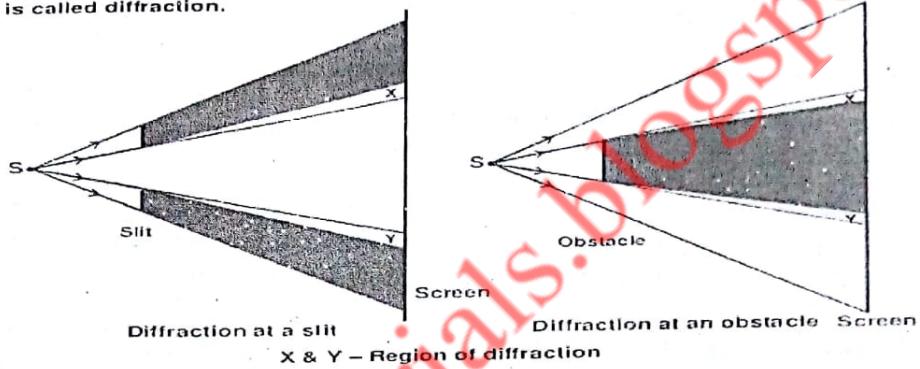


2. DIFFRACTION

"When light is incident on the obstacles or small apertures whose size is comparable to wavelength of light, then there is a departure from straight line propagation, the light bends round the corners of the obstacles and enters into geometrical shadow. This bending of light is called diffraction."

Diffraction of light:

The phenomenon of bending of light around the corners and the encroachment of light within the geometrical shadow of the opaque obstacles is called diffraction.

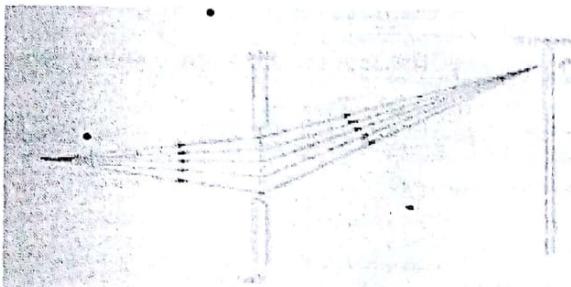
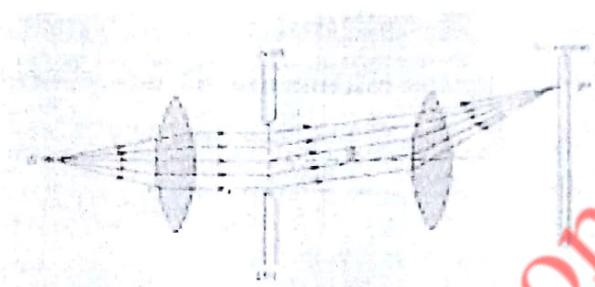


Differences between Interference and diffraction

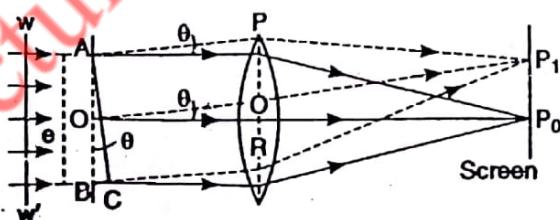
INTERFERENCE	DIFFRACTION
1. Superposition is due to two separate wave fronts originating from two coherent sources.	1. Superposition is due to secondary wavelets originating from different parts of same wave front.
2. Interference fringes may or may not be of same width.	2. Diffraction fringes are not of the same width
3. Points of minimum intensity are perfectly dark	3. Points of minimum intensity are not perfectly dark.
4. All bright bands are of uniform intensity	4. All bright bands are not of same intensity.

✓ There are two types of Diffractions are there, they are

1. Fresnel Diffraction
2. Fraunhofer Diffraction

Fresnel diffractionFraunhofer diffractionDifferences between Fresnel Diffraction and Fraunhofer Diffraction

Fresnel Diffraction	Fraunhofer Diffraction
1. Either a point source or an illuminated narrow slit is used.	1. Extended source at infinite distance is used.
2. The wave front undergoing diffraction is either spherical or cylindrical.	2. The wave front undergoing diffraction is plane wave front.
3. The source and screen are at finite distances from the obstacle.	3. The source and screen are at infinite distances from the obstacle.
4. No lens is used to focus the rays.	4. Converging lens is used to focus the rays.

FRAUNHOFER DIFFRACTION AT SINGLE SLIT:

(Consider a slit AB of width "e" and a plane wave front WW' of monochromatic light of wavelength " λ " is incident normally on the slit. The diffracted light through the slit is focused with the help of a convex lens on a screen. The screen is placed at the focal plane of the lens. Here the secondary wave lets spared out to the right in all directions.)

The waves travelling along OP_0 are brought out to focus at P_0 by the lens. Hence P_0 is the bright central image.

The secondary wavelets at angle " θ " with normal are focused at P_1 on the screen. Depending upon path difference, P_1 may be of maximum (or) minimum intensity point.

(To find intensity at P_1 we draw a normal AC from A to the light ray at B the path difference between the wave lets from A and B in the direction " θ " is given by

$$\text{From Triangle } ABC, \sin \theta = \frac{BC}{AB} \Rightarrow BC = AB \sin \theta = e \sin \theta$$

$$\therefore \text{Phase difference} = \frac{2\pi}{\lambda} e \sin \theta$$

Let us consider the width of the slit is divided into 'n' equal parts. Then the phase difference between any two consecutive waves from these parts would be.

$$\frac{1}{n} (\text{total phase}) = \frac{1}{n} \left(\frac{2\pi}{\lambda} \cdot e \sin \theta \right) = d \quad (\text{say})$$

$$\text{Resultant amplitude } R = \frac{a \sin \frac{nd}{2}}{\sin \frac{nd}{2}}$$

$$\therefore R = \frac{a \sin \left(\frac{n}{2} \times \frac{2\pi}{n\lambda} \cdot e \sin \theta \right)}{\sin \left(\frac{2\pi}{2n\lambda} \cdot e \sin \theta \right)}$$

$$= \frac{a \sin \left(\frac{\pi e \sin \theta}{\lambda} \right)}{\sin \left(\frac{\pi e \sin \theta}{n\lambda} \right)}$$

Let $\alpha = \frac{\pi e \sin \theta}{\lambda}$. Then

$$R = \frac{a \sin \alpha}{\sin \frac{\alpha}{n}}$$

As $\frac{\alpha}{n}$ is small, $\sin \frac{\alpha}{n} \approx \frac{\alpha}{n}$

$$\therefore R = \frac{a \sin \alpha}{\frac{\alpha}{n}} = na \frac{\sin \alpha}{\alpha}$$

Now the intensity

$$I = R^2 = A^2 \left(\frac{\sin \alpha}{\alpha} \right)^2 \dots\dots (1)$$

Principal Maximum:

$$R = A \frac{\sin \alpha}{\alpha} = \frac{A}{\alpha} \left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + \dots \right]$$

$$= A \left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + \dots \right]$$

The value of R will be maximum, when $\alpha=0$, i.e $\frac{\pi e \sin \theta}{\lambda} = 0$ or $\sin \theta = 0$

Or $\theta = 0$

$$\therefore \text{Phase difference} = \frac{2\pi}{\lambda} e \sin \theta$$

Let us consider the width of the slit is divided into 'n' equal parts. Then the phase difference between any two consecutive waves from these parts would be.

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$$= \frac{a \sin \left(\frac{\pi e \sin \theta}{\lambda} \right)}{\sin \left(\frac{\pi e \sin \theta}{n\lambda} \right)}$$

$$\text{Let } \alpha = \frac{\pi e \sin \theta}{\lambda}. \text{ Then}$$

$$R = \frac{a \sin \alpha}{\sin \frac{\alpha}{n}}$$

As $\frac{\alpha}{n}$ is small, $\sin \frac{\alpha}{n} \approx \frac{\alpha}{n}$

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Now the intensity

$$I = R^2 = A^2 \left(\frac{\sin \alpha}{\alpha} \right)^2 \dots\dots (1)$$

Principal Maximum:

$$R = A \frac{\sin \alpha}{\alpha} = A \frac{\alpha}{\alpha} \left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + \dots \right]$$

$$= A \left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + \dots \right]$$

The value of R will be maximum, when $\alpha=0$, i.e $\frac{\pi e \sin \theta}{\lambda} = 0$ or $\sin \theta = 0$

Or $\theta = 0$

\therefore Maximum intensity $I=R^2=A^2$, this is occurred at $\theta=0$, this maximum is known as principal maximum.

Minimum intensity Positions:

The intensity will be minimum, when $\sin \alpha=0$.

$$\therefore \alpha = \pm\pi, \pm 2\pi, \pm 3\pi, \dots, \pm m\pi$$

$$\alpha = \pm m\pi$$

$$\frac{\pi e \sin \theta}{\lambda} = \pm m\pi$$

$$e \sin \theta = \pm m\lambda$$

In this way, we obtain the points of min. inf. on either side of the principle maxima.

Secondary maxima:

In addition to principle maxima at $\alpha=0$. There are weak secondary maxima between equally spaced minima. The points of secondary maxima obtained as follows.

$$I = A^2 \left(\frac{\sin \alpha}{\alpha} \right)^2$$

$$\frac{dI}{d\alpha} = A^2 \cdot 2 \frac{\sin \alpha}{\alpha} \cdot \frac{\alpha \cos \alpha - \sin \alpha}{\alpha^2} = 0$$

From above either $\sin \alpha=0$, or $\alpha \cos \alpha - \sin \alpha = 0$ if $\sin \alpha=0$, it is min. intensity position.

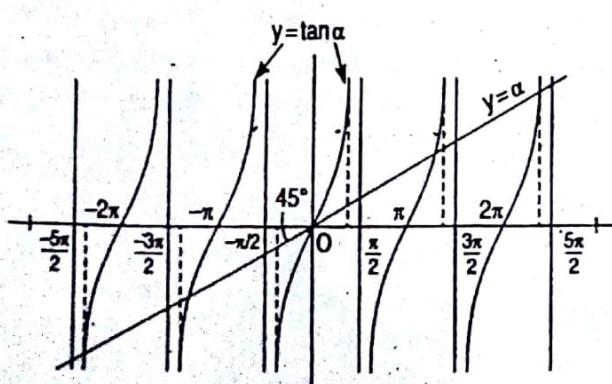
Hence positions of maximum are obtained by

$$\alpha \cos \alpha - \sin \alpha = 0$$

$$\alpha \cos \alpha = \sin \alpha$$

$$\alpha = \alpha \quad \text{----- (2)}$$

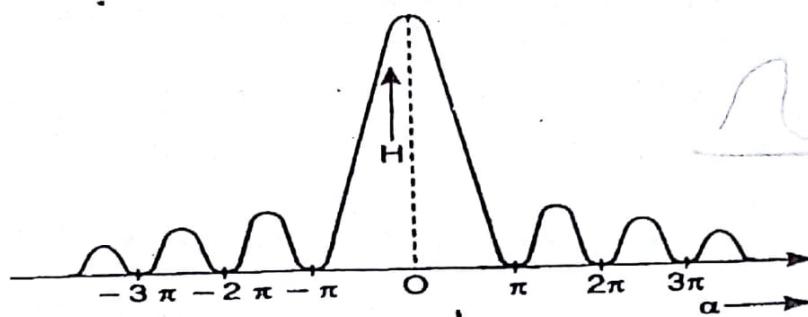
The values of α satisfying the above equation are obtained graphically by plotting curves $y=\alpha$, $y=\tan \alpha$ on the same graph. The points of intersection of two curves give the values of α which satisfy the equation (2)



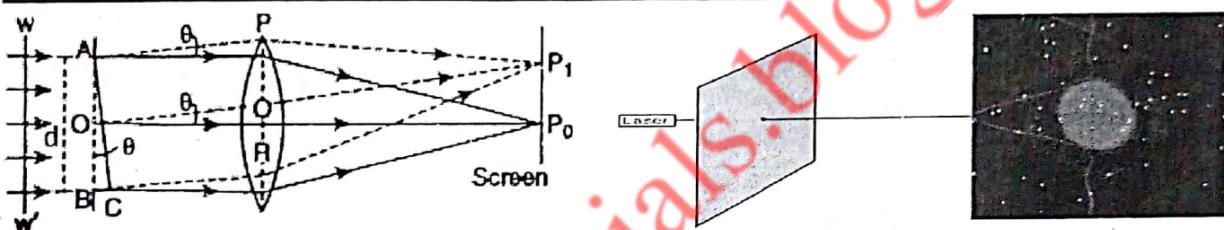
From fig

The points of intersections are $\alpha = 0, \pm 3\frac{\pi}{2}, \pm 5\frac{\pi}{2}, \dots$, at these points we get secondary maxima

Intensity distribution graph



Fraunhofer diffraction at circular aperture



✓ Consider an aperture AB of diameter "d" and a plane wave front WW' of monochromatic light of wavelength " λ " is incident normally on the circular aperture. The diffracted light through the aperture is focused with the help of a convex-lens on a screen. The screen is placed at the focal plane of the lens. Here the secondary wavelets spread out to the right in all directions.

The secondary wavelets at angle " θ " with normal are focused at P_1 on the screen. Depending upon path difference, P_1 may be of maximum (or) minimum intensity point.

✓ Let $P_0P_1 = x$. We draw a normal AC from A to the light ray at B. The path difference between the wavelets from A and B in the direction " θ " is given by

From Triangle ABC,

$$\sin \theta = \frac{BC}{AB} \Rightarrow BC = AB \sin \theta = d \sin \theta \quad \checkmark$$

* In the same way as in the case of single slit the point ' P_1 ' will be minimum intensity if the path difference is an integral multiple of ' λ ' and of maximum intensity if the path difference is odd multiple of ' $\lambda/2$ '. i.e;

$$d \sin \theta = m\lambda \dots \text{minima}$$

$$d \sin \theta = (2m+1)\lambda/2 \dots \text{maxima.}$$

The diffraction pattern may be considered by rotating intensity distribution graph of single slit about the central axis passing through ' P_0 ' about which the circular aperture is perfectly symmetrical. The point P_0 traces out a circular ring of uniform illumination.

Thus the diffraction pattern consists of a central bright disc, called Airy's disc surrounded by alternate dark and bright concentric rings, called Airy's rings.

The intensity of dark ring is zero and that of the bright rings decreases gradually outwards from P_0 .

The angular radius θ of the Airy's disk, i.e., the angular separation between the centre of the bright disc and the first dark ring is given by

$$\theta = \frac{1.22\lambda}{d}$$

Where λ is the wave length of the incident light and e be the diameter of the circular aperture. ✓

If the collecting lens is very near to the circular aperture and the screen is large distance x from the lens, then

$$\sin \theta = \theta = \frac{x}{f} \quad \dots \dots \dots \quad (1)$$

Here f is the focal length of lens

Further, for the first secondary minimum

$$d \sin\theta = 1.2$$

from eqs (1) and (2)

$$\frac{x}{f} = \frac{\lambda}{d}$$

$$x = \frac{f\lambda}{d}$$

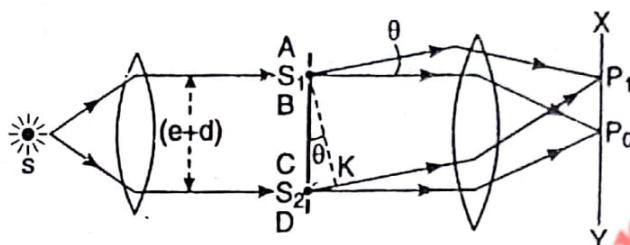
Where 'x' is the radius of the Airy's disc. It was shown by Airy, which exact value of x is given by

$$x = \frac{1.22f\lambda}{d}$$

Thus, if the diameter of the aperture is large, the radius of the central disc is small.

This is used to importance in the discussion of the resolving powers of telescope and microscope.

FRAUNHOFFER DIFFRACTION AT DOUBLE SLIT:



Let A B and CD be two parallel slits of equal width 'e' separated by an opaque distance d . The distance between the corresponding middle points of the two slits is $(e+d)$. Let a parallel beam of monochromatic beam of wave length λ be incident normally upon to the two slits.

When a wave front is incident normally on both slits all the points within the slits become the sources of secondary wavelets. The secondary waves traveling in the direction of incident light come to focus at P_0 while the secondary waves traveling in the direction making an angle with θ the incident light come to focus at P_1 .

According to the theory of diffraction at a single slit. The amplitude R due to all the wavelets diffracted from each slit in a dissection θ is given by.

$$R = A \frac{\sin \alpha}{\alpha} \text{ where } \alpha = \frac{\pi e \sin \theta}{\lambda}$$

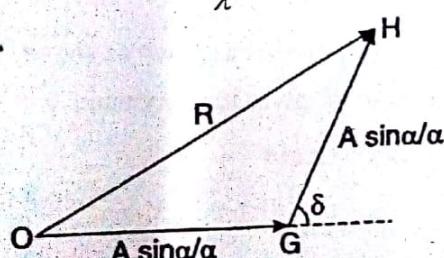
Thus for simplicity we can take two slits as equivalent to two sources S_1 and S_2 placed at mid points of the slits and each slit sending a wavelet of amplitude $\frac{A \sin \alpha}{\alpha}$ in the direction θ .

Resultant amplitude at a point P_1 on the screen will be a result of interference between two waves of amplitude $\frac{A \sin \alpha}{\alpha}$ and having a phase difference.

The path difference between the wavelets from S_1 and S_2 in the dissection $\theta = S_2 G$.

$$\text{path.difference} = (e+d) \sin \theta$$

$$\therefore \text{phase.difference}(\delta) = \frac{2\pi}{\lambda} (e+d) \sin \theta$$



From figure $R \cos \theta = \frac{A \sin \alpha}{\alpha} + \frac{A \sin \alpha}{\alpha} \cos \delta \dots\dots\dots(1)$

$$R \sin \theta = \frac{A \sin \alpha}{\alpha} \sin \delta \dots\dots\dots(2)$$

Squaring & adding eq(1)&(2)

$$\begin{aligned} I = R^2 &= \left(\frac{A \sin \alpha}{\alpha}\right)^2 + \left(\frac{A \sin \alpha}{\alpha}\right)^2 \cos^2 \delta + 2 \left(\frac{A \sin \alpha}{\alpha}\right)^2 \cos \delta \left(\frac{A \sin \alpha}{\alpha}\right)^2 \sin \delta \\ &= \left(\frac{A \sin \alpha}{\alpha}\right)^2 [2 + 2 \cos \delta] \\ &= 2 \left(\frac{A \sin \alpha}{\alpha}\right)^2 [1 + \cos \delta] \\ &= 2 \left(\frac{A \sin \alpha}{\alpha}\right)^2 2 \cos^2 \frac{\delta}{2} \\ I &= 4 \left(\frac{A \sin \alpha}{\alpha}\right)^2 \cos^2 \beta \dots\dots\dots(3), \text{ Where } \beta = \frac{\delta}{2} = \frac{\pi}{\lambda} (e+d) \sin \theta \end{aligned}$$

Discussion of Intensity:

From equation (3) the resultant intensity depending upon the following two factors.

1. $A^2 \frac{\sin^2 \alpha}{\alpha^2}$ Which is same as the intensity in the case of single slit diffraction thus it gives intensity distribution in the diffraction pattern.
2. $\cos^2 \beta$ Which gives the intensity pattern due to two waves interfere.

The resultant intensity at any point on the screen is given by the product of these two factors.

\therefore Diffraction term $\frac{\sin^2 \alpha}{\alpha^2}$ gives the

- (i) Central maximum at $\theta = 0$
- (ii) Minimum intensity positions $\alpha = \pm m\pi$

$$\frac{\pi e \sin \theta}{\lambda} = \pm m\pi$$

$$e \sin \theta = \pm m\lambda$$

- (iii) Secondary maxima obtained at $\alpha = \pm 3\frac{\pi}{2}, \pm 5\frac{\pi}{2}, \dots\dots\dots$

On taking these three points plotted as graph as shown in the fig(a).

The interference term $\cos^2 \beta$ gives the maximum

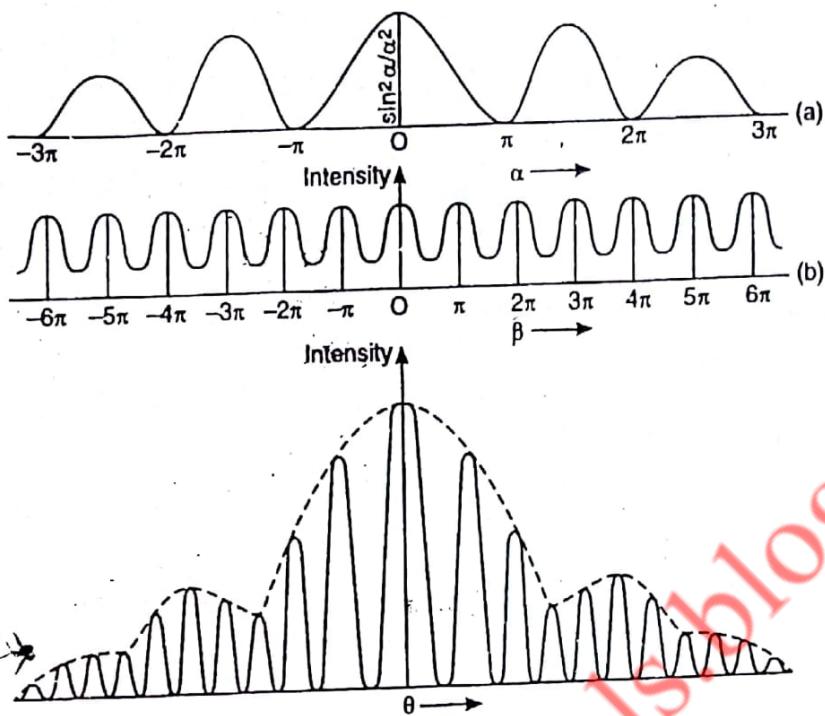
$$\cos^2 \beta = 1 \Rightarrow \beta = \pm m\pi$$

$$\frac{\pi}{\lambda} (e+d) \sin \theta = \pm m\pi$$

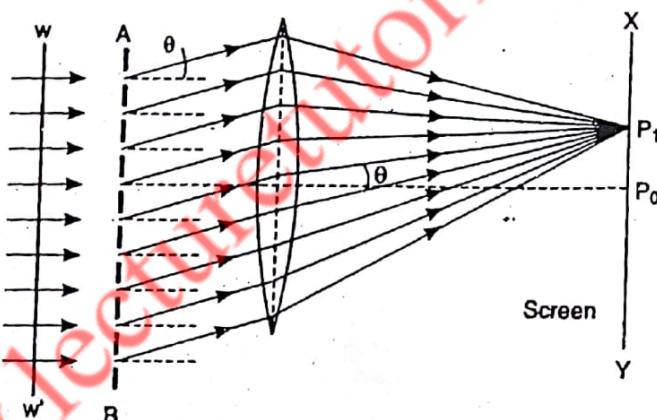
$$(e + d) \sin \theta = \pm m\lambda$$

This is plot as shown in fig.(b)

The resultant intensity graph is as shown in fig. (c)



Diffraction at N-Parallel slits [Diffraction grating]



An arrangement consists of large no. of parallel slits of same width and separated by equal opaque spaces is known as diffraction grating.

If there are N slits.

The path difference between any two consecutive slits is $= (e + d) \sin \theta$

$$\therefore \text{Phase difference} = \frac{2\pi}{\lambda} (e + d) \sin \theta = 2\beta$$

By the method of vector addition of amplitudes

$$R = \frac{a \sin \frac{nd}{2}}{\sin \frac{d}{2}}$$

$$\text{In this case } a = \frac{A \sin \alpha}{\alpha}, n = N \text{ and } d = 2\beta$$

$$\therefore R = \frac{A \sin \alpha}{\alpha} \cdot \frac{\sin N\beta}{\sin \beta}$$

$$I = R^2 = \left(\frac{A \sin \alpha}{\alpha} \right)^2 \frac{\sin \beta}{\beta} \left(\frac{\sin N\beta}{\sin \beta} \right)^2$$

The factor $\left(\frac{A \sin \alpha}{\alpha} \right)^2$ gives the distribution of intensity due to single slit. While the factor $\left(\frac{\sin N\beta}{\sin \beta} \right)^2$ gives the distribution of intensity as a combined effect of all the slits.

Principle maxima:

The intensity will be maximum when $\sin \beta = 0$

$$\beta = \pm n\pi, n = 0, 1, 2, 3, \dots$$

But at the same time $\sin N\beta = 0$. So that the factor $\left(\frac{\sin N\beta}{\sin \beta} \right)$ becomes indeterminate.

$$\therefore \lim_{\beta \rightarrow n\pi} \frac{\sin N\beta}{\sin \beta} = \lim_{\beta \rightarrow n\pi} \frac{N \cos N\beta}{\cos \beta} = \pm N$$

$$\lim_{\beta \rightarrow n\pi} \left(\frac{\sin N\beta}{\sin \beta} \right)^2 = N^2$$

$$\therefore \text{The Resultant intensity } I = \left(\frac{A \sin \alpha}{\alpha} \right)^2 N^2$$

i.e. The principle maxima obtained for $\beta = \pm n\pi$

$$\frac{\pi(e+d) \sin \theta}{\lambda} = \pm n\pi$$

$$(e+d) \sin \theta = \pm n\lambda$$

Minimum Intensity Positions:

Intensity I is the minimum when $\sin N\beta = 0$, but $\sin \beta \neq 0$

$$\therefore N\beta = \pm \pi, \pm 2\pi, \pm 3\pi, \dots$$

$$\frac{N\pi(e+d) \sin \theta}{\lambda} = \pm m\pi$$

$$N(e+d) \sin \theta = \pm m\lambda$$

Where m having all values except

$0, N, 2N, \dots, nN$.

i.e., $m = 1, 2, \dots, (N-1), (N+1), \dots, (2N-1), (2N+1), \dots$

Secondary maximum:

1 maximum when

$$\frac{dI}{d\beta} = 0$$

$$\frac{d}{d\beta} \left[\left(A \frac{\sin \alpha}{\alpha} \right)^2 \left(\frac{\sin N\beta}{\sin \beta} \right)^2 \right] = 0$$

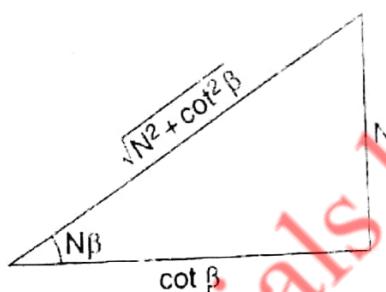
$$\left(\frac{A \sin \alpha}{\alpha} \right)^2 2 \left[\frac{\sin N\beta}{\beta} \right] \left[\frac{N \sin \beta \cos N\beta - \sin N\beta \cos \beta}{\sin^2 \beta} \right] = 0$$

$$N \sin \beta \cos N\beta - \sin N\beta \cos \beta = 0$$

$$N \sin \beta \cos N\beta = \sin N\beta \cos \beta$$

$$N \sin \beta = \cos \beta \left(\frac{\sin N\beta}{\cos N\beta} \right)$$

$$\tan N\beta = \frac{N}{\cot \beta}$$



$$\therefore \sin N\beta = \frac{N}{\sqrt{(N^2 + \cot^2 \beta)}}$$

$$\begin{aligned} \frac{\sin^2 N\beta}{\sin^2 \beta} &= \frac{N^2}{(N^2 + \cot^2 \beta) \sin^2 \beta} \\ &= \frac{N^2}{N^2 \sin^2 \beta + \cos^2 \beta} \\ &= \frac{N^2}{N^2 \sin^2 \beta + 1 - \sin^2 \beta} \end{aligned}$$

$$\therefore \frac{\sin^2 N\beta}{\sin^2 \beta} = \frac{N^2}{1 + (N^2 - 1) \sin^2 \beta}$$

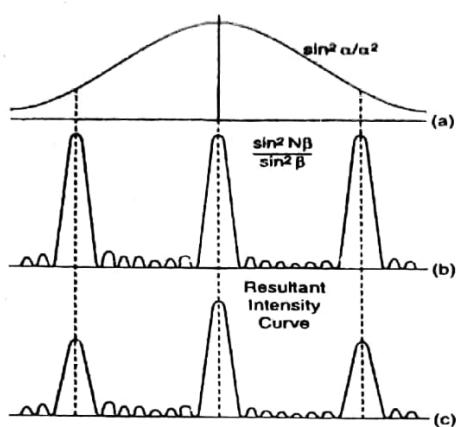
$$I_{\text{sec}} = \left(A \frac{\sin \alpha}{\alpha} \right)^2 \left[\frac{N^2}{(N^2 - 1) \sin^2 \beta + 1} \right]$$

intensity of secondary maxima
intensity of principle maxima

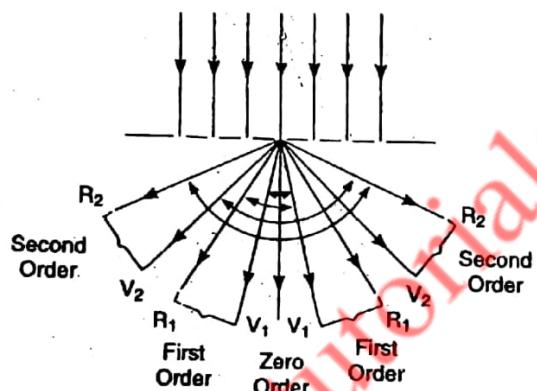
$$= \frac{N^2}{(1 + (N^2 - 1) \sin^2 \beta) \times N^2}$$

$$\therefore \frac{\text{intensity of secondary maxima}}{\text{intensity of principle maxima}} = \frac{1}{1 + (N^2 - 1) \sin^2 \beta}$$

From this we conclude that as the value of N increases the intensity of secondary maxima will decreases



GRATING SPECTRA



We know that the principle maxima in a grating are formed in a direction θ is given by

$$(e + d) \sin \theta = \pm n\lambda$$

Where $(e + d)$ grating element is θ is the angle diffraction and λ is Wave length

From the above equation, we conclude that

1. For a particular wave length λ , the angle of diffraction θ is different for different orders.
2. For white light and for an order n the light of different wave lengths will be diffracted in different directions. The longer the wavelength, greater is the angle of diffraction. So violet color being in the innermost position and red color in the outermost position.
3. Most of the intensity goes to zero order and rest is distributed among other orders thus the spectra become fainter as we go to higher orders.

Characteristics of grating spectra

1. Spectrum of different orders are situated symmetrically on both sides of zero order
2. Spectral lines are almost straight and quite sharp.
3. Spectral colors are in the order from violet to red.
4. Most of the intensity goes to zero order and rest is distributed among the other orders.

Maximum no. orders available with a grating

The principle maxima in grating satisfying the condition

$$(e + d) \sin \theta = n\lambda$$

$$n = \frac{(e+d)\sin\theta}{\lambda}$$

$$n_{\max} = \frac{(e+d)\sin 90^\circ}{\lambda}$$

$$n_{\max} = \frac{(e+d)}{\lambda}$$

RALEIGH'S CRITERION OF RESOLUTION:

According to the Rayleigh's criterion, two sources are resolvable by an optical instrument when the principle maximum in the diffraction pattern of one falls on the first minimum of the other and vice versa.

Let us consider the resolution of two wave lengths λ_1 and λ_2 by grating.

1. WELL RESOLVED

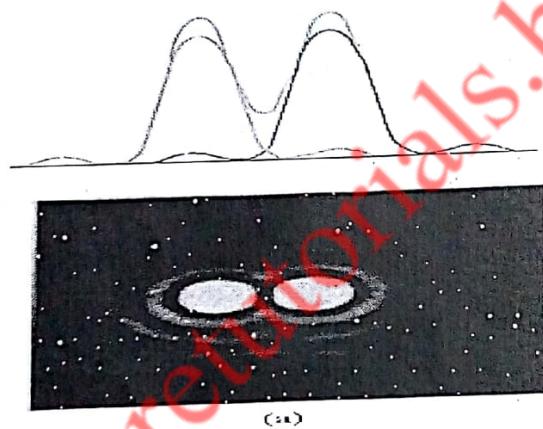
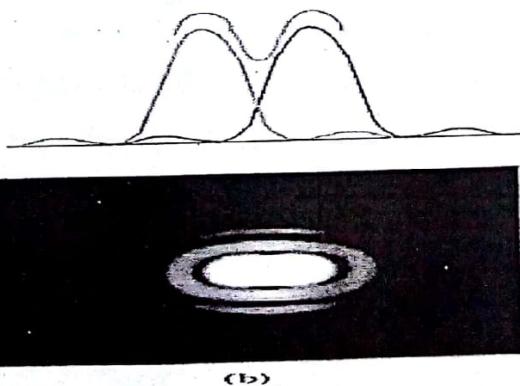


Fig (a) shows the diffraction patterns of two wavelengths. The difference in wavelengths is such that their principle maxima are separately visible. Hence the two wavelengths are well resolved.

2. JUST RESOLVED

Now consider the difference of wave lengths is smaller such that the principle maximum of one coincides with first minimum of the other as shown in the fig.

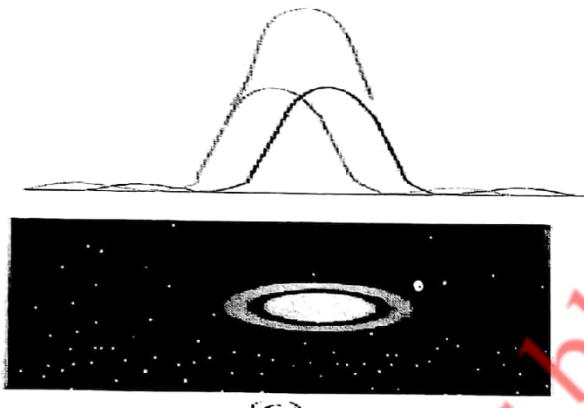
Principle maximum



The resultant density curve shows a distinct dip at the middle of the two central maxima. Thus two wave lengths can be distinguished from one another and according to Raleigh they are said to just resolved.

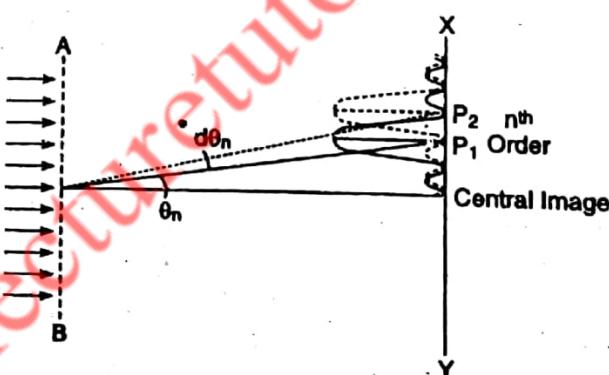
3.UN RESOLVED

Again consider the case when the difference in wave lengths is so small that central maxima corresponding to two wave lengths come closure. The resultant intensity curve is quite smooth without any dip. Thus it gives the impression as there is only one wave length. Hence the two wave lengths are not resolved.



RESOLVING POWER OF GRATING:

The resolving power of a grating is defined as the capacity to form separate diffraction maxima of two wave lengths which are very close to each other



Let A B be a plane grating having grating element $(e + d)$ and N be the total no. of slits. let a beam of wavelengths λ and $\lambda + d\lambda$ is normally incident on the grating in the fig P_1 is the n_{th} primary maximum of wavelength λ at an angle of diffraction θ_n and P_2 is the n_{th} primary maximum of wavelength $\lambda + d\lambda$ at an angle of diffraction $(\theta_n + d\theta_n)$.

According to Rayleigh's criterion, the two wave lengths will be resolved if the principle maximum of one falls on the first minimum of the other.

The principle maximum of λ in the direction θ_n is given by

$$(e + d) \sin \theta_n = \pm n\lambda \dots\dots\dots(1)$$

The wave length $(\lambda + d\lambda)$ form its n_{th} primary maxima in the direction $(\theta_n + d\theta_n)$

$$(e+d)\sin(\theta_n + d\theta_n) = \pm n(\lambda + d\lambda) \dots\dots\dots(2)$$

The first minimum of wave length λ from in the direction $(\theta_n + d\theta_n)$

$$N(e+d)\sin(\theta_n + d\theta_n) = (nN+1)\lambda \dots\dots\dots(3)$$

Multiplying eq(2) with N

$$N(e+d)\sin(\theta_n + d\theta_n) = \pm nN(\lambda + d\lambda) \dots\dots\dots(4)$$

From (3) & (4)

$$Nn(\lambda + d\lambda) = (nN+1)\lambda$$

$$nN\lambda + nNd\lambda = nN\lambda + \lambda$$

$$nNd\lambda = \lambda$$

$$\frac{\lambda}{d\lambda} = nN$$

$$\text{But from eq (1)} \quad n = \frac{(e+d)\sin\theta_n}{\lambda}$$

\therefore Resolving power of grating

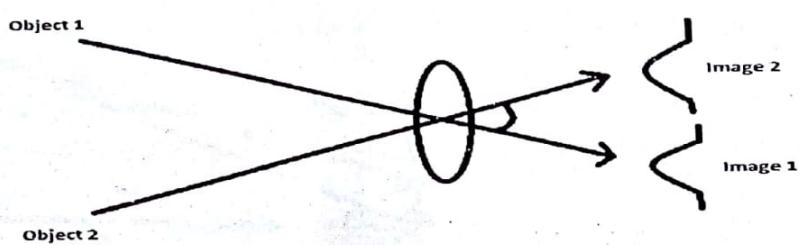
$$\frac{\lambda}{d\lambda} = \frac{N(e+d)\sin\theta_n}{\lambda}$$

RESOLVING POWER OF TELESCOPE

The resolving power of the telescope is defined as the inverse of the angular separation between two objects which can be just resolved when viewed through the telescope.

According Rayleigh criterion

$$\text{Resolving power of the telescope} = \frac{1}{\Delta\theta}$$



Consider two point sources subtended very small angle at the telescope objective lens of diameter D . each source produces a Fraunhofer diffraction pattern in the field of view of telescope. The diffraction pattern of each point source is an Airy pattern.

If Airy discs of two diffraction patterns do not overlap, the two images can be easily distinguished.

But if they overlap, it is not possible to see them separate.

According to Rayleigh criterion, if Airy disks corresponding to their images are such that, the first min. of one coincides with the first max. of the other and vice versa. Then the two images to be just resolved.

The minimum resolvable angle separation $\Delta\theta$ is given by

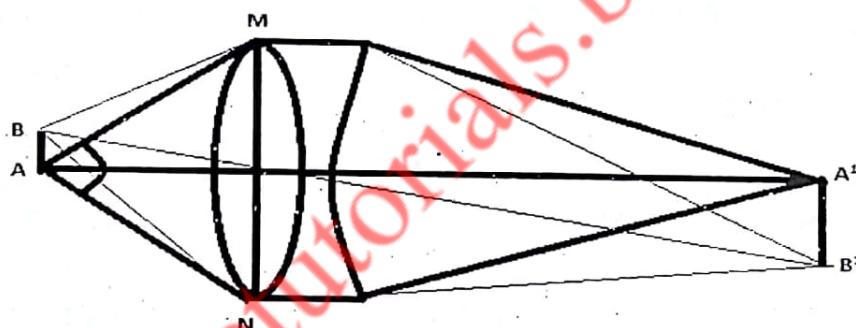
$$\Delta\theta = \frac{1.22\lambda}{D}$$

$$\text{Resolving power of telescope} = \frac{1}{\Delta\theta} = \frac{D}{1.22\lambda}$$

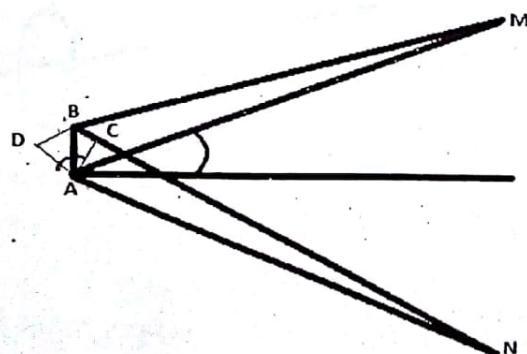
RESOLVING POWER OF MICROSCOPE

The resolving power of the microscope is defined as the reciprocal of the linear distance between two microscopic objects that can be just resolved when seen through the microscope.

$$\text{Resolving power} = \frac{1}{d}$$



The path difference between the light ray reaching from B to A' is given by



$$\Delta = (BN + NA') - (BM + MA')$$

$$\text{But } NA' = MA'$$

~~$$\Delta = BN - BM$$~~

$$= (BC + CN) - (DM - DB)$$

But $CN = AN = AM = DM$

$$\Delta = BC + DB$$

From triangle ACB & ADB

$$\sin \theta = \frac{BC}{AB} \Rightarrow BC = AB \sin \theta = d \sin \theta \quad , \quad \text{By } BD = d \sin \theta$$

$$\Delta = 2d \sin \theta$$

If $2d \sin \theta = 1.22\lambda$, then A' is the first minimum of the image B' . then two images resolved

$$d = \frac{1.22\lambda}{2 \sin \theta}$$

$$\frac{1}{d} = \frac{2 \sin \theta}{1.22\lambda}$$