National Cheng Kung University

Mechanical Engineering

Stochastic Dynamic Data - Analysis and Processing

White Signal Processing

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Contents

1 Introduction

We encountered white process, which is ideal Gaussian white process, and deal with it in the assignment. We define the process properties, analyze it and grab useful information from these data. By the way, the cover is a well-known movie which the title refers to electronic voice phenomena (EVP), where voices, which some believe to be from the "other side", can be heard on audio recordings.

2 Gaussian White Process

Based on last assignment, we are capable of generating Gaussian white process with MATLAB function. Now, we try to analyze it with an insightful tool, autocorrelation function.

2.1 Mathematical Properties

By mathematical definition, Gaussian random series can be varified by three properties.

- Autocorrelation function : $R_N(\tau) = \frac{N}{2}\delta(\tau), -\infty < \tau < \infty$
- Power Spectral density is flat: $S_N(\tau) = \frac{N}{2}, -\infty < f < \infty$

I would show property 1 and 2 but we can not verify infinity average power $P_N = E[N^2(t)]$ with numerical method.

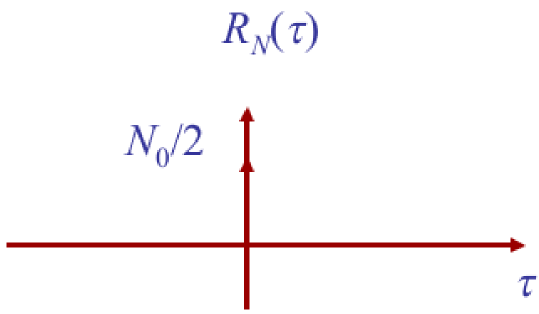


Figure 1: Autocorrelation function

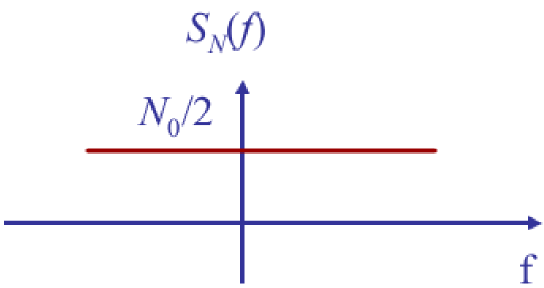


Figure 2: Power Spectrum

2.2 Data Verifying

I create a white signal with MATLAB. We guarantee it approximates ideal properties such as

- Mean value is 0
- Standard Deviation is 1

Now, we expect its autocorrelation function is an Dirac delta function.

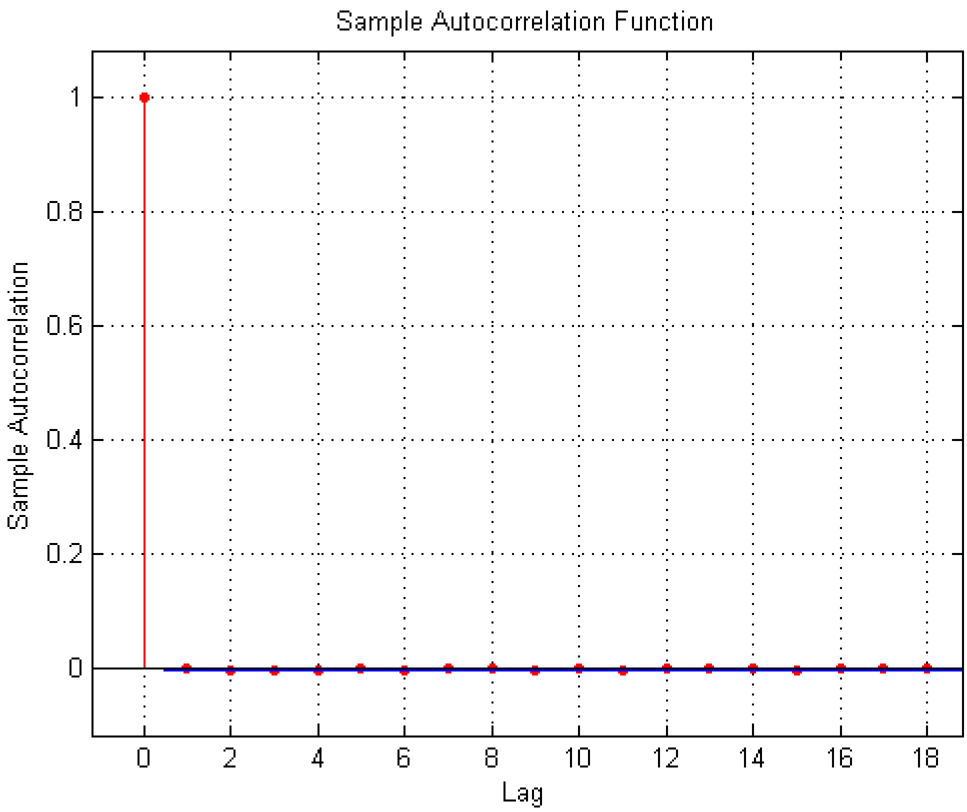


Figure 3: Autocorrelation function of Gaussian white process

MATLAB showed our guess when I number of data approaches 10^6 because high data length could guarantee its distribution properties which last assignment gave a deep digging.

3 Signal Processing with Low-pass Filter

Low-pass filter is widly-used tool when signal processing because high frequency signal usually can be recognized as noise. We apply 1st and 2nd order filter to analyze our white process.

3.1 System properties

The first order system is

$$T(s) = \frac{1}{2s+1}$$

the second order system is

$$T(s) = \frac{0.25}{s^2 + 0.7071s + 0.25}$$

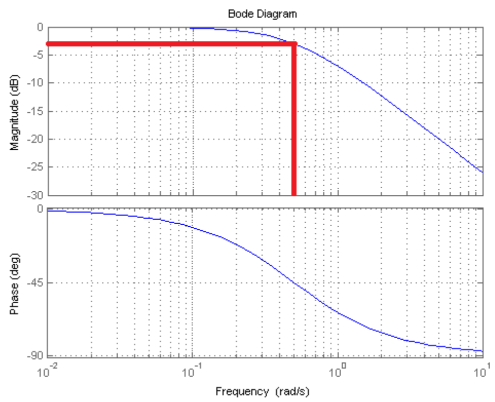


Figure 4: Bode plot of 1st order filter

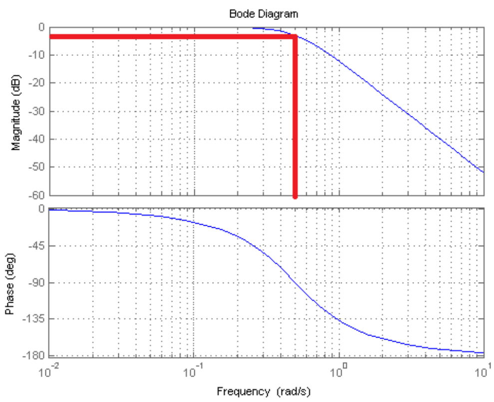


Figure 5: Bode plot of 2nd order filter

Using Bode plot, we easily find that the **bandwidth** of both filter are

$$\omega_c = 0.5\ rad/sec$$

3.2 Filtered Signal

I used Simulink to design a block diagram of measuring system and demonstrate the time response simultaneously. The Gaussian noise generator is set as

Gaussian noise generator	Parameters
Sampling frequency	10 Hz
Measuring time	1,000 sec
Data number	10,000

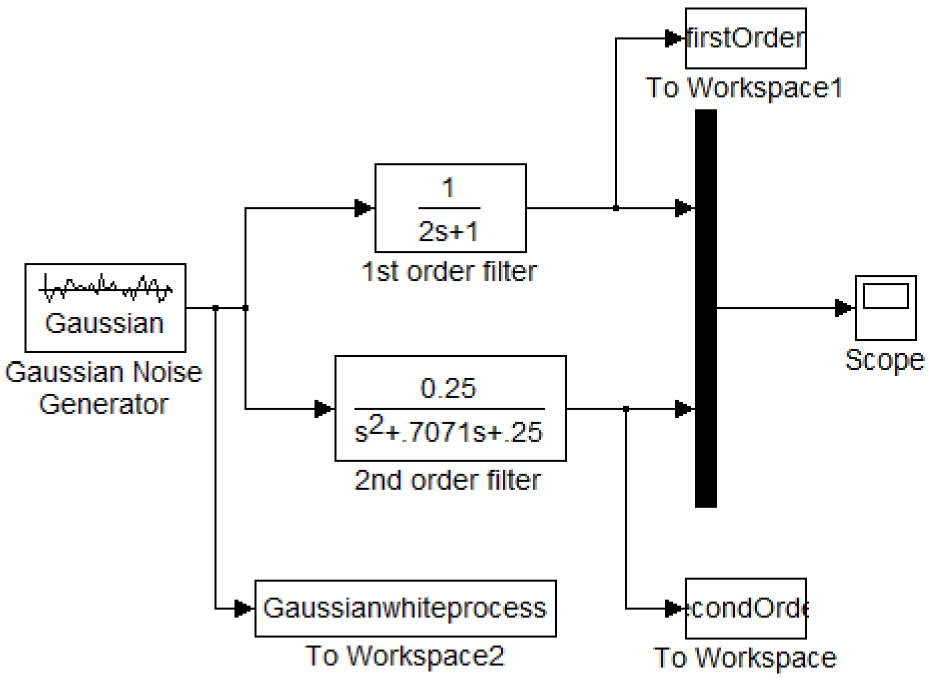


Figure 6: Block Diagram of Measurement System

The system captures the signal source from three independent data path. In simulation, we can confirm the source would not be contaminated. Idealy, we can get three kinds of processed signal from the identical sample.

We testify the non-stationary phenomenon with simple statistical properties, mean value.

$$\hat{\mu_x}(t_i) = \frac{1}{N_i} \sum_{i=1}^{N} x(t_i)$$

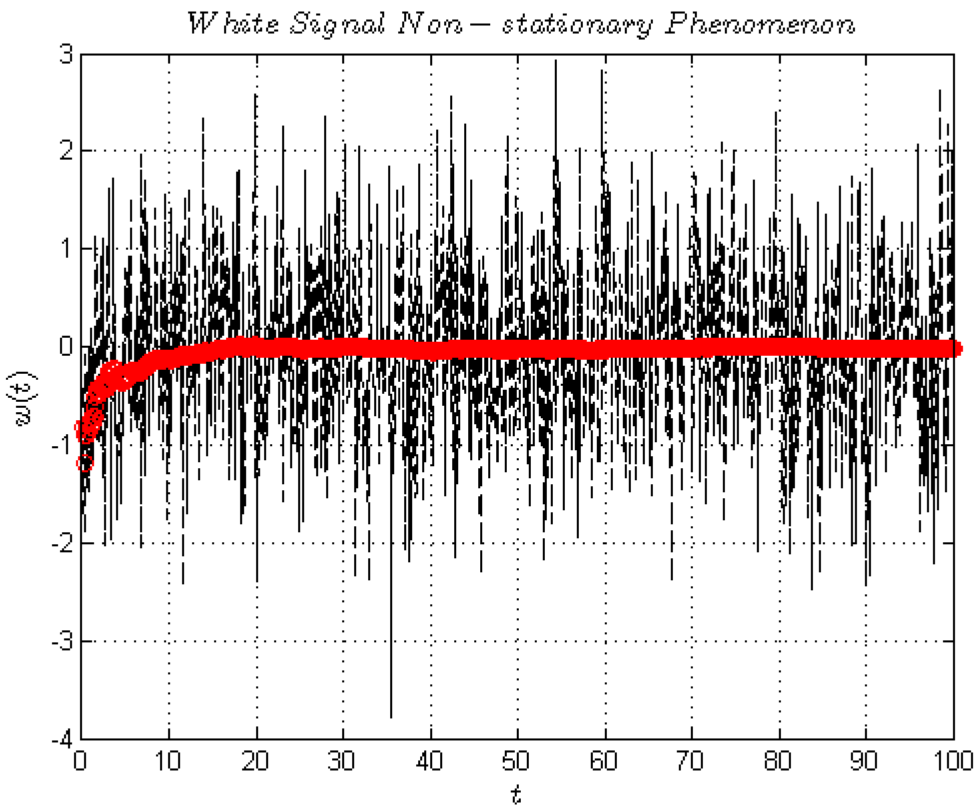


Figure 7: Red line is mean value of time-series

Non-stationary part exists in first 20 **seconds** . We can verify the fact with naked eyes. Then the mean value does not change significantly and approximates to 0.

To be conservative, I choose data far away from non-stationary region.

White signal $w(t)$	Parameters
Time space	400 sec $\sim$ 500 sec
Data length	1,000

Zooming in the time response, we still can find that siganl pass through 2nd order filter is much more sooth than those through 1st order.

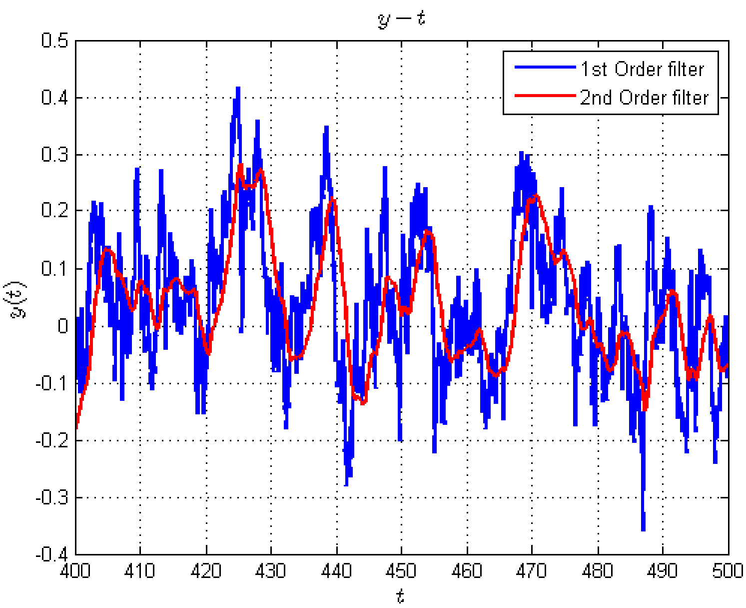


Figure 8: Filters Comapring

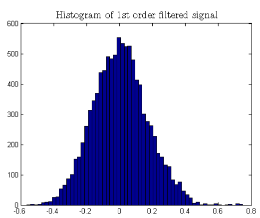


Figure 9: First Order Histogram

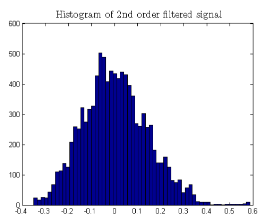


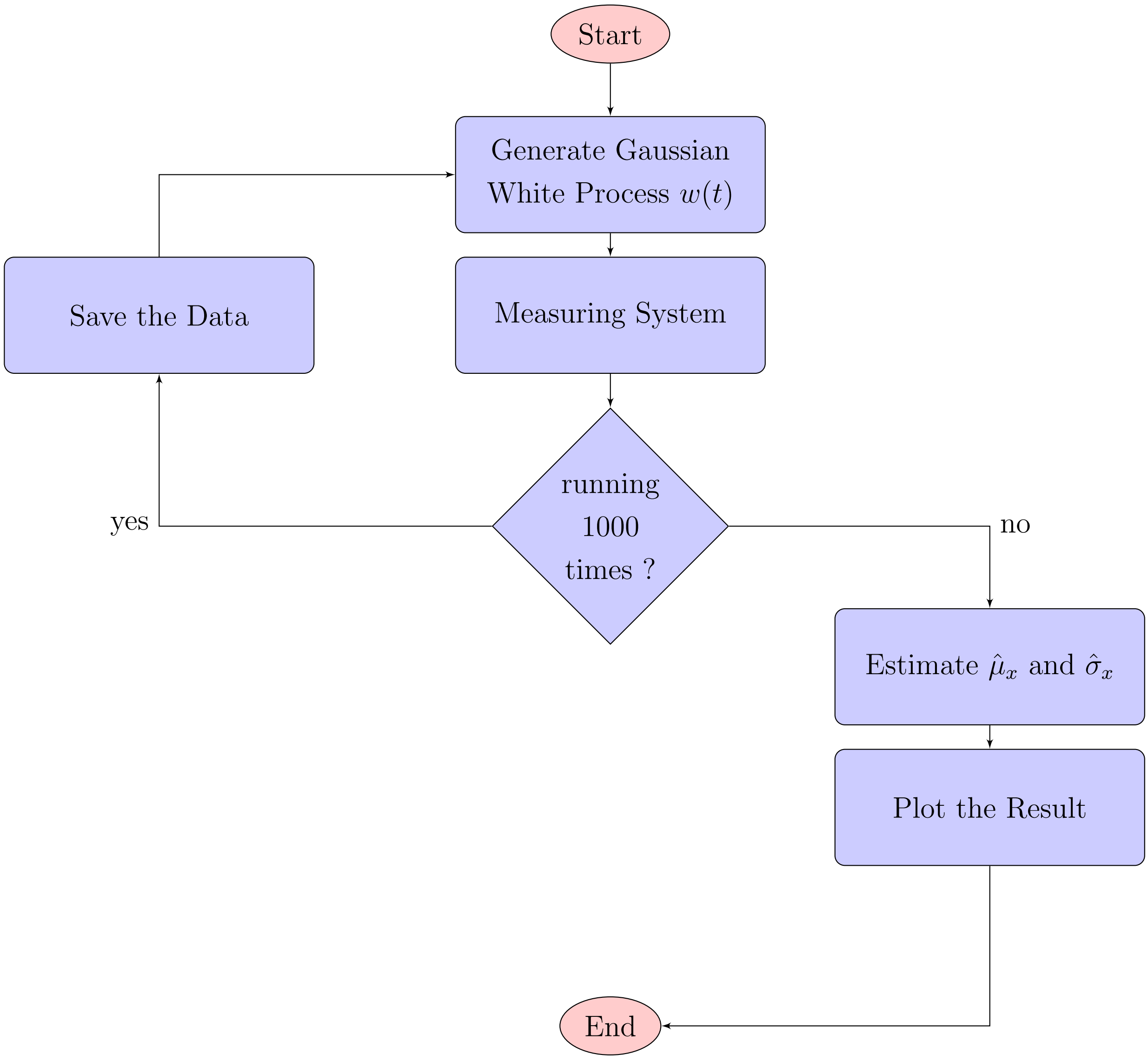
Figure 10: Second Order Histogram

However, histogram indicates the more high frequency parts we kill the more beautiful curves could be. The right-hand side histogram appears to jitter and shake more dramaticaly than the left one.

4 Ensembled Data

In simulation, it is facile to repeat each measurement and get independent samples. I repeat nearly a thousand time and find ensemble average and standard deviation of these data.

4.1 Flow Chart of Ensembling



4.2 Ensemble Mean Value

The average still oscillates but close to 0. The first order filtered error lies in the interval of $[-0.01, 0.01]$ and second order error lies in the interval of $[-0.005, 0.005]$.

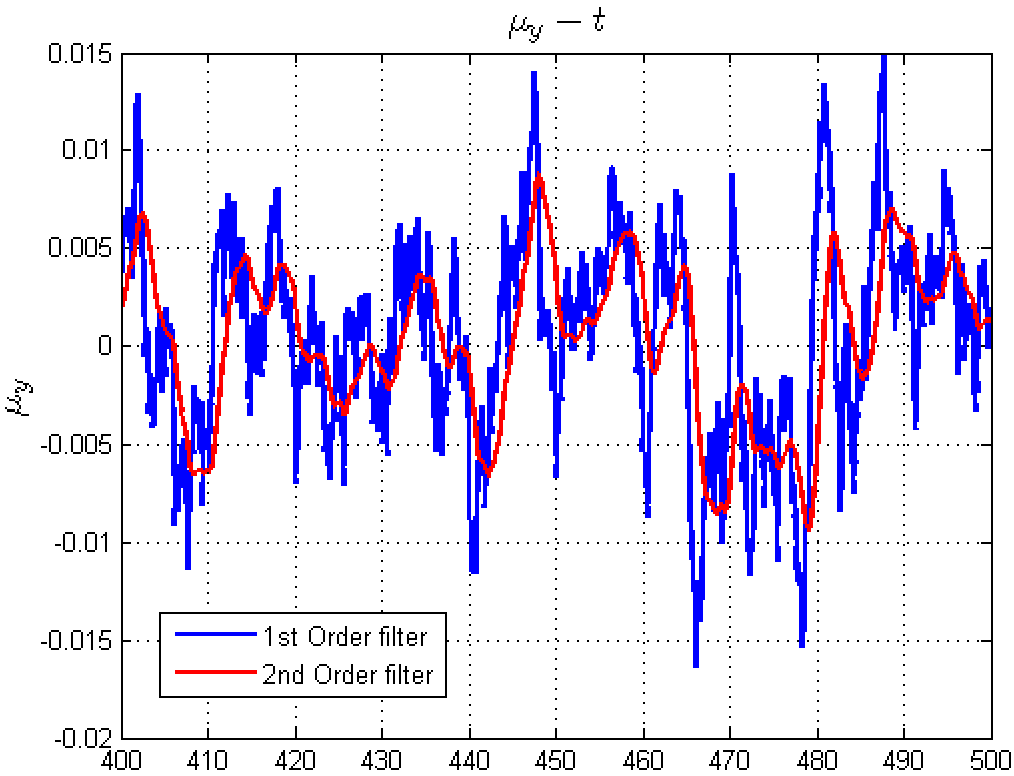


Figure 11: Ensemble average

System order	Mean Value
1st order	$\hat{\mu}_x \in [-0.01, 0.01]$
2nd order	$\hat{\mu}_x \in [-0.005, 0.005]$

4.3 Ensemble Standard Devation

Deviation includes a broader picture of signal. After filtering, the difference comes up.

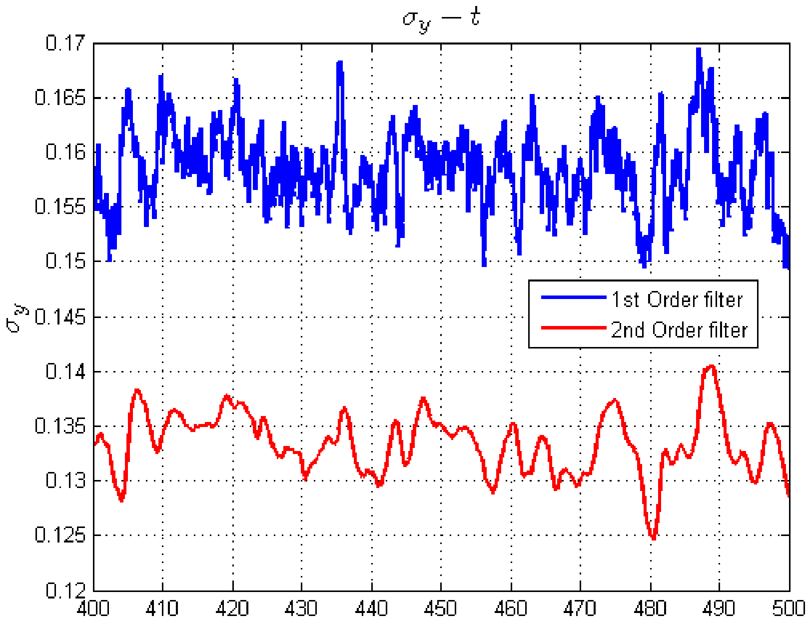


Figure 12: Ensemble Standard Deviation

The plot indicates that the more high frequency parts we kill, the more certain properties we could acquire. Since that small deviation implies less uncertainty and we could predict them with more confidence.

System order	Deviation
1st order	$\hat{\sigma}_x = 0.157$
2nd order	$\hat{\sigma}_x = 0.133$

In this case, we would choose 2nd order or higher order filter to anaylze and smooth the signal. But to consider computational efficiency (time cost), above 5th order filter should be second thought.

5 Discussion

5.1 Non-Stationary Phenomenon

It is really an important issue in random signal processing. We must get rid of non- stationary part before we doing anything.

5.1.1 FATAL MISTAKES

Here is a living-proof that the ensembled outcomes were polluted by non-stationary signals.

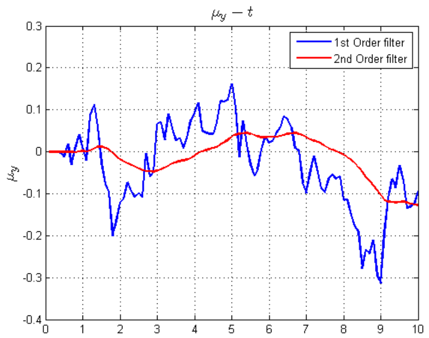


Figure 13: Polluted ensemble average

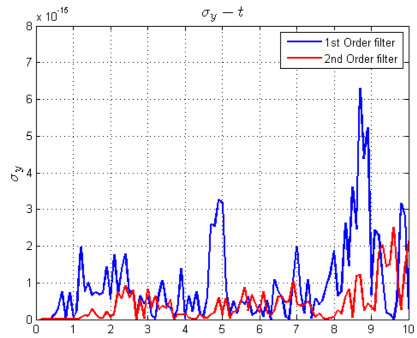


Figure 14: Polluted ensemble deviation

We can not get any useful message from these results. And sometimes we unfortunately get the opposite result. Second order deviation should be less than first order, I, however, get the opposite results.

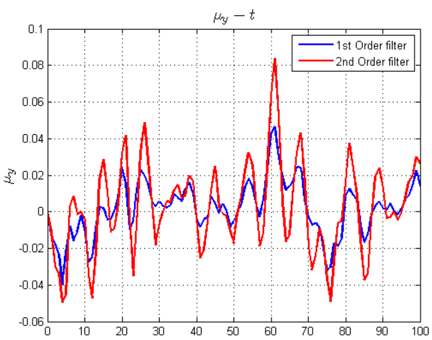


Figure 15: Distorted ensemble average

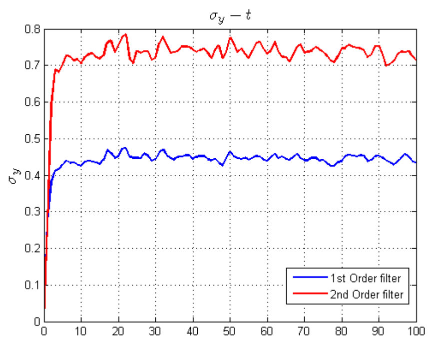


Figure 16: Distorted ensemble deviation

5.2 FFT

I want to dig deeper and thoroughly on the white process. So I choose two powerful tools, spectrum and autocorrelation function to analyze the data. Fast fourier transform is engineers' eyes. I analyze the process with it.

5.2.1 Setting of Algorithm

There are several properties should be determined before simulation. First I choose numbers of point to be

$$N = 2^{10} = 1024$$

beacause multiplication of 2 can reduce time complexity of algorithm to

$$\Theta(N \ln N)$$

rather than

$$\Theta(N^2)$$

It significantly reduce time consumption.

Then, in mathematical definition, specturm of white process is an inifinity. No matter how fast we sample, we definitely loss some information. In the numerical simulation, I choose sample frequency as high as we can. So,

$$f_s = 2000\ Hz$$

is a great number relative to our bandwith $\omega_c = 3\ rad/sec = 18.85\ Hz$. And each frequency step can be determined

$$freqstep = \frac{f_s}{N} = 1.95$$

I use both sides of (symmetric) spectrum to describe my result.

5.2.2 Result of Analysis

We swipe the Gaussian white process first, as below

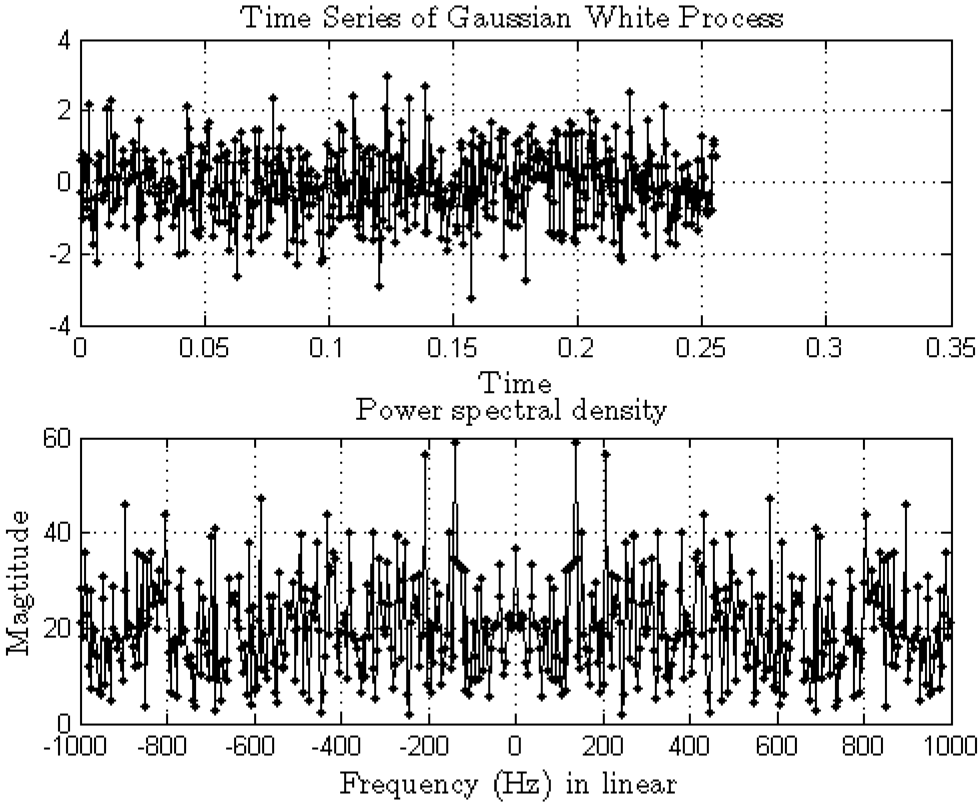


Figure 17: Specturm of White Signal

The frequencies stretches and covers the almost whole spectrum, undoubtly, it is "white".

Let we compare filtered data together. As we expected, as low-pass filters'

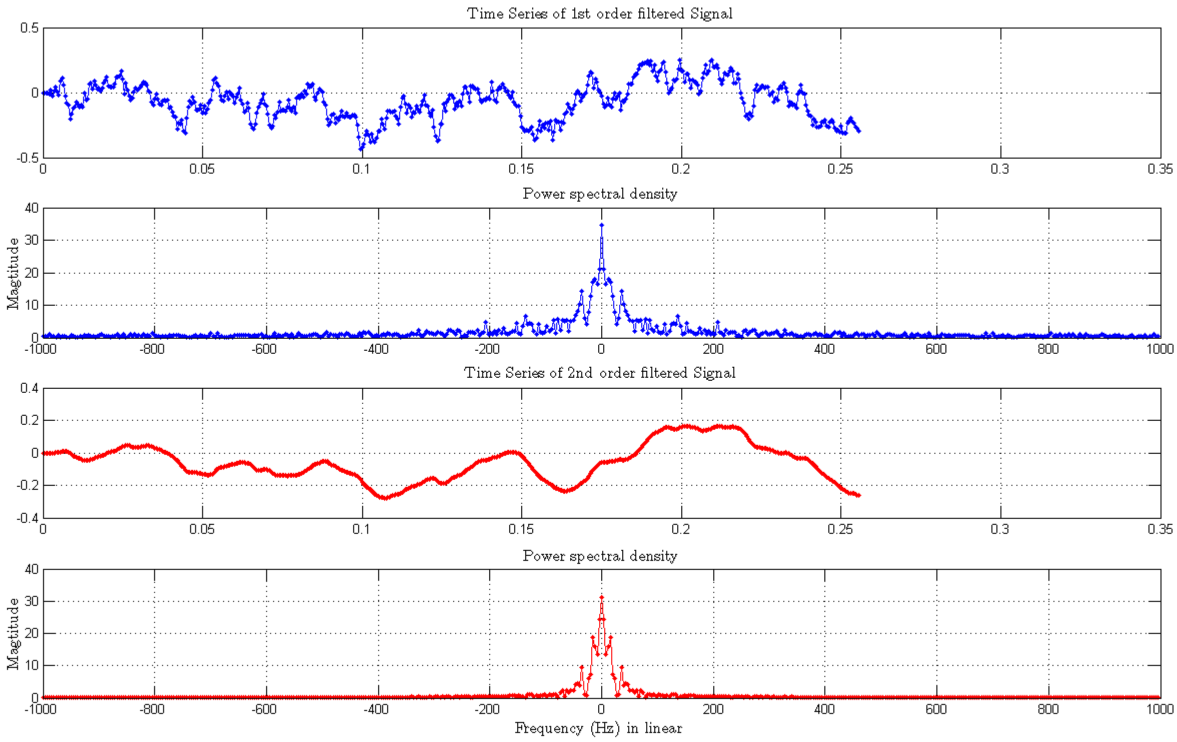


Figure 18: Specturm of White Signal

bandwith are 0.5 rad/sec, only low frequencies survive and higer filter is, more centered is in spectrum. It concludes that rapid oscillation represents high frequency, as we kill them the curve could be more smooth and easily and clearly indicates data tendency and properties. In other words, we can capture more information and more chance to predict the future. We will see in next section.

5.3 Autocorrelation function

In section 2.2 we numerically proved autocorrelation of white process should be an impulse function. We also expect that we could acquire more information after filtering the signal. In section 4, we did that. Using 2nd order filter, we find mean value with error ±4% and deviation with offset 0.3, equal error 33%. It's kind of large but informative than white. So, intuitively, we hope these data can be used to predict. That is why we find thier autocorrelation function.

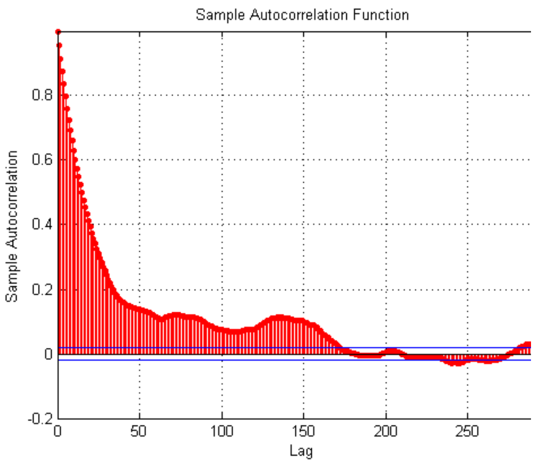


Figure 19: Fisrt order filtered data

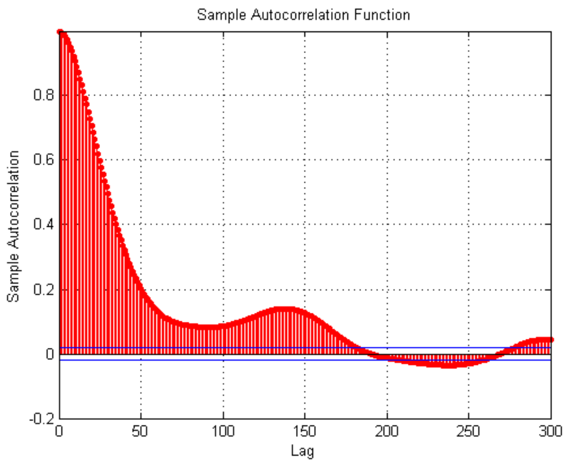


Figure 20: Second order filtered data

Amazingly, their autocorrelation do increase and it means we can use the past data and algorithm to predict what will happen!
The right-hand side plot is well-shaped but left one has some defects. It makes sence as we discussed in previous section. According to the book *Random Data – Analysis and Measurement Procedure*, that this form of autocorrelation comes from narrow bandwidth random noise. Verify our result again. I really acquire big pictures of random data processing and several powerful tools.

References

[1] Random Data:©: Analysis and Measurement Procedures,Julius S. Bendat, Allan G. Piersol

[2] Wikipedia : Autocorrelation, 2012