Code No: R161203 **R16** SET - 1

I B. Tech II Semester Supplementary Examinations, Nov/Dec - 2017
MATHEMATICS-III
(Com. to CE, CSE, IT, AE, AME, EIE, EEE, ME, ECE, Min. E, E Com. E, Agri. E, Chem. E, PE)

Time: 3 hours Max. Marks: 70

Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B)**
2. Answer **ALL** the question in **Part-A**
3. Answer any **FOUR** Questions from **Part-B**

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PART –A

1. a) Define Rank and write two properties of rank. (2M)

b) Find the Eigen values of A^{-1} if $A=\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}$ (2M)

c) Write the matrix corresponding to the Quadratic form. (2M)
$x^2+y^2+z^2+2xy+2yz+2xz$

d) Evaluate $\int_0^1\int_0^y xdxdy$ (2M)

e) Find $\beta(1,1)$ (2M)

f) Write physical interpretation of curl of vector. (2M)

g) State Stoke's theorem. (2M)

PART -B

2. a) Solve the system of equations $x+10y+z=6, 10x+y=6, x+y+10z=6$ by Gauss-Seidel method. (7M)

b) Find the Rank of the matrix $A=\begin{bmatrix}3 & -2 & 0 & -1\\ 0 & 2 & 2 & 1\\ 1 & -2 & -3 & 2\\ 0 & 1 & 2 & 1\end{bmatrix}$ by reduce into Normal form. (7M)

3. a) Reduce the quadratic form $x^2+3y^2+3z^2-2yz$ into canonical form and find its rank , index and signature. (7M)

b) Diagnolize the matrix $A=\begin{bmatrix}1 & 0 & -1\\ 1 & 2 & 1\\ 2 & 2 & 3\end{bmatrix}$ (7M)

4. a) Trace the curve $y=\dfrac{x^2+1}{x^2-1}$ (7M)

b) Using double integration, find the volume of the sphere $x^2+y^2+z^2=a^2$. (7M)
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5. a) Evaluate $\int_0^1 (x \log x)^4 dx$ using beta –gamma function. (7M)

b) Evaluate $\int_0^1 x^3 \sqrt{1-x}\,dx$ (7M)

6. a) Find the directional derivative of $\frac{1}{r}$ in the direction of $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ at . $(1,1,2)$ (7M)

b) Show that $\bar{f} = r^n \left(\bar{a} \times \bar{r}\right)$ is solenoidal where $\bar{a} = a_1\bar{\imath} + a_2\bar{\jmath} + a_3\bar{k}$ and $\bar{r} = x\bar{\imath} + y\bar{\jmath} + z\bar{k}$ (7M)

7. a) Evaluate $\iint_S \bar{F}.\bar{n}\,ds$ if $\bar{F} = 2xy\,\bar{i} + y\,z^2\,\bar{j} + xz\,\bar{k}$ over the parallelepiped $x = 0$, $y = 0$, $x = 2$, $y = 1$, $z = 3$. (7M)

b) Using Divergence theorem, evaluate $\iint_S \bar{F}.\bar{n}ds$ where s is the surface of the sphere $x^2 + y^2 + z^2 = b^2$ in the first octant where $\bar{F} = y\bar{i} + z\bar{j} + x\bar{k}$. (7M)

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Time: 3 hours Max. Marks: 70

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2. Answer **ALL** the question in **Part-A**
3. Answer any **FOUR** Questions from **Part-B**

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PART –A

1. a) Define Echelon form. (2M)

b) Find the Eigen values of A^{-1} if $A=\begin{bmatrix}2 & 0\\ 0 & 3\end{bmatrix}$ (2M)

c) Write the matrix corresponding to the Quadratic form. (2M)
$x^2+y^2+z^2+4xy-2yz+6xz$

d) Evaluate $\int_0^1\int_0^x y\,dy\,dx$ (2M)

e) Find $\beta(2,2)$ (2M)

f) Write the two properties of gradient. (2M)

g) State Gauss Divergence theorem. (2M)

PART -B

2. a) Solve the system of equations $2x+y+2z+w=6, 6x-6y+6z+12w=36, 4x+3y+3z-3w=-1, 2x+2y-z+w=10$ by Gauss Elimination method. (7M)

b) Find the Rank of the matrix $A=\begin{bmatrix}2 & -2 & 0 & 6\\ 4 & 2 & 0 & 2\\ 1 & -1 & 0 & 3\\ 1 & -2 & 1 & 2\end{bmatrix}$ by reduce into Normal form. (7M)

3. a) Reduce the quadratic form $10x^2+2y^2+5z^2-4xy-10xz+6yz$ into canonical form and find its rank , index and signature. (7M)

b) Diagnolize the matrix $A=\begin{bmatrix}3 & -1 & 1\\ -1 & 5 & -1\\ 1 & -1 & 3\end{bmatrix}$ (7M)

4. a) Trace the curve $r^2=a^2\sin 2\theta$. (7M)

b) Find by double Integration, the volume of solid bounded by $z=0$, $x^2+y^2=1$, and $x+y+z=3$. (7M)
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5. a) Evaluate $\int_0^{\infty} xe^{-ax}\sin bx dx$ (7M)

b) Evaluate $\int_0^1 x^5(1-x)^3 dx$ (7M)

6. a) Find the directional derivative of the function $\phi = xy^2 + yz^3$ at the point $(2,-1,1)$ in the direction of the normal to the surface $x\log z - y^2 + 4 = 0$ at $(-1,2,1)$ (7M)

b) Show that $\overline{f} = \overline{a}\times\overline{r}$ is solenoidal where $\bar{a} = a_1\bar{\imath} + a_2\bar{\jmath} + a_3\bar{k}$ and $\bar{r} = x\bar{\imath} + y\bar{\jmath} + z\bar{k}$ (7M)

7. a) Evaluate $\int_s \phi\,\bar{n}\,ds$ where s is the surface $x^2 + y^2 = 16$ included in the first octant between $z = 0$ $z = 5$ where $\varphi = \frac{3}{8}xyz$. (7M)

b) Evaluate $\oint_c (x^2 + y^2)dx + 3xy^2 dy$ where c is the circle $x^2 + y^2 = 4$ in xy plane using Green's theorem. (7M)

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Time: 3 hours Max. Marks: 70

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2. Answer **ALL** the question in **Part-A**
3. Answer any **FOUR** Questions from **Part-B**

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PART –A

1. a) Write the working procedure to reduce the given matrix into Echelon form. (2M)

b) Find the Eigen value of the matrix $A = \begin{bmatrix} 4 & 2 \\ 1 & 5 \end{bmatrix}$. (2M)

c) Find the point of the curve $r = a\,(1 + \cos\theta)$ where tangent coincide with the radius vector. (2M)

d) Evaluate $\int_1^2\int_3^4 (xy + e^y)\,dx\,dy$ (2M)

e) Show that $\Gamma(n+1) = n\Gamma(n)$ for ` $n > 0$ (2M)

f) Find grad ϕ where $\phi = x^3 + y^3 + 3xyz$ at $(1,1,-2)$ (2M)

g) Find the work done in moving particle in the force field $\bar{F} = 3x^2\,\bar{i} + \bar{j} + z\bar{k}$ along the straight line (0, 0, 0) to (2, 1, 3). (2M)

PART -B

2. a) Reduce the matrix $A = \begin{bmatrix} 3 & -2 & 0 & -1 \\ 0 & 2 & 2 & 1 \\ 1 & -2 & -3 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$ in to normal form hence find the rank. (7M)

b) If consistent, solve the system of equations. (7M)

$$x + y + z + t = 4$$
$$x - z + 2t = 2$$
$$y + z - 3t = -1$$
$$x + 2y - z + t = 3.$$

3. a) Determine the diagonal matrix orthogonally similar to the matrix. (7M)

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

b) Find the Nature , index and signature of the quadratic form $10x^2 + 2y^2 + 5z^2 - 4xy - 10xz + 6yz$ (7M)
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4. a) By change of order of integration evaluate $\int_0^a \int_x^a (x^2 + y^2)\, dy\, dx$ (7M)

b) Evaluate $\int_0^{\pi/2} \int_0^{a \sin\theta} \int_0^{(a^2 - r^2)/a} r\, dr\, d\theta\, dz$ (7M)

5. a) Evaluate $\int_0^{\infty} 3^{-4x^2} dx$ (7M)

b) Show that $\int_0^{\infty} \sin x^2 dx = \int_0^{\infty} \cos x^2 dx = \frac{1}{2}\sqrt{\frac{\pi}{2}}$ (7M)

6. a) Show that $\overline{f} = r^n \left(\overline{a} \times \overline{r}\right)$ is solenoidal where $\bar{a} = a_1\bar{\imath} + a_2\bar{\jmath} + a_3\bar{k}$ and $\bar{r} = x\bar{\imath} + y\bar{\jmath} + z\bar{k}$ (7M)

b) Prove that $\nabla\left(r\nabla\left(\frac{1}{r^3}\right)\right) = \frac{3}{r^4}$ (7M)

7. a) Verify stoke's theorem for $\overline{F} = y\,\overline{i} + z\,\overline{j} + x\,\overline{k}$ for the upper part of the sphere $x^2 + y^2 + z^2 = 1$. (7M)

b) Verify Green's theorem in the plane for $\oint_c (xy + y^2)dx + x^2 dy$. Where c is the closed curve of the region bounded by y=x & y=x^2 (7M)

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2. Answer **ALL** the question in **Part-A**
3. Answer any **FOUR** Questions from **Part-B**

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PART –A

1. a) Write the working procedure to reduce the given matrix into Normal form. (2M)

b) Write quadratic form $x^2+3y^2+3z^2-2yz$ (2M)

c) Write the tangents at the origin of the curve $a^2y^2 = x^2(a^2 - x^2)$. (2M)

d) Evaluate $\int_0^1\int_0^1\int_0^1 dx\,dy\,dz$ (2M)

e) Prove that $\beta(m,n) = \int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta \cos^{2n-1}\theta d\theta$ (2M)

f) Find the maximum value of the directional derivative of $\phi = 2x^2 - y - z^4$ at $(2,-1,1)$ (2M)

g) Write Stoke's theorem. (2M)

PART -B

2. a) For what value of k the matrix A = $\begin{bmatrix} 4 & 4 & -3 & 1 \\ 1 & 1 & -1 & 0 \\ k & 2 & 2 & 2 \\ 9 & 9 & k & 3 \end{bmatrix}$ has rank 3. (7M)

b) Solve the following system of equations $\begin{array}{l} 8x - 3y + 2z = 20 \\ 4x + 11y - z = 33 \\ 6x + 3y + 12z = 35 \end{array}$ by using. Gauss – Seidel method. (7M)

3. a) Determine the characteristic roots and the corresponding characteristic vectors of the matrix. (7M)

$$A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$$

b) Find the Nature , index and signature of the quadratic form $4x^2+3y^2+z^2-8xy+4xz-6yz$ (7M)
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4. a) Trace the curve $r^2 = a^2 \cos 2\theta$ (7M)

b) Evaluate $\int\int (x^2 + y^2)\, dx\, dy$ over the area bounded by the Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (7M)

5. a) Evaluate $\int_0^\infty a^{-bx^2} dx \;\; b > 0, a > 1$ (7M)

b) Show that $\int_0^1 x^m (\log x)^n\, dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$ (7M)

6. a) Find the constants 'a' and 'b' such that the surfaces $5x^2$-2yz-9x=0 and $ax^2y+bz^3=4$ cuts orthogonally at (1,-1,2) (7M)

b) Show that the vector $\left(x^2 - yz\right)\bar{i} + \left(y^2 - zx\right)\bar{j} + \left(z^2 - xy\right)\bar{k}$ is irrotational and find its scalar potential. (7M)

7. a) If $\tilde{f} = (3x^2 - 2z)\bar{i} - 4xy\,\bar{j} - 5x\bar{k}$ Evaluate $\int_V Curl\ \bar{F}\ dv$, where v is volume bounded by the planes x = 0; $y = 0$; $z = 0$ and $3x + 2y - 3z = 6$. (7M)

b) Evaluate $\oint_c \cos y\, dx + x(1 - \sin y)\, dy$ over a closed curve c given by $x^2 + y^2 = 1$; $z = 0$ using Green's theorem. (7M)

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I B. Tech II Semester Regular/Supplementary Examinations, April/May - 2018

MATHEMATICS-III

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Time: 3 hours Max. Marks: 70

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2. Answer **ALL** the question in **Part-A**
3. Answer any **FOUR** Questions from **Part-B**

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PART –A

1. a) Write the working procedure to find the inverse of the given matrix by Jordan method. (2M)

b) Find the Eigen value of Adj A if the 'λ' is the Eigen value of A. (2M)

c) Write the symmetry of the curve $y^2(2a-x)=x^3$ (2M)

d) Evaluate $\int_0^3\int_{-x}^{x} xy\,dx\,dy$ (2M)

e) Find the value of $\beta\left(\frac{1}{2},\frac{1}{2}\right)$ (2M)

f) Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point $(2,-1,2)$. (2M)

g) Write the physical interpretation of Gauss divergence theorem. (2M)

PART -B

2. a) Reduce the matrix to Echelon form and find its rank $\begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$ (7M)

b) Solve the equations $\begin{array}{l} 10x+y+z=12, \\ 2x+10y+z=13, \\ x+y+5z=7. \end{array}$ by Gauss – Jordan method. (7M)

3. a) Find the Natural frequencies and normal modes of vibrating system for which mass $M=\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ and stiffness $K=\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$ (7M)

b) Verify Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$. Hence find A^{-1} (7M)
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4. a) Find the volume of region bounded by the surface $z = x^2 + y^2$ and $z = 2x$. (7M)

b) Evaluate $\int_0^a \int_0^{\sqrt{a^2 - x^2}} \sqrt{x^2 + y^2}\, dy\, dx$ by changing in to polar co-ordinates. (7M)

5. a) Show that $\int_a^b (x-a)^{m-1}(b-x)^{n-1} dx = (b-a)^{m+n-1}\beta(m,n) m > 0, n > 0$ (7M)

b) Evaluate $\int_0^1 \left(x \log x\right)^4 dx$ (7M)

6. a) Find the directional derivative of $\phi = xyz$ at (1,-1, 1) along the direction which makes equal angles with the positive direction of x , y , z axes (7M)

b) Prove that div $curl\,\overline{f} = 0$ (7M)

7. a) Verify Green's theorem for $\int_c (3x^2 - 8y^2)dx + (4y - 6xy)\,dy$ where c is the boundary of the region enclosed by the lines. $x = 0$ $y = 0$ $x + y = 1$. (7M)

b) Find the flux of vector function $\overline{F} = (x - 2z)\overline{i}\ (x + 3y)\overline{j} + (5x + y)\,\overline{k}$ through the upper side of the triangle ABC with vertices (1,0,0), (0,1,0), (0,0,1). (7M)

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Time: 3 hours Max. Marks: 70

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2. Answer **ALL** the question in **Part-A**
3. Answer any **FOUR** Questions from **Part-B**

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PART –A

1. a) Find the Rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ (2M)

b) Prove the AB and BA has same Eigen values. (2M)

c) Write the Asymptote of the curve **y=** $\frac{x^2+1}{x^2-1}$ (2M)

d) Evaluate $\int_0^3 \int_1^2 xy\,(x+y)\,dx\,dy$ (2M)

e) Show that $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$ (2M)

f) Show that $\nabla\left(r^2\right)=2\bar{r}$ (2M)

g) Write Green's theorem. (2M)

PART -B

2. a) Reduce the matrix $A=\begin{bmatrix} 1 & 2 & 3 & -2 \\ 2 & -2 & 1 & 3 \\ 3 & 0 & 4 & 1 \end{bmatrix}$ into PAQ form and hence find the rank of the matrix. (7M)

b) Solve the equations $\begin{matrix} x+y+z=8, \\ 2x+3y+2z=19 \\ 4x+2y+3z=23 \end{matrix}$ by Gauss – Elimination method. (7M)

3. a) Diagonalize the matrix $A=\begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ if possible. (7M)

b) Find the Nature, index and signature of the quadratic form $3x^2+5y^2+3z^2-2xy+2xz-2yz$ by orthogonal reduction. (7M)
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4. a) Trace the curve $x = a\cos t + \frac{a}{2}\log\tan^2 t/2,\quad y = a\sin t$ (7M)

b) Find the area between the circles $r = a\cos\theta$ and $r = 2a\cos\theta$. (7M)

5. a) Prove that $\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(a+bx)^{m+n}}dx = \frac{\beta(m,n)}{(a+b)^m a^n}$ (7M)

b) Evaluate $\int_0^\infty e^{-x^6} x^4 dx$ (7M)

6. a) Find the directional derivative of the function $e^{2x}\cos yz$ at the origin in the direction to the tangent to the curve $x = a\sin t, y = a\cos t, z = at$ at $t = \frac{\pi}{4}$ (7M)

b) Show that $curl\ curl\ \overline{f} = \nabla\times\left(\nabla\times\overline{f}\right) = \nabla\left(\nabla.\overline{f}\right) - (\nabla.\nabla)\overline{f}$ if $\overline{f}(x,y,z)$ is vector point function. (7M)

7. a) Verify Gauss Divergence theorem for $\overline{F} = (x^2 - yz)\overline{i} + (y^2 - zx)\overline{j} + (z^2 - xy)\overline{k}$ taken over the rectangular parallelepiped $0 \le x \le a\,;\, 0 \le y \le b\,;\, 0 \le z \le c$. (7M)

b) Evaluate $\iint_s (\nabla\times\overline{F}).\overline{n}\,ds$ where $\overline{F} = (x^2 + y - 4)\overline{i} + 3xy\,\overline{j} + (2xy + z^2)\overline{k}$ and s in the surface of the paraboloid $z = 4 - x^2 - y^2$ above the xy plane. (7M)

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2. Answering the question in **Part-A** is Compulsory
3. Answer any **FOUR** Questions from **Part-B**

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PART –A

1. a) Find the rank of the matrix by reducing it to normal form $\begin{bmatrix} 3 & 2 & 1 & 5 \\ 2 & 3 & 4 & 6 \\ 3 & 5 & 6 & 10 \end{bmatrix}$. (2M)

b) What is the nature of the quadratic form $x^2+y^2+z^2-2xy$? (2M)

c) Write the physical significance of grad φ. (2M)

d) Find the area bounded by the upper half of the curve $r = a(1-\cos\theta)$. (2M)

e) Prove that the work done in moving an object from P_1 to P_2 in a conservative force field $\overline{F}$ is independent of the path joining the two points P_1 and P_2. (2M)

f) Show that $\int_0^1 \left(\log\frac{1}{x}\right)^{m-1} dx = \Gamma(m)$. (2M)

g) Prove that the eigenvalues of A^{-1} are the reciprocals of the eigenvalues of A. (2M)

PART -B

2. a) Use Gauss Seidel method to solve 25x + 2y + 2z = 69, 2x + 10y + z = 63, x + y +z = 43. (6M)

b) Reduce the quadratic form $x^2+4y^2+z^2+4xy+6yz+2zx$ to canonical form by linear transformation. Also find signature and rank of the quadratic form. (8M)

3. a) Find the natural frequencies and normal modes of vibrating system for which the mass matrix is $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ and stiffness K = $\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$. (7M)

b) Verify Cayley-Hamilton theorem for A = $\begin{bmatrix} 2 & -1 & 2 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and find A^{-1}. (7M)

4. a) Trace the curve $x^3+y^3+3axy=0$. (7M)

b) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{x^2+y^2}\,dydx$ by changing into polar coordinates. (7M)

5. a) Express the integral $\int_0^\infty \frac{x^c}{c^x}dx$ in terms of Gamma function. (7M)

b) Show that $\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(x+a)^{m+n}}dx = \frac{B(m,n)}{a^n(1+a)^m}$. (7M)
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6. a) Find the directional derivative of the function f = $x^2 - y^2 + 2z^2$ at the point P=(1,2,3) in the direction of the line PQ where Q = (5,0,4). (7M)

b) Prove that $\nabla \times \left(\frac{\overline{A} \times \overline{r}}{r^n} \right) = \frac{(2-n)\overline{A}}{r^n} + \frac{n(\overline{r}.\overline{A})\overline{r}}{r^{n+2}}$. (7M)

7. If $\overline{F} = 4xz\overline{i} - y^2\overline{j} + yz\overline{k}$, evaluate $\int_S \overline{F}.\overline{n}ds$ where S is the surface of the cube bounded by x = 0, x = a, y = 0, y = a, z = 0, z = a. (14M)

Code No: R161203 **R16** SET - 2

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MATHEMATICS-III

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Time: 3 hours Max. Marks: 70

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2. Answering the question in **Part-A** is Compulsory
3. Answer any **FOUR** Questions from **Part-B**

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PART –A

1. a) Find the rank of the matrix A = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & -2 & 1 \\ 3 & 0 & 4 \end{bmatrix}$ by reducing it into echelon form. (2M)

b) What is the nature of the quadratic form $x^2-2y^2+z^2-2zy$? (2M)

c) Evaluate $\int_0^1 \sqrt[3]{\log\frac{1}{x}}\,dx$. (2M)

d) If λ is eigenvalue of an orthogonal matrix, then show that $\frac{1}{\lambda}$ is also an eigenvalue. (2M)

e) Find the area bounded by the curves $y = x$ and $y = x^2$. (2M)

f) In what direction from the point $(1,-1,3)$ the directional derivative of $\phi = 2xy + z^2$ is maximum? What is the magnitude of this maximum? (2M)

g) State Gauss divergence theorem. (2M)

PART –B

2. a) Solve by Gauss – Seidal method, the equations. (6M)

$$9x - 2y + z - t = 50$$
$$x - 7y + 3z + t = 20$$
$$-2x + 2y + 7z + 2t = 22$$
$$x + y - 2z + 6t = 18$$

b) Verify Cayley-Hamilton theorem for A = $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 4 \\ 2 & 1 & 3 \end{bmatrix}$ and find A^{-1}. (8M)

3. a) Find the natural frequencies and normal modes of vibrating system for which the mass matrix is $\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$ and stiffness K = $\begin{bmatrix} 6 & -2 \\ -2 & 9 \end{bmatrix}$. (7M)

b) Reduce the quadratic form $3x_1^2 + 3x_2^2 + 3x_3^2 + 2x_1x_2 + 2x_1x_3 - 2x_2x_3$ to orthogonal transformation. Also find signature and rank of the quadratic form. (7M)

4. a) Trace the curve $y^2(a+x) = x^2(a-x)$. (7M)

b) By changing the order of integration, evaluate $\int_0^1 \int_1^{2-x} xy\,dx\,dy$. (7M)
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5. a) Evaluate $\int_0^1 (8-x^3)^{1/3}dx$ using β and γ functions. (7M)

b) Prove that $\Gamma\left(n+\frac{1}{2}\right)=\frac{\sqrt{\pi}\,\Gamma(2n+1)}{2^{2n}\Gamma(n+1)}$. (7M)

6. a) Find the directional derivative of $\phi = x^2yz+4xz^2$ at (1, -2, -1) in the direction of $2\bar{i}-\bar{j}-2\bar{k}$. (7M)

b) Prove that curl $(\bar{a}\times\bar{b}) = \bar{a}div\bar{b} - \bar{b}div\bar{a} + (\bar{b}.\nabla)\bar{a} - (\bar{a}.\nabla)\bar{b}$. (7M)

7. Verify Stoke's theorem for $\overline{F}=(2x-y)\bar{i}-yz^2\bar{j}-y^2z\bar{k}$ over the upper half of surface of sphere $x^2 + y^2 + z^2 = 1$ bounded by the projection of the xy- plane. (14M)

I B. Tech II Semester Regular/Supplementary Examinations, April/May - 2019

MATHEMATICS-III

(Com to AE.AME,CE,CSE,IT,EIE,EEE,ME,ECE,Metal E, Min E, E Com E, Agri E, Chem E, PCE,PE)

Time: 3 hours Max. Marks: 70

Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B)**
2. Answering the question in **Part-A** is Compulsory
3. Answer any **FOUR** Questions from **Part-B**

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PART –A

1. a) Find the rank of the matrix by reducing it to normal form $\begin{bmatrix} 1 & 7 & 8 & 1 \\ 1 & 3 & 4 & 2 \\ 3 & 5 & 6 & 10 \\ & & & \end{bmatrix}$. (2M)

b) What is the nature of the quadratic form $-2x^2+2y^2-z^2-2xy$? (2M)

c) Find the complete area of the curve $a^2y^2 = x^3(2a-x)$. (2M)

d) Evaluate $\int_0^{\frac{\pi}{2}} \sin^4\theta\cos^2\theta d\theta$. (2M)

e) If λ is an eigenvalue of a nonsingular matrix A, then show that $\frac{|A|}{\lambda}$ is an eigenvalue of adj A. (2M)

f) In what direction from the point (2, –1, 1) the directional derivative of $\phi = xy^2 + yz^3$ is maximum. What is the magnitude of this maximum? (2M)

g) State Stoke’s theorem. (2M)

PART -B

2. a) Apply Guass – Seidel method to solve the equations. (6M)

$$27x + 6y - z = 85$$
$$x + y + 54z = 110$$
$$6x + 15y + 2z = 72$$

b) Verify Cayley-Hamilton theorem for A = $\begin{bmatrix} 10 & 1 & 1 \\ 1 & 10 & -1 \\ 1 & -2 & 10 \end{bmatrix}$ and find A^{-1}. (8M)

3. a) Find the natural frequencies and normal modes of vibrating system for which the mass matrix is $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ and stiffness K = $\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$. (7M)

b) Reduce the quadratic form $4x^2+3y^2+z^2-8xy-6yz+4zx$ to orthogonal transformation. Also find signature and rank of the quadratic form. (7M)

4. a) Find the perimeter of the loop of the curve $3ay^2 = x(x-a)^2$. (7M)

b) By changing the order of integration, evaluate $\int_0^3 \int_1^{\sqrt{4-y}} (x+y)dxdy$. (7M)
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5. a) Evaluate $\int_0^\infty \frac{x^2}{1+x^4}dx$ using β and γ functions. (7M)

b) Show that $\Gamma(m)\Gamma\left(m+\frac{1}{2}\right)=\frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$. (7M)

6. a) Find the angle between the normal to the surface $x^2 = yz$ at the points (1, 1, 1) and (2, 4, 1). (7M)

b) Find the constants a, b, c so that $(x+2y+az)\bar{i}+(bx-3y-z)\bar{j}+(4x+cy+2z)\bar{k}$ is irrotational. Also find the scalar potential. (7M)

7. Verify Green's theorem for $\int_C (xy+y^2)dx+(x^2)dy$ where C is the curve bounded by y = x^2 and y = x. (14M)

Code No: R161203 **R16** SET - 4

I B. Tech II Semester Regular/Supplementary Examinations, April/May - 2019

MATHEMATICS-III

(Com to AE.AME,CE,CSE,IT,EIE,EEE,ME,ECE,Metal E, Min E, E Com E, Agri E, Chem E, PCE,PE)

Time: 3 hours Max. Marks: 70

Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B)**
2. Answering the question in **Part-A** is Compulsory
3. Answer any **FOUR** Questions from **Part-B**

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PART –A

1. a) Find the rank of the matrix by reducing it to normal form $\begin{bmatrix} 2 & 6 & 8 & 2 \\ 1 & 3 & 4 & 1 \\ 3 & 5 & 6 & 10 \end{bmatrix}$ (2M)

b) What is the nature of the quadratic form x^2-$3y^2$-z^2-zy ? (2M)

c) Prove that zero is an eigen value of a matrix if and only if it is singular. (2M)

d) In what direction from the point $(1,-2,-1)$ the directional derivative of $\phi = x^2yz + 4xz^2$ is maximum? What is the magnitude of the maximum? (2M)

e) Show that in an irrotational field, the value of a line integral between two points A and B will be independent of the path of integration and be equal to their potential difference. (2M)

f) Find the area bounded by the curves $y = x^2$ and $x = y^2$. (2M)

g) Show that $\int_0^\infty x^{n-1}e^{-kx}dx = \frac{\Gamma(n)}{k^n}$ (2M)

PART -B

2. a) Verify Cayley-Hamilton theorem for A = $\begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & 1 \\ 1 & 2 & 1 \end{bmatrix}$ and find A^{-1}. (6M)

b) Apply Guass – Seidel method to solve the equations (8M)

$$20x + y - 2z = 17,$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

3. a) Find the natural frequencies and normal modes of vibrating system for which the mass matrix is $\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$ and stiffness K = $\begin{bmatrix} 6 & -2 \\ -2 & 9 \end{bmatrix}$. (7M)

b) Using Lagrange's reduction, transform $x_1^2 + 2x_2^2 - 7x_3^2 - 4x_1x_2 + 8x_1x_3$. (7M)

4. a) Trace the curve $x^{2/3} + y^{2/3} = a^{2/3}$. (7M)

b) By changing the order of integration, evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{a^2 - x^2 - y^2}\, dydx.$ (7M)
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5. a) Show that $\Gamma(m)\Gamma\left(m+\frac{1}{2}\right)=\frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$. (7M)

b) Express the integral $\int_0^\infty \frac{x^c}{c^x}dx$ in terms of Gamma function. (7M)

6. a) Find the angle of intersection of the spheres $x^2 + y^2 + z^2 = 39$ and $x^2 + y^2 + z^2 + 4x - 6y - 8z + 52 = 0$ at the point (4, -3, 2). (7M)

b) Prove that $grad(\overline{a}.\overline{b}) = (\overline{b}.\nabla)\overline{a} + (\overline{a}.\nabla)\overline{b} + \overline{b}\times curl\overline{a} + \overline{a}\times curl\overline{b}$. (7M)

7. Verify Gauss divergence theorem for $\overline{F} = x^2\overline{i} + y^2\overline{j} + z^2\overline{k}$, over the cube formed by the planes x = 0, x = a, y = 0, y = b, z = 0, z = c. (14M)

Code No: R161203 **R16** SET - 1

I B. Tech II Semester Regular Examinations, April/May – 2017
MATHEMATICS-III
(Com. to CE, EEE, ME, ECE, CSE, CHEM, EIE, IT, ECC, AE, AME, MM, PE, PCE, MET, AGE)
Time: 3 hours Max. Marks: 70

Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B)**
2. Answering the question in **Part-A** is Compulsory
3. Answer any **FOUR** Questions from **Part-B**

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PART –A

1. a) Find the rank of a matrix $A = \begin{bmatrix} -1 & 2 & 1 & 8 \\ 2 & 1 & -1 & 0 \\ 3 & 2 & 1 & 7 \end{bmatrix}$ (2M)

b) Prove that if λ is an eigen value of a matrix A then λ^{-1} is an eigen value of the matrix A^{-1} if it exists. (2M)

c) Evaluate $\int_0^1 \int_0^1 \int_{\sqrt{x^2+y^2}}^{y} xyz\, dzdydx$. (2M)

d) Find the value of $\Gamma\left(\frac{5}{2}\right)$. (2M)

e) Find the angle between the surface $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at (2, -1, 2). (2M)

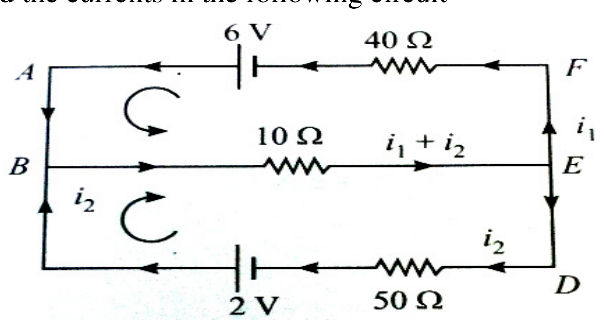
f) If $\bar{F} = (5xy - 6x^2)\bar{\imath} + (2y - 4x)\bar{\jmath}$ then evaluate $\int \bar{F}.\overline{dR}$ along the curve $y = x^3$ from the point (1, 1) to (2, 8). (2M)

g) Write the quadratic form corresponding to the symmetric matrix $\begin{bmatrix} 1 & 0 & 4 \\ 0 & -2 & -1 \\ 4 & -1 & 3 \end{bmatrix}$. (2M)

PART –B

2. a) Solve the system of equations $20x + y - 2z = 17, 3x + 20y - z = -18, 2x - 3y + 20z = 25$ by Gauss Jacobi method. (7M)

b) Find the currents in the following circuit (7M)



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3. a) Verify Cayley-Hamilton theorem and find the inverse of the matrix (7M)
$A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$

b) Reduce the quadratic form $2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx$ to canonical form by orthogonal transformation and hence find rank, index, signature and nature of the quadratic form. (7M)

4. a) Trace the curve $r^2 = a^2 cos2\theta$. (7M)

b) Evaluate $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} x\, y^2 dydx$ by changing the order of integration. (7M)

5. a) Express $\int_0^1 x^m (1 - x^n)^p\, dx$ in terms of Γ functions and hence evaluate $\int_0^1 x^5 (1 - x^3)^{10}\, dx$. 6M)

b) Evaluate $\int_0^{\frac{\pi}{2}} sin^5\theta\; cos^7\theta\;\; d\theta$ by using β, Γ functions. (4M)

c) Express $\int_0^4 \sqrt{x}(4 - x)^{3/2}\, dx$ in terms of β function. (4M)

6. a) Show that the vector field $\bar{F} = (x^2 - yz)\bar{\imath} + (y^2 - zx)\bar{\jmath} + (z^2 - xy)\,\bar{k}$ is conservative and find the scalar potential function corresponding to it. (7M)

b) Show that $\nabla.(\bar{F} \times \bar{G}) = \bar{G}.(\nabla \times \bar{F}) - \bar{F}.(\nabla \times \bar{G})$ (7M)

7. State Stoke's theorem and verify the theorem for $\overline{F} = (x + y)\bar{\imath} + (y + z)\bar{\jmath} - x\bar{k}$ and S is the surface of the plane $2x + y + z = 2$, which is in the first octant. (14M)

Code No: R161203 **R16** SET - 2

I B. Tech II Semester Regular Examinations, April/May – 2017
MATHEMATICS-III
(Com. to CE, EEE, ME, ECE, CSE, CHEM, EIE, IT, ECC, AE, AME, MM, PE, PCE, MET, AGE)
Time: 3 hours Max. Marks: 70

Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B)**
2. Answering the question in **Part-A** is Compulsory
3. Answer any **FOUR** Questions from **Part-B**

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PART –A

1. a) Determine the rank of a matrix $A = \begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$. (2M)

b) Use Cayley-Hamilton theorem to find A^8 if $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$. (2M)

c) Evaluate $\int_0^1 \int_0^1 \int_0^y xyz\, dxdydz$. (2M)

d) Find the value of $\Gamma\left(-\frac{5}{2}\right)$. (2M)

e) Find unit normal vector to the surface $x^2y + 2xz^2 = 8$ at the point (1, 0, 2). (2M)

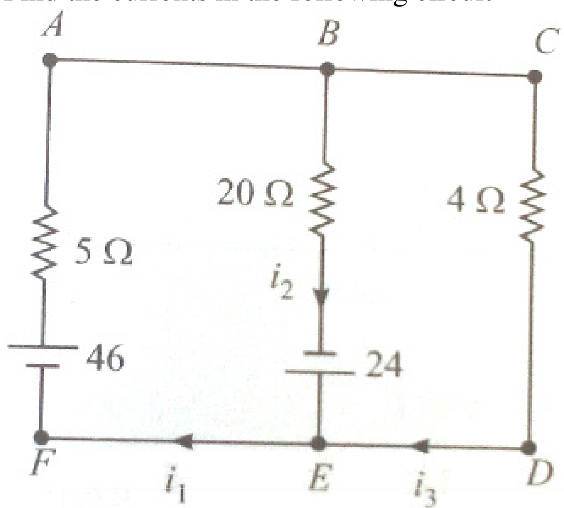
f) If $\bar{F} = (3x^2 + 6y)\bar{\imath} - 14yz\,\bar{\jmath} + 20xz\bar{k}$ then evaluate $\int \bar{F}.\overline{dR}$ from (0, 0, 0) to (1, 1, 1) along the path $x = t, y = t^2, z = t^3$. (2M)

g) Write the quadratic form corresponding to the symmetric matrix
$\begin{bmatrix} 0 & 5/2 & 3 \\ 5/2 & 7 & 1 \\ 3 & 1 & 2 \end{bmatrix}$ (2M)

PART –B

2. a) Show that the system of equations is consistent $2x - y - z = 2, x + 2y + z = 2, 4x - 7y - 5z = 2$ and solve. (7M)

b) Find the currents in the following circuit (7M)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3\. a) Reduce the quadratic form $6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_1x_3$ to canonical form and hence state nature, rank, index and signature of the quadratic form. (7M)

b) Determine the natural frequencies and normal modes of a vibrating system for which mass $m = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ and stiffness $k = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$. (7M)

4\. a) Trace the curve $y^2(2a - x) = x^3$. (7M)

b) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\, dxdy$ by changing in to polar coordinates and hence deduce $\int_0^\infty e^{-x^2}\, dx = \frac{\sqrt{\pi}}{2}$. (7M)

5\. a) Show that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. 6M)

b) Evaluate $\int_0^{\frac{\pi}{2}} sin^4\theta\ cos^2\theta\ d\theta$ by using β, Γ functions. (4M)

c) Express $\int_0^1 \frac{1}{(1-x^3)^{1/3}} dx$ in terms of β function. (4M)

6\. a) Show that the vector field $\bar{F} = (x^2 + xy^2)\bar{\imath} + (y^2 + x^2y)\bar{\jmath}$ is conservative and find the scalar potential function. (7M)

b) Show that $\nabla(\nabla.\bar{F}) = \nabla \times (\nabla \times \bar{F}) + \nabla^2\bar{F}$. (7M)

7\. State Greens theorem in plane and verify the theorem for $\oint_C [(y - sinx)dx + cosx\ dy]$, where C is the plane triangle formed by the lines $y = 0, x = \frac{\pi}{2}$, $y = \frac{2}{\pi}x$. (14M)

I B. Tech II Semester Regular Examinations, April/May – 2017
MATHEMATICS-III
(Com. to CE, EEE, ME, ECE, CSE, CHEM, EIE, IT, ECC, AE, AME, MM, PE, PCE, MET, AGE)
Time: 3 hours Max. Marks: 70

Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B)**
2. Answering the question in **Part-A** is Compulsory
3. Answer any **FOUR** Questions from **Part-B**

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PART –A

1. a) Determine the rank of a matrix $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$. (2M)

b) Use Cayley-Hamilton theorem and find A^{-1} if $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$. (2M)

c) Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{a\sin\theta} \int_0^{\frac{(a^2-r^2)}{a}} r\, dzdr\, d\theta$. (2M)

d) Show that $\beta\left(\frac{1}{2},\frac{1}{2}\right) = \pi$. (2M)

e) Find directional derivative of $\phi = xy^2 + yz^2$ at the point (2,-1,1) in the direction of the vector $\bar{\imath} + 2\bar{\jmath} + 2\bar{k}$. (2M)

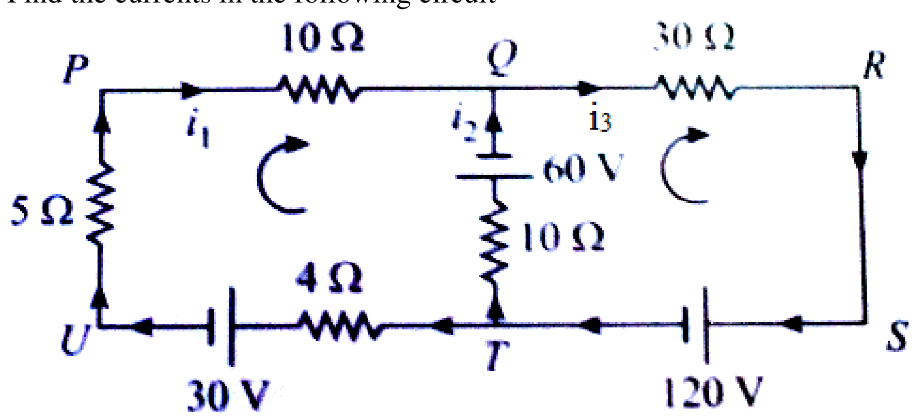
f) If $\bar{F} = (x^2 - y)\bar{\imath} + (2xz - y)\bar{\jmath} + z^2\bar{k}$ then evaluate $\int \bar{F}.\overline{dR}$ where C is the straight line joining the points (0, 0, 0) to (1, 2, 4). (2M)

g) Write the quadratic form corresponding to the symmetric matrix $\begin{bmatrix} 3 & 5 & 0 \\ 5 & 5 & 4 \\ 0 & 4 & 7 \end{bmatrix}$. (2M)

PART –B

2. a) Solve the system of equations $10x + y + z = 12,\ 2x + 10y + z = 13, 2x + 2y + 10z = 14$ by Gauss Seidel method. (7M)

b) Find the currents in the following circuit (7M)



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3. a) Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2xy - 2yz + 2zx$ to canonical form by orthogonal transformation and hence find the rank, index signature and nature of the quadratic form. (7M)

b) Find the natural frequencies and normal modes of a vibrating system $mx'' + kx = 0$ for mass $m = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ and stiffness $k = \begin{bmatrix} 9 & -3 \\ -3 & 3 \end{bmatrix}$. (7M)

4. a) Trace the curve $a^2y^2 = x^2(a^2 - x^2)$. (7M)

b) Evaluate $\int_0^1 \int_{\sqrt{y}}^{2-y} xy\, dxdy$ by changing the order of integration. (7M)

5. a) Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. 6M)

b) Evaluate $\int_0^{\frac{\pi}{2}} sin^5\theta \ d\theta$ by using β, Γ functions. (4M)

c) Express $\int_0^1 \frac{x\, dx}{\sqrt{1+x^4}}$ in terms of β function. (4M)

6. a) Find the constants a, b such that the surfaces $5x^2 - 2yz - 9x = 0$ and $ax^2y + bz^3 = 4$ cut orthogonally at (1,-1,2). (7M)

b) Show that $\nabla \times (\nabla \times \bar{F}) = \nabla(\nabla . \bar{F}) - \nabla^2 \bar{F}$. (7M)

7. State Gauss divergence theorem in plane and verify the theorem for $\bar{F} = 4xz\bar{\imath} - y^2\,\bar{\jmath} + zy\bar{k}$ over the cube $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. (14M)

I B. Tech II Semester Regular Examinations, April/May – 2017
MATHEMATICS-III
(Com. to CE, EEE, ME, ECE, CSE, CHEM, EIE, IT, ECC, AE, AME, MM, PE, PCE, MET, AGE)
Time: 3 hours Max. Marks: 70

Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B)**
2. Answering the question in **Part-A** is Compulsory
3. Answer any **FOUR** Questions from **Part-B**

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PART –A

1. a) Find inverse of the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$ by elementary operations. (2M)

b) Prove that if λ is an eigen value of a matrix A then $\frac{|A|}{\lambda}$ is an eigen value of adjA. (2M)

c) Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x\, dzdxdy$. (2M)

d) Determine the value of $\beta(2,3)$. (2M)

e) Show that $\nabla fg = f\,\nabla g + g\,\nabla f$. (2M)

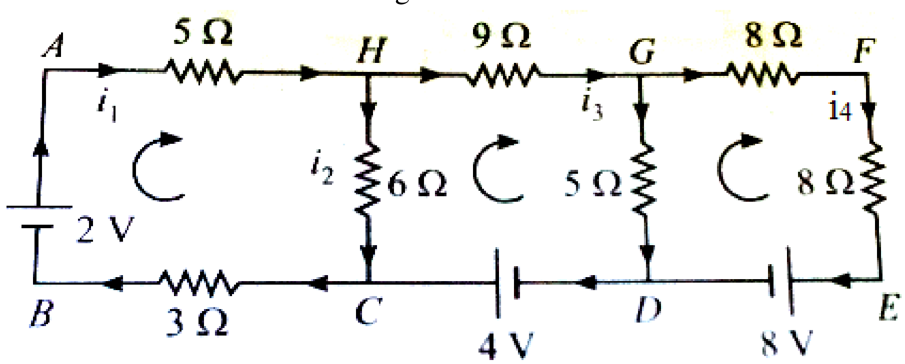
f) If $\bar{F} = x^2y^2\bar{\imath} + y\,\bar{\jmath}$ then evaluate $\int_C \bar{F}.\overline{dR}$ where C is the curve $y^2 = 4x$ in the XY plane from (0, 0) to (4, 4). (2M)

g) Write the quadratic form corresponding to the symmetric matrix (2M)
$\begin{bmatrix} 2 & -3 & 5 \\ -3 & 2 & -2 \\ 5 & -2 & 2 \end{bmatrix}$

PART –B

2. a) Solve the system of equations (7M)
$x + 10y + z = 6,\ 10x + y + z = 6, x + y + 10z = 6$ by Gauss Seidel method.

b) Find the currents in the following circuit (7M)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3. a) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ and hence find A^4 . (7M)

   b) Reduce the quadratic form $3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_1x_2 - 2x_1x_3 + 2x_2x_3$ to canonical form and hence state nature, rank, index and signature of the quadratic form. (7M)

4. a) Trace the curve $r = a\, sin3\theta$. (7M)

   b) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} y\sqrt{x^2+y^2}\; dydx$ by transforming to polar coordinates. (7M)

5. a) Establish a relation between β and Γ functions. 6M)

   b) Evaluate $\int_0^{\frac{\pi}{2}} cos^7\theta\; d\theta$ by using β, Γ functions. (4M)

   c) Express $\int_0^1 \frac{x\,dx}{\sqrt{1-x^5}}$ in terms of β function. (4M)

6. a) Find the angle between the surfaces $ax^2 + y^2 + z^2 - xy = 1$ and conservative $bx^2y + y^2z + z = 1$ at (1, 1, 0). (7M)

   b) Show that $\bar{F} = (y^2 - z^2 + 3yz - 2x)\,\bar{\imath} + (3xz + 2xy)\bar{\jmath} + (3xy - 2xz + 2z)\bar{k}$ is both solenoidal and irrotational. (7M)

7. a) State Greens theorem in plane and apply the theorem to evaluate $\oint_C x^2y\,dx + y^3dy$, where C is the closed path formed by $y = x,\; y = x^3$ from (0, 0) to (1, 1). (7M)

   b) Evaluate $\int_S \bar{F}.\overline{ds}$ using Gauss divergence theorem, where $\bar{F} = 2xy\,\bar{\imath} + yz^2\,\bar{\jmath} + z\,\bar{k}$ and S is the surface of the region bounded by $x = 0,\; y = 0,\; z = 0,$ $x + 2z = 6$. (7M)