National Cheng Kung University

Mechanical Engineering

Stochastic Dynamic Data - Analysis and Processing

Spectrum

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1 Introduction

Spectrum acts like eyes of engineers giving insightful perspective in different ways. We first discuss some issues in spectrum, then use it to measure stochastic process.

2 FFT and Window functions

The section, we discussed some properties and issues of FFT, use simple sine wave to show the result of test.

2.1 Limited Measurement Time

In reality, we can not take infinite length of time series. If our measure interval is not (and always not) exact the period, it causes sharp discontinuties and the spectrum to be spread out.

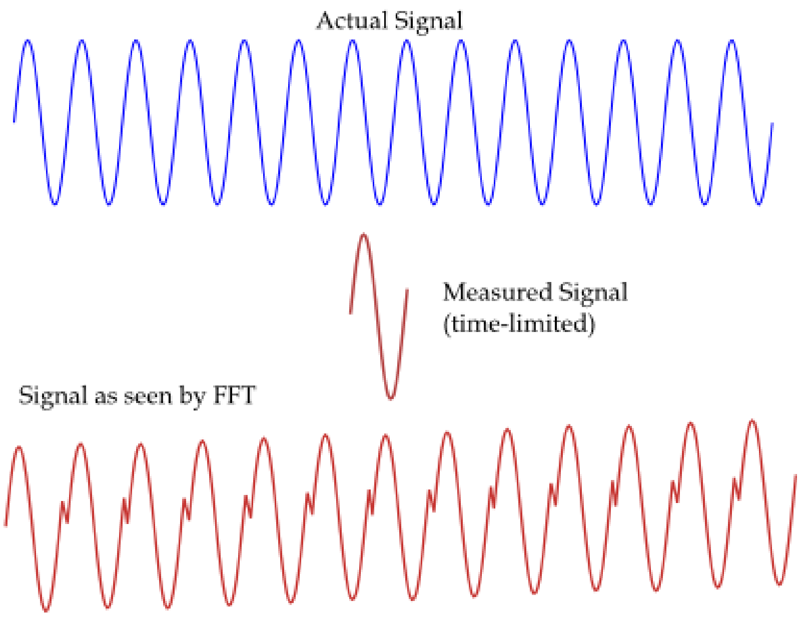

Figure 1: Example of limited time measurement

2.2 Spectrum Leakage

As previous section, shrap corner would be interpreted to high frequencies components in FFT spectrum. We could see the phenomenum that spectrum spread out.

Source	Data Length	Sampling Rate	Time Span	Frequency	Amplitude
Matlab $sin()$	512	100 Hz	5 sec	1 Hz	1

Table 1: Specs of the Testing Sine Wave

If we take a slice of sine which is not continuous can cause spectrum leakage.

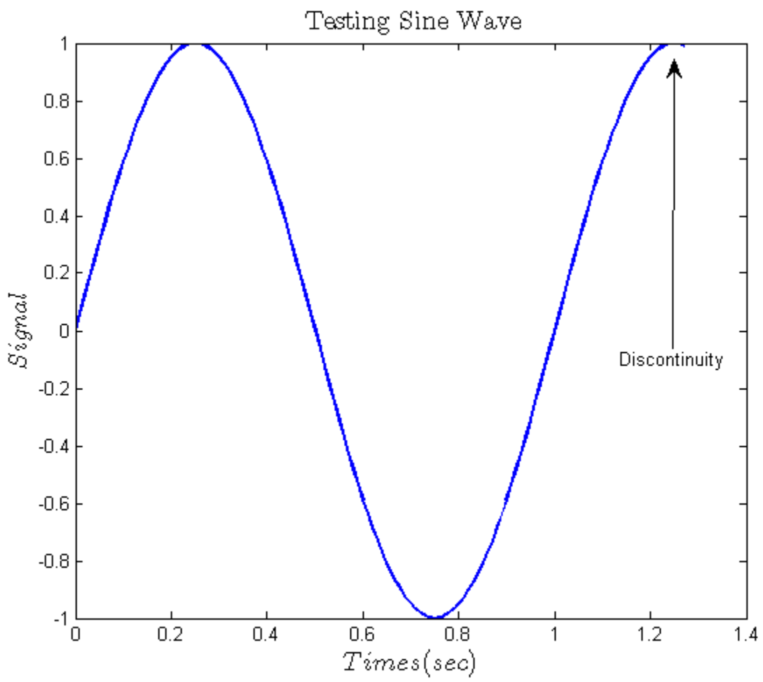

Figure 2: Take an Aperiodic Sine

Spectrum as below.

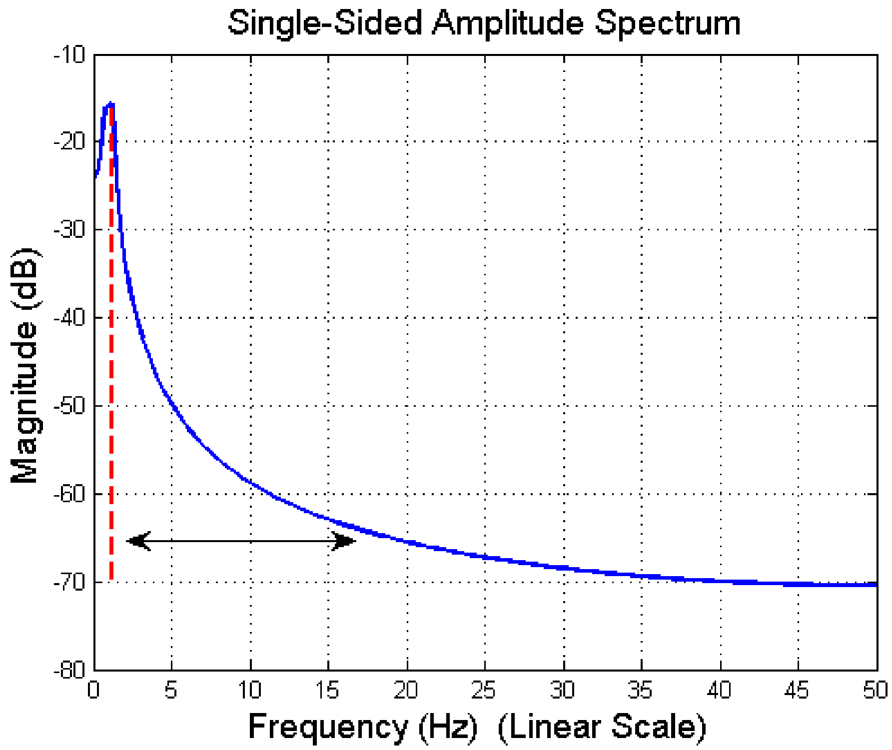

Figure 3: Spectrum Leakage

2.3 Window functions

Window function helps us "trim" the signal, smooth them and make them continuous. However, window functions have several specs.

- Sidelobe Level
- Fall off
- Coherent gain
- Equivalent noise bandwidth
- Worst case processing loss

Window function	Sidelobe level (dB)	Fall off (dB per octave)	Coherent gain	Equivalent noise bandwidth (bins)	6 dB bandwidth (bins)	Worst case processing loss (dB)
Rectangular	-13	-6	1.00	1.00	1.21	3.92
Triangular	-27	-12	0.50	1.33	1.78	3.07
Hanning	-32	-18	0.50	1.50	2.00	3.18
Hamming	-43	-6	0.54	1.36	1.81	3.10
Poisson (3.0)	-24	-6	0.32	1.65	2.08	3.64
Poisson (4.0)	-31	-6	0.25	2.08	2.58	4.21
Cauchy (4.0)	-35	-6	0.33	1.76	2.20	3.83
Cauchy (5.0)	-30	-6	0.28	2.06	2.53	4.28
Gaussian (3.0)	-55	-6	0.43	1.64	2.18	3.40
Kaiser-Bessel (3.0)	-69	-6	0.40	1.80	2.39	3.56
Kaiser-Bessel (3.5)	-82	-6	0.37	1.93	2.57	3.74

Figure 4: Various Window functions

The noun of sidelobe comes from antenna engineering. In signal processing, we use it to describe the strength of ripples of minor frquency. As the table tells us rectancular window has the highest sidelobe because its sharp corner causes numerous ripples and high frequency components. In some level, we can say sidelobe indicates specturm leakage.

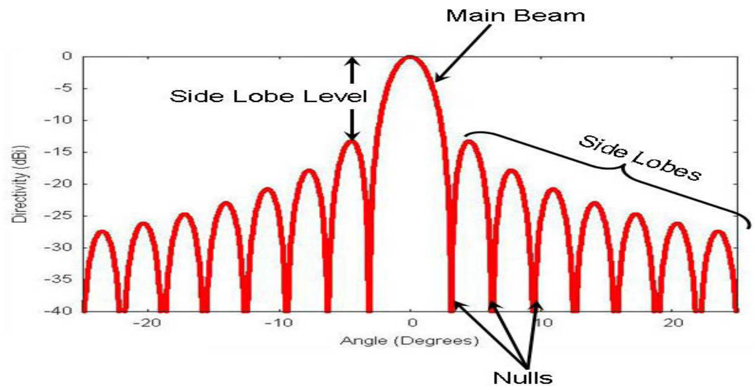

Figure 5: Sidelobe effect

2.3.1 Hamming Window

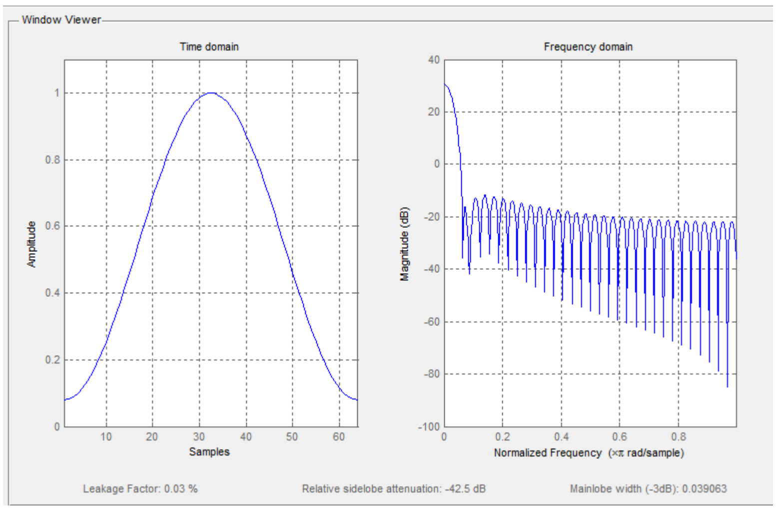

Figure 6: Time Domain and Frequency Domain of window

Hamming window is widely used because its good performance. It has both low sidelobe and less processing loss when worst situation.

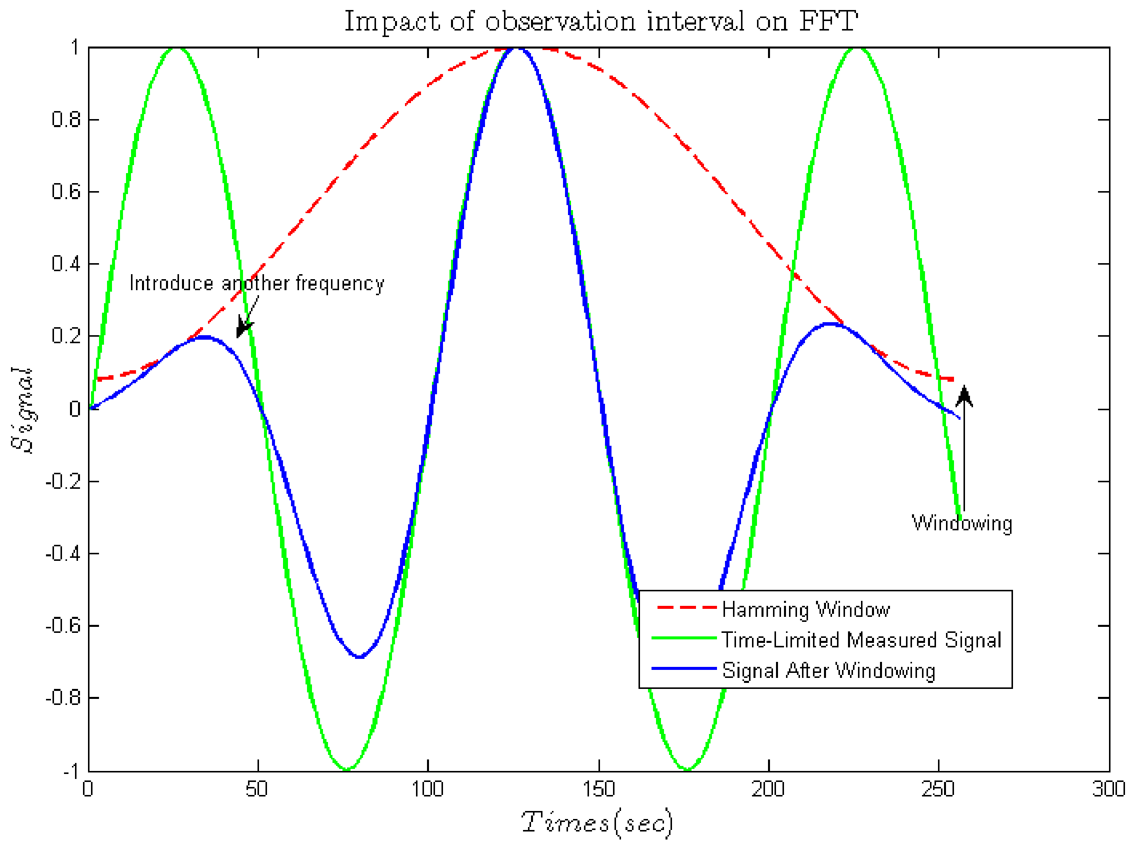

Figure 7: Hamming Window

Hamming window fixes the end of signal and make them meet together and eliminate discontinuity. However, due to its curve shape, portion of the original signal would be distorted. The distortion can bring a new frequency component to the spectrum.

We can see the result of spectrum. The leakage is less than before.

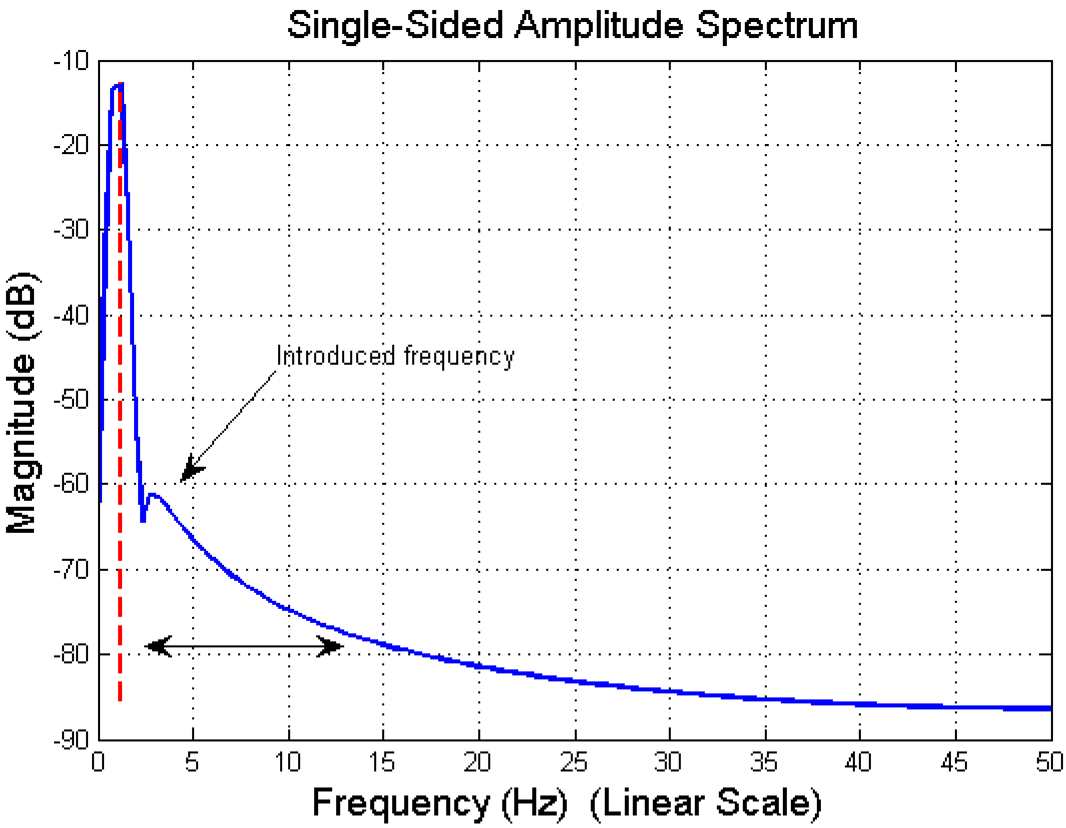

Figure 8: Spectrum of Sine Signal with Hamming window

But there is a new frequenct that might be misleading.

2.3.2 Triangular Window

Still we try another window. Triangular one can be powerful in this case.

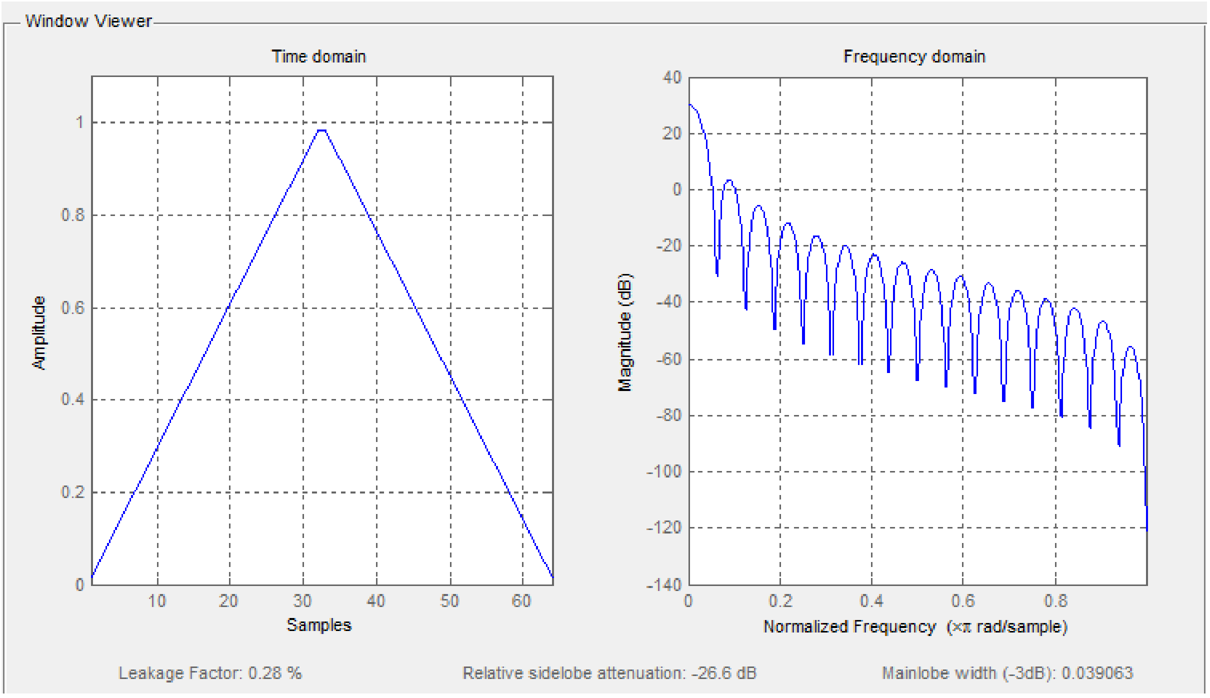

Figure 9: Time Domain and Frequency Domain of window

In the fellowing case, our signal is cut in worst case. It has the largest discontinuity. However, triangular window works perfect in the case. Of course, we have another tradeoff to use this window. We cause fluctuation because of triangle's peak.

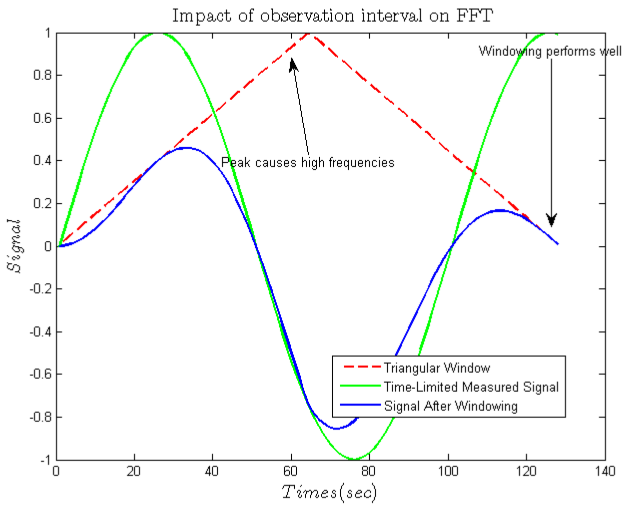

Figure 10: Triangular Window

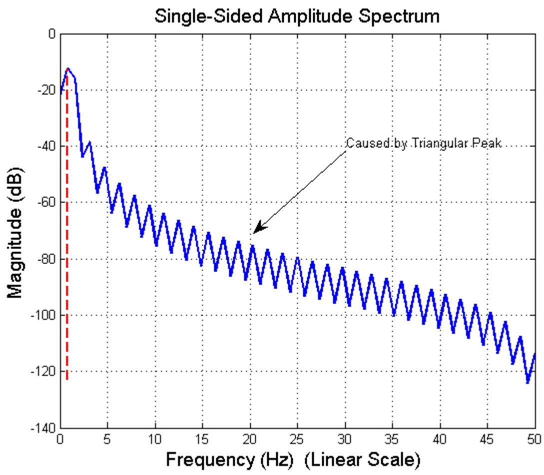

Figure 11: Spectrum of Sine Signal with Hamming window

3 Random Process

3.1 System and Data source

As we applied before, random process could be treated as a linear system. The low-pass filter is a useful form in processing. We take

$$H(s) = \frac{0.25}{s^2 + 0.7071s + 0.25}$$

as our random procedure.

Order	Cutoff frequency	Settling Time
n = 2	0.4994 rad/sec	16 sec

Table 2: Specs of the process

Source	Statistic Property	Data Length	JB Test($\alpha = 0.05$)
Matlab func normrnd()	$\mu = 0, \sigma = 1$	10^3	Accept

Table 3: The Specs of Raw Data

Sample Rate	Reject Data Length	Stationarity Test($\alpha = 0.05$)
10 Hz (TimeStep = 0.1 sec)	first 400 data	Accept

Table 4: The Specs of Processed Data

Input data

$$x(t) = w(t) + A\sin(2\pi f_0 t)$$

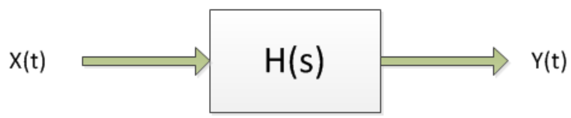

Figure 12: Block Diagram

3.2 Spectrum Analysis

We get the results from linear system and take FFT

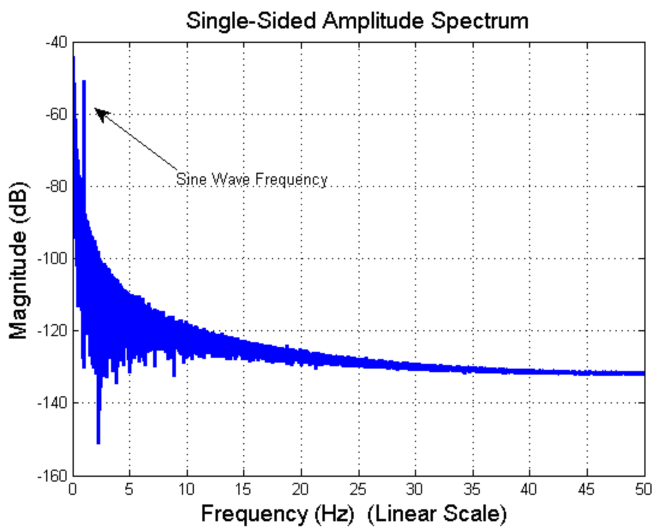

Figure 13: Spectrum of Random Data in Linear Scale

However, the linear scale is not easy to observe the frequency of sine.

We change it to Log scale and it is easier to get the frequency. Beacuse the log operation could stretch the low frequencies portion and spread the spectrum.

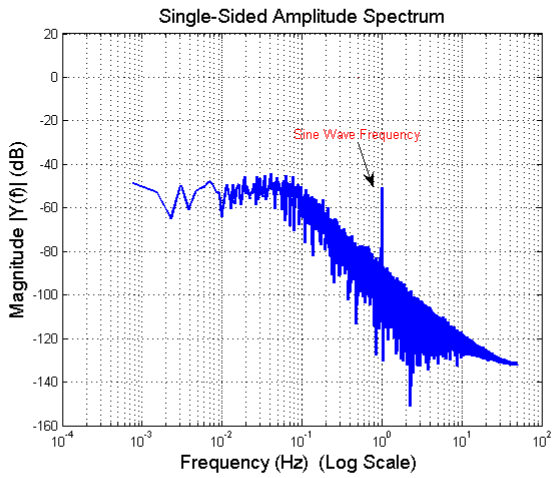

Figure 14: Spectrum of Random Data in Log scale

In this case, we still try to apply windows on it.

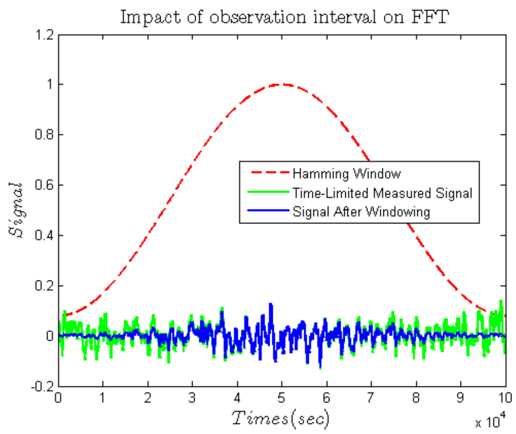

Figure 15: Hamming Window

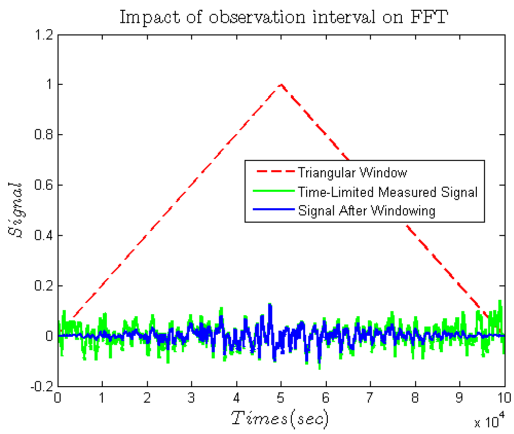

Figure 16: Triangular Window

And their spectrum look so familiar.

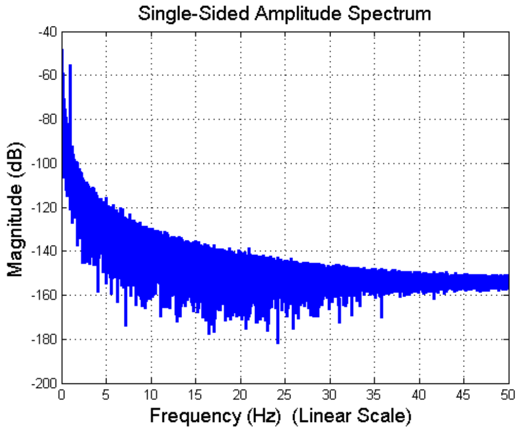

Figure 17: Spectrum with Hamming Window

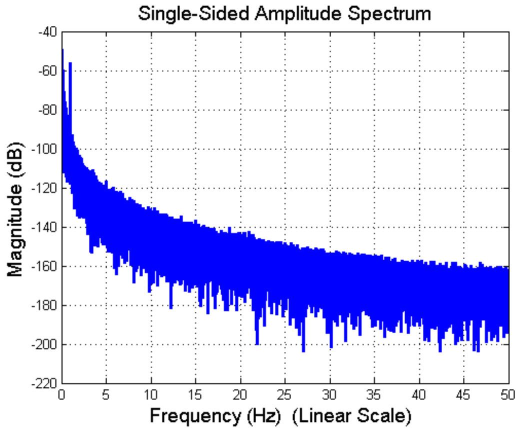

Figure 18: Spectrum with Triangular Window

The only different is that spectrum with Hamming is more "clean" than the other. However, I try with Matlab function to find power density spectrum. It seems very clear and shows some perodicity. It could be a useful tool in the following analysis.

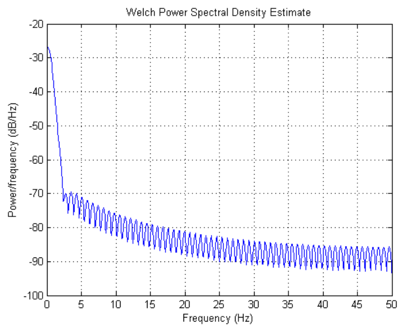

Figure 19: Powe Density Spectrum

References

[1] Matlab : Window function, 2012

[2] Wikipedia : Spectral leakage, 2012

[3] Website : Bores Signal Processing, 2012