9x_3 - 36(3) = -99$

$-9x_3 = -99 + 108$

$-9x_3 = 9 \Rightarrow \boxed{x_3 = -1}$

$-9x_2 + 9x_4 = 18$

$-9x_2 + 9(3) = 18$

$-9x_2 = 18 - 27$

$= -9$

$\boxed{x_2 = 1}$

$2x_1 + x_2 + 2x_3 + x_4 = 6$

$\Rightarrow 2x_1 + 1 + (-2) + 3 = 6$

$2x_1 + 2 = 6$

$2x_1 = 4 \Rightarrow \boxed{x_1 = 2}$

$\therefore x_1 = 2, \; x_2 = 1, \; x_3 = -1, \; x_4 = 3$

4) $5x_1 + x_2 + x_3 + x_4 = 4$

$x_1 + 7x_2 + x_3 + x_4 = 12$

$x_1 + x_2 + 6x_3 + x_4 = -5$

$x_1 + x_2 + x_3 + 4x_4 = -6$

A) Augmented matrix of the system of eq is

$$[A \quad B] : \begin{bmatrix} 5 & 1 & 1 & 1 & 4 \\ 1 & 7 & 1 & 1 & 12 \\ 1 & 1 & 6 & 1 & -5 \\ 1 & 1 & 1 & 4 & -6 \end{bmatrix}$$

$R_1 \leftrightarrow R_4$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 4 & -6 \\ 1 & 7 & 1 & 1 & 12 \\ 1 & 1 & 6 & 1 & -5 \\ 5 & 1 & 1 & 1 & 4 \end{bmatrix}$$

$R_2 \to R_2 - R_1, \quad R_3 \to R_3 - R_1, \quad R_4 \to R_4 - 5R_1$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 4 & -6 \\ 0 & 6 & 0 & -3 & 18 \\ 0 & 0 & 5 & -3 & 1 \\ 0 & -4 & -4 & -19 & 34 \end{bmatrix}$$

$R_4 \to 3R_4 + 2R_2$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 4 & -6 \\ 0 & 6 & 0 & -3 & 18 \\ 0 & 0 & 5 & -3 & 1 \\ 0 & 0 & -12 & -63 & 138 \end{bmatrix}$$

$R_4 \to 5R_4 + 12R_3$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 4 & -6 \\ 0 & 6 & 0 & -3 & 18 \\ 0 & 0 & 5 & -3 & 1 \\ 0 & 0 & 0 & -351 & 702 \end{bmatrix}$$

This is in echelon form, by back word sub

$-351 x_4 = 702 \Rightarrow x_4 = \frac{702}{-351} = -2 \Rightarrow \boxed{x_4 = -2}$

$\Rightarrow 5x_3 - 3x_4 = 1$

$\Rightarrow 5x_3 - 3(-2) = 1 \Rightarrow 5x_3 + 6 = 1 \Rightarrow \boxed{x_3 = -1}$

$\Rightarrow 6x_2 - 3x_4 = 18$

$6x_2 - 3(-2) = 18 \Rightarrow 6x_2 + 6 = 18$

$\boxed{x_2 = 2}$

$\Rightarrow x_1 + x_2 + x_3 + 4x_4 = -6$

$x_1 + 2 - 1 + 4(-2) = -6 \Rightarrow x_1 + 1 - 8 = -6$

$\Rightarrow x_1 - 7 = -6$

$x_1 = -6 + 7$

$\boxed{x_1 = 1}$

$\therefore x_1 = 1,\quad x_2 = 2,\quad x_3 = -1,\quad x_4 = -2$

$\Rightarrow$ Gauss - Jordan method :- consider a linear system of three eq in three variables reduce the augmented matrix in to the form $\begin{bmatrix} 1 & 0 & 0 & \alpha \\ 0 & 1 & 0 & \beta \\ 0 & 0 & 1 & \gamma \end{bmatrix}$ using elementary row transformations than the solution is given by $x = \alpha,\ y = \beta,\ z = \gamma$

solve problems:-

(i) $2x + y + z = 10$

$3x + 2y + 3z = 18$

$x + 4y + 9z = 16$

Augmented Matrix of the system of eq is

$$[A \quad B] = \begin{bmatrix} 2 & 1 & 1 & 10 \\ 3 & 2 & 3 & 18 \\ 1 & 4 & 9 & 16 \end{bmatrix}$$

$R_2 \to 2R_2 - 3R_1,\quad R_3 \to 2R_3 - R_1$

$$\sim \begin{bmatrix} 2 & 1 & 1 & 10 \\ 0 & 1 & 3 & 6 \\ 0 & 7 & 17 & 22 \end{bmatrix}$$

$R_3 \to R_3 - 7R_2,\quad R_1 \to R_1 - R_2$

$$\sim \begin{bmatrix} 2 & 0 & -2 & 4 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & -4 & -20 \end{bmatrix}$$

$R_1 \to R_1/2,\quad R_3 \to R_3/-4$

$$\sim \begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$R_1 \to R_1 + R_3, \quad R_2 \to R_2 - 3R_3$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

Solution is $x = 7, \; y = -9, \; z = 5$

(2) $$\begin{cases} 10x + y + z = 12 \\ 2x + 10y + z = 13 \\ x + y + 5z = 7 \end{cases}$$

Augmented Matrix of system of eq is

$$[A \;\; B] = \begin{bmatrix} 10 & 1 & 1 & 12 \\ 2 & 10 & 1 & 13 \\ 1 & 1 & 5 & 7 \end{bmatrix}$$

$R_1 \leftrightarrow R_3$

$$\sim \begin{bmatrix} 1 & 1 & 5 & 7 \\ 2 & 10 & 1 & 13 \\ 10 & 1 & 1 & 12 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1, \quad R_3 \to R_3 - 10R_1$

$$\sim \begin{bmatrix} 1 & 1 & 5 & 7 \\ 0 & 8 & -9 & -1 \\ 0 & -9 & -49 & -58 \end{bmatrix}$$

$R_1 \to 8R_1 - R_2, \quad R_3 \to 8R_3 + 9R_2$

$$\sim \begin{bmatrix} 8 & 0 & 49 & 57 \\ 0 & 8 & -9 & -1 \\ 0 & 0 & -473 & -473 \end{bmatrix}$$

$R_3 \to \dfrac{R_3}{-473}$

$$\sim \begin{bmatrix} 8 & 0 & 49 & 57 \\ 0 & 8 & -9 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$R_1 \to R_1 - 49R_3, \quad R_2 \to R_2 + 9R_3$

$$\sim \begin{bmatrix} 8 & 0 & 0 & 8 \\ 0 & 8 & 0 & 8 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$R_1 \to R_1/8, \quad R_2 \to R_2/8$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Solution is $x = 1, \ y = 1, \ z = 1$

(3) $x + y + z - w = 2$

$7x + y + 3z + w = 12$

$8x - y + z - 3w = 5$

$10x + 5y + 3z + 2w = 20$

Augmented Matrix

$$[A \ \ B] = \begin{bmatrix} 1 & 1 & 1 & -1 & 2 \\ 7 & 1 & 3 & 1 & 12 \\ 8 & -1 & 1 & -3 & 5 \\ 10 & 5 & 3 & 2 & 20 \end{bmatrix}$$

$R_2 \to R_2 - 7R_1, \quad R_3 \to R_3 - 8R_1, \quad R_4 \to R_4 - 10R_1$

$$\sim \begin{bmatrix} 1 & 1 & 1 & -1 & 2 \\ 0 & -6 & -4 & 8 & -2 \\ 0 & -9 & -7 & 5 & -11 \\ 0 & -5 & -7 & 12 & 0 \end{bmatrix}$$

$R_2 \to \dfrac{R_2}{-2}, \quad R_3 \to \dfrac{R_3}{-1}, \quad R_4 \to \dfrac{R_4}{-1}$

$$\sim \begin{bmatrix} 1 & 1 & 1 & -1 & 2 \\ 0 & 3 & 2 & -4 & 1 \\ 0 & 9 & 7 & -5 & 11 \\ 0 & 5 & 7 & -12 & 0 \end{bmatrix}$$

$R_1 \to 3R_1 - R_2$, $R_3 \to R_3 - 3R_2$, $R_4 \to 3R_4 - 5R_2$

$$\sim \begin{bmatrix} 3 & 0 & 1 & 1 & 5 \\ 0 & 3 & 2 & -4 & 1 \\ 0 & 0 & 1 & 7 & 8 \\ 0 & 0 & 11 & -16 & -5 \end{bmatrix}$$

$R_1 \to R_1 - R_3$, $R_2 \to R_2 - 2R_3$, $R_4 \to R_4 - 11R_3$

$$\sim \begin{bmatrix} 3 & 0 & 0 & -6 & -3 \\ 0 & 3 & 0 & -18 & -15 \\ 0 & 0 & 1 & 7 & 8 \\ 0 & 0 & 0 & -93 & -93 \end{bmatrix}$$

$R_1 \to \frac{R_1}{3}$, $R_2 \to \frac{R_2}{3}$, $R_4 \to \frac{R_4}{-93}$

$$\sim \begin{bmatrix} 1 & 0 & 0 & -2 & -1 \\ 0 & 1 & 0 & -6 & -5 \\ 0 & 0 & 1 & 7 & 8 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$R_1 \to R_1 + 2R_4$, $R_2 \to R_2 + 6R_4$, $R_3 \to R_3 - 7R_4$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

solution is $x = 1$, $y = 1$, $z = 1$

Gauss - Jacobi Iteration method :-

Consider the system of linear equations in three variables,

$$a_{11}x + a_{12}y + a_{13}z = b_1$$
$$a_{21}x + a_{22}y + a_{23}z = b_2$$
$$a_{31}x + a_{32}y + a_{33}z = b_3$$

are such that the diagonal elements are large compared with other coefficients such that systems of eq^n is called Diagonally dominent system. If it is not diagonally dominent interchang the eq^n to get such system.

The above eq^n can be retum as

$$\left.\begin{aligned} x &= \frac{1}{a_{11}}\left[b_1 - a_{12}y - a_{13}z\right] \\ y &= \frac{1}{a_{22}}\left[b_2 - a_{21}x - a_{23}z\right] \\ z &= \frac{1}{a_{33}}\left[b_3 - a_{31}x - a_{32}y\right]. \end{aligned}\right\} \text{——} ①$$

consider the initial values $x=0, y=0, z=0$.

1st Iteration :-

from ①, $x = \dfrac{b_1}{a_{11}}$, $y = \dfrac{b_2}{a_{22}}$, $z = \dfrac{b_3}{a_{33}}$

2nd Iteration :-

$$x^{(2)} = \frac{1}{a_{11}}\left[b_1 - a_{12}y^{(1)} - a_{13}z^{(1)}\right]$$

$$y^{(2)} = \frac{1}{a_{22}}\left[b_2 - a_{21}x^{(1)} - a_{23}z^{(1)}\right]$$

$$z^{(2)} = \frac{1}{a_{33}}\left[b_3 - a_{31}x^{(1)} - a_{32}y^{(1)}\right].$$

Repeat the process upto two successive eq' Iterations are equal.

Solve the following system of equations using gauss-jacobi Iteration method:-

1. $10x+y-z=11.19$, $x+10y+z=28.08$, $-x+y+10z=35.61$

Clearly, the given system of equations are diagonally dominant.

These eq's can be written as:

$$x=\frac{1}{a_{11}}[b_1-a_{12}y-a_{13}z]$$

$$x=\frac{1}{10}[11.19-y+z]$$

$$y=\frac{1}{a_{22}}[b_2-a_{21}x-a_{23}z]$$

$$y=\frac{1}{10}[28.08-x-z]$$

$$z=\frac{1}{a_{33}}[b_3-a_{31}x-a_{32}y] = \frac{1}{\cancel{35.61}}[$$

$$=\frac{1}{10}[35.61+x-y]$$

1st Iteration:-

$$x=\frac{b_1}{a_{11}}=\frac{11.19}{10}=1.119$$

$$y=\frac{b_2}{a_{22}}=\frac{28.08}{10}=2.808$$

$$z=\frac{b_3}{a_{33}}=\frac{35.61}{10}=3.561.$$

2nd Iteration:-

$$x^{(2)}=\frac{1}{10}[11.19-y^{(1)}+z^{(1)}]$$

$$=\frac{1}{10}[11.19-2.808+3.561]=1.1943$$

$$y^{(2)} = \frac{1}{10}\left[28.08 - x^{(1)} - z^{(1)}\right]$$

$$= \frac{1}{10}\left[28.08 - 1.119 - 3.561\right] = 2.34.$$

$$z^{(2)} = \frac{1}{10}\left[35.61 + x^{(1)} - y^{(1)}\right]$$

$$= \frac{1}{10}\left[35.61 + 1.119 - 2.808\right] = 3.3921.$$

<u>3rd</u> <u>Iteration</u> :-

$$x^{(3)} = \frac{1}{10}\left[11.19 - y^{(2)} + z^{(2)}\right]$$

$$= \frac{1}{10}\left[11.19 - 2.34 + 3.3921\right] = 1.22421.$$

$$y^{(3)} = \frac{1}{10}\left[28.08 - 1.1943 - 3.3921\right] = 2.34936$$

$$z^{(3)} = \frac{1}{10}\left[35.61 + 1.1943 - 2.34\right] = 3.44643.$$

<u>4th</u> <u>Iteration</u> :-

$$x^{(4)} = \frac{1}{10}\left[11.19 - 2.34936 + 3.44643\right] = 1.2287$$

$$y^{(4)} = \frac{1}{10}\left[28.08 - 1.22421 - 3.44643\right] = 2.3409$$

$$z^{(4)} = \frac{1}{10}\left[35.61 + 1.22421 - 2.34936\right] = 3.448$$

From 3rd & 4th Iterations solution is

$$x = 1.2287,\ y = 2.3409,\ z = 3.4484.$$

2. $\underset{②}{x + y + 54z = 110}$, $\underset{③}{6x + 15y + 2z = 72}$, $\underset{①}{27x + 6y - z = 85}$

The given system of equations is not diagonally dominant.

By interchanging the 2nd & 3rd eq^n we get a diagonally dominent system.

$\therefore$ The system of eq^ns become

$27x + 6y - z = 85$, $6x + 15y + 2z = 72$, $x + y + 54z = 110$.

$$x = \frac{1}{a_{11}}\left[b_1 - a_{12}y - a_{13}z\right]$$

$$x = \frac{1}{27}\left[85 - 6y + z\right]$$

$$y = \frac{1}{15}\left[72 - 6x - 2z\right], \quad z = \frac{1}{54}\left[110 - x - y\right].$$

1st Iteration:-

$$x = \frac{b_1}{a_{11}} = \frac{85}{27} = 3.1481$$

$$y = \frac{b_2}{a_{22}} = \frac{72}{15} = 4.8.$$

$$z = \frac{b_3}{a_{33}} = \frac{110}{54} = 2.037$$

2nd Iteration:-

$$x^{(2)} = \frac{1}{27}\left[85 - 6(4.8) + 2.037\right]$$

$$= \frac{1}{27} \times 58.237 = 2.1569.$$

$$y^{(2)} = \frac{1}{15}\left[72 - 6(3.1481) - 2(2.037)\right]$$

$$= 3.26916.$$

$$z^{(2)} = \frac{1}{54}\left[110 - 3.1481 - 4.8\right].$$

$$= 1.8898.$$

3rd Iteration:-

$$x^{(3)} = \frac{1}{27}\left[85 - 6(3.26916) + 1.8898\right] = 2.4916.$$

$$y^{(3)} = \frac{1}{15}\left[72 - 6(2.1569) - 2(1.8898)\right] = 3.6852$$

$$z^{(3)} = \frac{1}{54}\left[110 - 2.1569 - 3.2691\right] = 1.9365$$

4th Iteration:-

$$x^{(4)} = \frac{1}{27}[85 - 6(3.6852) + 1.9365] = 2.4009.$$

$$y^{(4)} = \frac{1}{15}[72 - 6(2.4916) - 2(1.9365)] = 3.5451.$$

$$z^{(4)} = \frac{1}{54}[110 - 2.4916 - 3.6852] = 1.9226.$$

5th Iteration:-

$$x^{(5)} = \frac{1}{27}[85 - 6(3.5451) + 1.9226] = 2.4315$$

$$y^{(5)} = \frac{1}{15}[72 - 6(2.4009) - 2(1.9226)] = 3.5832$$

$$z^{(5)} = \frac{1}{54}[110 - 2.4009 - 3.5451] = 1.9269.$$

6th Iteration:-

$$x^{(6)} = \frac{1}{27}[85 - 6(3.5832) + 1.9269] =$$

From, 4th, 5th Iterations solutions are

$x = 2.4315$, $y = 3.5832$, $z = 1.9269$.

3. $5x - y + z = 10$, $2x + 4y = 12$, $x + y + 5z = -1$. starts with solution $(2, 3, 0)$.

clearly, the given system of eqn are diagonally dominent

These eqns can be writtern as.

$$x = \frac{1}{5}[10 + y - z]$$

$$y = \frac{1}{4}[12 - 2x], \quad z = \frac{1}{5}[-1 - x - y].$$

consider, the initial values $(2, 3, 0)$.

$$x = \frac{1}{5}[10 + 3 - 0] = 2.6$$

$$y = \frac{1}{4}[12 - 2(2)] = 4$$

$$z = \frac{1}{5}[-1 - 2 - 3] = -1.2$$

2nd Iteration:-

$$x^{(2)} = \frac{1}{5}[10+2+1.2] = 2.164.$$

$$y^{(2)} = \frac{1}{4}[12-2(2.6)] = 1.7$$

$$z^{(2)} = \frac{1}{5}[-1-2.6-2] = -1.12$$

3rd Iteration:-

$$x^{(3)} = \frac{1}{5}[10+1.7+1.12] = 2.564$$

$$y^{(3)} = \frac{1}{4}[12-2(2.64)] = 1.68$$

$$z^{(3)} = \frac{1}{5}(-1-2.64-1.7) = -1.068$$

4th Iteration:-

$$x^{(4)} = \frac{1}{5}[10+1.68+1.068] = 2.5496$$

$$y^{(4)} = \frac{1}{4}[12-2(2.564)] = 1.718$$

$$z^{(4)} = \frac{1}{5}[-1-2.564-1.68] = -1.0488.$$

5th Iteration:-

$$x^{(5)} = \frac{1}{5}[10+1.718+1.0488] = 2.5533$$

$$y^{(5)} = \frac{1}{4}[12-2(2.5496)] = 1.7252$$

$$z^{(5)} = \frac{1}{5}[-1-2.5496-1.718] = -1.05352$$

from, 4th & 5th Iterations the solutions are $x = 2.5533$, $y = 1.7252$, $z = -1.05352$.

Gauss - Seidal Iteration method :- Consider the system of linear equations

$$a_{11}x + a_{12}y + a_{13}z = b_1$$
$$a_{21}x + a_{22}y + a_{23}z = b_2$$
$$a_{31}x + a_{32}y + a_{33}z = b_3$$

such that the diagonal elements are large as compared with the other coefficients, such system is called diagonally dominant. The given equations can be writtern as

$$x = \frac{1}{a_{11}}\left[b_1 - a_{12}y - a_{13}z\right].$$

$$y = \frac{1}{a_{22}}\left[b_2 - a_{21}x - a_{23}z\right].$$

$$z = \frac{1}{a_{33}}\left[b_3 - a_{31}x - a_{32}y\right].$$

let the initial values $x=0, y=0, z=0$

1st Iteration:-

$$x^{(1)} = \frac{1}{a_{11}}\left[b_1 - 0 - 0\right]$$

$$y^{(1)} = \frac{1}{a_{22}}\left[b_2 - a_{21}x^{(1)} - 0\right]$$

$$z^{(1)} = \frac{1}{a_{33}}\left[b_3 - a_{31}x^{(1)} - a_{32}y^{(1)}\right].$$

2nd Iteration:-

$$x^{(2)} = \frac{1}{a_{11}}\left[b_1 - a_{12}y^{(1)} - a_{13}z^{(1)}\right]$$

$$y^{(2)} = \frac{1}{a_{22}}\left[b_2 - a_{21}x^{(2)} - a_{23}z^{(1)}\right].$$

$$z^{(2)} = \frac{1}{a_{33}}\left[b_3 - a_{31}x^{(2)} - a_{32}y^{(2)}\right].$$

Repeat the process upto two successive Iterations are equal.

Solve the following system of equations using Gauss - Seidal Iteration method.

1. $25x+2y+2z=69$, $2x+10y+z=63$, $x+y+z=43$

clearly the given system of equations is diagonally dominant.

these eq^{n}s can be written as

$$x = \frac{1}{25}[69-2y-2z]$$

$$y = \frac{1}{10}[63-2x-z]$$

$$z = [43-x-y].$$

let the initial values $x=0, y=0, z=0$.

1st Iteration:-

$$x^{(1)} = \frac{1}{25}[69] = 2.76$$

$$y^{(1)} = \frac{1}{10}[63-2(2.76)-0] = 5.749.$$

$$z^{(1)} = 43-2.76-5.749 = 34.491.$$

2nd Iteration:-

$$x^{(2)} = \frac{1}{25}[69-2(5.749)-2(34.491)] = -0.459$$

$$y^{(2)} = \frac{1}{10}[63-2(-0.4592)-34.491] = 2.9427$$

$$z^{(2)} = 43-(-0.4592)-2.9427 = 40.5165.$$

3rd Iteration:-

$$x^{(3)} = \frac{1}{25}[69-2(2.9427)-2(40.5165)] = -0.716$$

$$y^{(3)} = \frac{1}{10}\left[63 - 2(-0.7167) - 40.5165\right] = 2.3916.$$

$$z^{(3)} = 43 - (-0.7167) - (2.3916) = 41.3251.$$

4th Iteration:-

$$x^{(4)} = \frac{1}{25}\left[69 - 2(2.3916) - 2(41.3251)\right] = -0.7373.$$

$$y^{(4)} = \frac{1}{10}\left[63 - 2(-0.7373) - 41.3251\right] = 2.3149$$

$$z^{(4)} = 43 - (-0.7373) - 2.3149 = 41.4224.$$

5th Iteration:-

$$x^{(5)} = \frac{1}{25}\left[69 - 2(2.3149) - 2(41.4224)\right] = -0.7389$$

$$y^{(5)} = \frac{1}{10}\left[63 - 2(-0.7389) - 41.4224\right] = 2.3055$$

$$z^{(5)} = 43 - (-0.7389) - 2.3055 = 41.4334.$$

6th Iteration:-

$$x^{(6)} = \frac{1}{25}\left[69 - 2(2.3055) - 2(41.4334)\right].$$

$$= -0.7391.$$

$$y^{(6)} = \frac{1}{10}\left[63 - 2(-0.7391) - 41.4334\right] = 2.30448$$

$$z^{(6)} = 43 - (-0.7391) - 2.3044 = 41.4347$$

$\therefore$ From 5th & 6th Iterations, the solutions are, $x = -0.7391$, $y = 2.3044$, $z = 41.4347$.

2. $8x_1 - 3x_2 + 2x_3 = 20$,

$4x_1 + 11x_2 - x_3 = 33$,

$6x_1 + 3x_2 + 12x_3 = 36$.

Clearly, the given equations are diagonally dominant

The equations can be written as,

$$x_1 = \frac{1}{8}\left[20 + 3x_2 - 2x_3\right]$$

$$x_2 = \frac{1}{11}\left[33 - 4x_1 + x_3\right]$$

$$x_3 = \frac{1}{12}\left[36 - 6x_1 - 3x_2\right].$$

let the initial values $x_1 = 0, y = 0, z = 0$.
$x_1 = 0, x_2 = 0, x_3 = 0$

$$x_1^{(1)} = \frac{1}{8}\left[20 + 0 + 0\right] = \frac{20}{8} = 2.5$$

$$x_2^{(1)} = \frac{1}{11}\left[33 - 4(2.5) + 0\right] = 2.0909$$

$$x_3^{(1)} = \frac{1}{12}\left[36 - 6(2.5) - 3(2.0909)\right] = 1.2272$$

2nd Iteration :-

$$x_1^{(2)} = \frac{1}{8}\left[20 + 3(2.0909) - 2(1.2272)\right] = 2.9772$$

$$x_2^{(2)} = \frac{1}{11}\left[33 - 4(2.9772) + 1.2272\right] = 2.0289$$

$$x_3^{(2)} = \frac{1}{12}\left[36 - 6(2.9772) - 3(2.0289)\right] = 1.0041.$$

3rd Iteration :-

$$x_1^{(3)} = \frac{1}{8}\left[20 + 3(2.0289) - 2(1.0041)\right] = 3.0098$$

$$x_2^{(3)} = \frac{1}{11}\left[33 - 4(3.0098) + 1.0041\right] = 1.9968$$

$$x_3^{(3)} = \frac{1}{12}\left[36 - 6(3.0098) - 3(1.9968)\right] = 0.9959$$

4th Iteration :-

$$x_1^{(4)} = \frac{1}{8}\left[20 + 3(1.9968) - 2(0.9959)\right] = 2.9998$$

$$x_2^{(4)} = \frac{1}{11}\left[33 - 4(2.9998) + 0.9959\right] = 1.9997$$

$$x_3^{(4)} = \frac{1}{12}\left[36 - 6(2.9998) - 3(1.9997)\right] = 1.0001$$

5th Iteration:-

$$x_1^{(5)} = \frac{1}{8}\left[20+3(1.9997)-2(1.0001)\right] = 2.9998.$$

$$x_2^{(5)} = \frac{1}{11}\left[33-4(2.9998)+1.0001\right] = 2.0000.$$

$$x_3^{(5)} = \frac{1}{12}\left[36-6\times(2.9998)-3(2.0000)\right] = 1.0001.$$

6th Iteration:-

$$x_1^{(6)} = \frac{1}{8}\left[20+3(2.0000)-2(1.0001)\right] = 2.9999.$$

$$x_2^{(6)} = \frac{1}{11}\left[33-4(2.9999)+1.0001\right] = 2.0000.$$

$$x_3^{(6)} = \frac{1}{12}\left[36-6\times(2.9999)-3(2.0000)\right] = 1.0000$$

From 5th & 6th Iterations, the solutions are

$x_1 = 2.9999$, $x_2 = 2.0000$, $x_3 = 1.0000$.

Consistente and Inconsistande of the equation:-

If the given system of equations has a solution then it is called consistent system of equations. If the given system of equations has no solution then it is called inconsistent system of equations.

Non-Homogeneous equations:- let $AX = B$ is the given system of non-Homogeneous equations. This system of equations is consistent. If rank of A = rank of AB that i.e is "n".

$$\rho(A) = \rho(AB),$$

If $\rho(A) = \rho(AB) = n$ then it has unique solution.

If $\rho(A) = \rho(AB) < n$ then it has infinite no of solution.

The given system of equations is inconsistent if rank of $A \neq$ rank of (AB).

Solve the following equations if they are consistent.

1. $x+2y+2z=2$, $3x-2y-z=5$, $2x-5y+3z=-4$, $x+4y+6z=0$.

$$[A \quad B] = \begin{bmatrix} 1 & 2 & 2 & 2 \\ 3 & -2 & -1 & 5 \\ 2 & -5 & 3 & -4 \\ 1 & 4 & 6 & 0 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - 2R_1$, $R_4 \to R_4 - R_1$

$$\sim \begin{bmatrix} 1 & 2 & 2 & 2 \\ 0 & -8 & -7 & -1 \\ 0 & -9 & -1 & -8 \\ 0 & 2 & 4 & -2 \end{bmatrix}$$

$R_3 \to 8R_3 - 9R_2$, $R_4 \to 4R_4 + R_2$

$$\sim \begin{bmatrix} 1 & 2 & 2 & 2 \\ 0 & -8 & -7 & -1 \\ 0 & 0 & 55 & -55 \\ 0 & 0 & +9 & -9 \end{bmatrix}$$

$R_3 \to R_3/55$

$R_4 \to R_4/9$.

$$\sim \begin{bmatrix} 1 & 2 & 2 & 2 \\ 0 & -8 & -7 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$R_4 \to R_4 - R_3$

$$\sim \begin{bmatrix} 1 & 2 & 2 & 2 \\ 0 & -8 & -7 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This is in echelon form $\Rightarrow$

rank of $A = 3$, rank of $AB = 3$.

∴ The rank of A = rank of AB = n = 3

The given system of equations is consistent & it has unique solution.

By Backward substitution,

$z = -1$, $\quad -8y - 7z = -1$, $\quad x + 2y + 2z = 2$

$-8y - 7(-1) = -1$ $\qquad x + 2(1) + 2(-1) = 2$

$-8y = -1 - 7$ $\qquad x + 2 - 2 = 2$

$-8y = -8$ $\qquad x = 2$

$y = 1$

2. $x+y+z+t = 4$, $x - z + 2t = 2$, $y + z - 3t = -1$, $x + 2y - z + t = 3$

→

$$[A \quad B] = \begin{bmatrix} 1 & 1 & 1 & 1 & 4 \\ 1 & 0 & -1 & 2 & 2 \\ 0 & 1 & 1 & -3 & -1 \\ 1 & 2 & -1 & 1 & 3 \end{bmatrix}$$

$R_2 \to R_2 - R_1$, $R_4 \to R_4 - R_1$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 1 & 4 \\ 0 & -1 & -2 & 1 & -2 \\ 0 & 1 & 1 & -3 & -1 \\ 0 & 1 & -2 & 0 & -1 \end{bmatrix}$$

$R_3 \to R_3 + R_2$, $R_4 \to R_4 + R_2$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 1 & 4 \\ 0 & -1 & -2 & 1 & -2 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & -4 & 1 & -3 \end{bmatrix}$$

$R_4 \to R_4 - 4R_3$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 1 & 4 \\ 0 & -1 & -2 & 1 & -2 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 0 & 9 & 9 \end{bmatrix}$$

$R_4 \to R_4 / 9$.

$$\sim \begin{bmatrix} 1 & 1 & 1 & 1 & 4 \\ 0 & -1 & -2 & 1 & -2 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

This is in echelon form

rank of A = 4, rank of AB = 4, n = 4.

the rank of A = rank of AB = n.

The given system of equations ~~are~~ consistent it has unique solution.

By Backward substitution,

$t = 1$, $-z - 2t = -3$, $-y - 2z + t = -2$, $x + y + z + t = 4$

$-z - 2(1) = -3$; $-y - 2(1) + 1 = -2$; $x + 1 + 1 + 1 = 4$

$-z = -3 + 2$; $-y - 2 + 1 = -2$; $x = 1$.

$-z = -1$; $-y = -2 + 1$

$z = 1$; $-y = -1$

$y = 1$

∴ The Solutions,

$x = 1, y = 1, z = 1, t = 1$.

3. $3x + 2y + z = 3$, $2x + y + z = 0$, $6x + 2y + 4z = 6$.

$$[A \ B] = \begin{bmatrix} 3 & 2 & 1 & 3 \\ 2 & 1 & 1 & 0 \\ 6 & 2 & 4 & 6 \end{bmatrix}$$

$R_2 \rightarrow 3R_2 - 2R_1$, $R_3 \rightarrow R_3 - 2R_1$

$$\sim \begin{bmatrix} 3 & 2 & 1 & 3 \\ 0 & -1 & 1 & -6 \\ 0 & -2 & 2 & 0 \end{bmatrix}$$

$R_3 \rightarrow R_3 - 2R_2$

$$\sim \begin{bmatrix} 3 & 2 & 1 & 3 \\ 0 & -1 & 1 & -6 \\ 0 & 0 & 0 & 12 \end{bmatrix}$$

This is in echelon form

rank of A = 2, rank of AB = 3, n = 3.

The given system of equations are inconsistent

$\therefore$ It has no solution.

4. $5x+3y+7z=4$, $3x+26y+2z=9$, $7x+2y+10z=5$.

$$[AB] = \begin{bmatrix} 5 & 3 & 7 & 4 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{bmatrix}$$

$R_2 \to 5R_2-3R_1, R_3 \to 5R_3-7R_1$

$$\sim \begin{bmatrix} 5 & 3 & 7 & 4 \\ 0 & 121 & -11 & 33 \\ 0 & -11 & 1 & -3 \end{bmatrix}$$

$R_3 \to 11R_3 + R_2$

$$\sim \begin{bmatrix} 5 & 3 & 7 & 4 \\ 0 & 121 & -11 & 33 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This is in echelon form

rank of $A = 2$, rank of $AB = 2$, $n=3$.

the given system of equations are consistent

it has infinite solutions.

$$121y - 11z = 33$$

$$11y - z = 3$$

let, $z = k$.

$$11y - k = 3 \Rightarrow 11y = 3+k \Rightarrow y = \frac{3+k}{11}$$

$$5x+3y+7z=4.$$

$$5x + 3\left(\frac{3+k}{11}\right) + 7k = 4.$$

$$5x + \frac{9}{11} + \frac{3k}{11} + 7k = 4.$$

$$5x = 4 - \frac{9}{11} - 7k - \frac{3k}{11} = \frac{35}{11} - \frac{80k}{11}$$

$\therefore x = \frac{7-16k}{11}$. $k \in R$.

5. For what values of a, the equations $3x-y+4z=3$, $x+2y-3z=-2$, $6x+5y+az=-3$ will have infinite no. of solutions and find the solution

$$[A\ B] = \begin{bmatrix} 3 & -1 & 4 & 3 \\ 1 & 2 & -3 & -2 \\ 6 & 5 & a & -3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 3 & -1 & 4 & 3 \\ 0 & 7 & -13 & -9 \\ 0 & 7 & a-8 & -9 \end{bmatrix} \quad \begin{matrix} R_2 \to 3R_2 - R_1 \\ R_3 \to R_3 - 2R_1 \end{matrix}$$

$R_3 \to R_3 - R_2$

$$\sim \begin{bmatrix} 3 & -1 & 4 & 3 \\ 0 & 7 & -13 & -9 \\ 0 & 0 & a+5 & 0 \end{bmatrix} \quad \text{--- (1)}$$

This is in echelon form

rank of $A = 3$, rank of $AB = 3$.

To get infinite no. of solutions we must have rank of A = rank of $AB < n$ this is possible when $a+5=0 \Rightarrow a=-5$.

from (1),

$$[A\ B] \sim \begin{bmatrix} 3 & -1 & 4 & 3 \\ 0 & 7 & -13 & -9 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$7y - 13z = -9$

let, $z = k$

$7y = 13z - 9 = 13k - 9$

$y = \frac{13k-9}{7}$

$$3x - y + 4z = 3$$

$$3x = 3 + y - 4z = 3 + \frac{13k-9}{7} - 4k.$$

$$3x = \frac{12}{7} - \frac{15k}{7}$$

$$x = \frac{4}{7} - \frac{5k}{7} = \frac{4-5k}{7}$$

$\therefore$ The solution,

$$x = \frac{4-5k}{7}, \quad y = \frac{13k-9}{7}, \quad z = k.$$

6. For what values of a, the equations, $x+y+z=1$, $x+2y+4z=a$, $x+4y+10z=a^2$ has a solution and find the solution.

$$[A \quad B] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & a \\ 1 & 4 & 10 & a^2 \end{bmatrix}$$

$$R_2 \to R_2 - R_1, \; R_3 \to R_3 - R_1$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & a-1 \\ 0 & 3 & 9 & a^2-1 \end{bmatrix}$$

$$R_3 \to R_3 - 3R_2$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & a-1 \\ 0 & 0 & 0 & a^2-1-3a+3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & a-1 \\ 0 & 0 & 0 & a^2-3a+2 \end{bmatrix} \quad -(1)$$

This is in echelon form.

Rank of $A = 2$ to get a solution we have have rank of $AB = 2$. This is possible when

$a^2 - 3a + 2 = 0$.

$a^2-2a-a+2=0$

$a(a-2)-1(a-2)=0$

$(a-1)(a-2)=0$

$a=1,\ a=2.$

The system of equations has infinite no. of solutions If $a=1,2$

case (i):-

$= \quad a=1.$

from ①,

$$[AB]=\begin{bmatrix}1 & 1 & 1 & 1\\ 0 & 1 & 3 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

$y+3z=0, \quad x+y+z=1.$

let $z=k_1$

$y=-3k_1, \quad x+(-3k_1)+k_1=1.$

$x-2k_1=1.$

$x=1+2k_1$

$\therefore x=1+2k_1, \quad y=-3k_1, \quad z=k_1, \quad k_1\in R$

case (ii):- $\quad a=2.$

from ①,

$$[A\ B]=\begin{bmatrix}1 & 1 & 1 & 1\\ 0 & 1 & 3 & 1\\ 0 & 0 & 0 & 0\end{bmatrix}$$

$y+3z=1, \quad x+y+z=1.$

let, $z=k_2$

$y=1-3k_2, \quad x+k_2+1-3k_2=1.$

$x=+2k_2$

The solutions are,

$x = 2k_2,\ y = 1-3k_2,\ z = k_2$.

7. Investigate for what of λ, μ for equations,

$2x+3y+5z=9,\ 7x+3y-2z=8,\ 2x+3y+\lambda z=\mu$, have

(i) no solution (ii) unique solution (iii) Infinite no. of solu

$$[A\ B] = \begin{bmatrix} 2 & 3 & 5 & 9 \\ 7 & 3 & -2 & 8 \\ 2 & 3 & \lambda & \mu \end{bmatrix}$$

$R_2 \to 2R_2 - 7R_1,\ R_3 \to R_3 - R_1$

$$\sim \begin{bmatrix} 2 & 3 & 5 & 9 \\ 0 & -15 & -39 & -47 \\ 0 & 0 & \lambda-5 & \mu-9 \end{bmatrix}$$

This is in echelon form.

(i) No solution:-

If $\lambda = 5,\ \mu \neq 9$ then the rank of $A = 2$, rank of $AB = 3$.

$\therefore$ The given system of eq^{n}s has no solution

(ii) unique solution:-

If $\lambda \neq 5$, then rank of $A = 3$, rank of $AB = 3$, $n = 3$.

$\therefore$ The given system of equations has unique solution for $\lambda \neq 5,\ \mu \in R$.

(iii) Infinite no. of solutions:-

If $\lambda = 5,\ \mu = 9$ then rank of $A = 2$, rank of $AB = 2$

$n = 3$.

$\therefore$ rank of A = rank of AB < n

The given system of eq^ns has infinite no. of solutions.

Homogeneous System of equations:- The system of homogeneous equation $AX = 0$ is always consistent since $x=0, y=0, z=0, \ldots$ satisfies the equations. This solution is called zero solution or trivial solution. Any other solution if exist it is called Non-zero or non-trivial solution.

Consider the coefficient matrix A and reduce into echelon form. If rank of $A = n$ then the system of sol equations have unique solution i.e zero soln. If the rank is less than n, then the system of equations have infinite no. of solutions. i.e non-zero solutions.

1. Solve the following system of equations.

1. $x_1 + 2x_3 - 2x_4 = 0$, $2x_1 - x_2 - x_4 = 0$, $x_1 + 2x_3 - x_4 = 0$, $4x_1 - x_2 + 3x_3 - x_4 = 0$.

$$A = \begin{bmatrix} 1 & 0 & 2 & -2 \\ 2 & -1 & 0 & -1 \\ 1 & 0 & 2 & -1 \\ 4 & -1 & 3 & -1 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - R_1$, $R_4 \to R_4 - 4R_1$

$$\sim \begin{bmatrix} 1 & 0 & 2 & -2 \\ 0 & -1 & +4 & +3 \\ 0 & 0 & +0 & 1 \\ 0 & -1 & -5 & +7 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 2 & -2 \\ 0 & -1 & -4 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & -1 & -5 & 7 \end{bmatrix}$$

$R_4 \to R_4 - R_2$

$$\sim \begin{bmatrix} 1 & 0 & 2 & -2 \\ 0 & -1 & -4 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 4 \end{bmatrix}$$

$R_3 \leftrightarrow R_4$.

$$\sim \begin{bmatrix} 1 & 0 & 2 & -2 \\ 0 & -1 & -4 & 3 \\ 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This is in echelon form

rank of $A = 4$, $n = 4$.

$\therefore$ Given system of equation has unique soln

$\therefore x_1 = 0,\ x_2 = 0,\ x_3 = 0,\ x_4 = 0$.

2. $2x - 2y + 5z + 3\omega = 0,\ 4x - y + z + \omega = 0,\ 3x - 2y + 3z + 4\omega = 0$

$x - 3y + 7z + 6\omega = 0$.

$$A = \begin{bmatrix} 2 & -2 & 5 & 3 \\ 4 & -1 & 1 & 1 \\ 3 & -2 & 3 & 4 \\ 1 & -3 & 7 & 6 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1,\ R_3 \to 2R_3 - 3R_1,\ R_4 \to 2R_4 - R_1$

$$\sim \begin{bmatrix} 2 & -2 & 5 & 3 \\ 0 & 3 & -9 & -5 \\ 0 & 2 & -9 & -1 \\ 0 & -4 & 9 & 9 \end{bmatrix}$$

$R_3 \to 3R_3 - 2R_2,\ R_4 \to 3R_4 + 4R_2$

$$\sim \begin{bmatrix} 2 & -2 & 5 & 3 \\ 0 & 3 & -9 & -5 \\ 0 & 0 & -9 & 7 \\ 0 & 0 & -9 & 7 \end{bmatrix}$$

$R_4 \to R_4 - R_3$

$$\sim \begin{bmatrix} 2 & -2 & 5 & 3 \\ 0 & 3 & -9 & -5 \\ 0 & 0 & -9 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This is in Echelon form.

rank of $A = 3$, $n = 4$.

rank of $A < n$

given system of equations has infinite no. of solutions.

let $\omega = 9k$.

$-9z + 7\omega = 0$

$3y - 9z - 5\omega = 0$

$2x - 2y + 5z + 3\omega = 0$

$-9z = -7k \times 9 \Rightarrow z = \frac{7k \times 9}{9}$

$z = 7k$.

$3y - 9(7k) - 5(9k) = 0$.

$+3y = 63k + 45k$.

$+3y = 108k$.

$y = +36k$.

$2x - 2(36k) + 5(7k) + 3(9k) = 0$.

$2x - 72k + 35k + 27k = 0$.

$2x = -62k + 72k$.

$2x = 10k \Rightarrow x = 5k$.

$\therefore x = 5k, y = 36k, z = 7k, \omega = 9k$

$(\lambda^2 - 2\lambda - 5)(\lambda^2 - 2\lambda - 5)$

$\lambda^4 - 2\lambda^3 - 5\lambda^2 - 2\lambda^3 + 4\lambda^2 + 10\lambda$
$- 5\lambda^2 + 10\lambda + 25$

$\lambda^4 - 4\lambda^3 - 6\lambda^2 + 20\lambda + 25$
$- 6\lambda^2 - 2\lambda - 21\lambda - 7$

$\lambda^4 - 4\lambda^3 - 12\lambda^2 - 3\lambda + 18 = 0$

$1 - 4 - 12 - 3 + 18$

$(2\lambda + 7)(-3\lambda - 1)$

$-6\lambda^2 - 2\lambda - 21\lambda - 7$

3. Find the value of λ for which the system of equations $x+2y+3z=\lambda x$, $3x+y+2z=\lambda y$, $2x+3y+z=\lambda z$ has non-zero solution.

Given equations can be written as

$$(1-\lambda)x+2y+3z=0$$
$$3x+(1-\lambda)y+2z=0$$
$$2x+3y+(1-\lambda)z=0.$$

$2-2\lambda-9$

$1+\lambda^2-2\lambda-6$

12

$1+\lambda^2-2\lambda-6$

$\lambda^2-2\lambda-5$

$3-3\lambda-4$

$$A=\begin{bmatrix}(1-\lambda) & 2 & 3\\ 3 & (1-\lambda) & 2\\ 2 & 3 & (1-\lambda)\end{bmatrix}$$

$$R_2\to(1-\lambda)R_2-3R_1,\quad R_3\to(1-\lambda)R_3-2R_1$$

$$\sim\begin{bmatrix}1-\lambda & 2 & 3\\ 0 & \lambda^2-2\lambda-5 & -2\lambda-7\\ 0 & -3\lambda-1 & \lambda^2-2\lambda-5\end{bmatrix}$$

$$R_3\to(\lambda^2-2\lambda-5)R_3-(-3\lambda-1)R_2.$$

$$\sim\begin{bmatrix}1-\lambda & 2 & 3\\ 0 & \lambda^2-2\lambda-5 & -2\lambda-7\\ 0 & 0 & \lambda^4-4\lambda^3-12\lambda^2-3\lambda+18\end{bmatrix}$$

Given This is in echelon form,

Given, the system of equations are non-zero solution. So,

$$\lambda^4-4\lambda^3-12\lambda^2-3\lambda+18=0.$$

$$\lambda^3-3\lambda^2-15\lambda-18=0.$$

$$\begin{array}{r|rrrrr} 1 & 1 & -4 & -12 & -3 & 18\\ & 0 & 1 & -3 & -15 & -18\\ \hline \end{array}$$

$$\lambda^2 + 3\lambda + 3 = 0$$

$$\begin{array}{c|cccc} 6 & 1 & -3 & -15 & -18 \\ & 0 & 6 & 18 & 18 \\ \hline & 1 & 3 & +3 & \boxed{0} \end{array}$$

$$\lambda = \frac{-3 \pm \sqrt{9 - 4(3)}}{2(1)}$$

$$= \frac{-3 \pm \sqrt{9-12}}{2} = \frac{-3 \pm \sqrt{-3}}{2} = \frac{-3 \pm \sqrt{3}i}{2}$$

$\therefore$ The root is 6.

* Applications (current in electric circuit):- Using linear system of equations, we can find the currents passing in the given electric circuit.

Circuit:- A closed connection of batteries, resistance and wires is called as circuit.

Junction (or) Node:- If three or more components are connected at a point then that point is called a junction

Loop:- Any closed part in a circuit is called a loop.

current:- Flow of electrons in a circuit is called current it is denoted by i. It is measured in Amperes.

Resistor:- Resistor opposes the flow of current It is (measured) denoted by R. It is measured in Volts.

current law (or) Kirchoff's 1^{st} law:- The algebric sum of the currents at a junction in a circuit is zero.

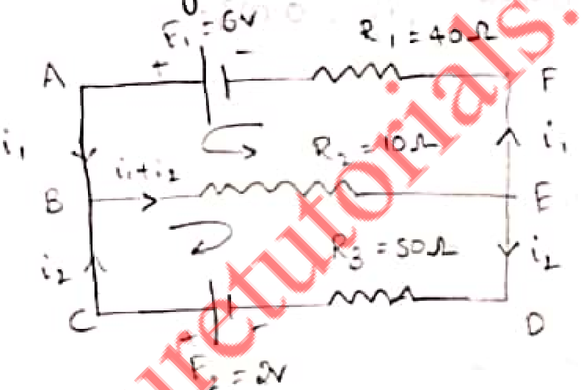
The sign convension when the current passes towards (or) into the junction is positive (+ve) out side to the junction is negative (-ve).

Voltage law: (or) Kirchoff's 2^{nd} law:- The algebric sum of the potential differences in a loop is zero.

problems:-

1. Find the currents in a each cell considering the circuit given below.

1.



Apply the voltage law in the loop BEFAB.

$$-(i_1+i_2)R_2 - i_1R_1 + E_1 = 0.$$

$$-(i_1+i_2)10 - 40i_1 + 6 = 0.$$

$$-50i_1 - 10i_2 = -6.$$

$$25i_1 + 5i_2 = 3 \quad \text{--- (1)}.$$

Apply the voltage law in the loop BEDCB.

$$-(i_1+i_2)R_2 - i_2R_3 + E_2 = 0.$$

$$-(i_1+i_2)10 - 50i_2 + 2 = 0.$$

$$-10i_1 - 60i_2 = -2 \Rightarrow 5i_1 + 30i_2 = 1 \quad \text{--- (2)}.$$

Augmented matrix for ① & ②.

$$[A\ \ B] = \begin{bmatrix} 25 & 5 & 3 \\ 5 & 30 & 1 \end{bmatrix} \quad R_2 \to 5R_2 - R_1$$

$$\sim \begin{bmatrix} 25 & 5 & 3 \\ 0 & 145 & 2 \end{bmatrix}$$

This is in echelon form

$145 i_2 = 2 \Rightarrow i_2 = \frac{2}{145} = 0.013$

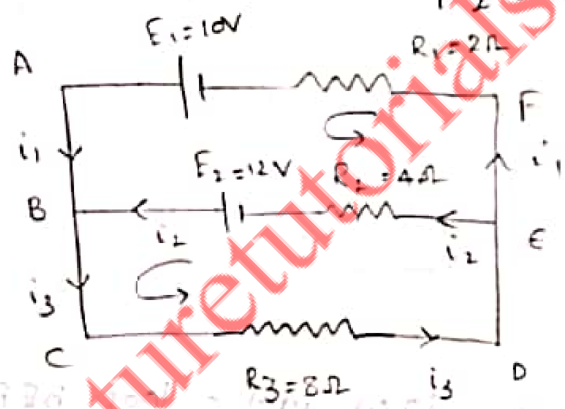
$25 i_1 + 5 i_2 = 3$

$25 i_1 = 3 - 5(0.013)$

$i_1 = \frac{3 - 5(0.013)}{25} = \frac{2.935}{25}$

$i_1 = 0.1174.$ $\quad i_2 = 0.013.$

2.



Apply current law at junction B.

$i_1 + i_2 - i_3 = 0.$ — ①

Apply current law at junction

Apply Voltage law ~~at~~ in loop BEFAB,

$i_2 R_2 - E_2 - i_1 R_1 + E_1 = 0.$

$4 i_2 - 12 - 2 i_1 + 10 = 0.$

$-2 i_1 + 4 i_2 = 2$

$2 i_1 - 4 i_2 = -2 \Rightarrow i_1 - 2 i_2 = -1$ — ②

Apply voltage law in loop BEDEB,

$$-i_3R_3 - R_2 i_2 + E_2 = 0.$$

$$-8i_3 - 4i_2 + 12 = 0.$$

$$i_2 + 2i_3 = -3 \quad \text{--- (3)}.$$

Augmented matrix for (1), (2), (3)

$$[A \;\; B] = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 1 & -2 & 0 & -1 \\ 0 & 1 & 2 & -3 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$$\sim \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & -3 & 1 & -1 \\ 0 & 1 & 2 & -3 \end{bmatrix}$$

$R_3 \to 3R_3 + R_2$

$$\sim \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & -3 & 1 & -1 \\ 0 & 0 & 7 & -10 \end{bmatrix}$$

$7i_3 = -10$, $-3i_2 + i_3 = -1$, $i_1 + i_2 - i_3 = 0.$

$i_3 = \frac{-10}{7}$, $-3i_2 - \frac{10}{7} = -1$, $i_1 - \frac{1}{7} + \frac{10}{7} = 0$

$$-3i_2 = -1 + \frac{10}{7} \qquad i_1 + \frac{9}{7} = 0$$

$$-3i_2 = \frac{-7+10}{7} \qquad i_1 = -\frac{9}{7}$$

$$-3i_2 = \frac{3}{7}$$

$$i_2 = -\frac{1}{7}$$

$\therefore i_1 = \frac{-9}{7}$, $i_2 = \frac{-1}{7}$, $i_3 = \frac{-10}{7}$.