National Cheng Kung University

Mechanical Engineering

Stochastic Dynamic Data - Analysis and Processing

Power Spectral Density

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1 Introduction

The report will test three methods of finding power spectral density. Compare them with two kinds of signal.

- Gaussian white process $w[n]$
- White process + Sinusoidal wave $y[n]$

Then we try several different MATLAB function and discuss one of them. It is the Welch's method.

2 Data Source and Specifications

2.1 Measuring System

The measuring system is a second order low-pass filter.

$$H(s) = \frac{0.25}{s^2 + 0.7071s + 0.25}$$

Bandwidth	Damping Ratio	Natural Frequency
0.4979 Hz	0.7071	0.5 Hz

Table 1: Specs of the Measuring System

2.2 Signal

Type	Characteristics	Data length	Sampling Rate
Gaussian White Process	mean = 0, deviation = 1	1,000	1 Hz
Sinusoidal Wave	Amplitude=1, frequency =0.3 Hz	1,000	1 Hz

Table 2: Specs of the Signal used in the report

3 Spectral Density Function

The section, we implement three methods to find power spectral density.

3.1 Narrow Bandwith Filter Method

Apply the narrow band filter as

$$H(s) = \frac{sk}{(s-\omega_0)(s-\omega_0^*)}$$

where $\omega_0 = -\pi B + j2\pi f_0$ and $\omega_0^* = -\pi B - j2\pi f_0$. We choose The procedure the

Bandwidth B	Static Gain k	Cutoff frequency f_0
0.001	0.01	0.5 Hz

Table 3: Specs of Narrow Bandpass filter

algorithm is showed below.

Figure 1: NBPF Method

In analytical form

$$\hat{G_{xx}}(f_k) = \frac{1}{B_e M}\sum_{i=0}^{M} |y_i^{(k)}|^2$$

where

$$y_i^{(k)} = \sum_{l=0}^{N} b(l)^{(k)} x_{i-l} - \sum_{i=0}^{N} a(l)^{(k)} y_{i-l}^{(k)}$$

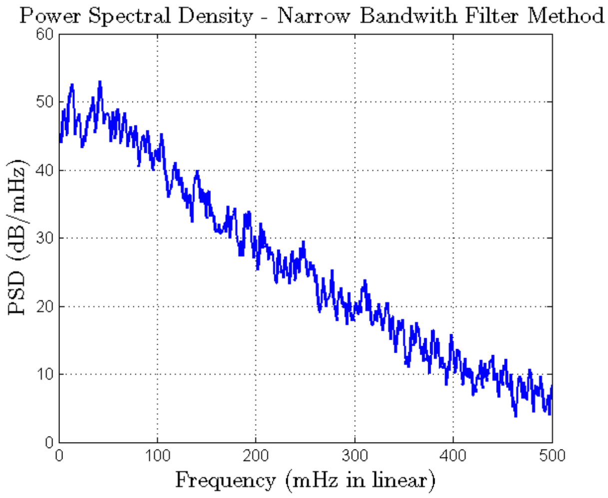

Figure 2: PSD of Gaussian White Process by NBPF Method

3.2 Blackman-Turkey Method

The Blackman-Turkey method is based on Wiener-Khinchin theorem which states that if the Fourier transform of a series $x(t)$ is $X(f)$, and if the autocorrelation function of the series is R, then the Fourier transform of R yields $P_X(f) = |X(f)|^2$ or the power spectrum of x(t).
In discrete form $\hat{G}_{xx}(f) = 2\Delta t[R(0) + 2\sum_{r=1}^{m-1} R(r)\cos(\frac{\pi r f}{f_c}) + R_m \cos(\frac{\pi m f}{f_c})]$
In discrete from we have

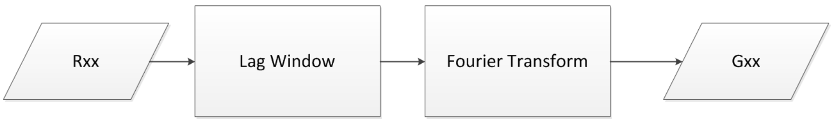

Figure 3: Blackman-Turkey Method

Autocorrelation	Time Lag	Cutoff frequency f_c
Unbiased	500 sec	0.5 Hz

Table 4: Specs of BT's parameters

3.2.1 Biased or Unbiased?

I choose unbiased autocorrelation for a reason. That I found in sinusoidal signal case, biased autocorrelation could cause more obvious leakage problem.

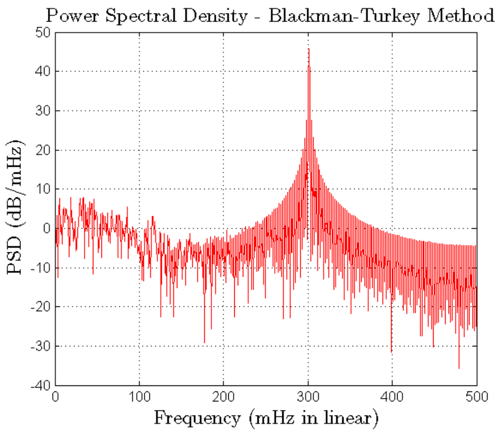

Figure 4: Biased Case

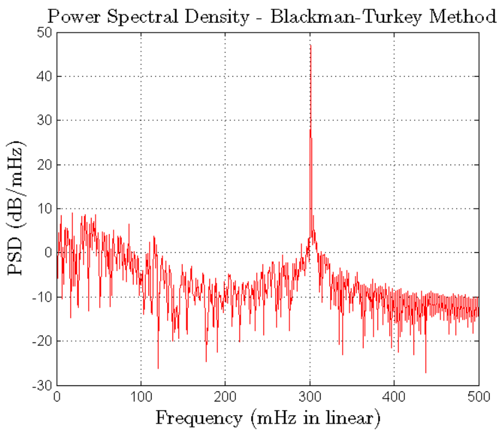

Figure 5: Unbiased Case

Then we can observe the spectrum of Gaussian signal $w[n]$.

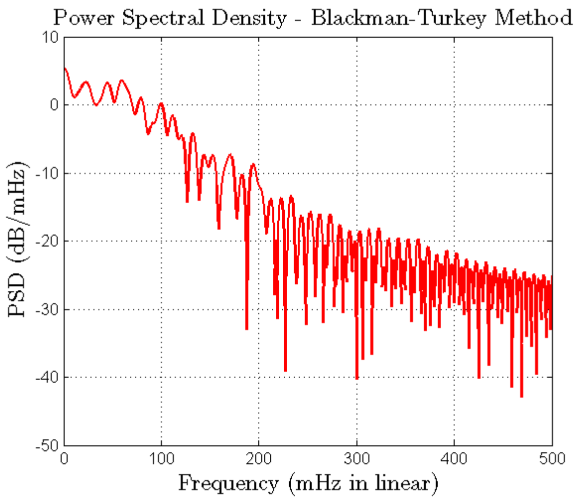

Figure 6: Blackman-Turkey Method

3.3 Discrete Fourier Transform Method

In this method, we directly take Fourier transform with raw data.

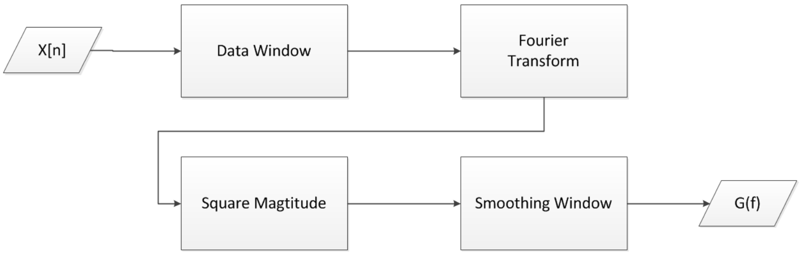

Figure 7: Direct Method

The form could be derived as s

$$\hat{G}_{xx}(f) = \frac{2}{N\Delta t}|\bar{X}_m|^2$$

The result of Gaussian process spectrum.

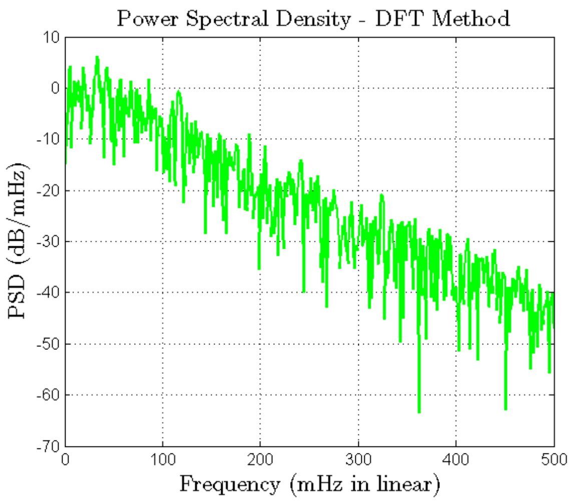

Figure 8: Direct Method

3.4 Comparision of three methods

We discuss these methods with Gaussian and sinusoidal signal.

3.4.1 Gaussian White Process

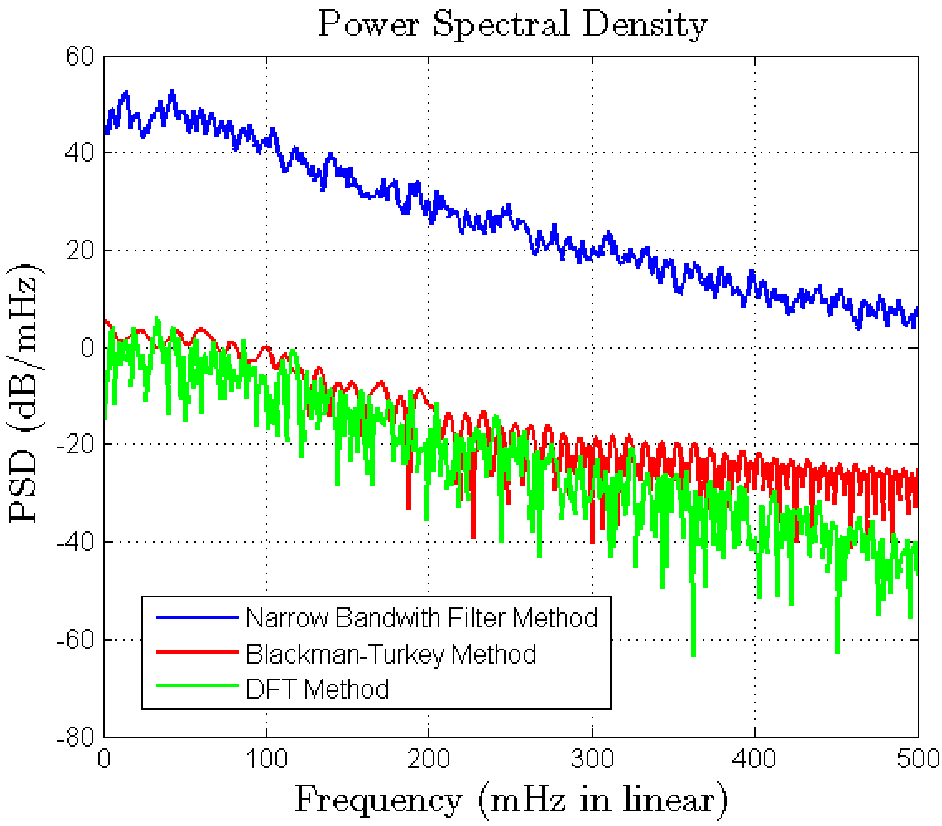

Figure 9: Three Methods Comparison

We observe that narrow bandpass method provides a gain due to its filter. And its spectrum seem to be less fluctuation. The rest two methods are similar but Blackman-Turkey method seem to sustain the energy which is more like Gaussian process.

3.4.2 Sinusodial Wave with noise

I prefer this case because we could capture the main lobe and compare the power leakage to judge the performance.

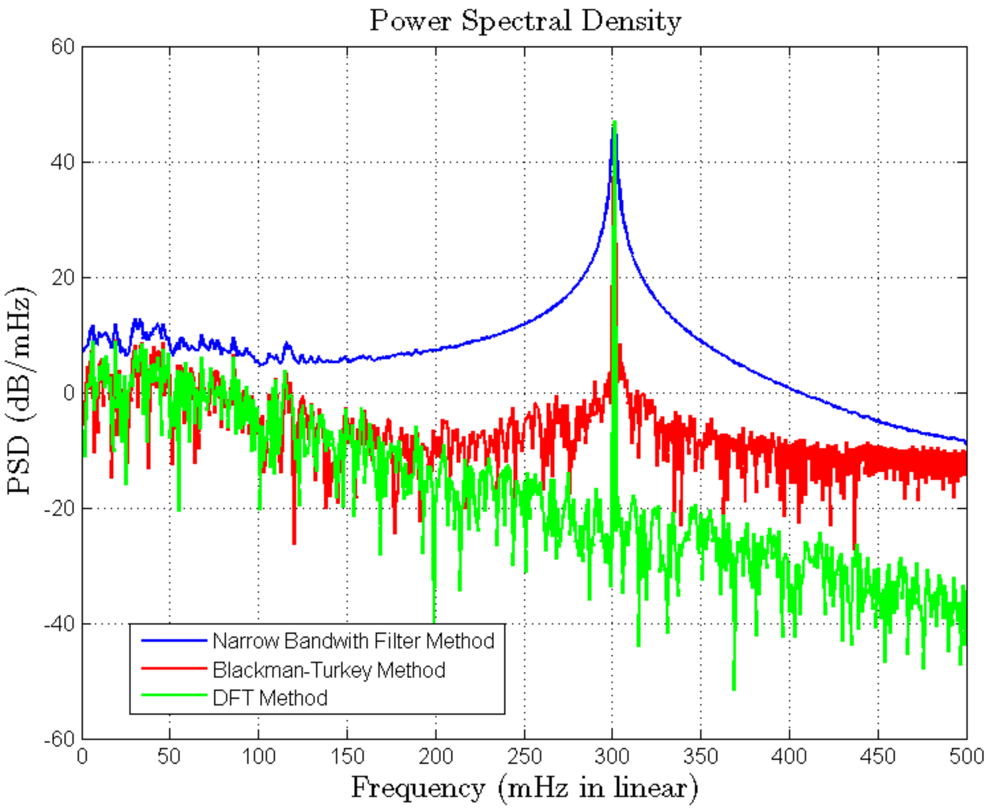

Figure 10: Three Methods Comparison (Sine with Gaussian noise)

The graph indicates that narrow bandpass method gives the most smooth spectrum. But it is awkward and have significant power leakage. Three of them are capable of catching the main frequency. DFT method "neatly" show the frequency and Blackman-Terkey method can be observed some leakage and less spectrum decay.

4 MATLAB Function

The section states a well-known algorithm, Welch-Bartlett method, finding power spectral density.

4.1 Algorithm

4.1.1 Bartlett's method

It is also called periodogram averaging method. The spectrum is estimated with several different segmentation without overlapping. It is understandable with illustration below.

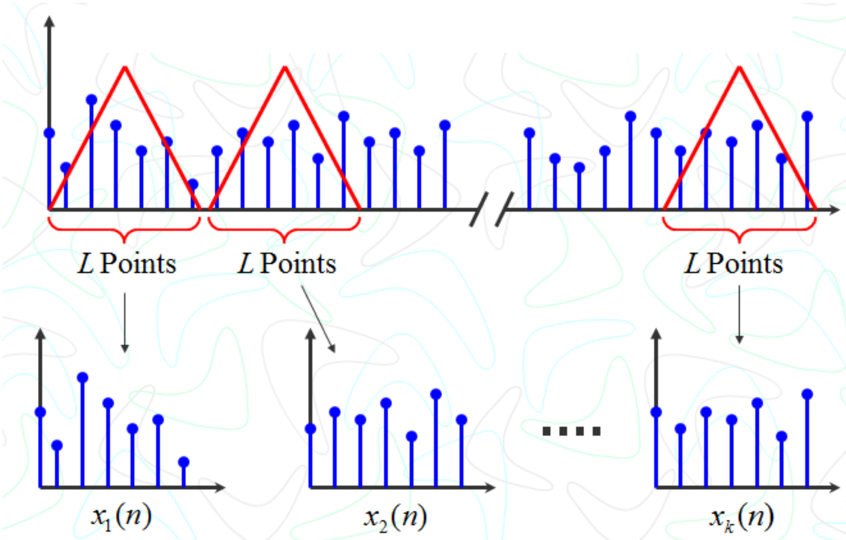

Figure 11: Data partition of Bartlett's method

4.1.2 Welch's method

Welch's method is so similar to the Bartlett's method, so some call it Welch-Bartlett method. And it is also called M-periodogram averaging.

We have the illustration below. In the simulation, we set Welch's parameters

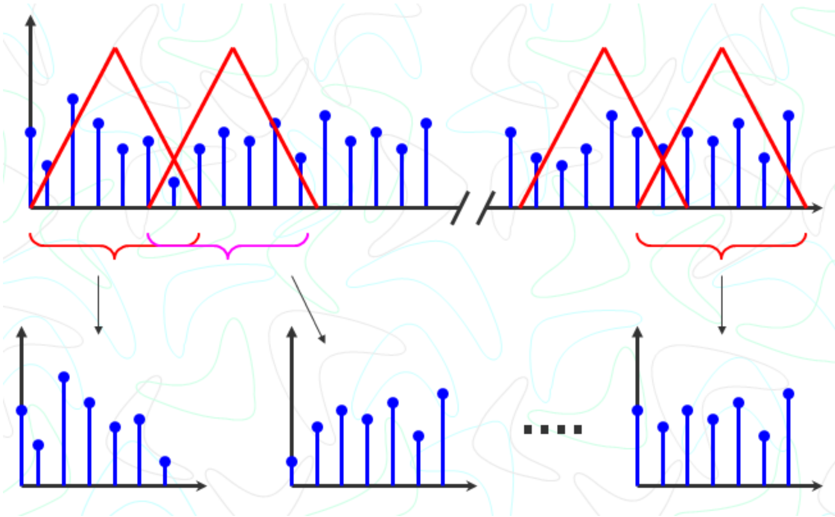

Figure 12: Data partition of Welch's method

as below.

Signal Type	Window function	Overlapping Percentage
1,000	Hamming	50%

Table 5: Specs of the Signal used in the report

- The vector is segmented into eight sections of equal length, each with 50% overlap.
- Any remaining (trailing) entries in vector that cannot be included in the eight segments of equal length are discarded.
- Each segment is windowed with a Hamming window that is the same length as the segment.

4.2 Results and Comparison

The Welch's method gives the result like below And compare with previous results

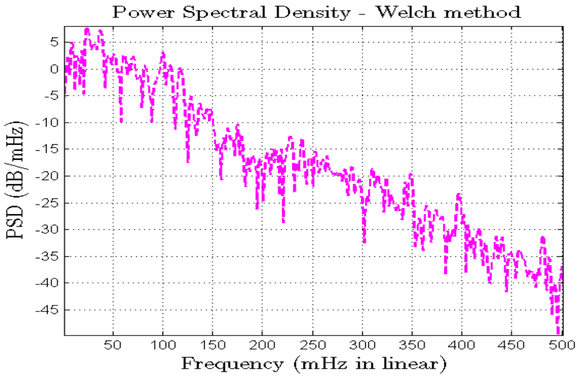

Figure 13: Spectral Density of White noise by Welch's method

could conclude that Welch's method provides a smoother spectrum.

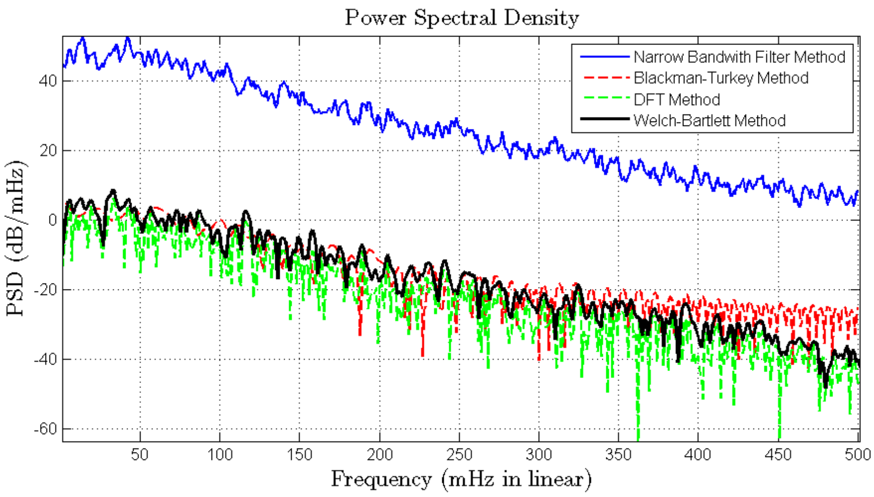

Figure 14: Spectral Density of White noise by Welch's method

5 Extention Reading

The section is listed several topics that I am intrested and would do some survey after this course. This part serves as my quick review notes.

- **The Burg Algorithm for Segments**
  The Burg algorithm for segments, both the variance and the bias in the estimated parameters are reduced by fitting a single model to all segments simultaneously.

- **Modified Periodogram**

- **Welch Method Revisited: Nonparametric Power Spectrum Estimation Via Circular Overlap**
  It is a paper published on IEEE TRANSACTIONS ON SIGNAL PROCESSING, VOL. 58, NO. 2, FEBRUARY 2010

- **AR Model to estimate Spectral**

- **PSD using Yule-Walker AR method**

References

[1] A ppt from some university : Power Spectrum Estimation

[2] Wikipedia : Spectral Density, 2012