1. ELECTROMAGNETIC FIELDS

THE FIELD:

The field is a representation of a physical quantity which is a continuous function of position and varies from point to point in space. The region of space in which each point specifies the physical quantity is called a field.

SCALAR FIELD:

A scalar field is a function that gives a single value for a scalar physical quantity at every point in space.

Suppose $f(x, y, z)$ is a scalar function. we know the function at any point say (x, y, z) in space. Then such a function f is called scalar field. Examples are distribution of temperature in a metal strip T(x, y), magnetostatic and electrostatic potentials, etc. A scalar field is shown in Fig. It shows a topographical map in which contour lines shows various elevations. Suppose the lines represent the pressure, a scalar quantity, it gives meteorological report which tells about the air pressure over a continent.

Fig : Representation of a Scalar field

In any scalar field, we can draw level surface in which scalar function has single value and is uniform at every point in that surface. Example are isothermal, equidensity or equipotential surfaces.

VECTOR FIELD:

It is a continuous vector function with definite magnitude and direction. Both magnitude and direction may change continuously from point to point in the field region. Such a vector function is called a vector field. This is illustrated in Fig.

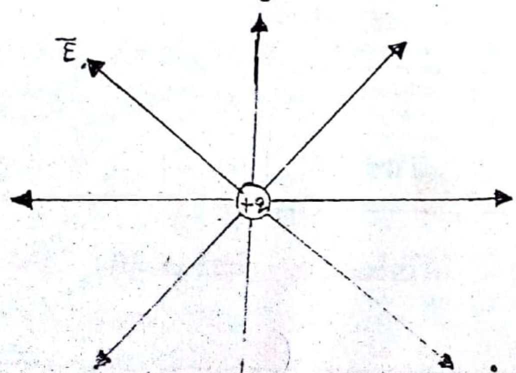

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Fig vector Field permeats around the charge.

Suppose F (x, y, z) is vector function. We know the function F at any point (x, y, z) in the field and then F is called as vector field. Examples of vector field are force acting on a body, electric field, magnetic field, gravitational field, the distribution of velocity in a fluid etc.

$$\boxed{\vec{F}(x, y, z) = \hat{i} F_x + \hat{j} F_y + \hat{k} F_z}$$

where F_x, F_y and F_z are the magnitudes or components of the vector along the x, y and z directions respectively.

ELECTRIC POTENTIAL :-

The electric potential at any point P in the electric field is defined as the work done in moving a unit positive charge from infinity to that point or energy per unit charge, then we have

$$\boxed{V(r) = \frac{q}{4\pi\varepsilon_0 . r}}$$

where r is the distance between the charge q and point P.

The workdone in moving a unit positive charge A to B in the electric field $\vec{E}\cdot\vec{dr}$, in the limits from infinity to r

$$\boxed{V(r) = -\int_{\infty}^{r} \vec{E}\cdot\vec{dr}} \quad \text{——— (1)}$$

Where, the negative sign is due to the fact that the motion of test charge is from A to B which is opposite in the direction of the electric field

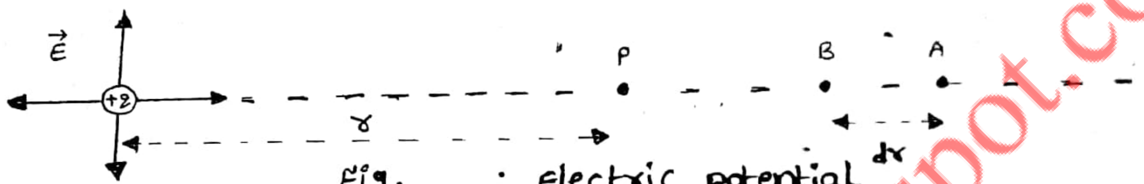

Fig. : electric potential

Electrical Potential Due to Point Charge

The electric field $\vec{E}$ at a distance r from a point charge q is given by:

$$\boxed{\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}}$$

Where, $\hat{r}$ is a unit vector along the vector $\vec{r}$. Now, $\vec{E}.\vec{dr}$ is expressed as:

$$\boxed{\vec{E}\cdot\vec{dr} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}\cdot\vec{dr} = \frac{q}{4\pi\varepsilon_0 r^2}dr} \quad \text{——— (2)}$$

Substituting Eq. (2) in Eq.(1)

$$V(r) = -\frac{q}{4\pi\varepsilon_0}\int_{\infty}^{r}\frac{dr}{r^2} = -\left[\frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{r}\right]_{\infty}^{r} = -\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$= -\frac{q}{4\pi\varepsilon_0 r}$$

Eq. gives the electric potential at a distance r away from a point charge q and it shows that the electric potential is inversely proportional to r.

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\Sigma_i \frac{q_i}{r_i}}$$

If the charge distribution is continuous, the electric potential V at any point in the electric field can be found by summing over the contributions from individual differential elements of charge q.

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}}$$

ELECTRICAL POTENTIAL GRADIENT : (Gradient of a scalar is a vector)

Differentiating the vector potential with respect to r gives the magnitude of electric field.

$$|\vec{E}| = \frac{d}{dr}V(r) = -\frac{d}{dr}\frac{q}{4\pi\varepsilon_0 r} = \frac{q}{4\pi\varepsilon_0 r^2}$$

Since the electric field is directed along $\bar{r}$, the electric field at point p is expressed as:

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$$

Thus, in general, an electric field can be expressed in a differential form from the known potential as:

$$\bar{E} = \frac{d}{dr}V(r) = \vec{\nabla}V(r)$$

Where, $\vec{\nabla}$ is a vector operator known as gradient which will be discussed in the next section. Thus, the electric field is the gradient of a scalar potential. Thus, the gradient of a scalar is a vector.

DIVERGENCE OF A VECTOR FIELD: is a Scalar

The divergence of a vector field is defined as :

$$\vec{\nabla}\cdot\vec{F}(r) = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

Where, $\vec{F}(r)$ is a vector field and $\vec{\nabla}$ is a vector operator and the dot product ~~of them~~ is a scalar quantity.

Let $\vec{F}(r)$ is a vector field defined as:

$$\vec{F}(r) = \hat{i}F_x + \hat{j}F_y + \hat{z}F_z$$

The vector operator $\vec{\nabla}$ is defined as:

$$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

Now, the dot product $\vec{\nabla}\cdot\vec{F}(r)$ is written as:

$$\vec{\nabla}\cdot\vec{F}(r) = \left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]\cdot(\hat{i}F_x + \hat{j}F_y + \hat{z}F_z)$$

$$\vec{\nabla}\cdot\vec{F}(r) = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

Therefore, a dot product of del operator with a vector field gives a scalar field and it is called ivergence of $\vec{F}$ It is often written as div $\vec{F}$.

Example: Electric field

Let $\vec{E}(r)$ be defined as:

$$\boxed{\bar{E}(r) = \hat{i}E_x + \hat{j}E_y + zE_z}$$

The divergence of the electric field $\bar{E}(r)$, according to the Equation is:

$$\boxed{\vec{\nabla}\cdot\bar{E}(r) = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}}$$

The divergence or convergence of electric field lines depends on the sign of the charge and hence the electric field has either positive or negative divergence. If $\vec{\nabla}\cdot\vec{E}(r) = 0$, then the vector field is uniform.

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GAUSS'S DIVERGENCE THEOREM:

The integral of the divergence of a vector field F over a volume V is equal to the surface integral of the normal component of that vector over that surface bounded by V. Mathematically, it can be written as:

$$\boxed{\int_V \vec{\nabla}\cdot\vec{F}\,dv = \oint_S \vec{F}\cdot\vec{ds}}$$

To understand this statement, we assume an arbitrary volume V bounded by the surface S as shown in Fig

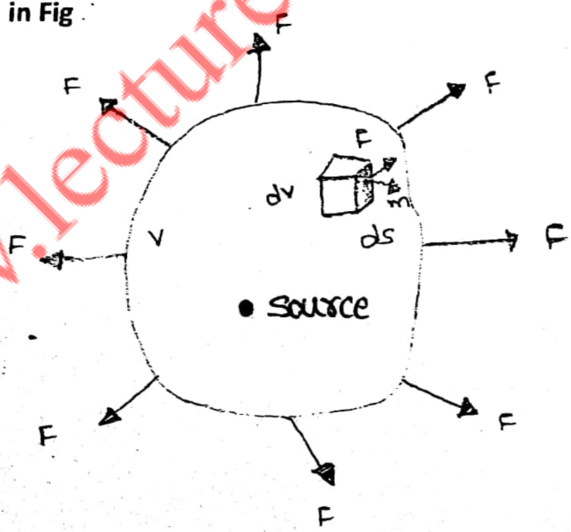

Fig : Illustration for Gauss's theorem

consider a small elemental area ds enclosing the volume dv The normal component of the divergence of the vector field $\vec{F}$ produced by the source through $\vec{ds}$ is given by :

$$\boxed{d\Phi = \vec{F} \cdot \vec{ds}} \longrightarrow (1)$$

The total flux through the surface s enclosing the volume v is obtained by taking the closed surface integral to the Equation (1) then we get

$$\boxed{\Phi = \oint d\Phi = \oint_s \vec{F} \cdot \vec{ds}} \longrightarrow (2)$$

The divergence of the vector $\vec{F}$ normally through a small volume dv or flux through the volume dv is given by :

$$\boxed{d\Phi = (\vec{\nabla} \cdot \vec{F}) \cdot dv} \longrightarrow (3)$$

The integration of Equation over the whole volume v gives the total flux diverging normally out of the volume.

$$\boxed{\Phi = \int_v d\Phi = \int_v (\vec{\nabla} \cdot \vec{F})\, dv.} \longrightarrow (4)$$

Since the total flux of the vector through the ~~surface~~ volume or through the surface enclosing

∵ the volume must be the same, equating the Eqs (2) and (4) we have :

$$\boxed{\int_V \vec{\nabla} \cdot \vec{F} \, dv = \oint_S \cdot \vec{F} \cdot \vec{ds}}$$

∴ This is known as the divergence theorem and using this Equations we can convert a volume integral into a surface integral and vice versa.

GENERAL INTERPRETATION :-

The divergence of a vector field at a $\vec{F}(r)$. point r is equal to the volume of the flux flowing out of a small volume dv divided by the infinitesimal volume in the limit dv goes to zero.

$$\text{div } \vec{F}(r) = \lim_{dv \to 0} \frac{\text{flux of } \vec{F}(r) \text{ out of } dv}{dv}$$

or For each volume element dv, we have

$$\text{div } \vec{F}(r) \, dv = \text{flux of } \vec{F}(r) \text{ out of } dv.$$

line surface

STOKES' THEOREM :- R

The line integral of the vector field $\vec{F}$. around a closed curve is equal to the integral of the normal component of its curl over any surface bounded by the curve Mathematically, it can be written as :

$$\boxed{\int_s (\vec{\nabla} \times \vec{F}) \cdot \hat{n}\, ds = \oint_c \vec{F} \cdot \vec{dl}}$$

consider a surface area that is divided into small surface elemental areas ds as shown in Fig

Fig: A surface area divided into small elemental areas showing circulation:

Let us consider a small elemental area $\vec{ds}$. then the circulation of vector field $\vec{F}$ around elemental area ds is :

$$\boxed{(\vec{\nabla} \times \vec{F}) \cdot \hat{n} \cdot ds}$$

The circulation around closed loop or closed curve.

∴ l can be obtained by integrating the Eq. over the closed loop l enclosing an area ds, then we have :

$$\boxed{\int_s (\vec{\nabla} \times \vec{F}) \cdot \hat{n} \, ds} \longrightarrow (1) \quad (9.74)$$

Let $\vec{dl}$ be the elemental length of the closed loop l, then the vector field $\vec{F}$ along this length is :

$$\boxed{\vec{F} \cdot \vec{dl}}$$

The circulation around the loop or closed curve l enclosing an area s is obtain by taking line integral over closed loop is :

$$\boxed{\oint_c \vec{F} \cdot \vec{dl}} \longrightarrow (2) \quad (9.76)$$

Equating Eqs. (9.74) (1) and (9.76) (2) we get

$$\boxed{\int_s (\vec{\nabla} \cdot \vec{F}) \cdot \hat{n} \, ds = \oint_c \vec{F} \cdot \vec{dl}}$$

From this Equation we conclude that the line integral of the vector field $\vec{F}$ around a closed loop or curve is equal to the integral of the normal component of its curl over any surface bounded by the loop or curve. This is called stokes theorem using this equation, we can convert a surface integral into a line integral and vice versa.

General Interpretation :-

To understand the theorem , let us consider a surface area s divided into small surface - elements as shown in the Fig .

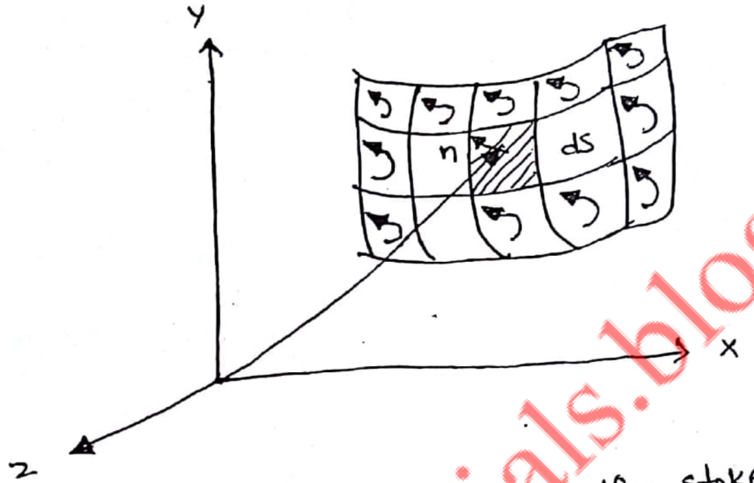

Fig :- Diagram illustrates the stokes' theorem Arrows indicate the sense of line integral.

Let us consider a small element of surface Area ds and calculate the line integral $\vec{F}(r) \cdot \vec{dr}$ around the perimeter. The value of the integral divided by the area ds in the limit as the area tends to zero will turns out to be the component of curl $\vec{F}(r)$ in the direction of unit vector $\hat{n}$ normal to the surface.

This can be expressed as :

$$(\text{curl } \vec{F}(r) \cdot \hat{n} = \lim_{\vec{ds} \to 0} \frac{\text{line integral } \vec{F}(r)\vec{dr} \text{ around ds}}{\vec{ds}}$$

or

for each surface element in the limit $\vec{ds}$ goes to zero

then we have :

$$(\text{curl } \vec{F}(r) \cdot \hat{n} \cdot ds = \text{curl } \vec{F}(r) \cdot \vec{ds} = \int_{dr} \vec{F}(r) \cdot \vec{dr}.$$

Dielectric Medium :-

- It is a non-conducted medium with permittivity ε and permeability μ because of non-conducting medium, current density $\vec{J} = 0$ and also $\rho = 0$ due to homogeneous isotropic medium in which there is no charge distribution. Then the Maxwell's equations in the dielectric medium are :

Maxwell equations

$$\vec{\nabla} \cdot \vec{E} = 0 \longrightarrow (1)$$

$$\vec{\nabla} \cdot \vec{B} = 0 \longrightarrow (2)$$

$$\vec{\nabla} \times \vec{E} = \frac{\partial \vec{B}}{\partial t} \longrightarrow (3)$$

$$\vec{\nabla} \times \vec{B} = \mu\varepsilon \frac{\partial \vec{E}}{\partial t} \longrightarrow (4)$$

for free

$\nabla \times (\nabla \times E) = \nabla(\nabla \cdot E) - \nabla^2 E$

$\nabla \times \left(\frac{-\partial B}{\partial t}\right) = -\nabla^2 E$

$\frac{\partial}{\partial t}(\nabla \times B) = \nabla^2 E$

To understand the propagation of an electromagnetic wave in a charge free medium. i.e., the dielectric medium, we need to derive wave equations for electric field $\vec{E}$ and magnetic field $\vec{B}$ respectively. The wave equations can be derived from the above said maxwell's equations.

PROPAGATION OF ELECTROMAGNETIC WAVES THROUGH DIELECTRIC MEDIUM :—

The wave equation for electric field ($\vec{E}$).

consider a dielectric medium . To obtain the equation of propagation of electromagnetic waves in the dielectric medium , first we find the curl of eq (3)

$$\boxed{\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\vec{\nabla} \times \frac{\partial \vec{B}}{\partial t} = -\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{B})} \rightarrow (5)$$

From Eq. (4) and the equation (5) becomes :

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{B}) = -\frac{\partial}{\partial t}\left[\mu\varepsilon \frac{\partial \vec{E}}{\partial t}\right]$$

$$= -\mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

Using the vector identity ,

$$\boxed{\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E}}$$

Since using $\vec{\nabla} \cdot \vec{E} = 0$

$$-\nabla^2 \cdot \vec{E} = -\mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

(or) $$\nabla^2 \vec{E} = \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} \longrightarrow (6)$$

This is called wave equation for the electric field $\vec{E}$.

The wave Equation for Magnetic field $\vec{B}$.

we can follow the same procedure as for the wave equation for electric field, taking the curl of Equation (4) we have:

$$\boxed{\vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = \vec{\nabla} \times \mu\varepsilon \frac{\partial \vec{E}}{\partial t}}$$

For an isotropic medium, μ and ε are constants

So that:

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = \mu\varepsilon\, \vec{\nabla} \times \frac{\partial \vec{E}}{\partial t} = \mu\varepsilon \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{E})$$

using the vector identity,

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla} (\vec{\nabla} \cdot \vec{B}) - \nabla^2 \vec{B}.$$

Since $\vec{\nabla} \cdot \vec{B}$, $\quad -\nabla^2 \vec{B} = -\mu\varepsilon \frac{\partial^2 \vec{B}}{\partial t^2}$

$$\nabla^2 \vec{B} = \mu \varepsilon \frac{\partial^2 \vec{B}}{\partial t^2} \longrightarrow (7)$$

This is the wave equation for the magnetic field $\vec{B}$.

THE VELOCITY OF THE ELECTROMAGNETIC WAVE IN A DIELECTRIC MEDIUM :-

The wave equation for an oscillating quantity f is given by

$$\boxed{\nabla^2 f = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2}} \longrightarrow (8)$$

when v is the velocity of the wave. since the electric field vector is also an oscillating Quantity, comparing Eqs (6) and (7) with Eq. (8). gives the velocity v of the electromagnetic wave in the dielectric medium.

$$\boxed{v^2 = \frac{1}{\mu \varepsilon}}$$

$$\boxed{v = \frac{1}{\sqrt{\mu \varepsilon}}}$$

This Eq. gives the velocity of the electromagnetic wave in a medium.

REFRACTIVE INDEX OF THE MEDIUM :-

Problems

we know that

$$\varepsilon = \varepsilon_0 \varepsilon_r$$

$$\mu = \mu_0 \mu_r$$

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \frac{1}{\sqrt{\mu_r \varepsilon_r}}$$

But we have $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ the velocity of light in free space and hence

$$v = \frac{c}{\sqrt{\mu_r \varepsilon_r}}.$$

Since most of materials have the relative permeability $\mu_r = 1$, the above equation becomes :

$$\boxed{V = \frac{c}{\sqrt{\varepsilon_r}}}$$

or
$$\nabla^2 H = \mu\varepsilon \frac{\partial^2 H}{\partial t^2} + \mu\sigma \frac{\partial H}{\partial t} \quad \text{.........(5.58)}$$

This is the electromagnetic wave equation for conducting medium in terms of magnetic intensity vector H.

In the case of non-conducting medium $\sigma = 0$, therefore,

$$\nabla^2 H = \mu\varepsilon \frac{\partial^2 H}{\partial t^2} \quad \text{.........(5.59)}$$

The eqns.(5.56)& (5.58) are more general wave equations for conducting media and are involving first and second order time derivatives which are well known equations for damped or attenuated waves in an absorbing medium of homogeneous, isotropic and source free such as metallic conductor.

5.13 Poynting Theorem and Poynting Vector

(i) Poynting theorem

When electromagnetic waves propagate through the space from their origin to a distant receiving point, there is a transfer of energy from the source to the receivers. There is a simple and direct relation between the rate of this energy transfer and the electromagnetic wave. In order to find the power in a uniform plane wave it is necessary to develop a power theorem for the electromagnetic field known as poynting theorem. It states that, the vector product of electric field intensities E and magnetic field intensity H at any point is a measure of the rate of energy flow per unit area at that point. It was originally postulated by an English physicist, John.H. Poynting and hence the name poynting theorem. It can be developed with the help of Maxwell's equations. Mathematically this theorem can be expressed as

$$P = E \times H \quad \text{.........(5.60)}$$

The direction of flow is perpendicular to E and H in the direction of the vector.

(ii) Poynting Vector

By poynting theorem, the vector product of electric field intensity E and magnetic field intensity H is another vector P. This vector is called poynting vector. It can be defined as, *the amount of field energy passing through unit area of the surface perpendicular to the direction of propagation of energy is called poynting vector.* It is one of the important characteristic of electromagnetic waves. It measures the rate of flow of energy or the intensity of the wave as it travels along. The poynting vector points along the direction of flow of radiation.