14.384 Time Series Analysis, Fall 2007
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Recitation 2

# Time Series in Matlab

## Simulating an ARMA Model

$a(L)y_t = b(L)e_t$

```
clear;
a = [1 0.5]; % AR coeffs
b = [1 0.4 0.3]; % MA coeffs
T = 1000;
e = randn(T,1); % generate gaussian white noise
y = filter(b,a,e); % generate y
```

The `filter` function can be used to generate data from an ARMA model, or apply a filter to a series.

## Impulse-Response

To graph the impulse response of an ARMA, use `fvtool`

```
% create an impulse response
fvtool(b,a,'impulse');
```

## Sample Covariances

```
[c lags]=xcov(y,'biased');
figure;
plot(lags,c);
title('Sample Covariances');
```

The option, `'biased'`, says to use $\hat{\gamma}_k = \frac{1}{T}\sum_{t=1}^{T-k}(y_t - \bar{y})(y_{t-k} - \bar{y})$; `'unbiased'` would use $\hat{\gamma}_k = \frac{1}{T-k}\sum_{t=1}^{T-k}(y_t - \bar{y})(y_{t-k} - \bar{y})$

# Spectral Analysis

Population spectral density for an ARMA:

```
% population density
w = 0:0.1:pi;      % frequencies to compute density at
h = freqz(b,a,w);  % returns frequency response = b(e^{iw})/a(e^{iw})
sd = abs(h).^2./sqrt(2*pi); % make into density
```

## Estimating the Spectral Density

**Parametric methods** These estimate an $AR(p)$ and use it to compute the spectrum.

```
[sdc wc] = pcov(y,8); % estimate spectral density by fitting AR(8)

[sdy wy] = pyulear(y,8); % estimate spectral density by fitting AR(8)
                         % using
                         % the Yule-walker equations
```

### Non-parametric methods

**Definition 1.** The *sample periodogram* of $\{x_t\}_{t=1}^T$ is $\hat{S}(\omega) = \frac{1}{T}|\sum_{t=1}^T e^{-i\omega t} x_t|^2$

*Remark* 2. The sample periodogram is equal to the Fourier transform of the sample autocovariances

$$\hat{S}(\omega) = \frac{1}{T}|\sum_{t=1}^{T} e^{-i\omega t} x_t|^2 = \sum_{k=-T-1}^{T-1} \hat{\gamma}_k e^{-i\omega k}$$

```
[sdp wp] = periodogram(y,[],'onesided'); % estimate  using sample
```

**Definition 3.** A *smoothed periodogram* estimate of the spectral density is

$$\hat{S}(\omega) = \int_{-\pi}^{\pi} h_T(\lambda - \omega) \frac{1}{T}|\sum_{t=1}^{T} e^{-i\lambda t} x_t|^2 d\lambda$$

where $h_T()$ is some kernel weighting function.

A smoothed periodogram is a weighting moving average of the sample periodogram.

The following code estimates a smoothed periodogram using a Parzen kernel with bandwidth $\sqrt{T}$.

```
rT = round(sqrt(T));
[sdw ww] = pwelch(y,parzenwin(rT),rT-1,[],'onesided'); % smoothed periodogram
```

**Definition 4.** A *weighted covariance* estimate of the spectrum is:

$$\hat{S}(\omega) = \sum_{k=-S_T}^{S_T} \hat{\gamma}_k g_T(k) e^{-i\omega k}$$

where $g_T(k)$ is some kernel.

```
% bartlett weighted covariances
wb = 0:0.1:pi;
rT = sqrt(T);
[c t]=xcov(y,'biased');
weight = 1-abs(t)/rT;
weight(abs(t)>rT) = 0;
for j=1:length(wb)
  sdb(j) = sum(c.*weight.*exp(-i*wb(j)*t));
end
sdb = sdb / sqrt(2*pi);
```

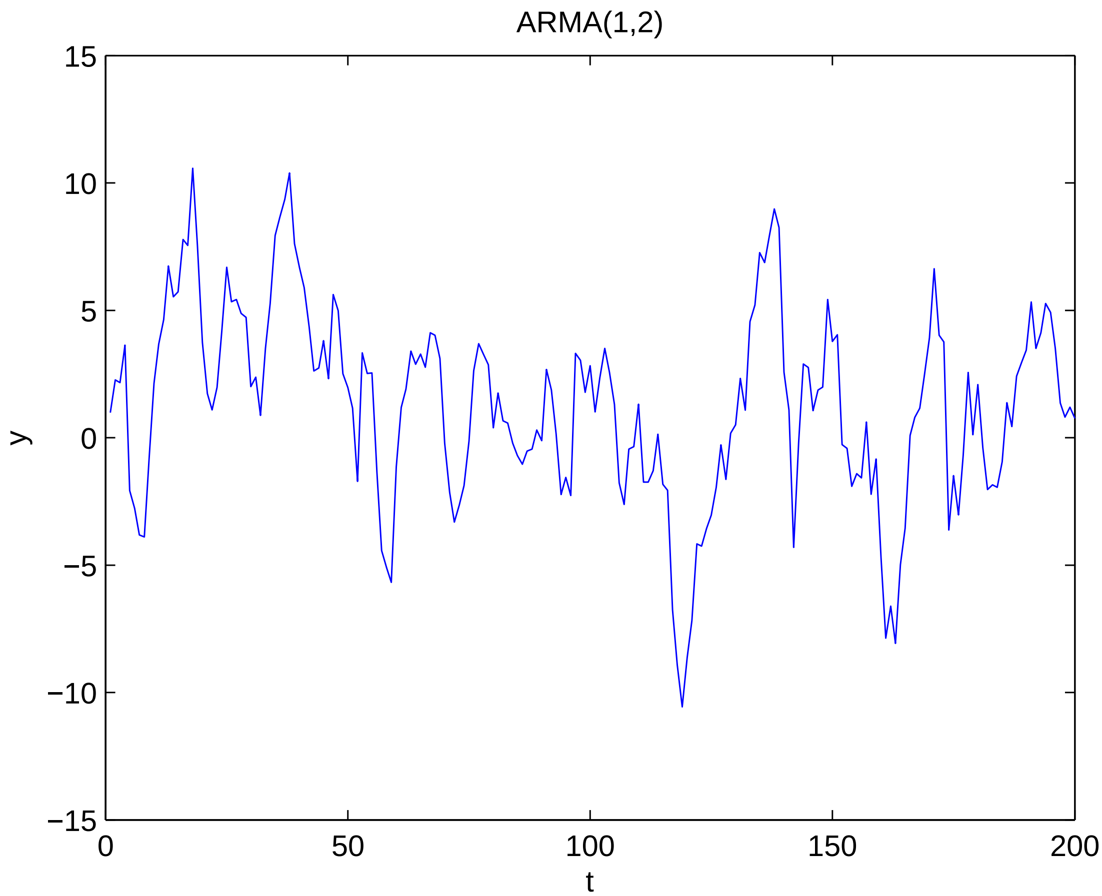
## Filtering

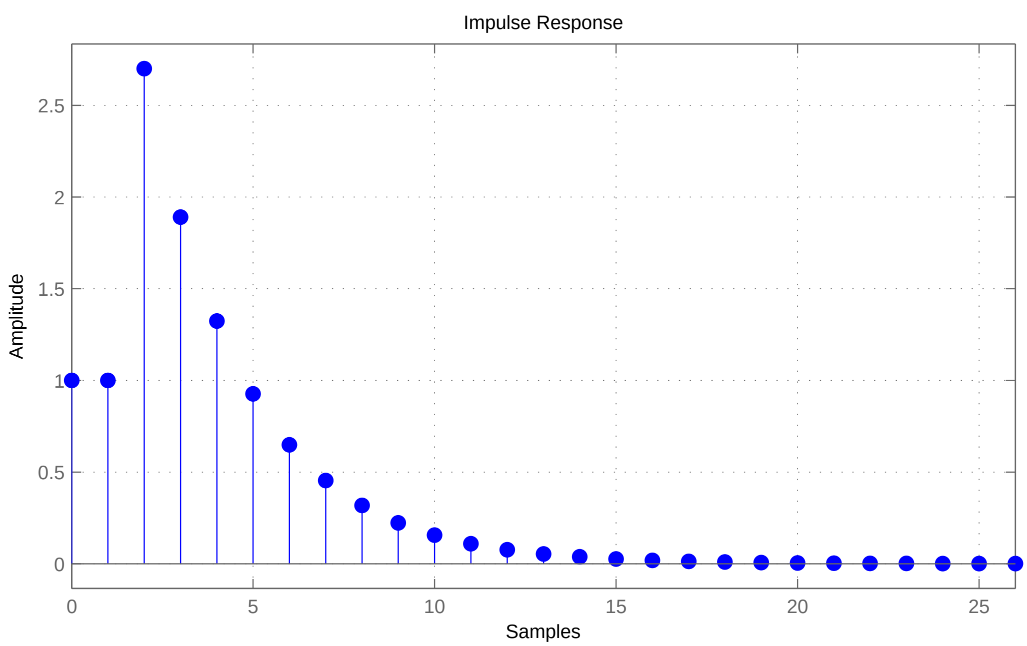
See: http://ideas.repec.org/c/wpa/wuwppr/0507001.html for code for common filters.

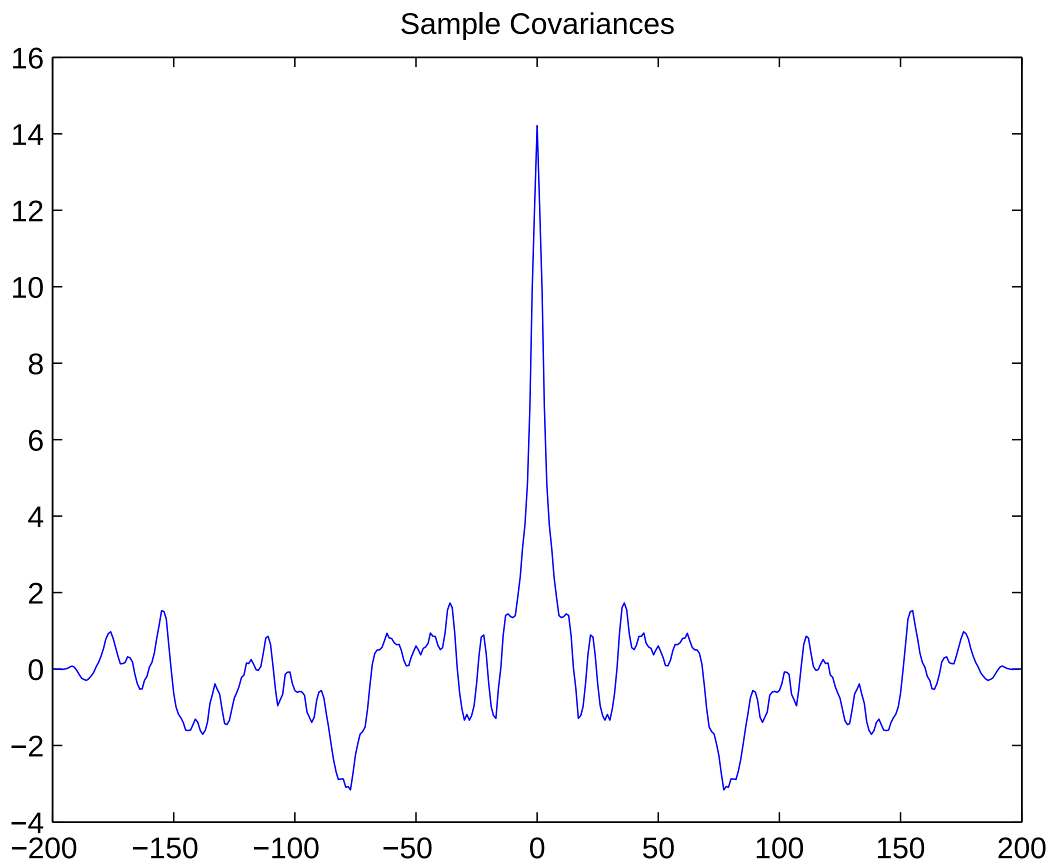
## Example: Simulating an ARMA and estimating the spectrum

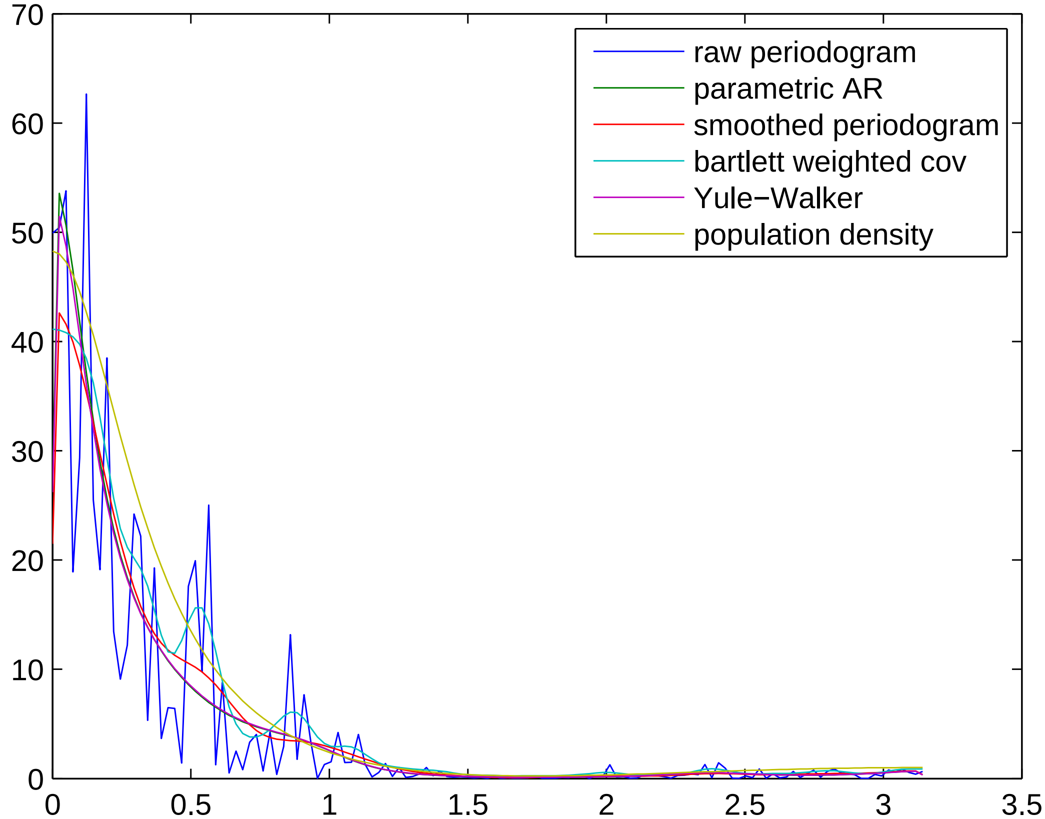
```
clear;
close all; % closes all open figure windows

% model: y_t = 0.9 y_{t-1} + b(L) e_t
a = [1 -0.7]; % AR coeffs
b = [1 0.3 2]; % MA coeffs
T = 200;
e = randn(T,1); % generate gaussian white noise
y = filter(b,a,e); % generate y

% plot y
figure;
plot(y);
xlabel('t');
ylabel('y');
title('ARMA(1,2)');
```

```

% create an impulse response
fvtool(b,a,'impulse');

% calculate and plot sample auto-covariances
[c lags]=xcov(y,'biased');
figure;
plot(lags,c);
title('Sample Covariances');

% estimate spectral density

% parametric
[sdc wc] = pcov(y,8); % estimate spectral density by fitting AR(8)
[sdy wy] = pyulear(y,8); % estimate spectral density by fitting AR(8)
                         % using
                         % the Yule-walker equations

% nonparametric
[sdp wp] = periodogram(y,[],'onesided'); % estimate  using unsmoothed
                                         % periodogram
rT = round(sqrt(T))*3;
[sdw ww] = pwelch(y,parzenwin(rT),rT-1,[],'onesided'); % smoothed
                                         periodogram

% bartlett weighted covariances
[c lags]=xcov(y,'biased');
t = -(T-1):(T-1);
weight = 1-abs(t')/rT;
weight(abs(t')>rT) = 0;
wb = ww;
for j=1:length(wb)
  sdb(j) = sum(c.*weight.*exp(-i*wb(j)*(-(T-1):(T-1))'));
end
sdb = sdb / sqrt(2*pi);

% population density
w = wb;
h = freqz(b,a,w);
sd = abs(h).^2./sqrt(2*pi);

figure;
plot(wp,sdp,wc,sdc,ww,sdw,wb,sdb,wy,sdy,w,sd);
legend('raw periodogram','parametric AR','smoothed periodogram', ...
       'bartlett weighted cov','Yule-Walker','population density');
```


ARMA(1,2)
y
t
15
10
5
0
−5
−10
−15
0
50
100
150
200


Impulse Response
Amplitude
Samples
2.5
2
1.5
1
0.5
0
0
5
10
15
20
25


Sample Covariances
16
14
12
10
8
6
4
2
0
−2
−4
−200
−150
−100
−50
0
50
100
150
200


70
60
50
40
30
20
10
0
0
0.5
1
1.5
2
2.5
3
3.5
raw periodogram
parametric AR
smoothed periodogram
bartlett weighted cov
Yule−Walker
population density