National Cheng Kung University

Mechanical Engineering

Stochastic Dynamic Data - Analysis and Processing

Butterworth Filter

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Contents

1 Introduction

The Butterworth filter is a type of signal processing filter designed to have as flat a frequency response as possible in the passband. It is also referred to as a maximally flat magnitude filter.

2 Filter Design

The frequency response gain is

$$G(\omega) = \sqrt{\frac{1}{1+\omega^{2n}}}$$

The transfer function of the filter is

$$|H(j\omega)| = \frac{G_0^2}{1+(\omega/\omega_c)^2}$$

where n is order of filter, ω_c is cutoff frequency, and G_0 is DC gain.
In matlab, we generate it by intrinsic function. We designate the cut-off frequency as 1 rad/sec(fig2) and implment for order 1, 2 and 5.(fig1)

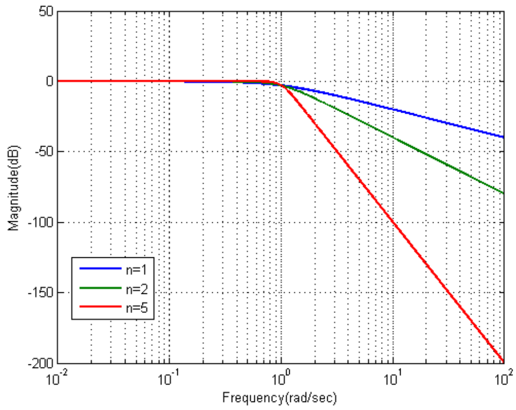


Figure 1: Different Order Filters

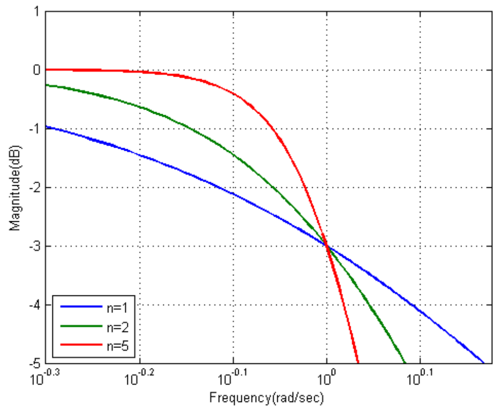


Figure 2: Cut-off freq is 1 rad/sec

3 Frequency Spectrum Analysis

3.1 Non-Stationary Data

After system identification, we could determine the time constant of filters.

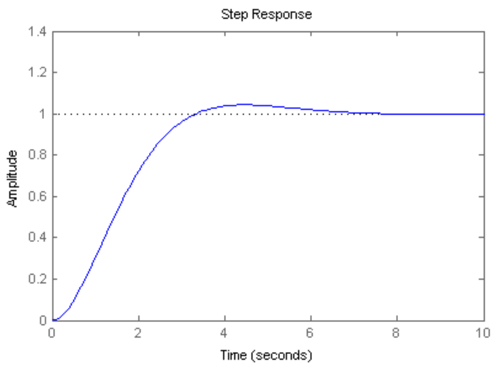


Figure 3: $n = 2$

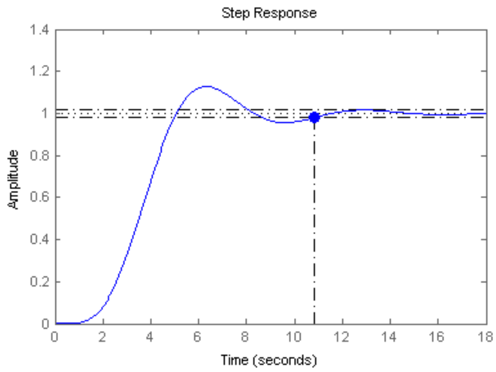
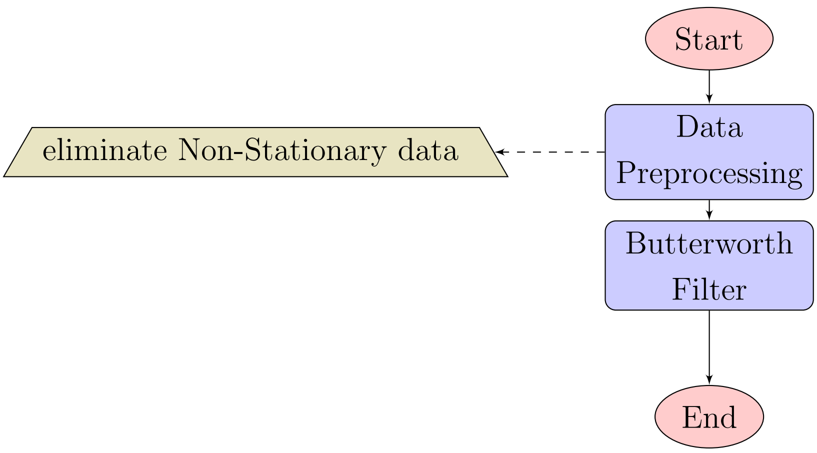


Figure 4: $n = 5$

Filter Order	$n = 1$	$n = 2$	$n = 5$
Settling Time	3.91 sec	5.96 sec	10.8 sec

For convience, we keep same data length and get rid of data before 12 sec. It is easier to do signal processing.

3.2 Filtering Procedure



3.3 FFT

The environment setting and data infomation.

Sampling Rate	Total Data length	N for FFT
5 Hz	10,000	1024

The original normal-distributed data in frequency domain like

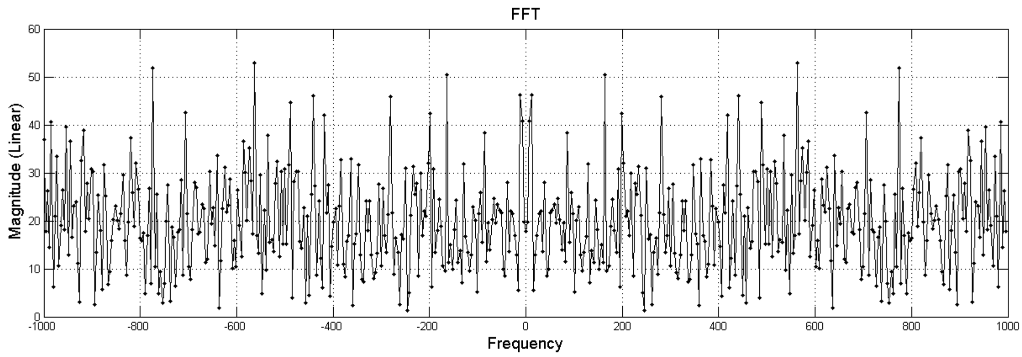


Figure 5: Gasussian Data in Spectrum

It contains rich frequencies information. The filter helps us kill the high frequency part

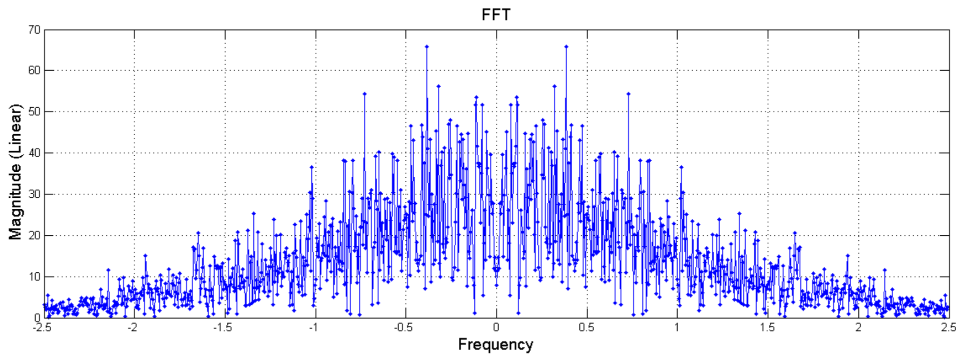


Figure 6: first Order Butterworth filter

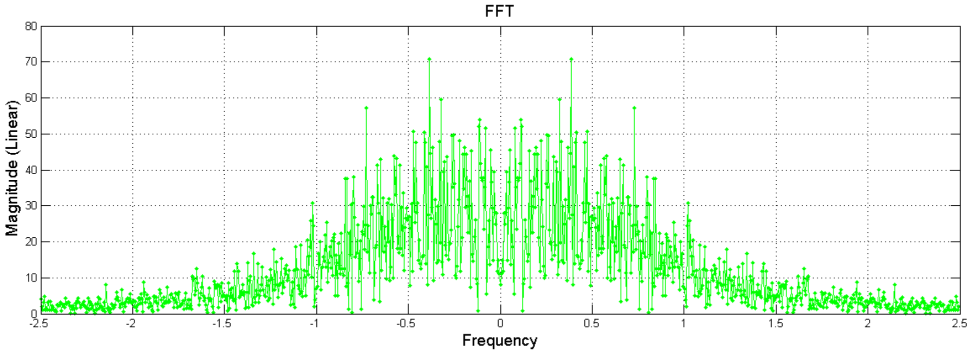


Figure 7: second Order Butterworth filter

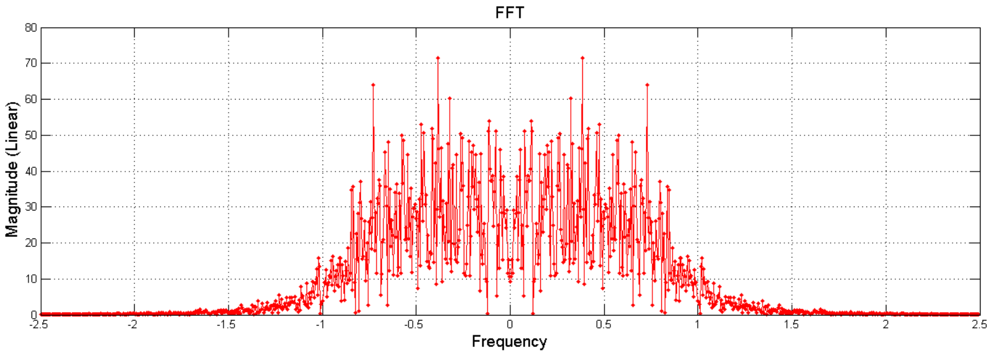


Figure 8: fifth Order Butterworth filter

4 Discussion

According to our design, the cutoff frequency is the same unity. So the spectrum looks similar but the high frequency portion is different. The higher order filter can wipe them out thoroughly. We can find this by plot spectrum and Bode plot together.

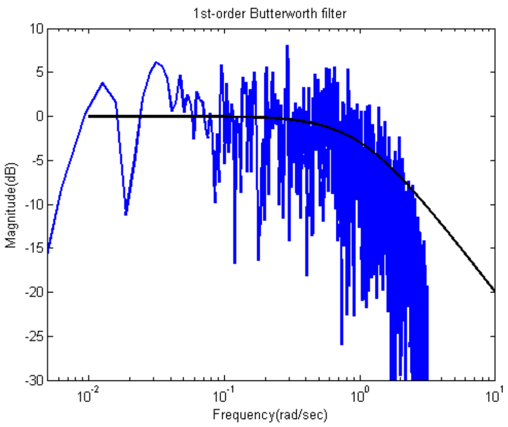


Figure 9: 1st order

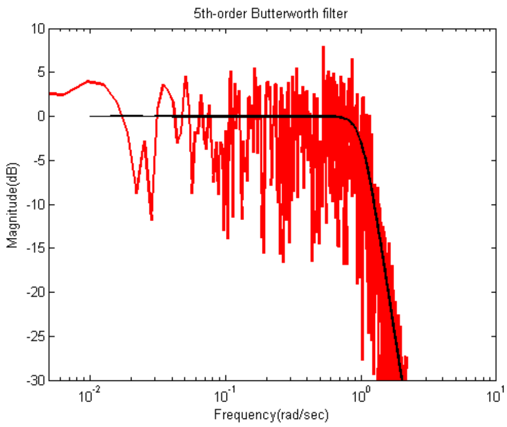


Figure 10: 5th order

References

[1] Matlab : User guide

[2] Wikipedia : Butterworth filter, 2012

[3] Filter Design - Audio signal processing , 2012