National Cheng Kung University

Mechanical Engineering

Stochastic Dynamic Data - Analysis and Processing

Narrow Band Random Signal

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Contents

1 Introduction

The assignement would design a narrow band filter to generate the signal. Then verify the NB signal could be modeled by $y(t_j) = R(t_i)\cos[2\pi f_c t_j + \theta(t_i)]$.

2 Filter Design

2.1 Narrow Band-Pass Filter Design

Band filter is a second order filter with two poles and one specific zero at the origin of pole-zero map.(fig 1)

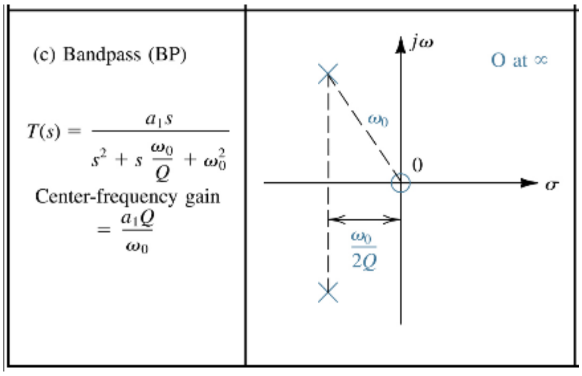


Figure 1: Pole Zero Diagram

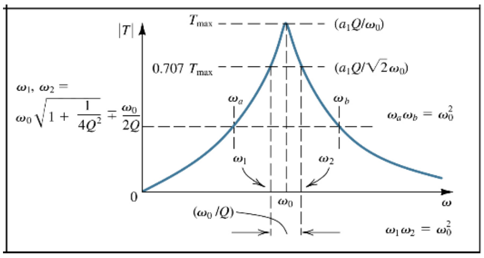
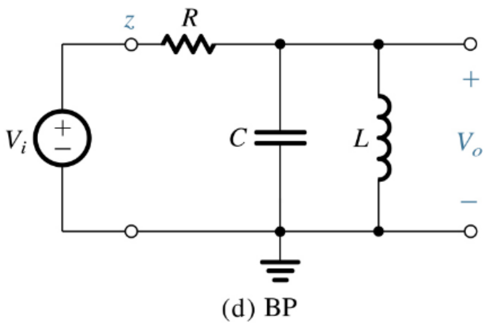


Figure 2: Bode Plot of NB filter

And the filter can be implemented by a simple RLC circuit.



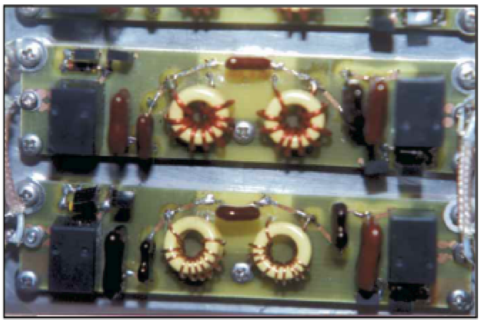


Figure 3: Schematic of NBP filter Circuit

Figure 4: Circuit Realization

2.2 Filter Simulation

We design filter with assigned specifications. The Bode plot we designed in matlab

Sepcs	filter gain	Center freq	Desired Bandwidth
Value	1	0.0795 rad/sec (0.5 Hz)	0.01

Table 1: Specs of the filter

as below. We use gain as 1 in following simulation.

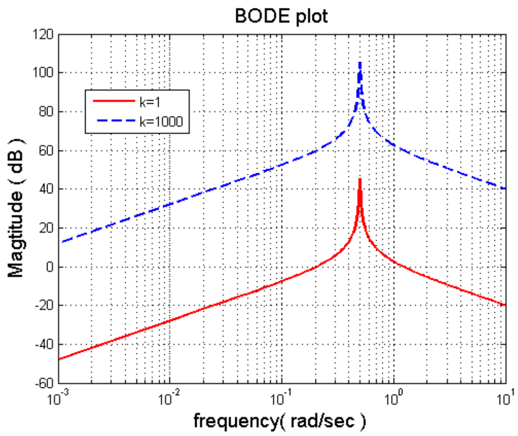


Figure 5: Bode plot comparing with two different filter gains

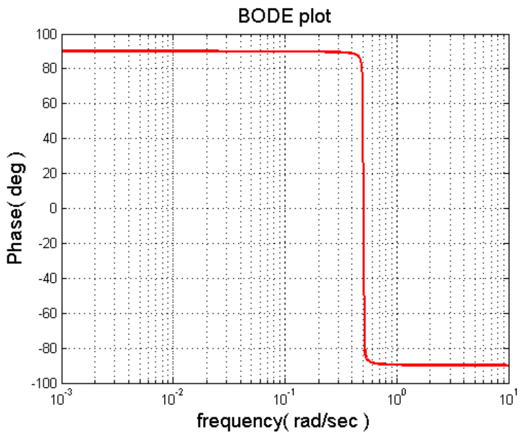


Figure 6: Phase Plot

2.3 Dynamic Response Specifications

We find the deterministic property of dynamic system to help us eliminating nonstationary singal.

Since that two symmetrical poles are quite close to the image axis, the phenomena of oscillation is significant. We would take data with 2000 sec delay base on the setting time is 1570 sec.

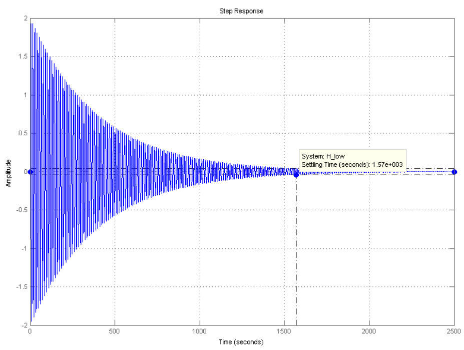
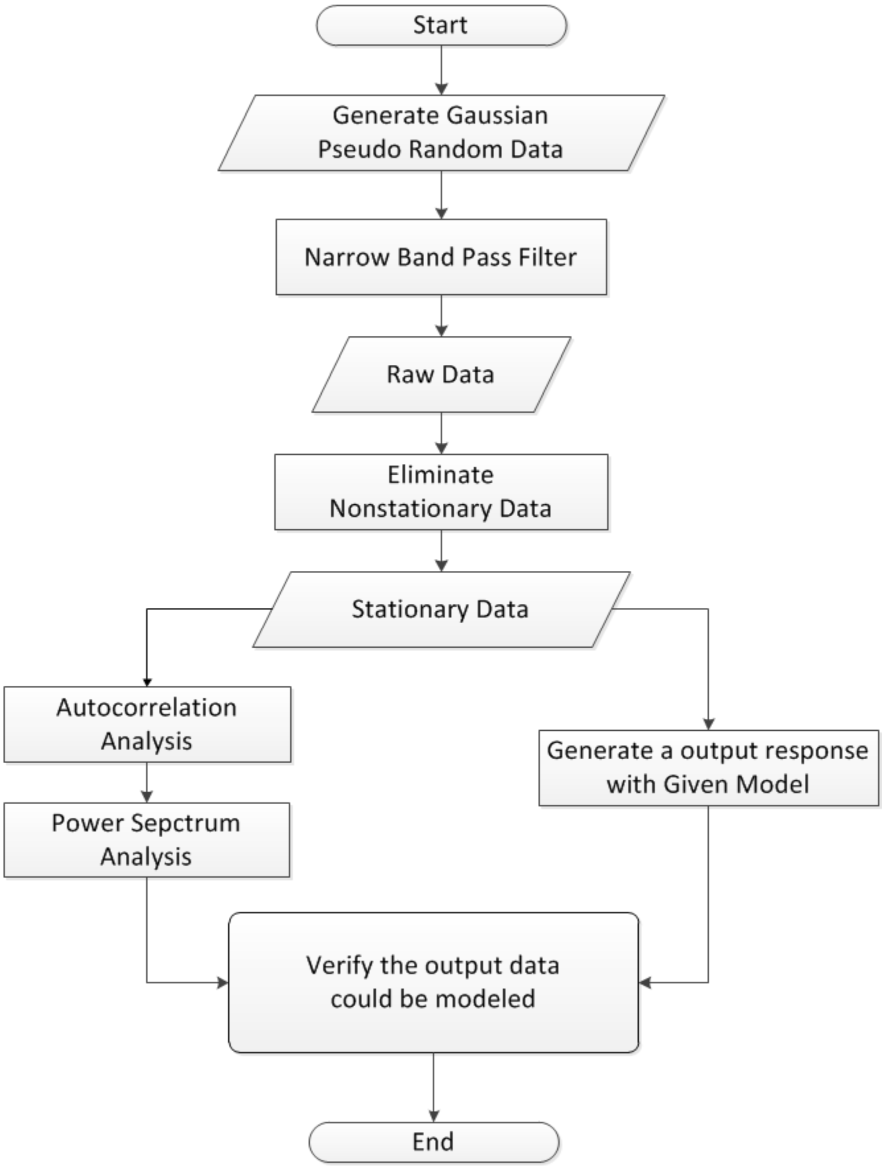


Figure 7: Step Response

3 Signal Processing

3.1 Flow Chart of Verification



3.2 Data and Processing Specifications

3.2.1 Raw Data

Method	Data Length	Sampling Rate
matlab function - wgn()	$2^{12} = 4096$	1 Hz (time step = 1)

Table 2: Specification of raw data

3.2.2 Stationary Data

Timespan	Data Length	Sampling Rate
$2049 \sim 4096$ sec	2048	1 Hz (time step = 1)

Table 3: Specification of stationary data

3.2.3 Spectrum

Sepctrum Type	Data Source	Data Length	Sampling Rate
FFT	Stationary Data	power of 2	10 Hz
One-sided Power Specs	Autocorrelation Data	power of 2	10 Hz

Table 4: Specification of Spectrum used in Data Processing

4 Results and Verifications

The following results are acquired by the procedure

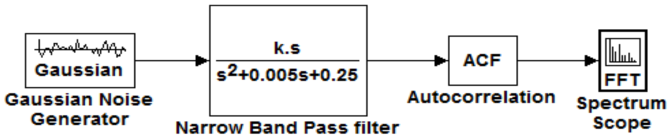


Figure 8: Signal Processing Diagram

4.1 Time Domain

The narrow band stationary time series are below. The envelop follows Rayleigh distribution.

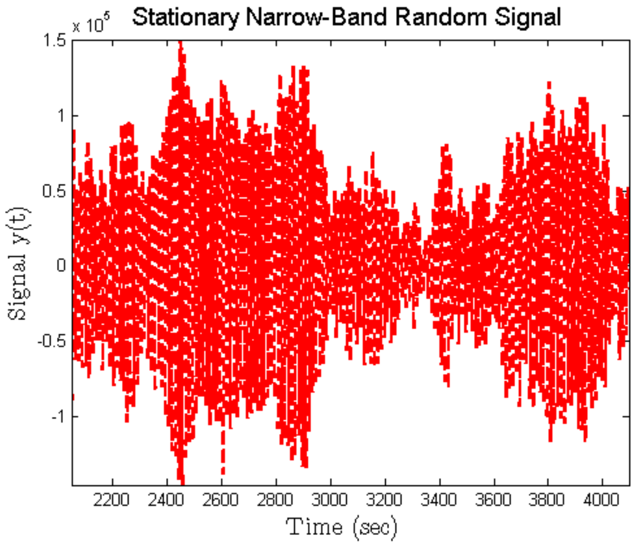


Figure 9: Narrow Band Signal in time domain

4.2 Autocorrelation and FFT Analysis

The autocorrelation function indicates periodic and it envelop decays with lag.

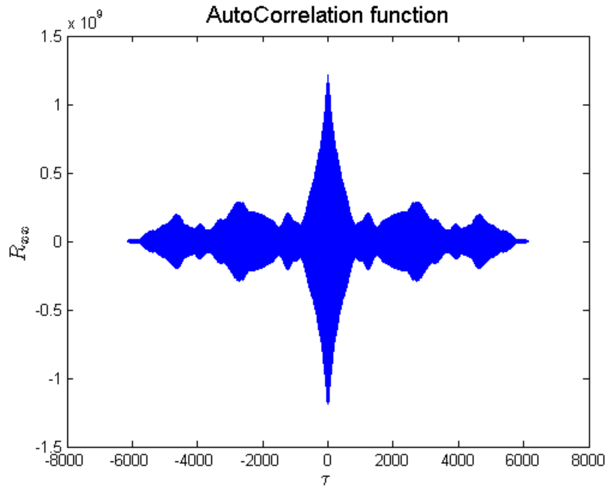


Figure 10: Autocorrelation function of Stationary Data

In frequency domain, the spectrum advocates the center frequency which is obviously a peak. On the right-hand side, the phase diagram show that it distributes uniformly.

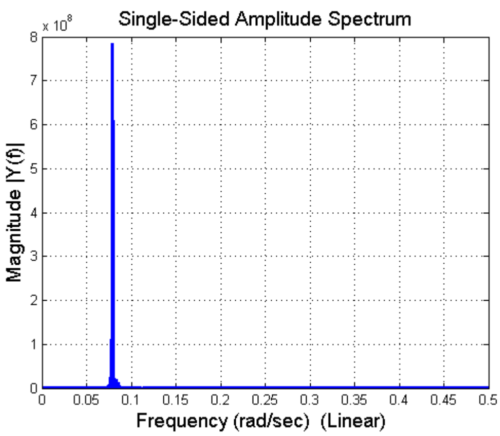


Figure 11: FFT Specturm

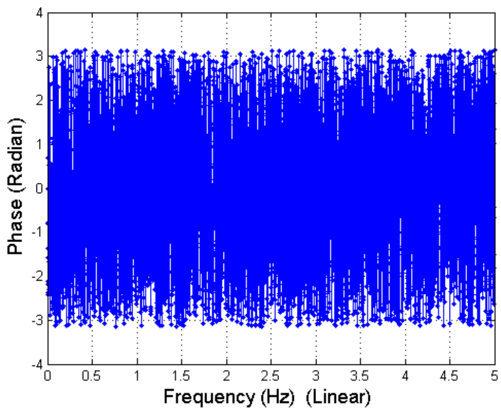


Figure 12: Phase Diagram

4.3 Modeling the Signal

We follow the procedure steps by steps to generate the signal which is

$$y(t_j) = R(t_i)\cos[2\pi f_c t_j + \theta(t_i)]$$

- Find standard deviation from stationary data, and use matlab function to create random variable $R(t_i)$
- Generate random variable $\theta(t_i)$ which is uniformly distributed
- Find $y(t_j)$ used by time t_j which is much less than t_i

4.3.1 Rayleigh Distribution

I generate the Rayleigh distribution, expressed with random variable R, by the standard deviation of stationary data and check the shape of distribution. The

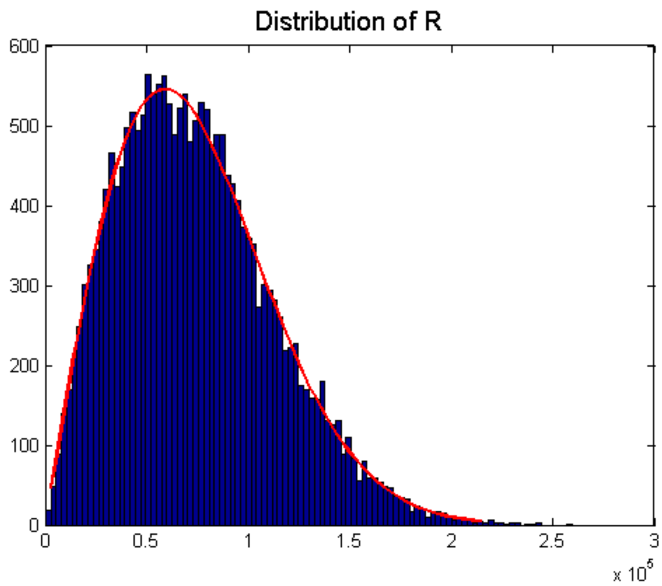


Figure 13: Rayleigh Distribution fitting

Rayleigh distribution is often observed when the overall magnitude of a vector is related to its directional components.

Rayleigh distribution is a significant distribution and dominant in fluid and stress problem. I once took a project which is an optimization problem of placing wind turbine. In such case, wind speed is often modeled as Rayleigh Distribution, and somtimes Weibull.

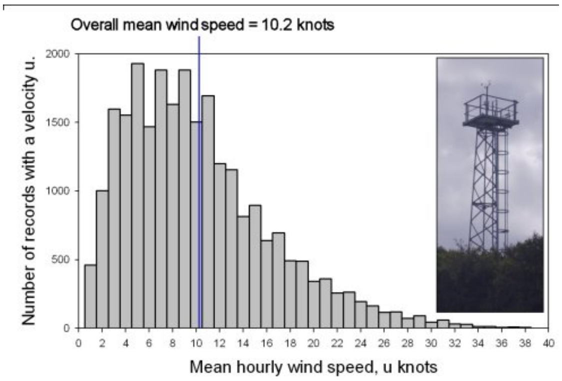


Figure 14: Wind speed distribution

4.3.2 Autocorrelation Comparision

We, first, compare the autocorrelation with simulated data and modeled data.

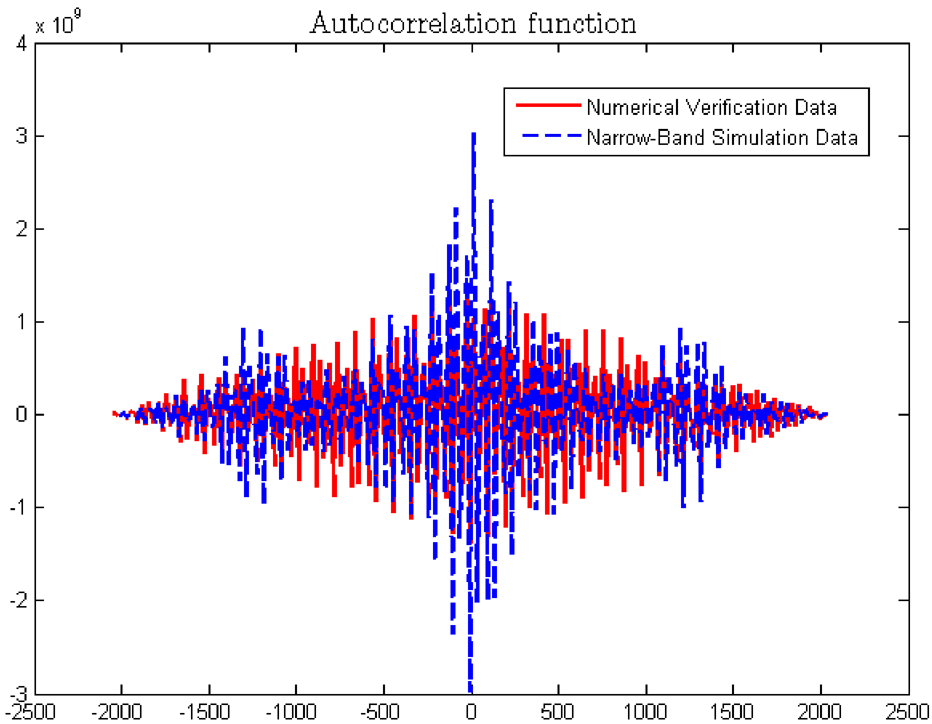


Figure 15: Autocorrelation function follows the same pattern

4.3.3 Power Sepctrum Comparision

Then, we take power spectrum from autocorrelation function and shows below.

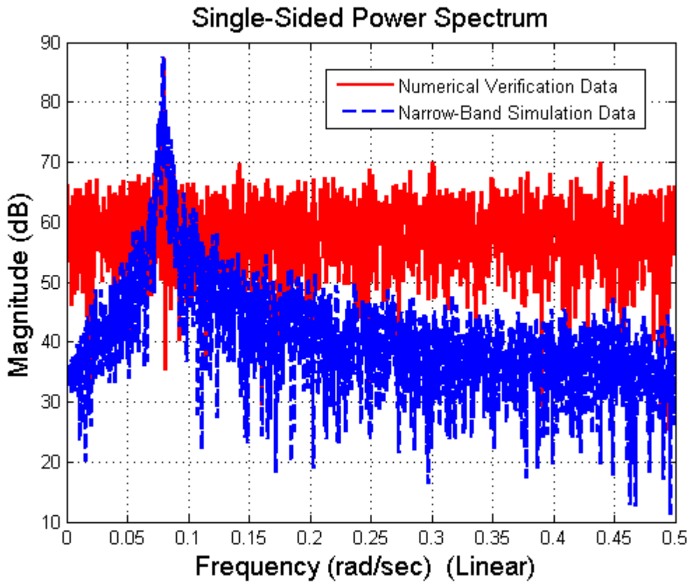


Figure 16: Power Spectrum with the same peak frequency

The modeled data, however, posess ”stronger” signal which evenly distribute on every frequencies. We still can find they have the same center frequency.

References

[1] Microelectronics, Sedra Smith

[2] Random Data, Allan Piersol and Julius Bendat