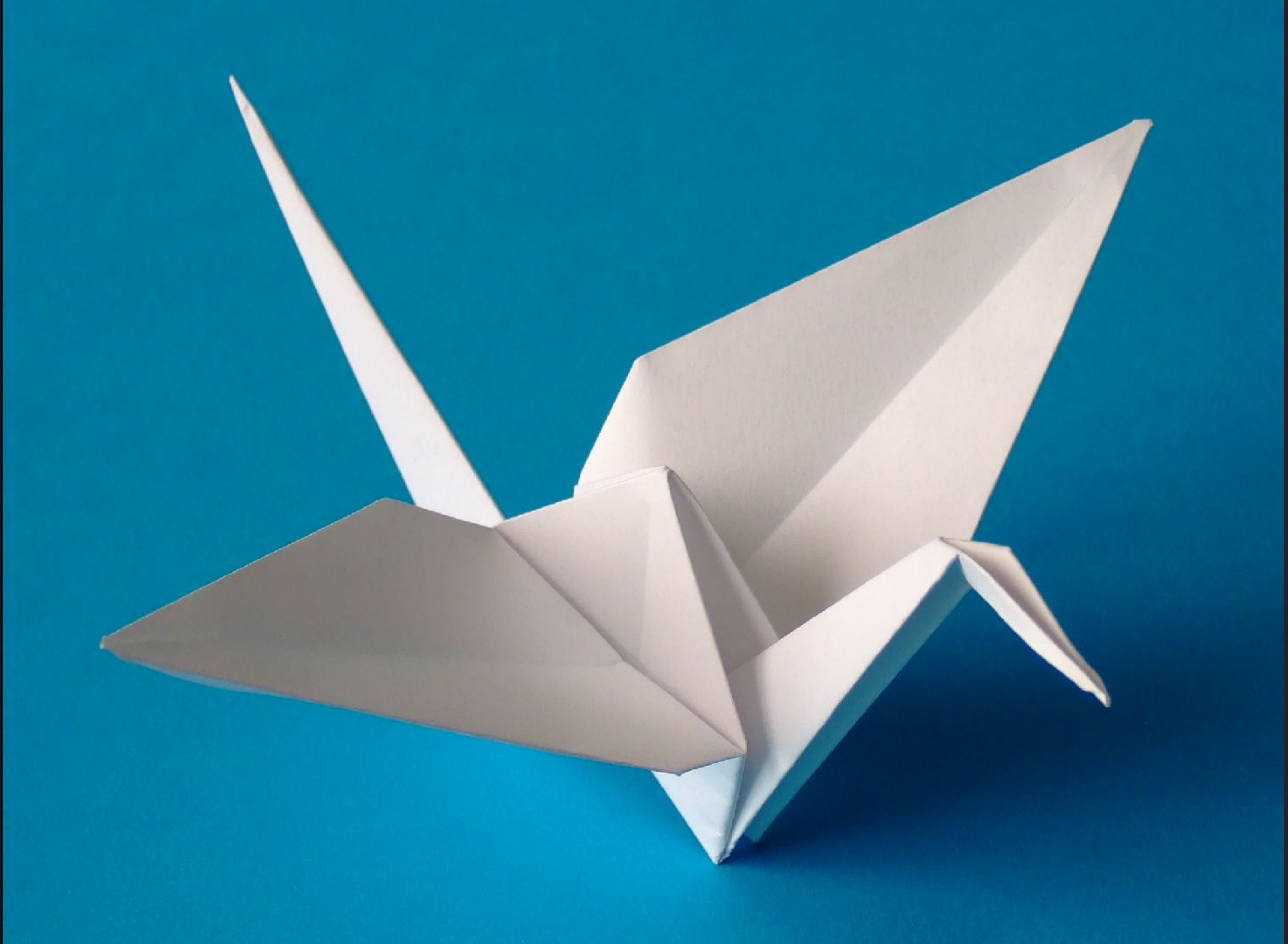
Origami




ORIGAMI NAZGUL


ORIGAMI NAZGUL
NAILED IT

2003

2004

2006

2008

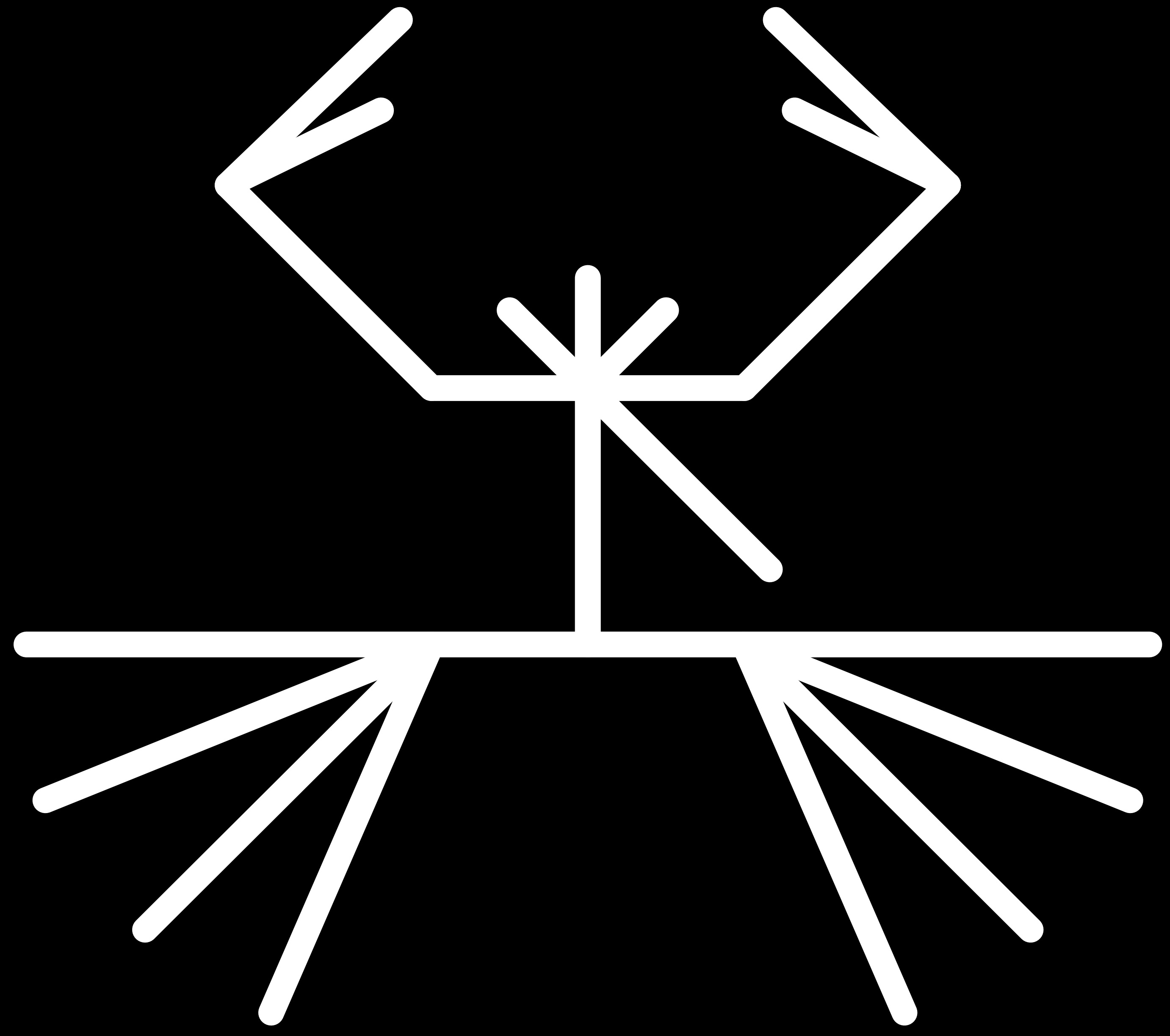
2009

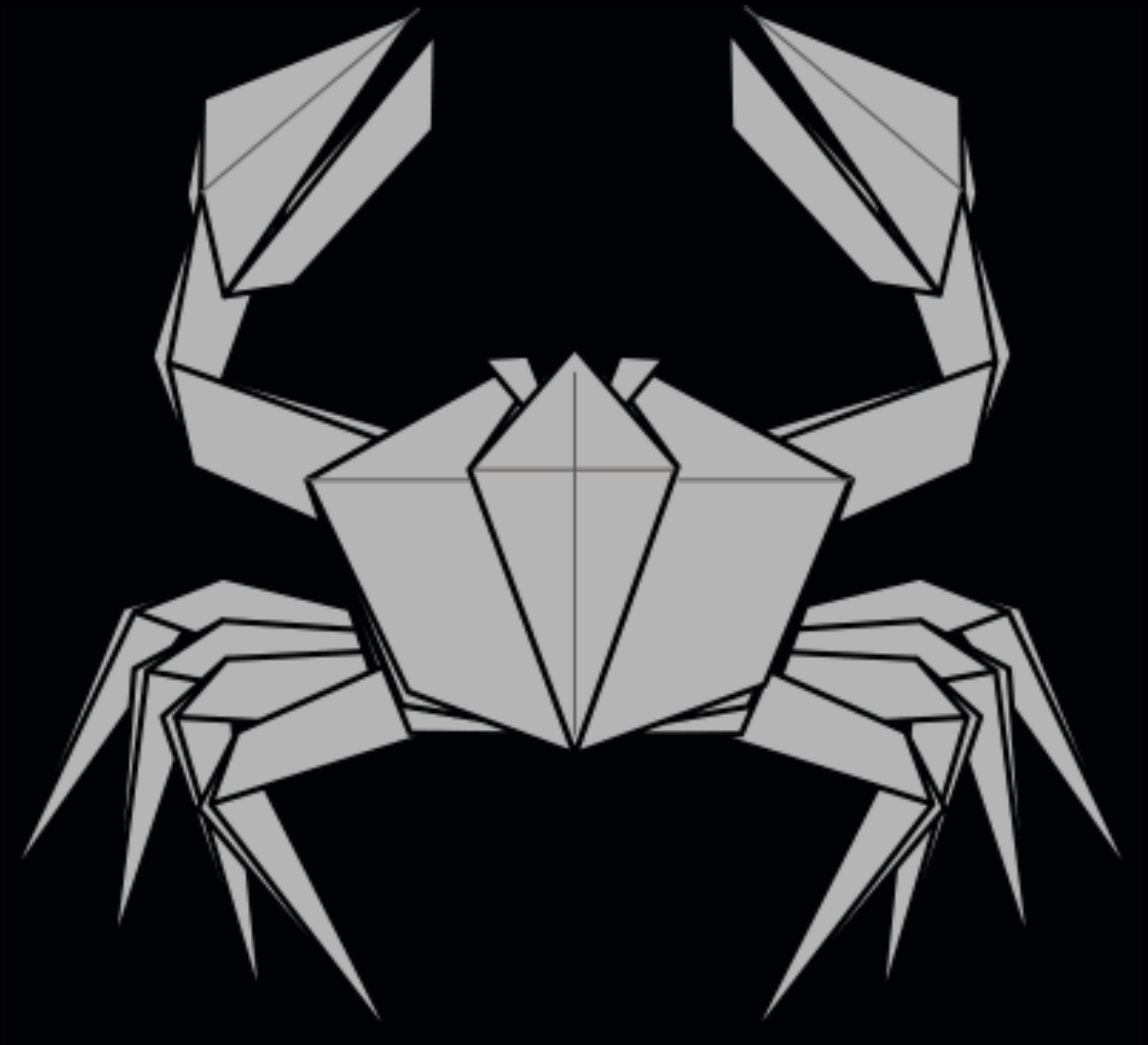
2011

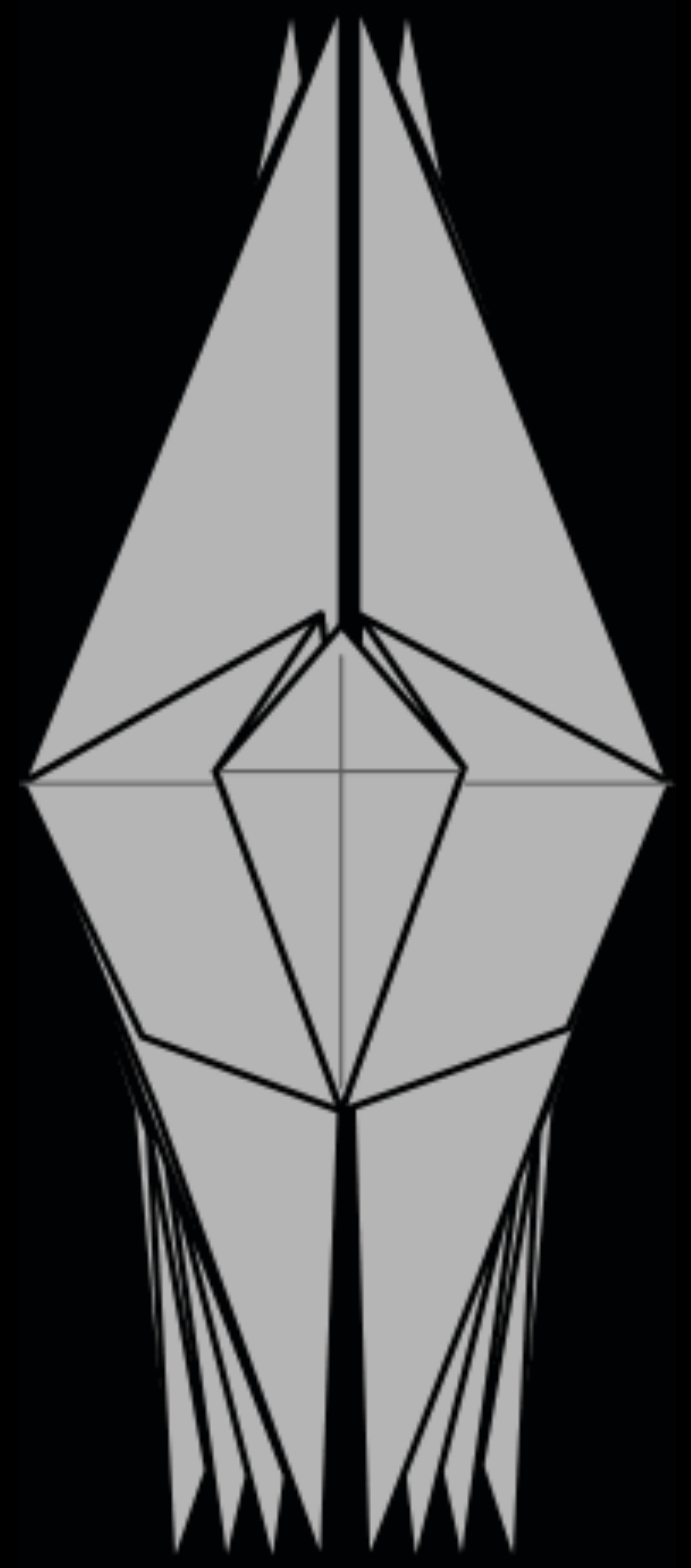
2012

2015

2018

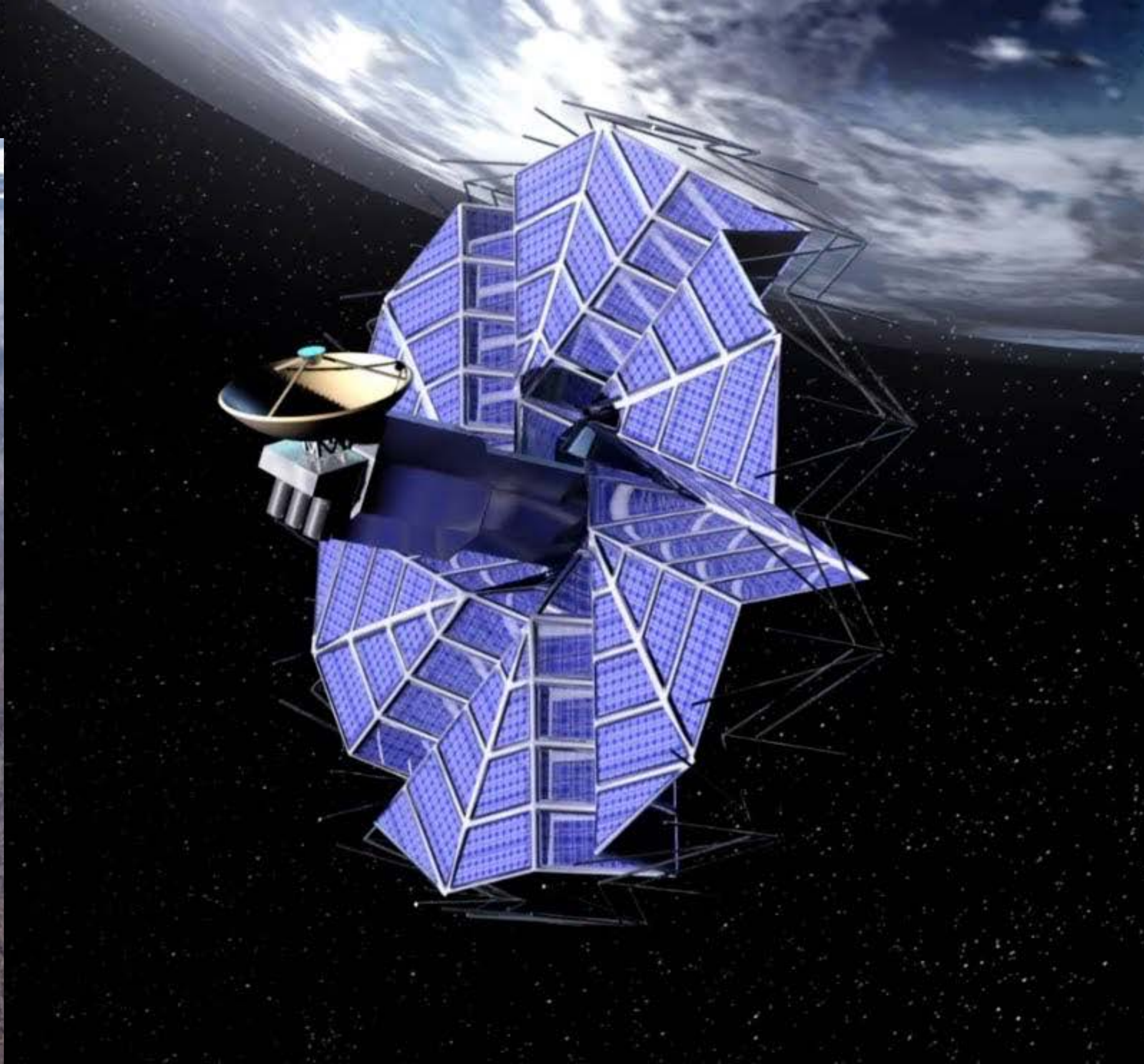




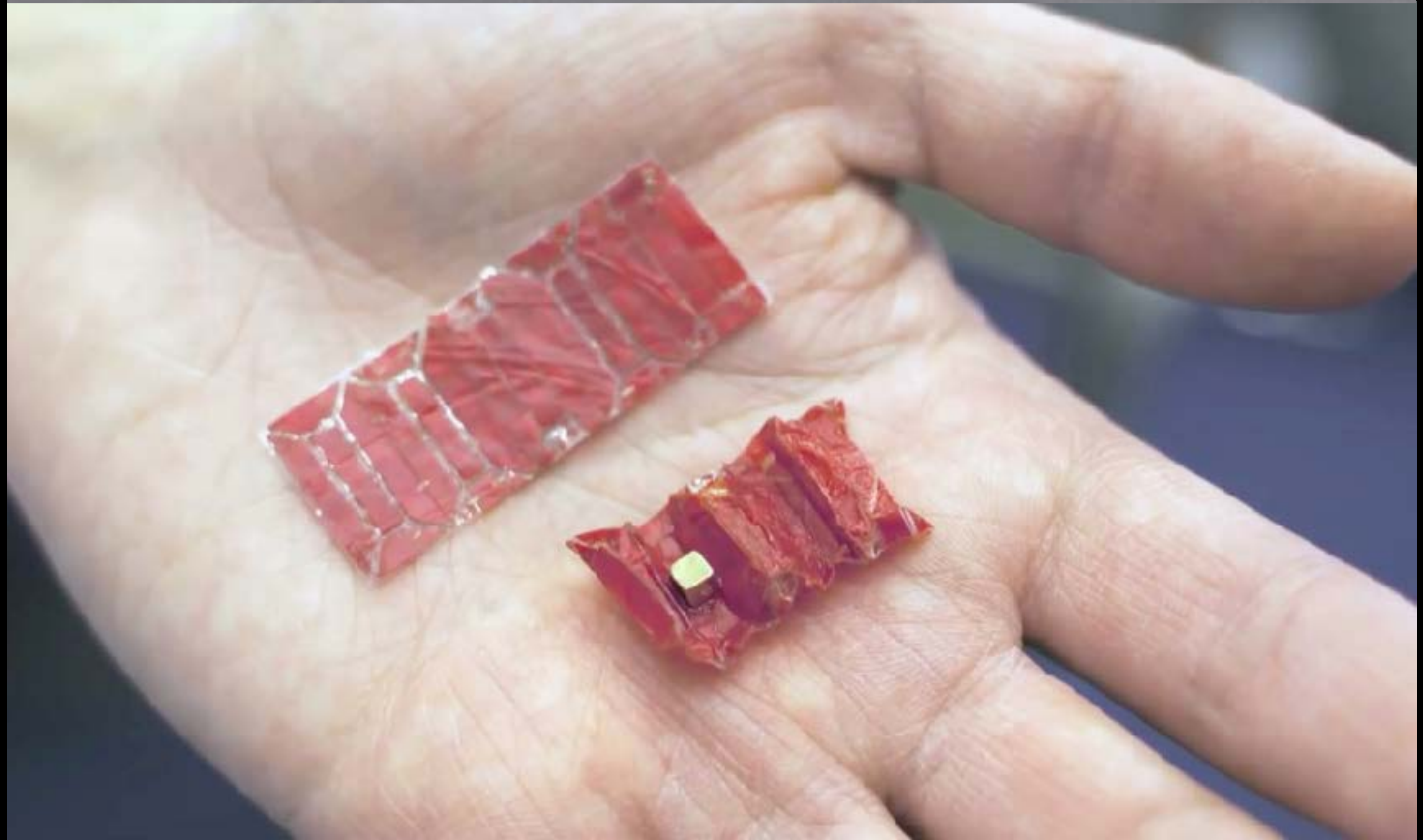


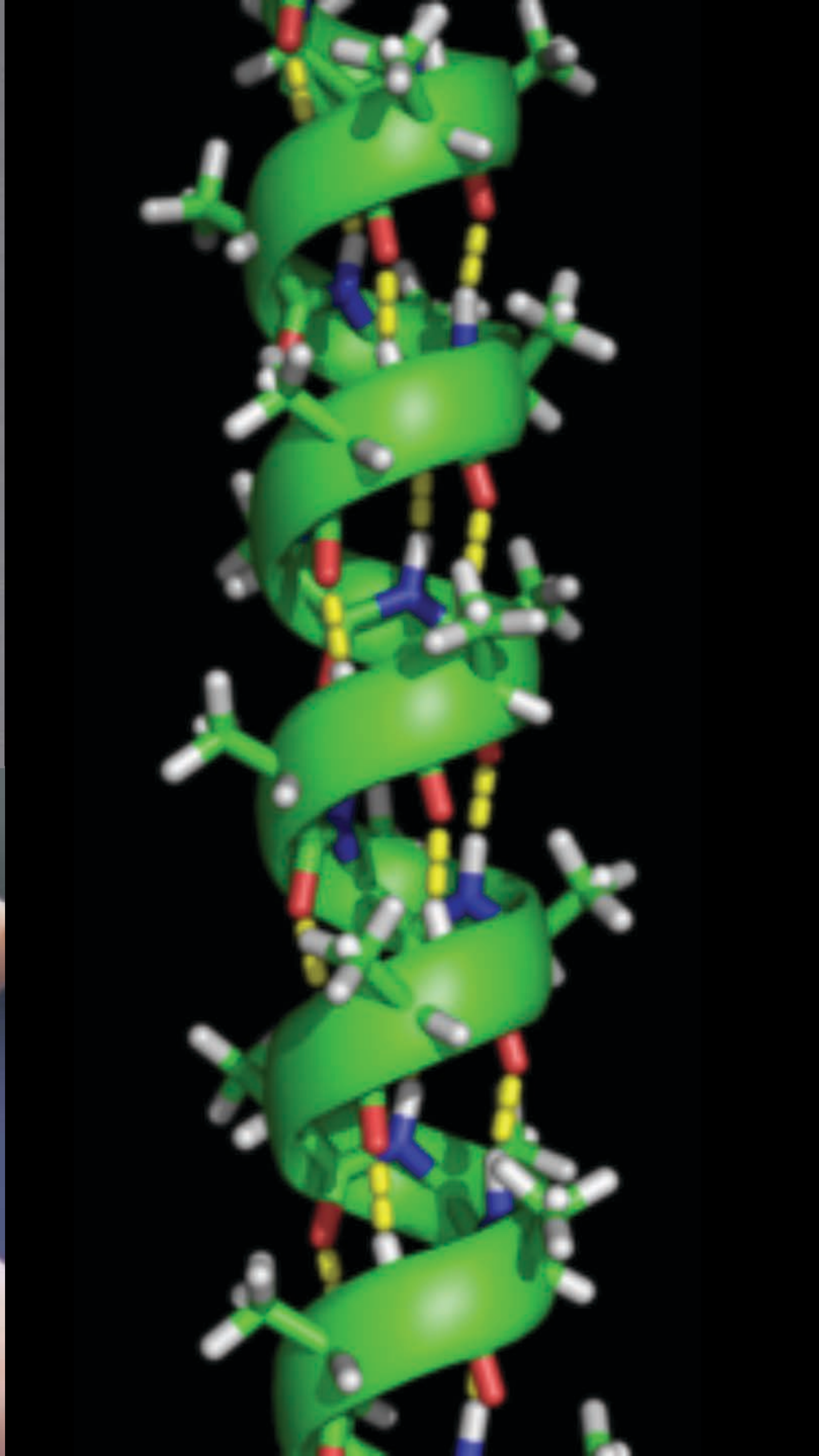
Applications

Space

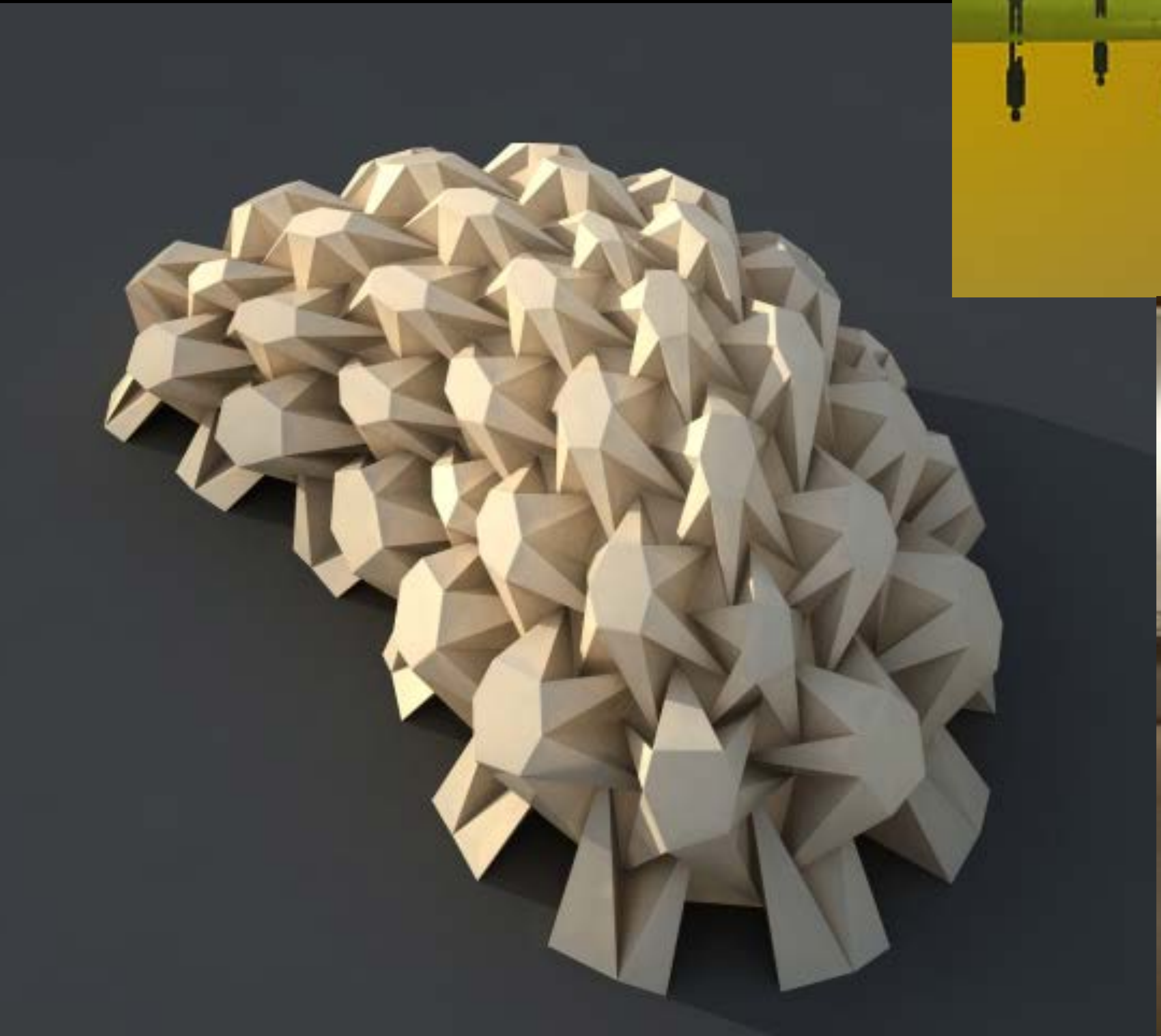


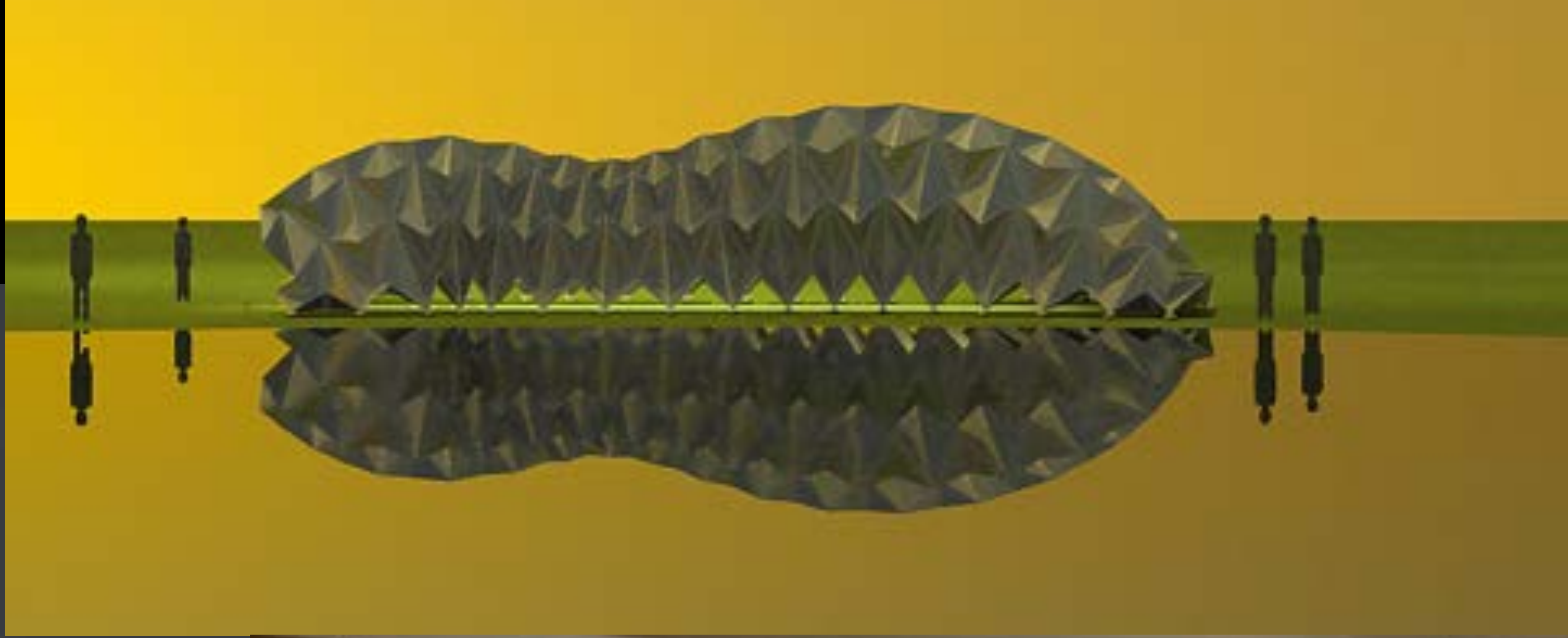
Medicine



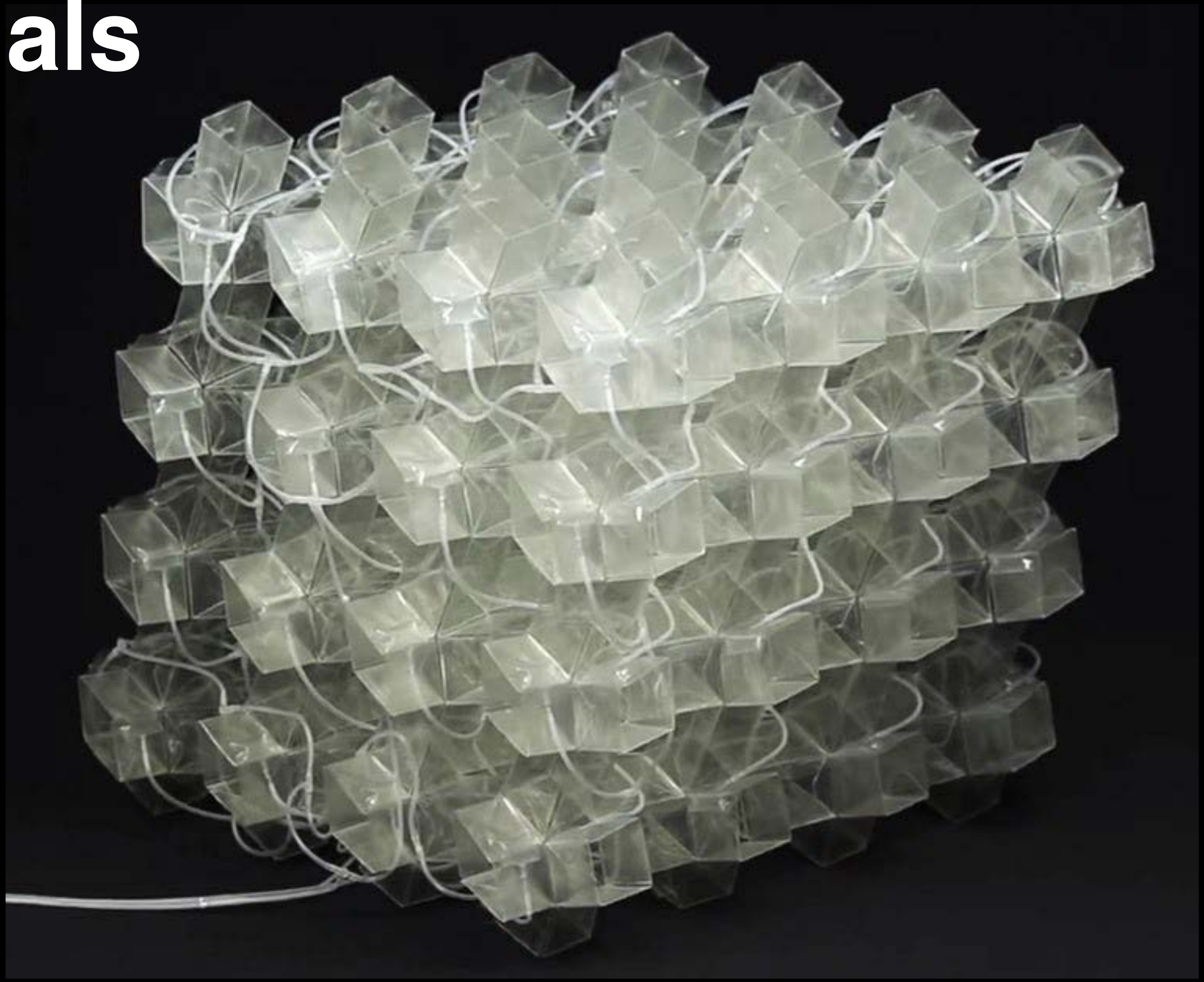


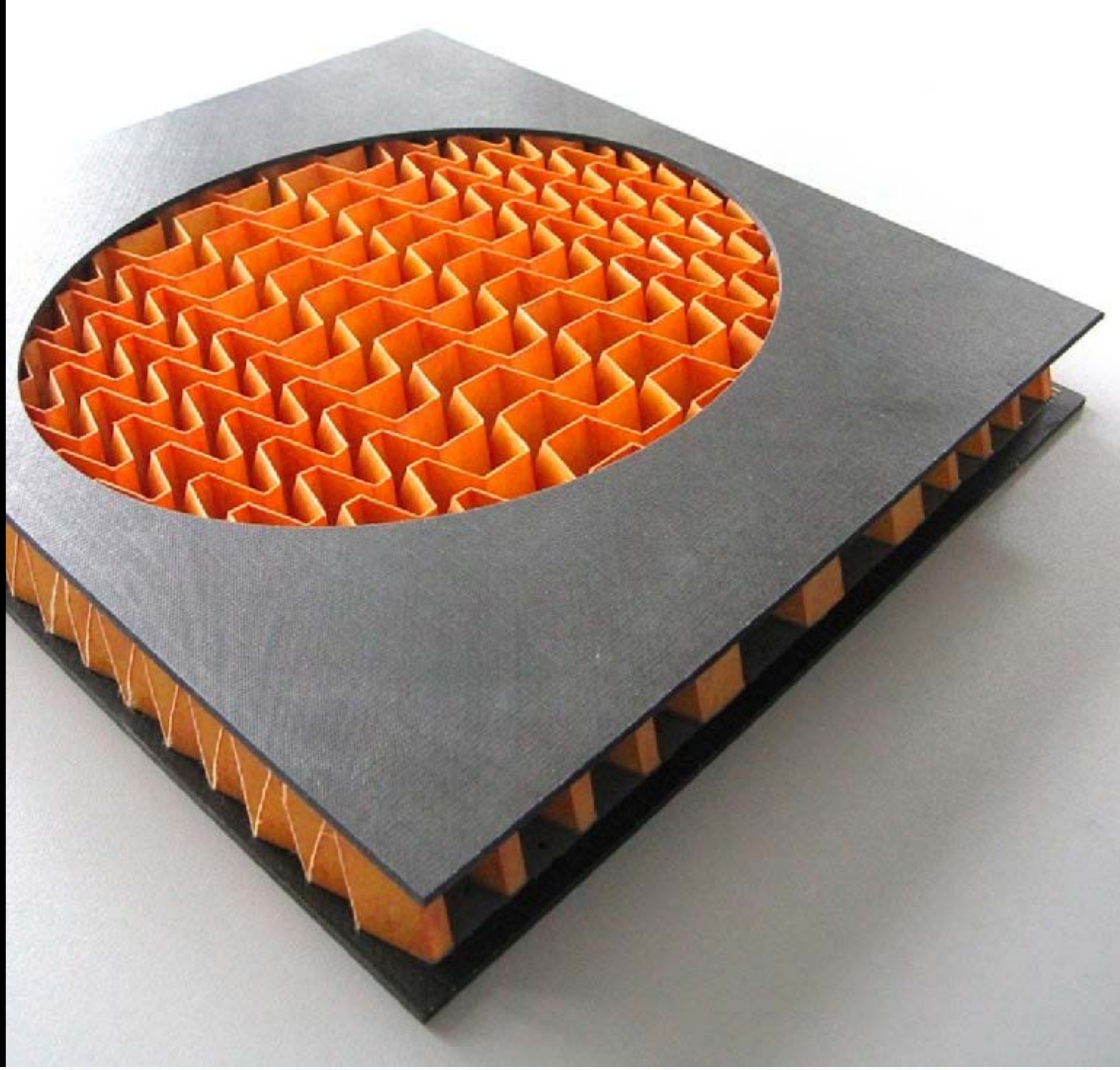
Architecture

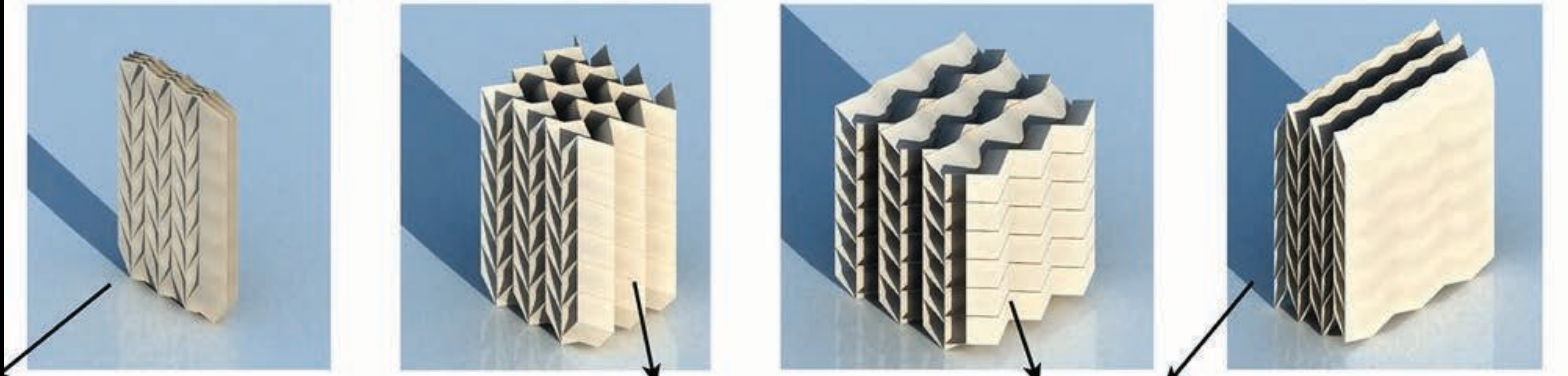




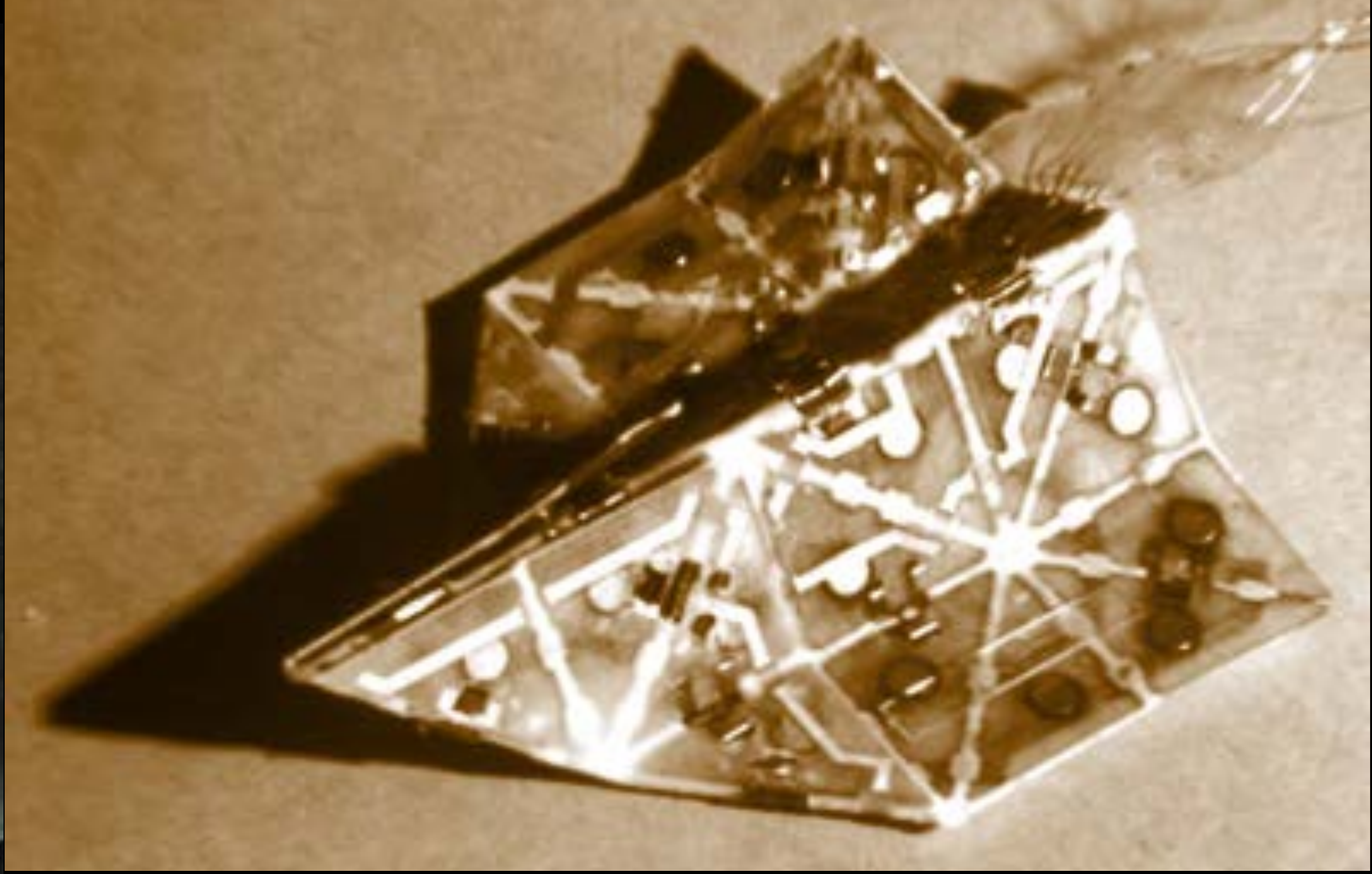
Materials

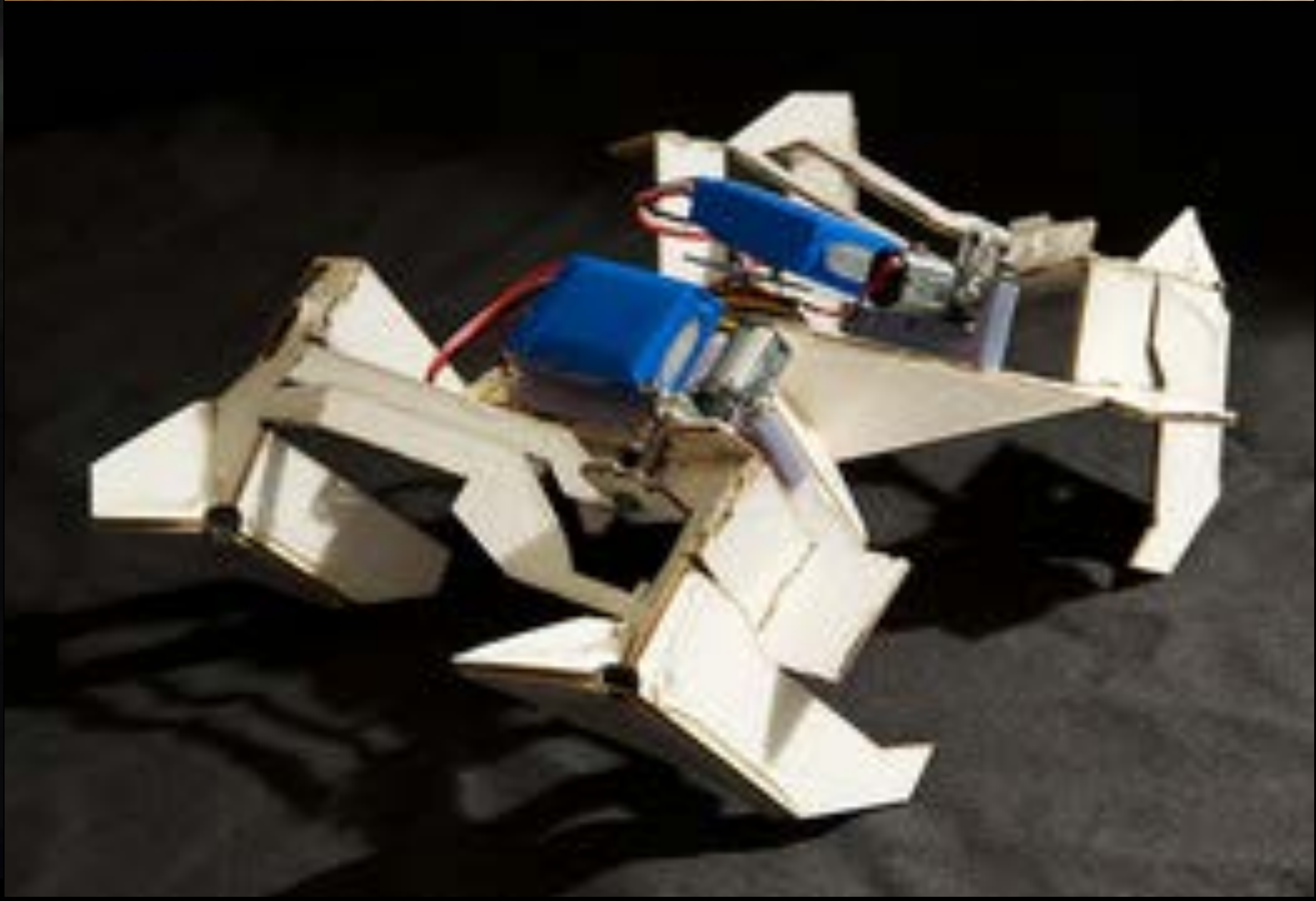




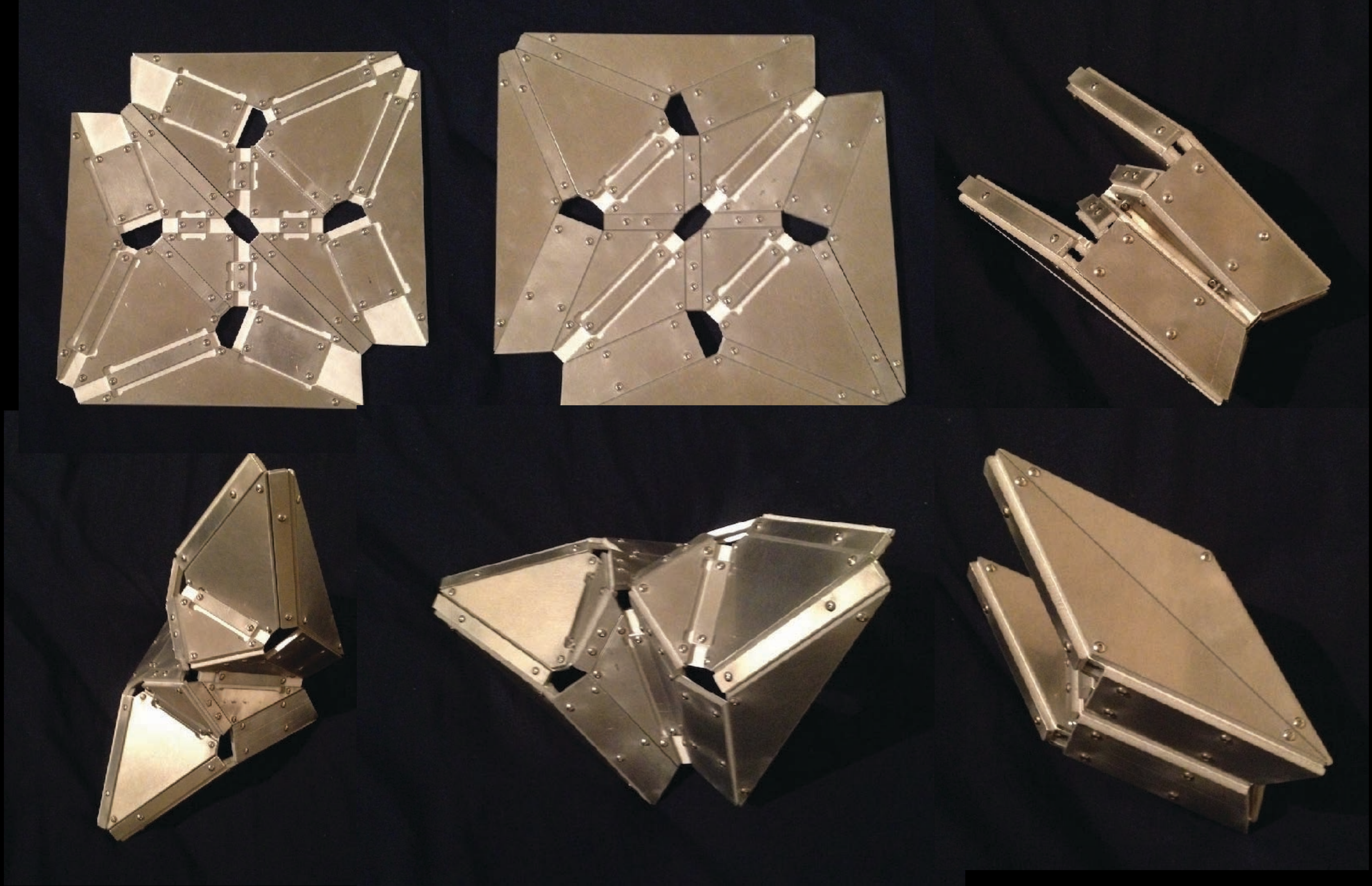


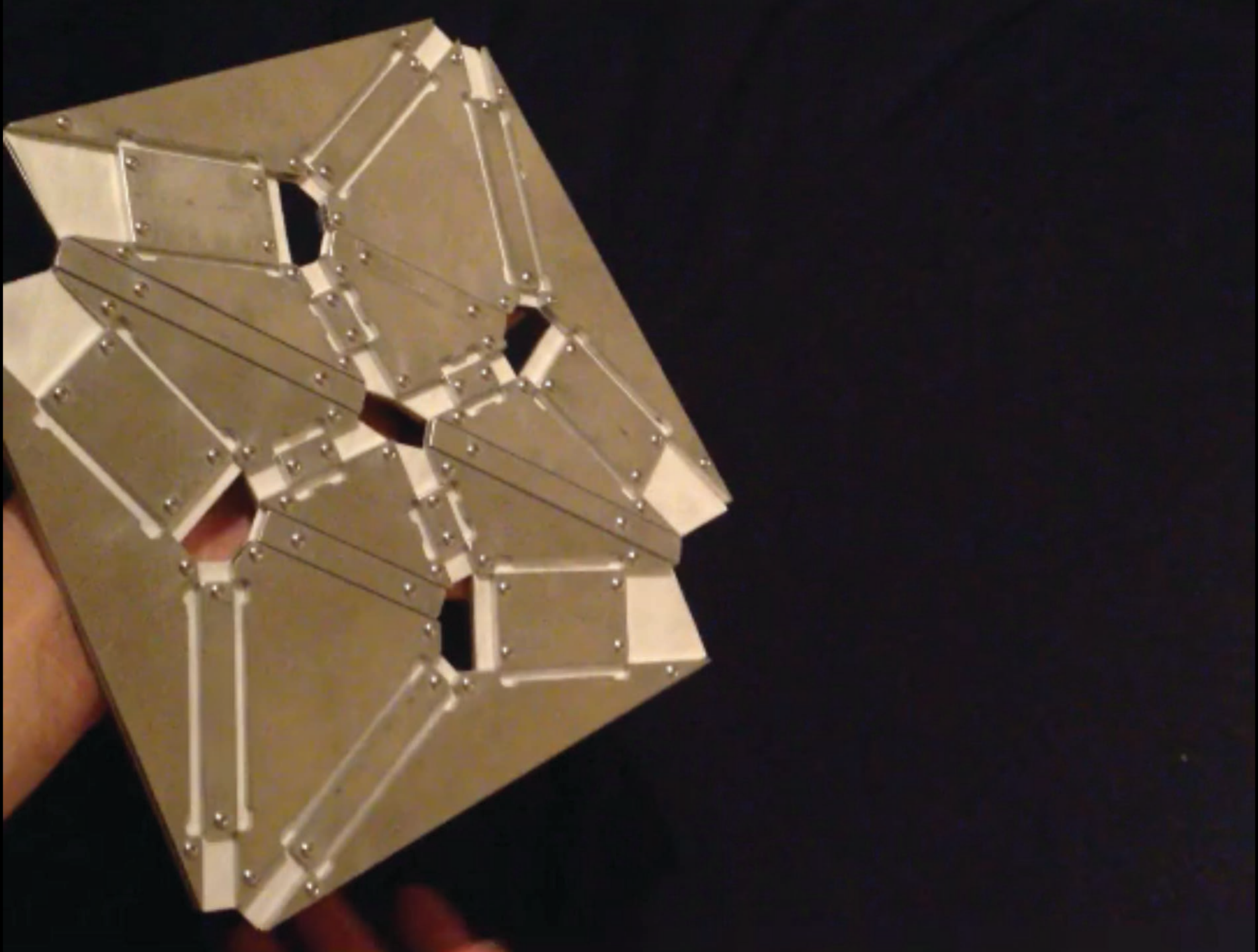
Transformers





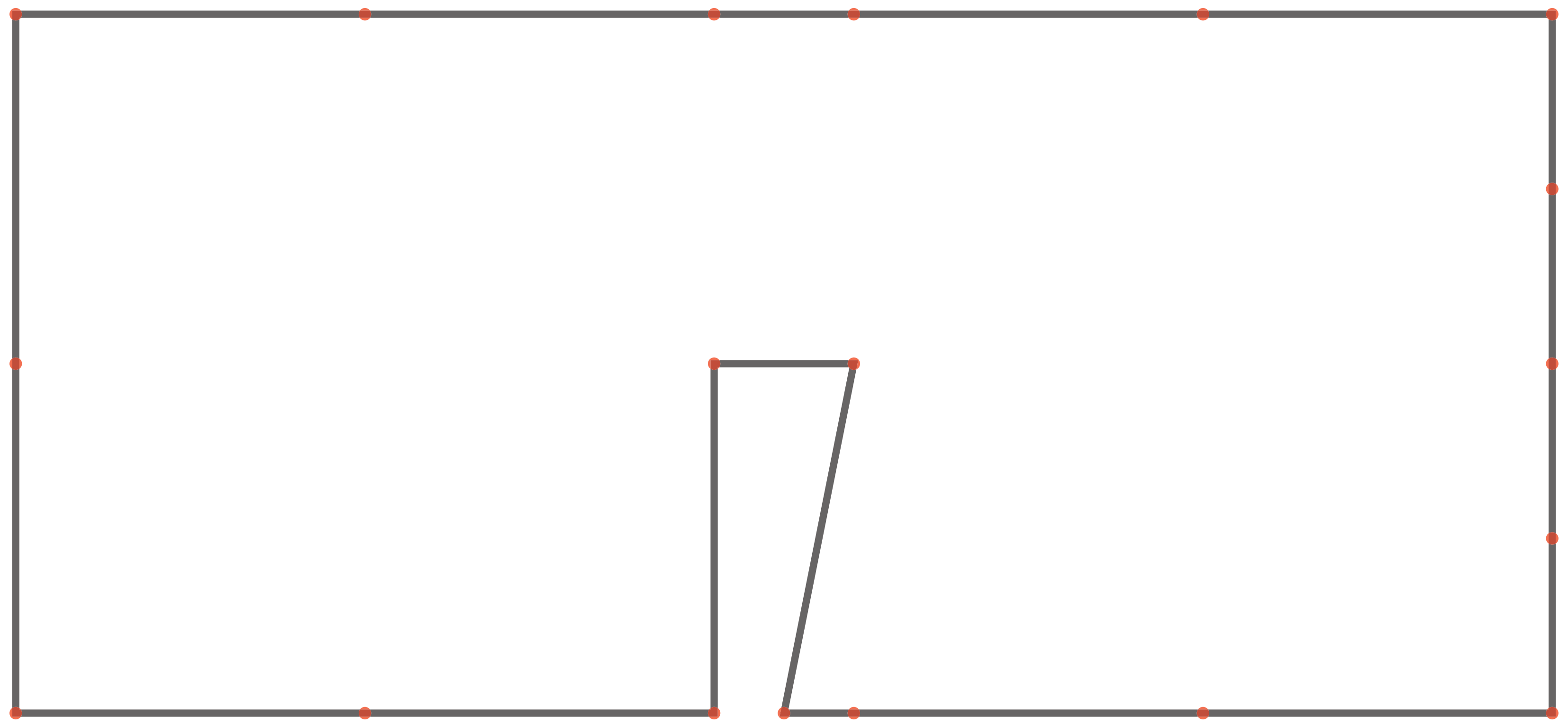
Research

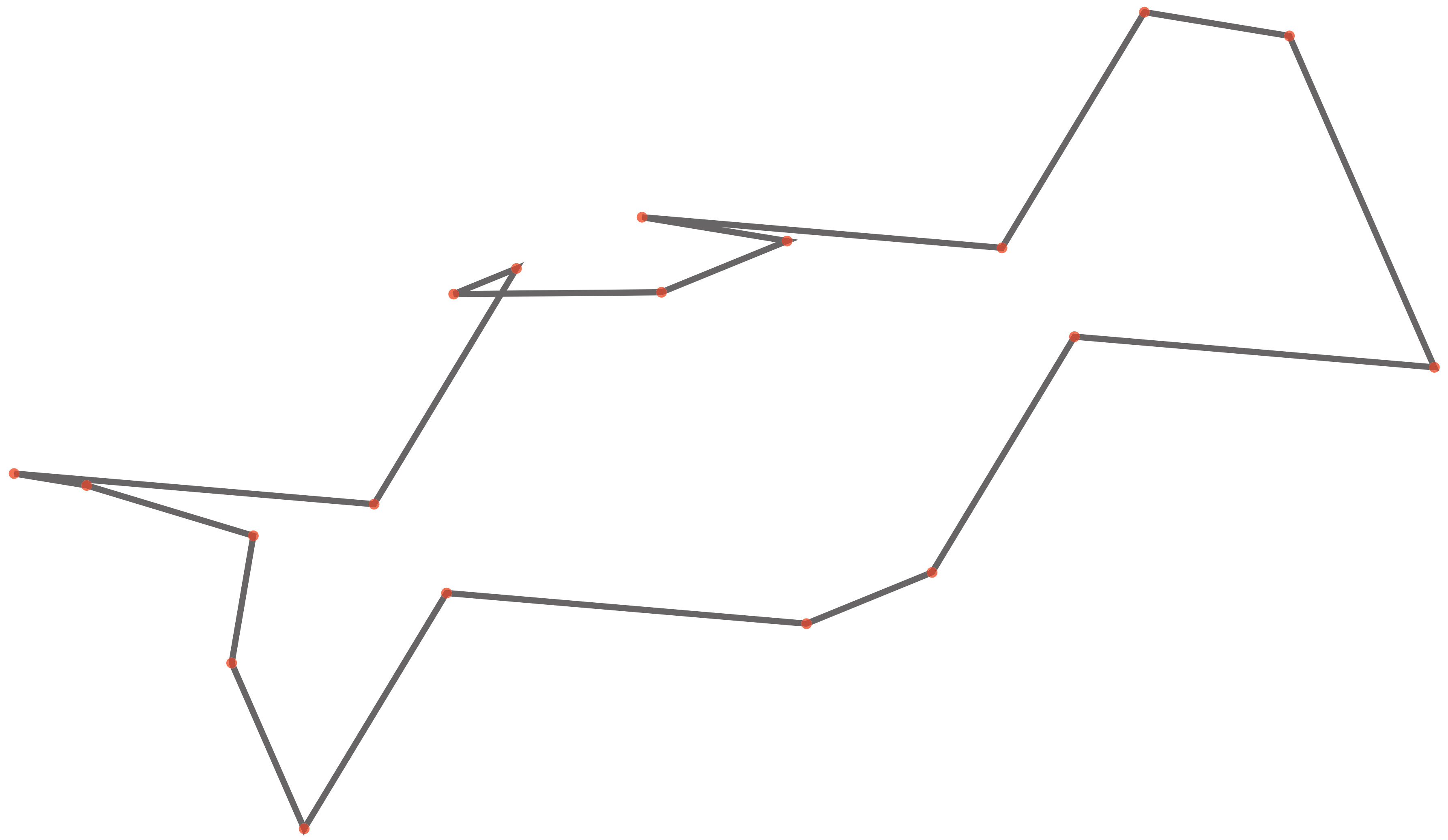




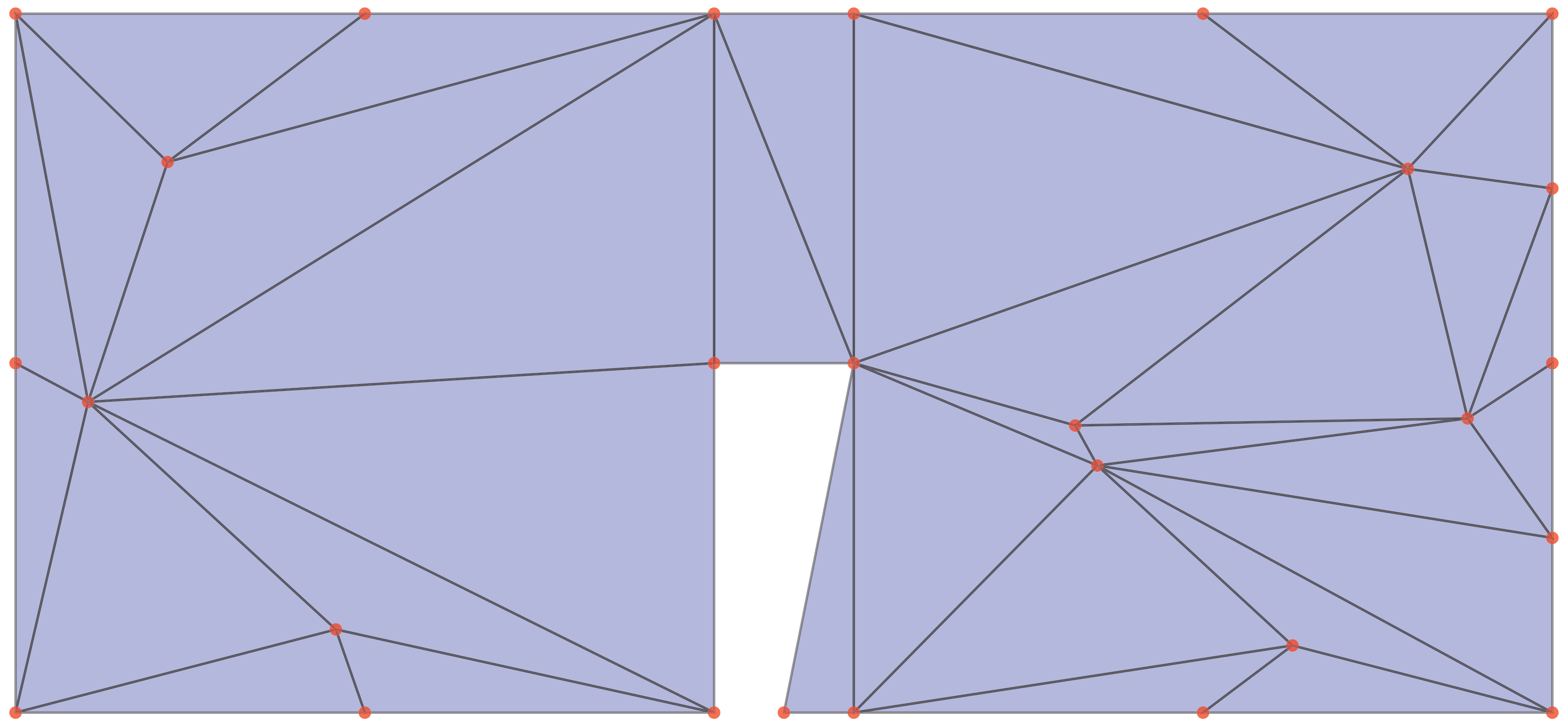
Boundary Value Problem

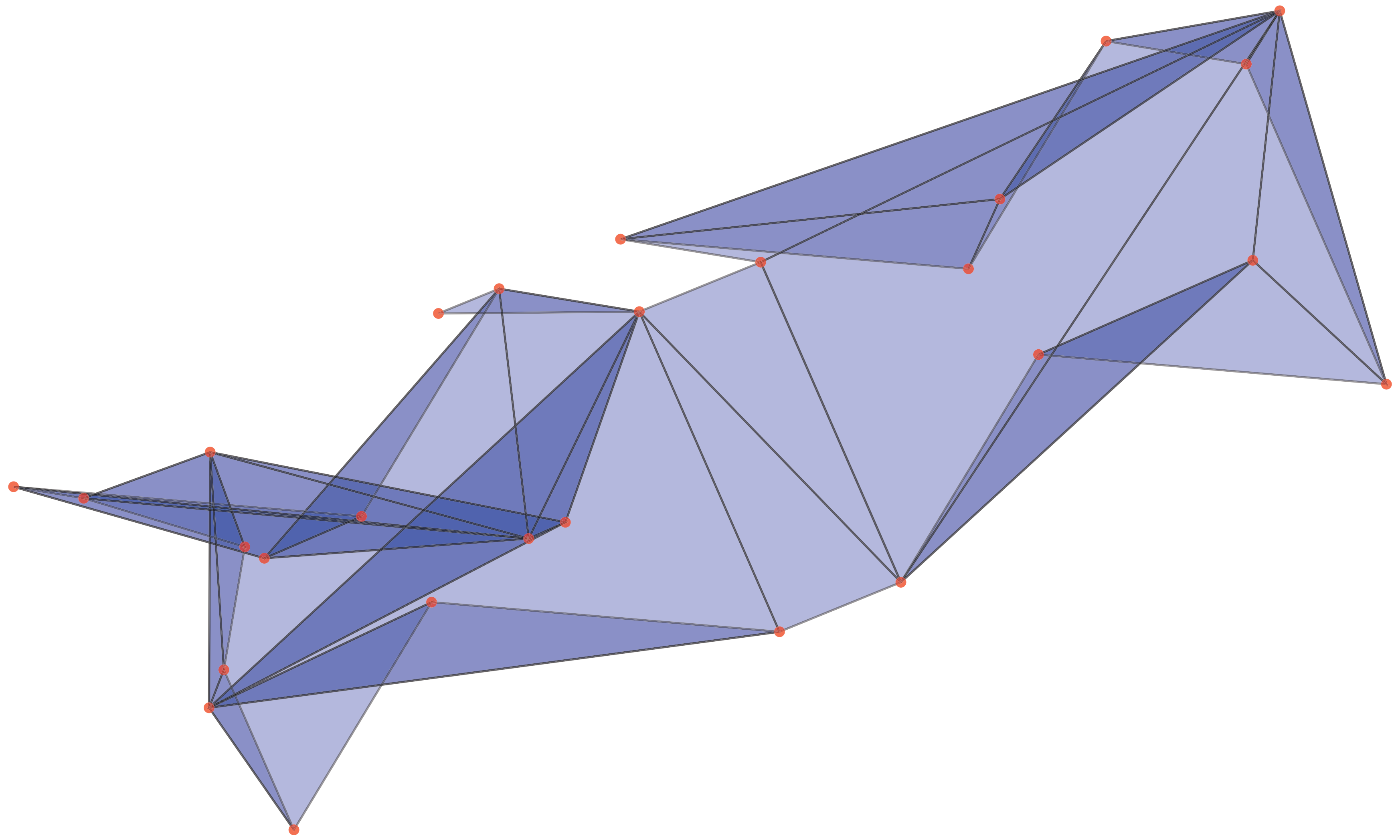
Input



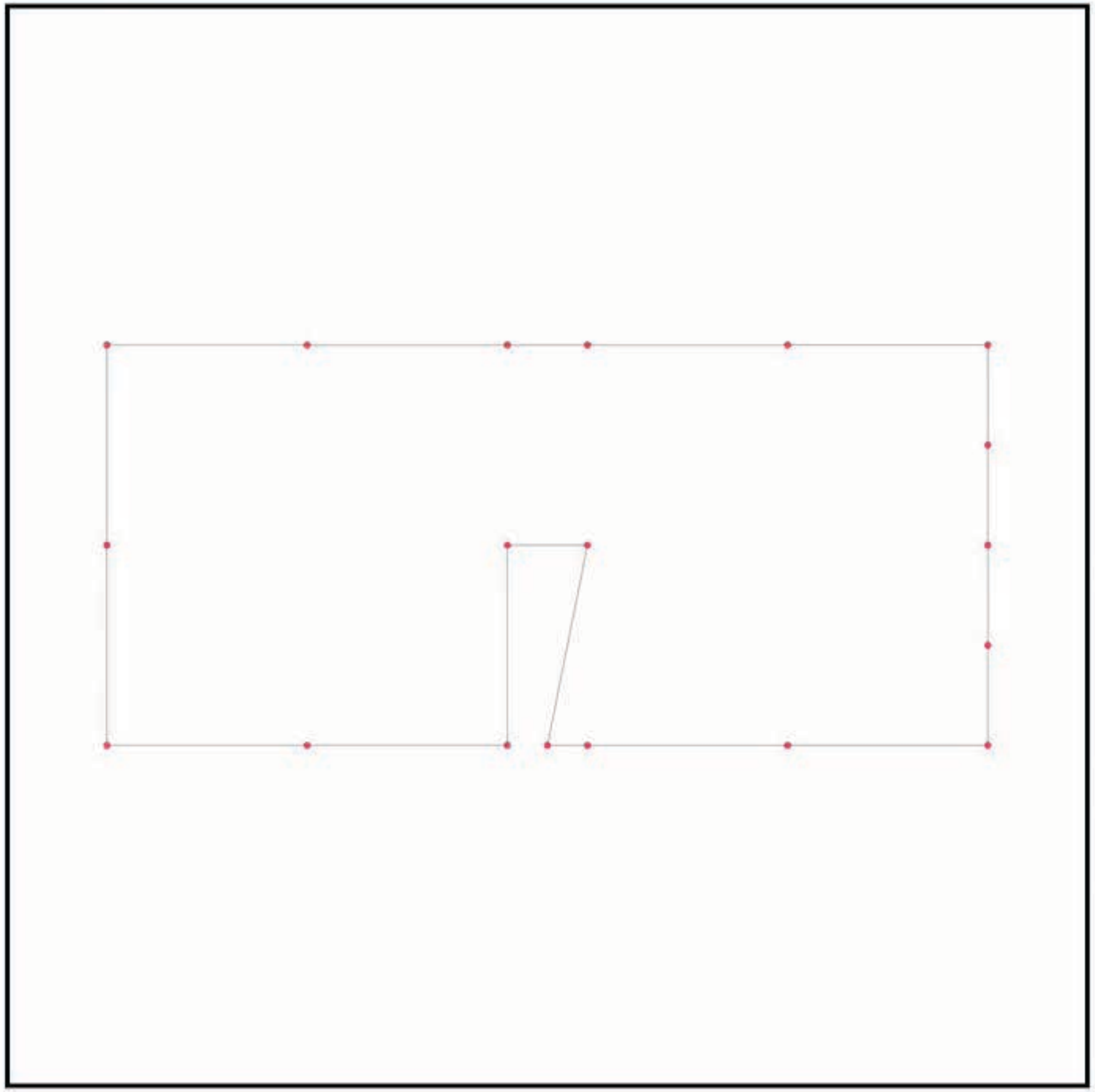


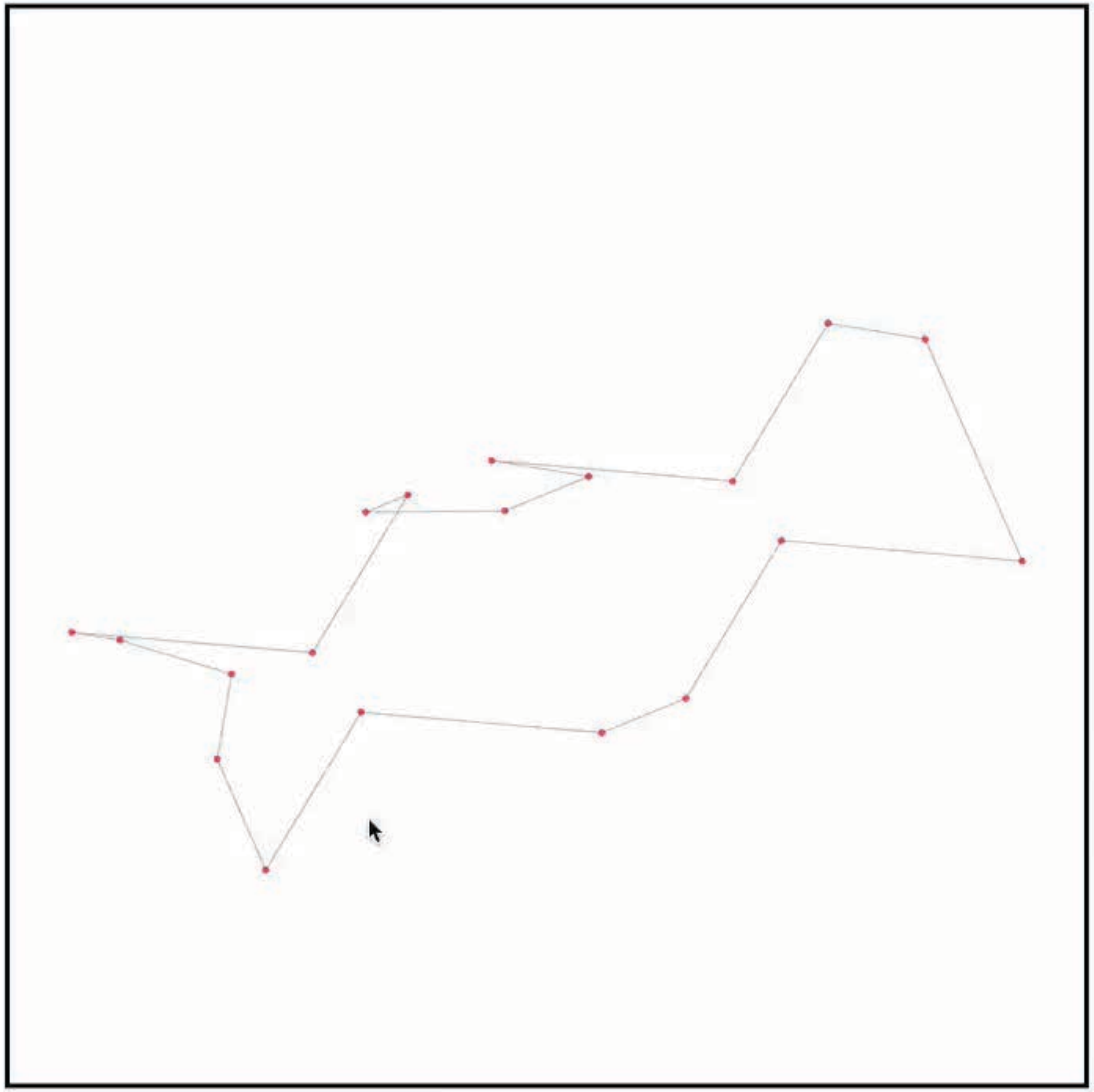
Output





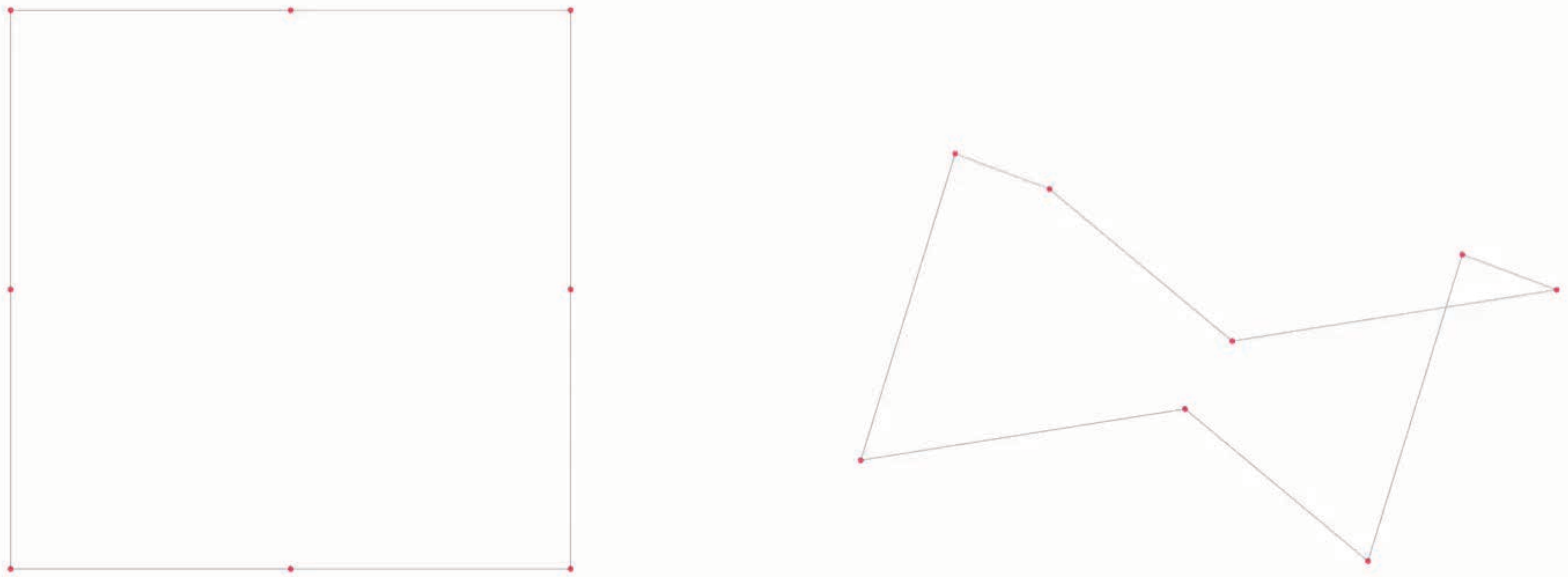
Boundary Value Problem



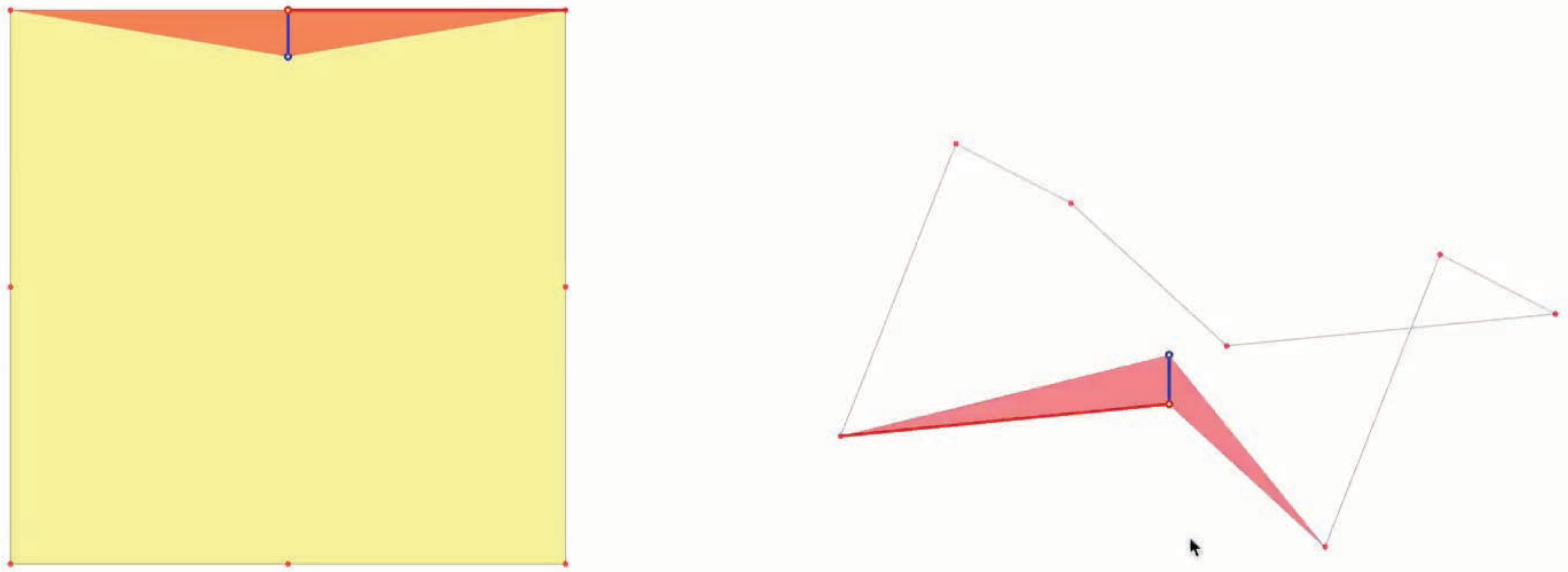


Boundary Value Problem

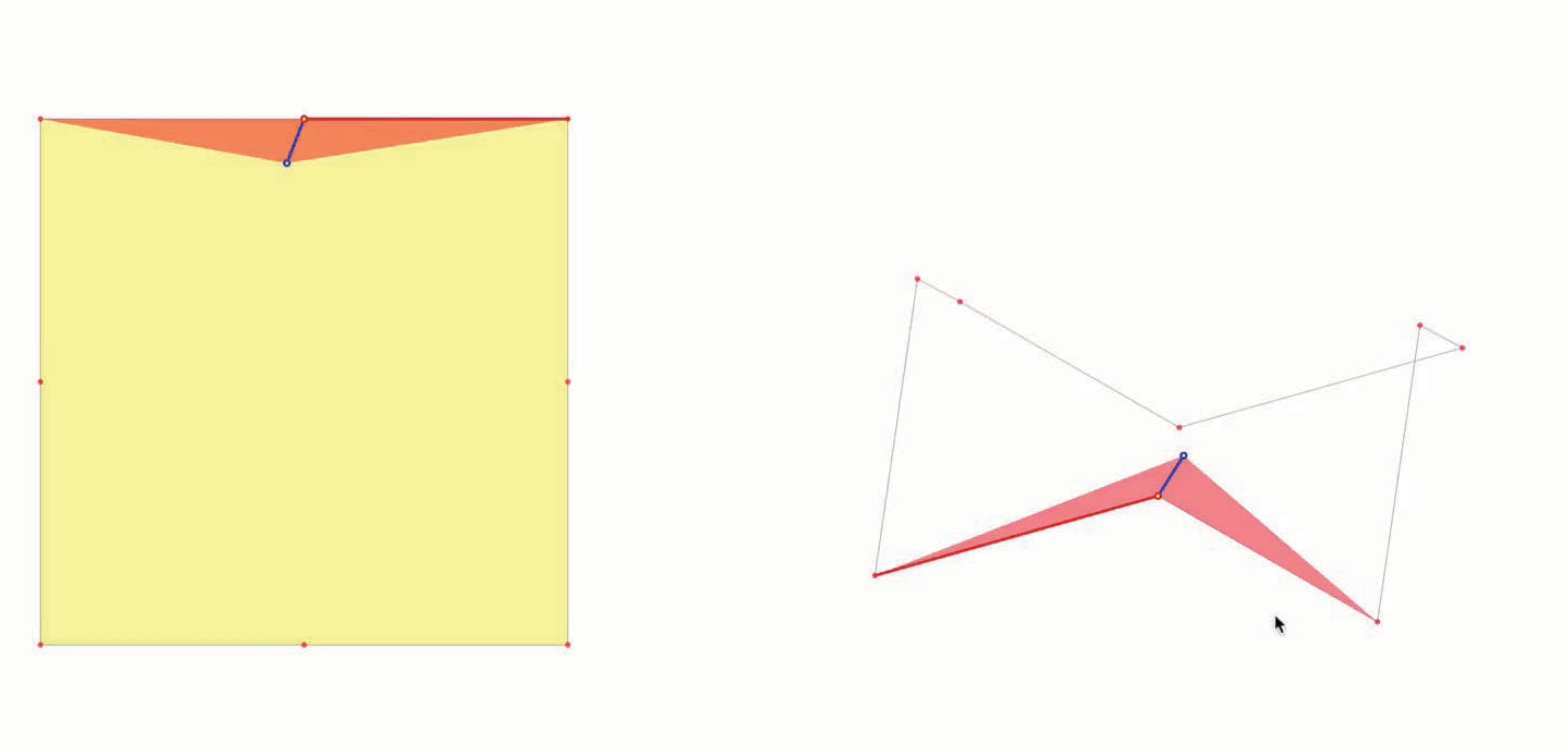
Boundary Value Problem



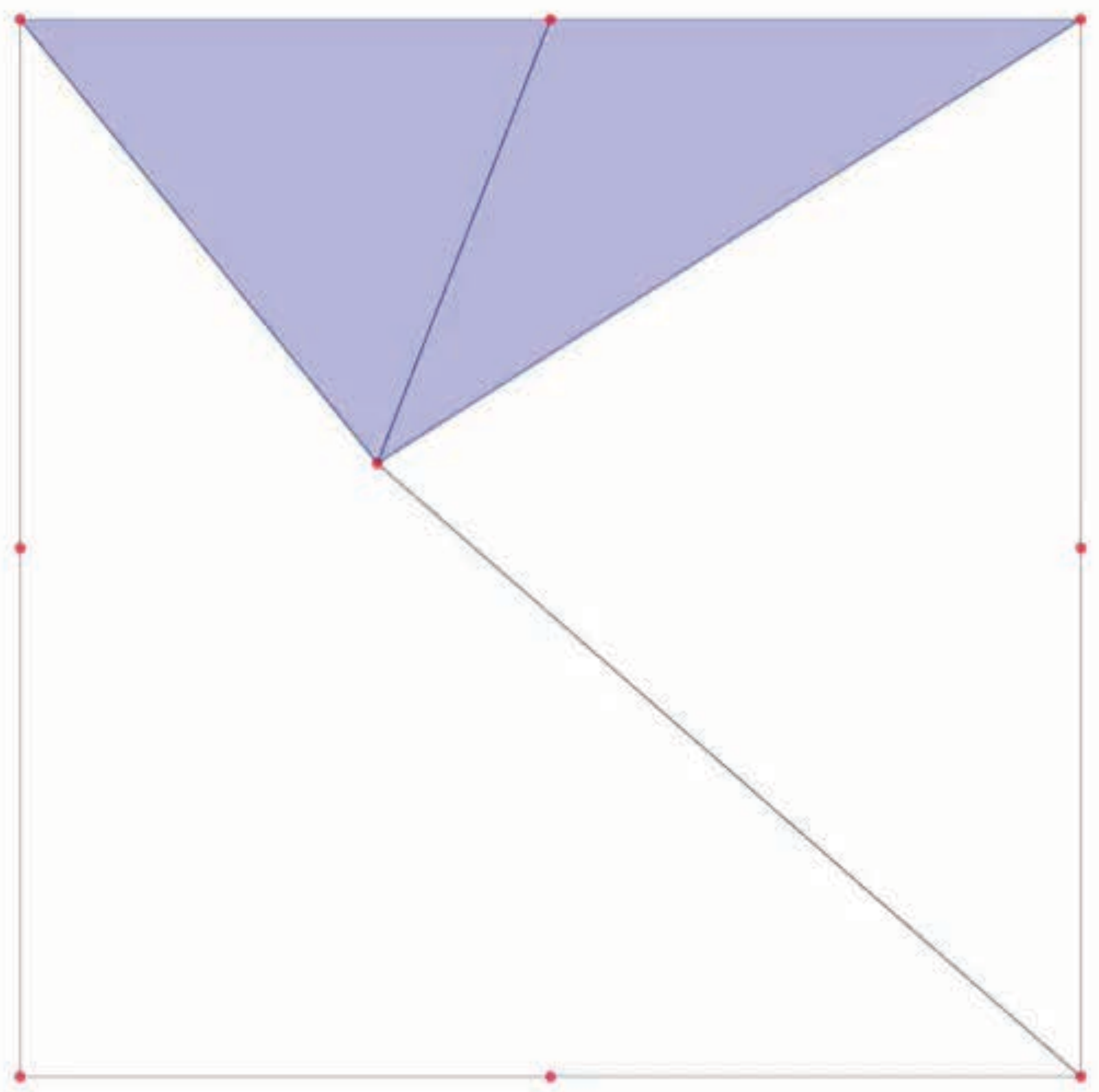
Boundary Value Problem

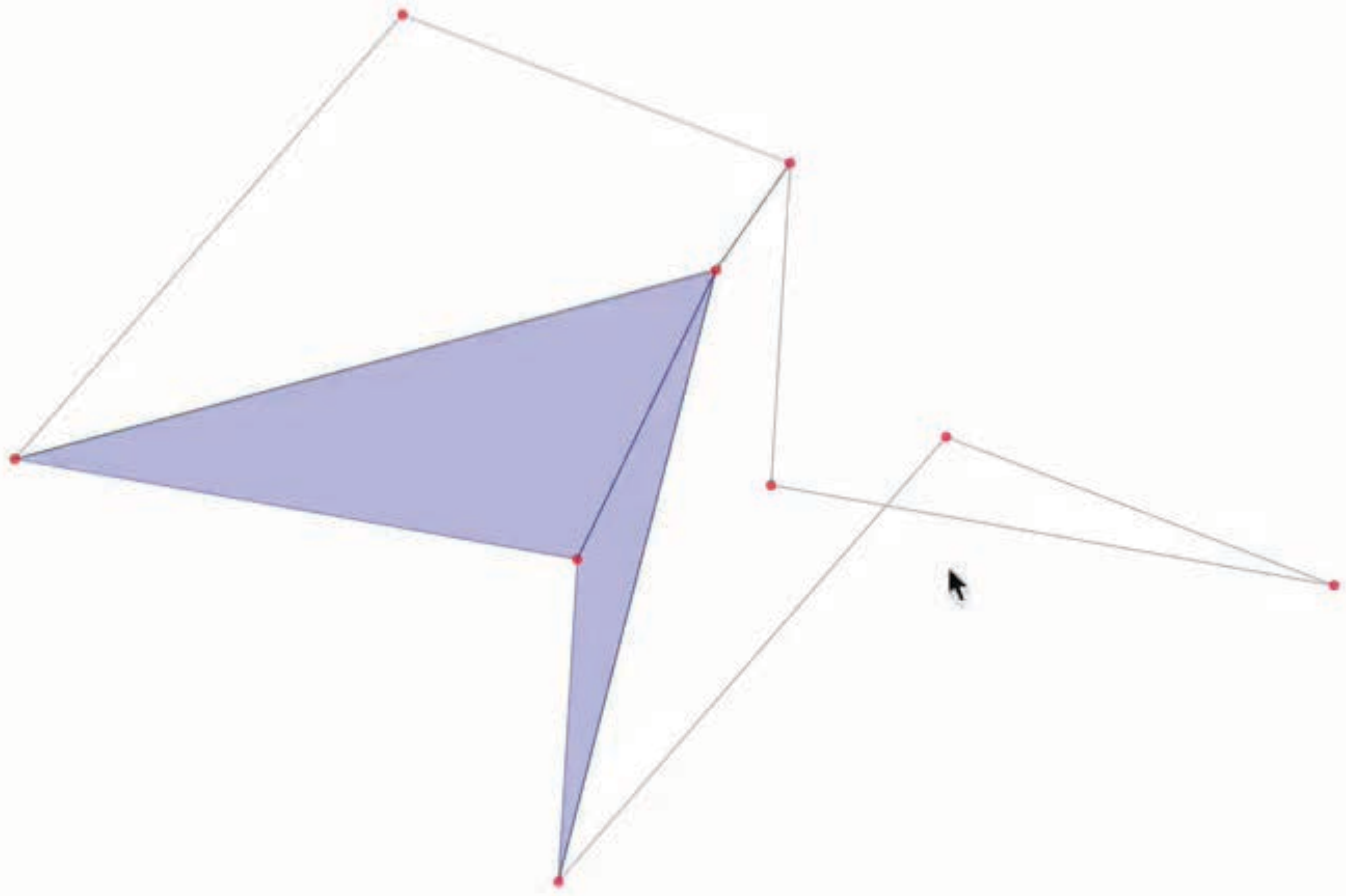


Boundary Value Problem

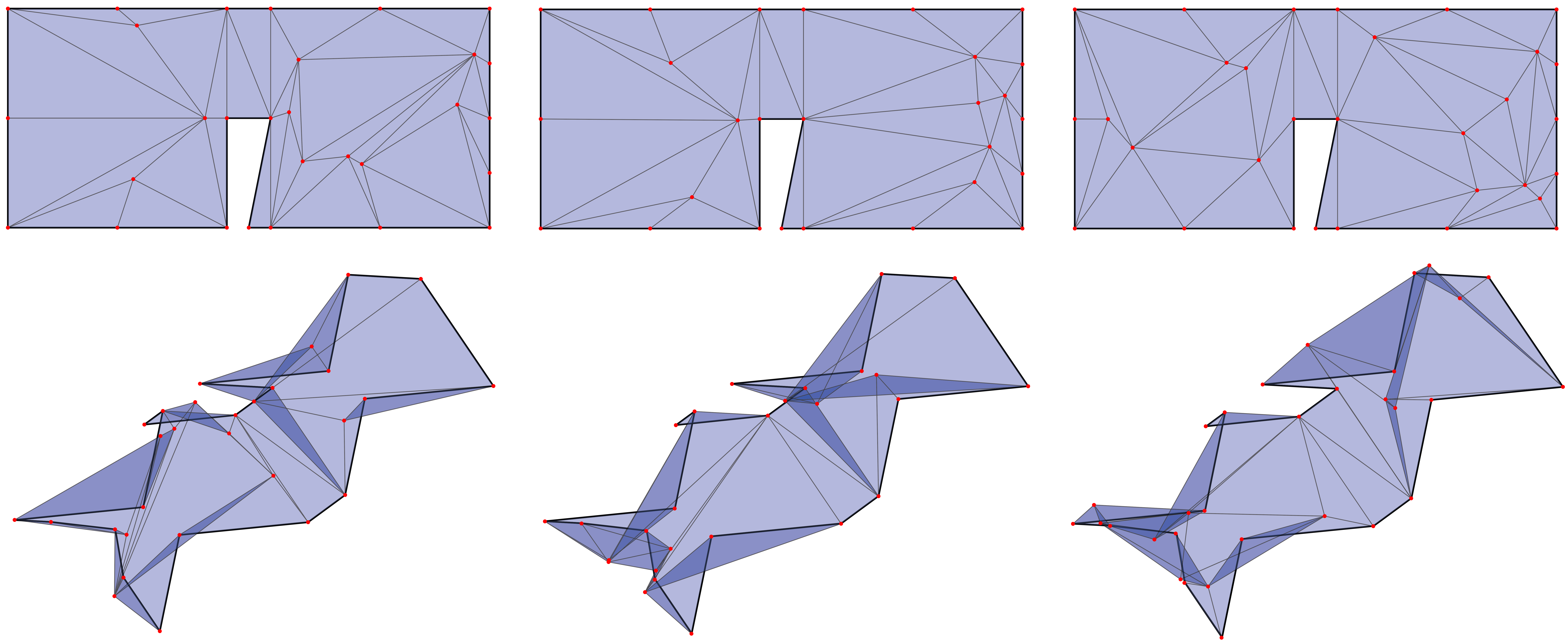


Boundary Value Problem

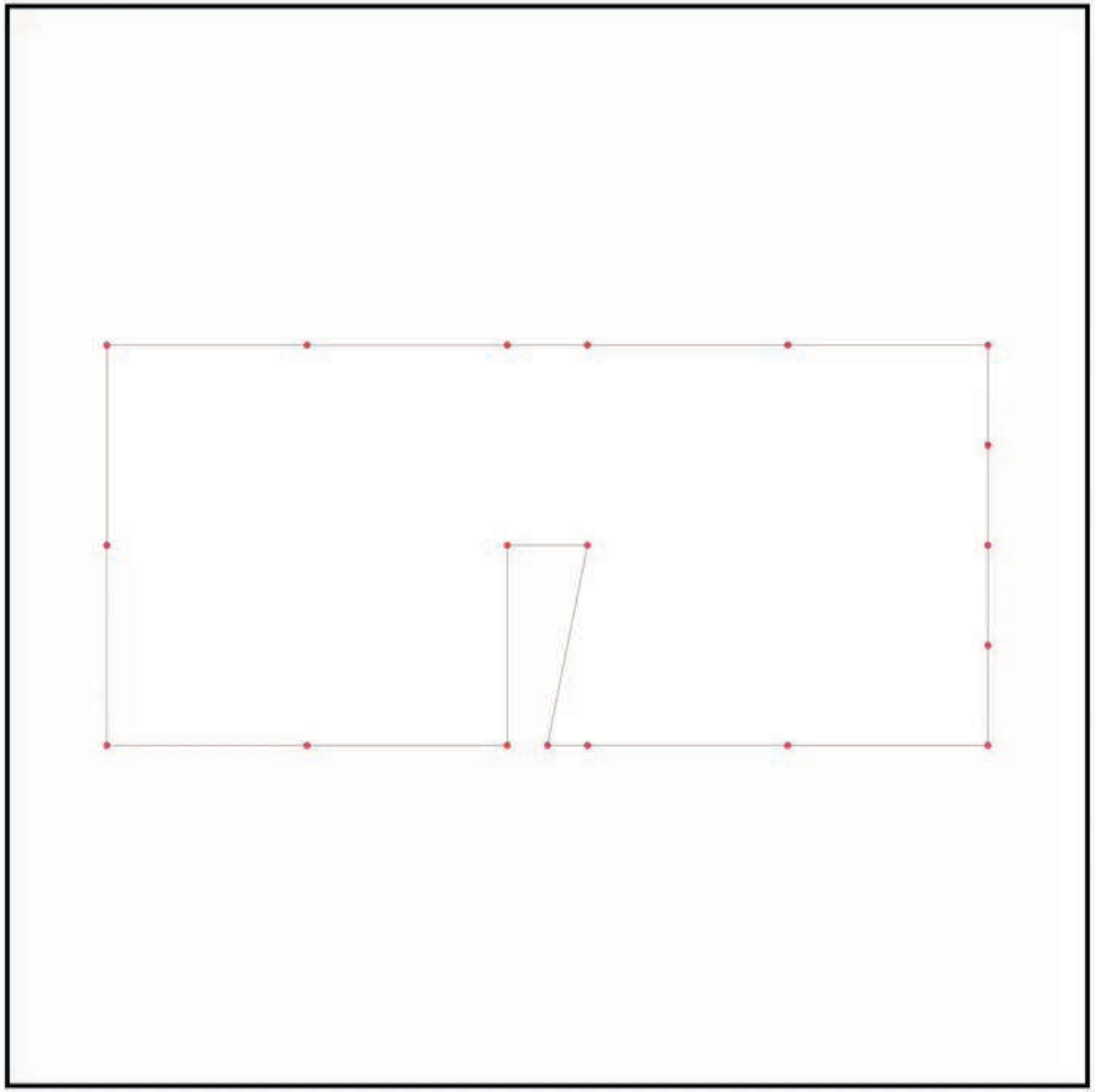


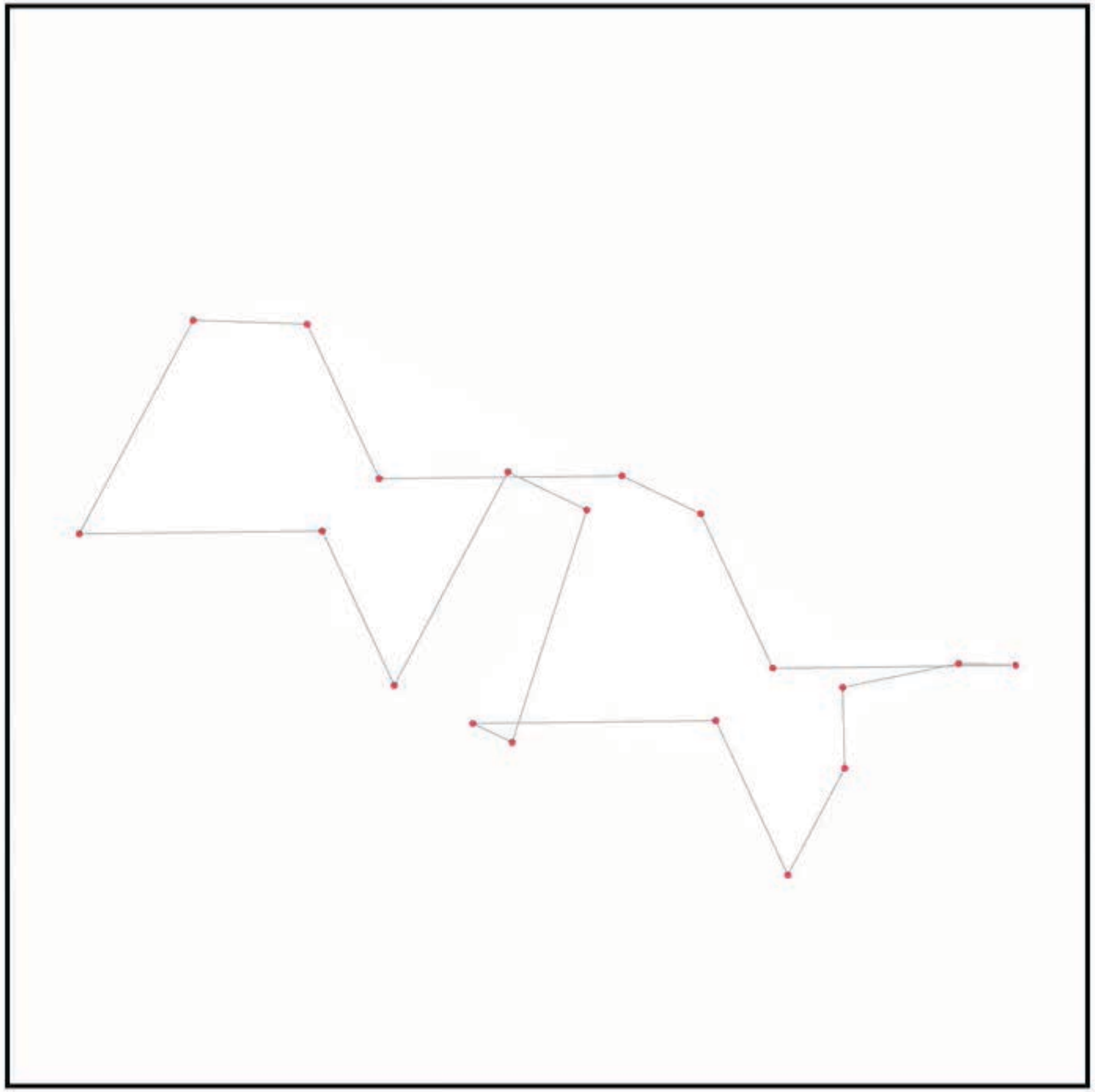


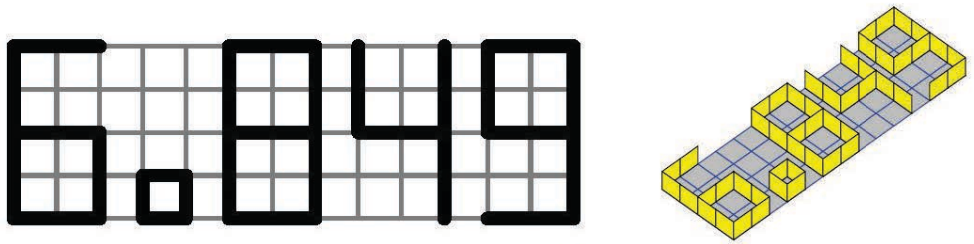
Boundary Value Problem



Boundary Value Problem

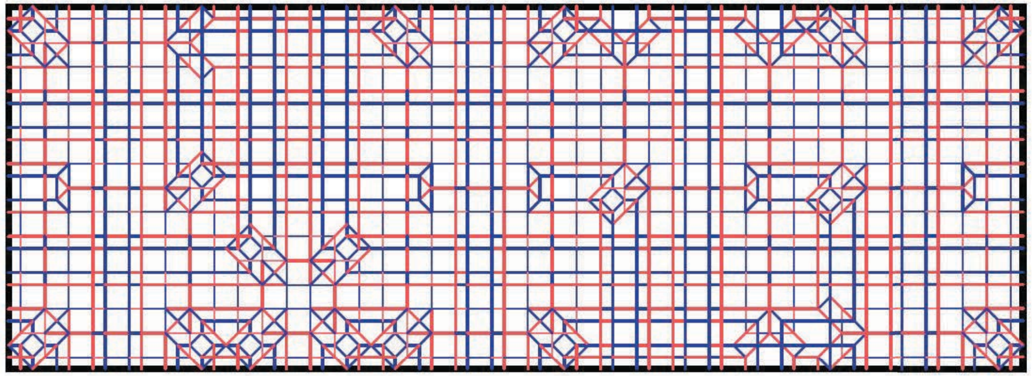






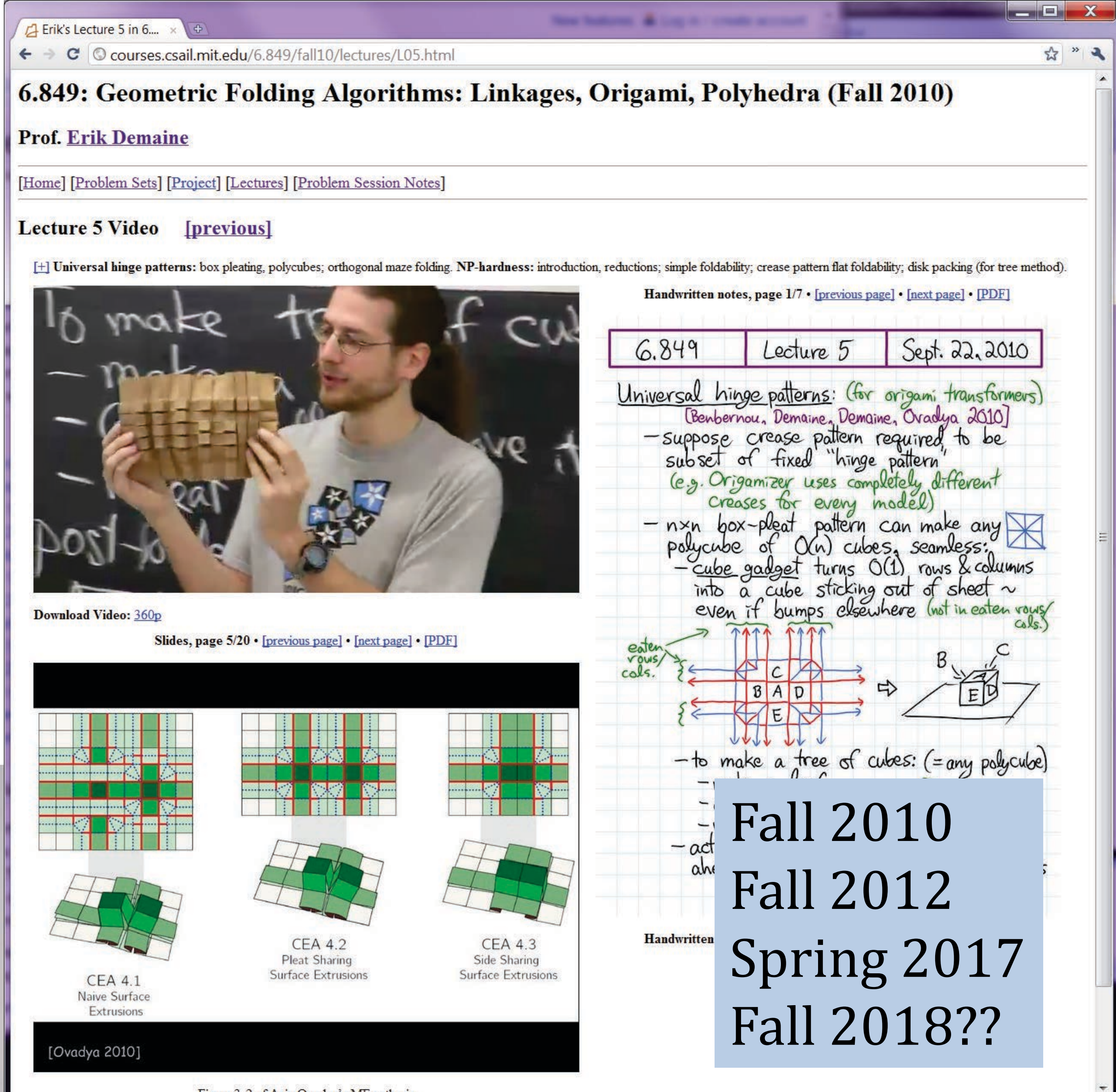
conceptual design

maze folding algorithm
[demaine, demaine, & jason ku, 2010]



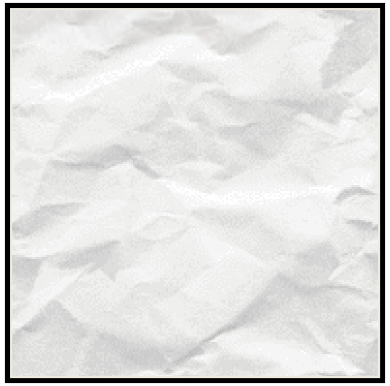
crease pattern

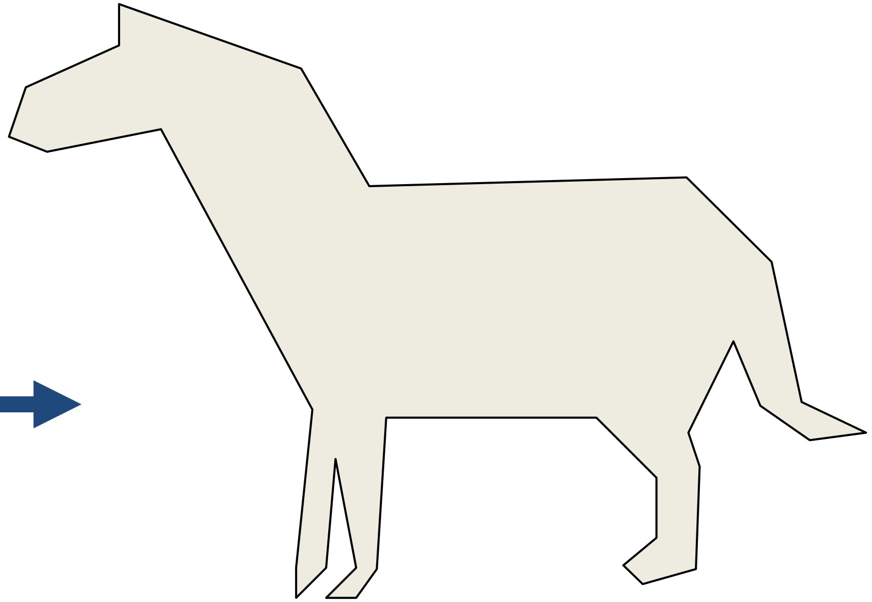
actual folding
[jenny ramseyer & eli davis, 2012]

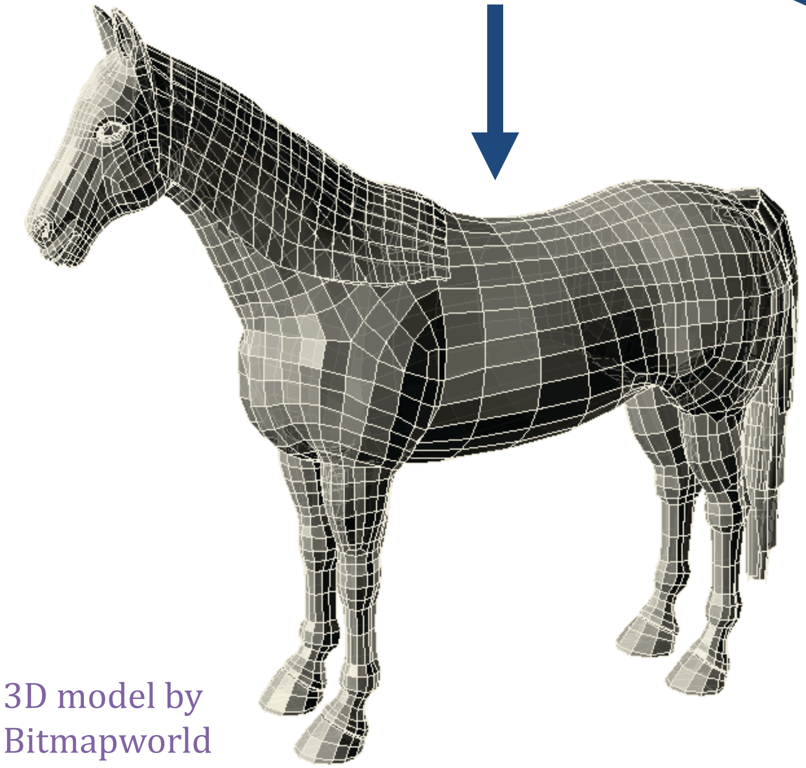


Fall 2010
Fall 2012
Spring 2017
Fall 2018??

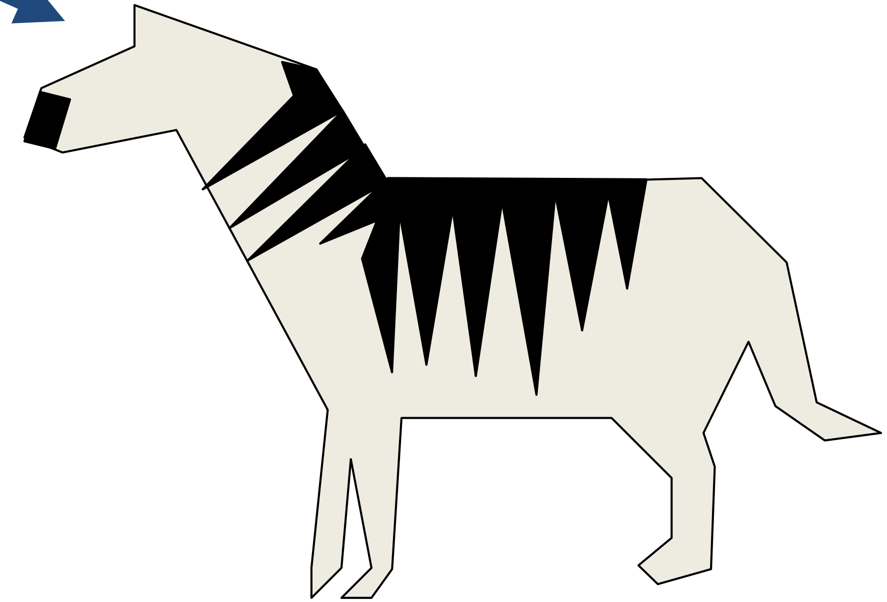
What Shapes Can Be Folded?







3D model by Bitmapworld



[Demaine, Demaine, Mitchell 1999]

Folding any shape: [Demaine, Demaine, Mitchell 2000]

(a.k.a. silhouette [Bern & Hayes 1998] / gift wrapping [Akiyama/Gardner])

Every connected union of polygons in 3D, each with a specified visible color (on each side), can be folded from a sufficiently large piece of bicolor paper of any shape (e.g., square).

Proof: fold paper down to long narrow strip (!)

- triangulate the polygons
- choose a path visiting each triangle at least once
- cover each triangle along the path by zig-zag parallel to next edge, starting at opposite corner:

choose parity of zig-zag to arrive at correct corner for next triangle

- turn gadget implements zig-zags & vertex turns:

perpendicular mountain — fold bottom layer — hide excess (many folds)

Proof of folding any shape: (cont'd)

- hide excess paper underneath each triangle: (more generally, can hide under any convex polygon)

repeatedly mountain fold along lines extending desired edges

- if paper is unicolor (or don't care) can use valley folds ⇒ simple folds
- if mountain folds, might colide with other Δs ⇒ not really simple folds (but still works as origami fold)

- color-reversal gadget along transition between triangles of opposite colors:

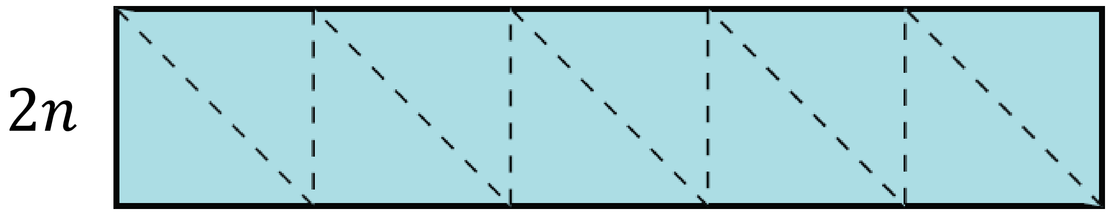
fold bottom layer

□

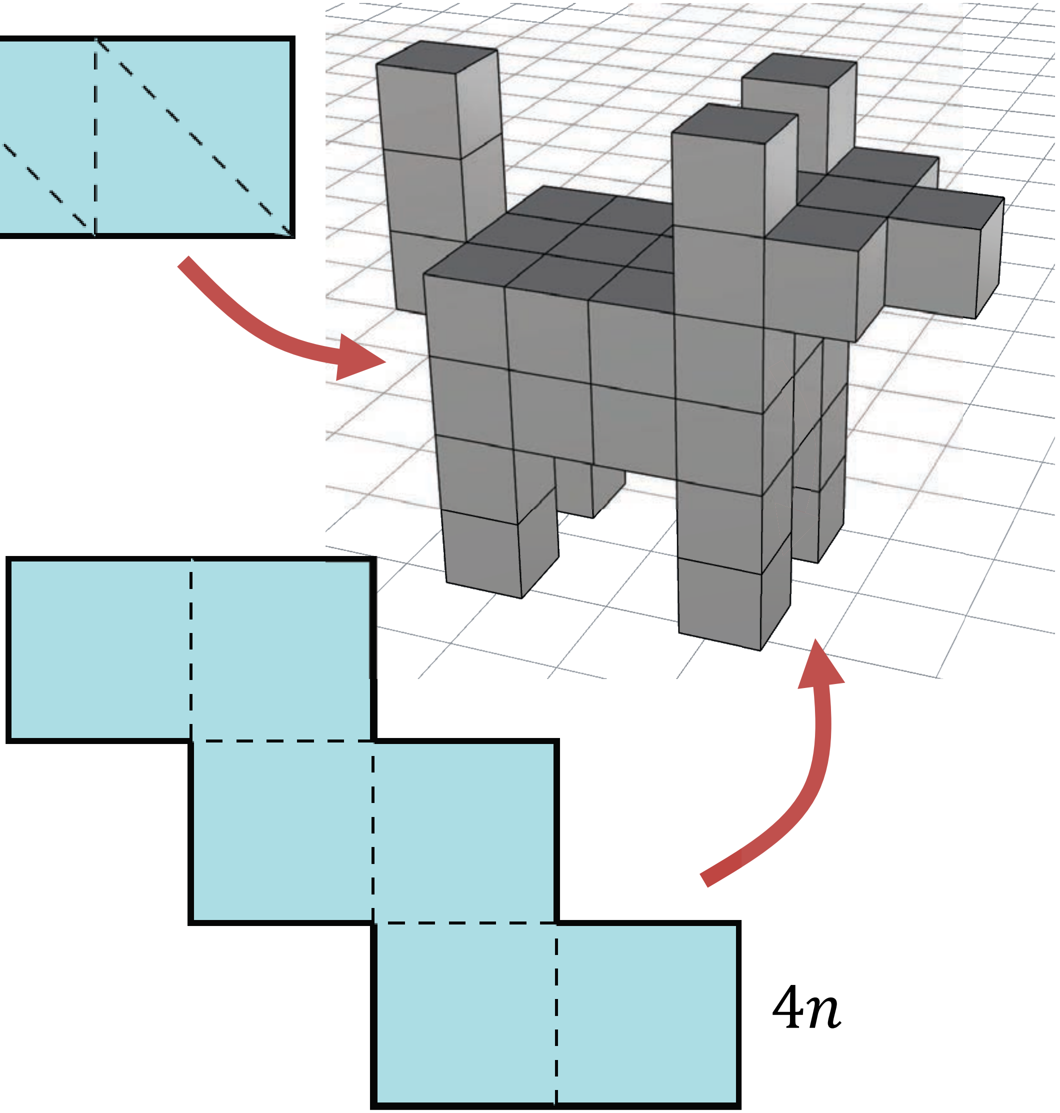
http://courses.csail.mit.edu/6.849/spring17/lectures/L01.html?notes=3&images=3

Universal Strip Folding

[Benbernou, Demaine, Demaine, Lubiw 2017]

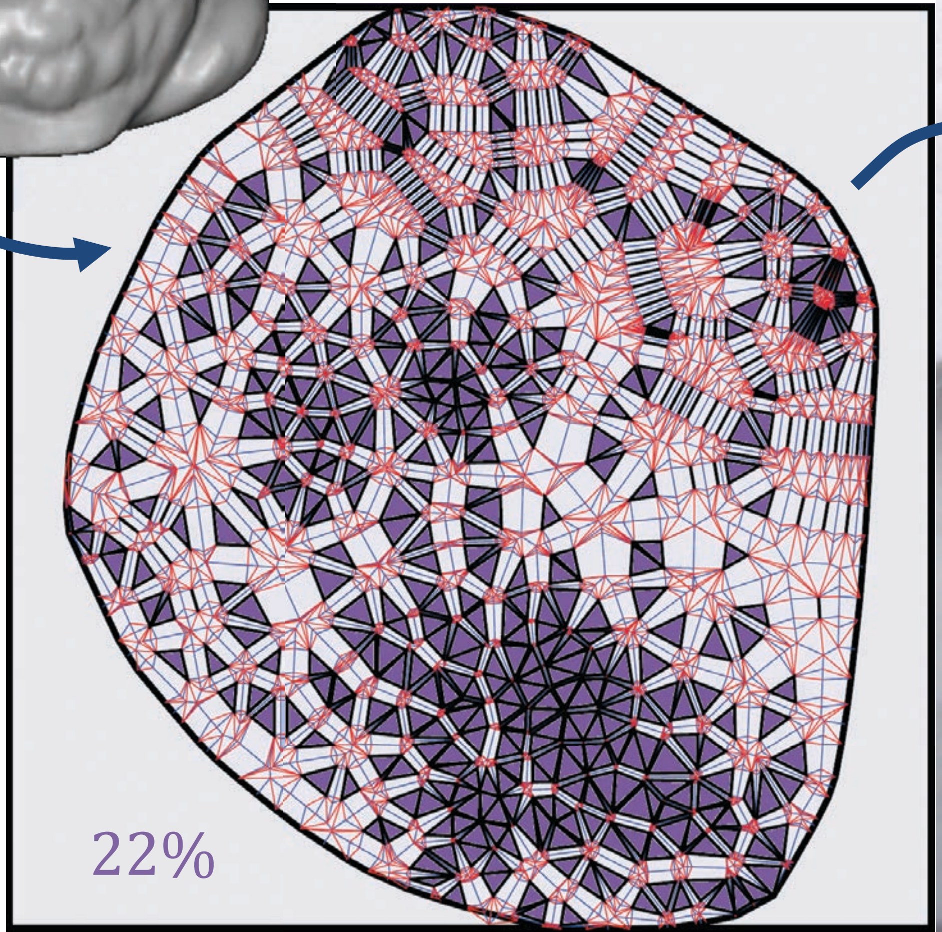


- Strip of length $2n$ folds into any grid polyhedron of surface area n
  - ≤ 2 layers everywhere
  - Rigid motion without collisions

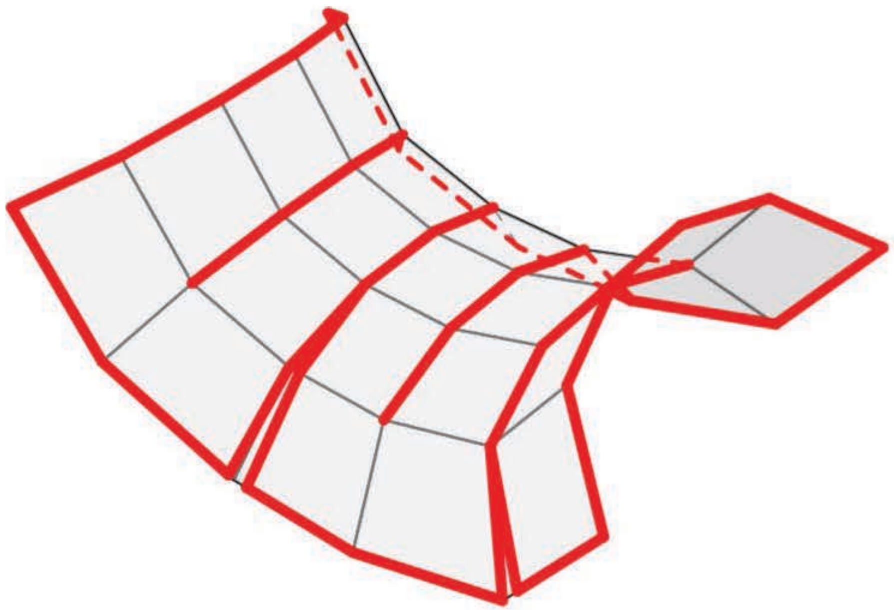


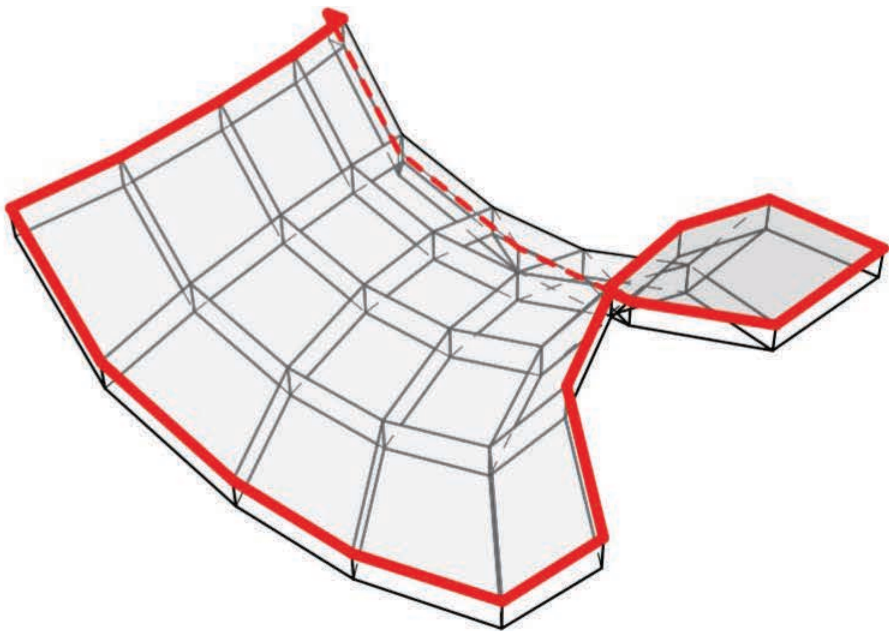
Origamizer

[Tachi 2006;
Demaine &
Tachi 2017]



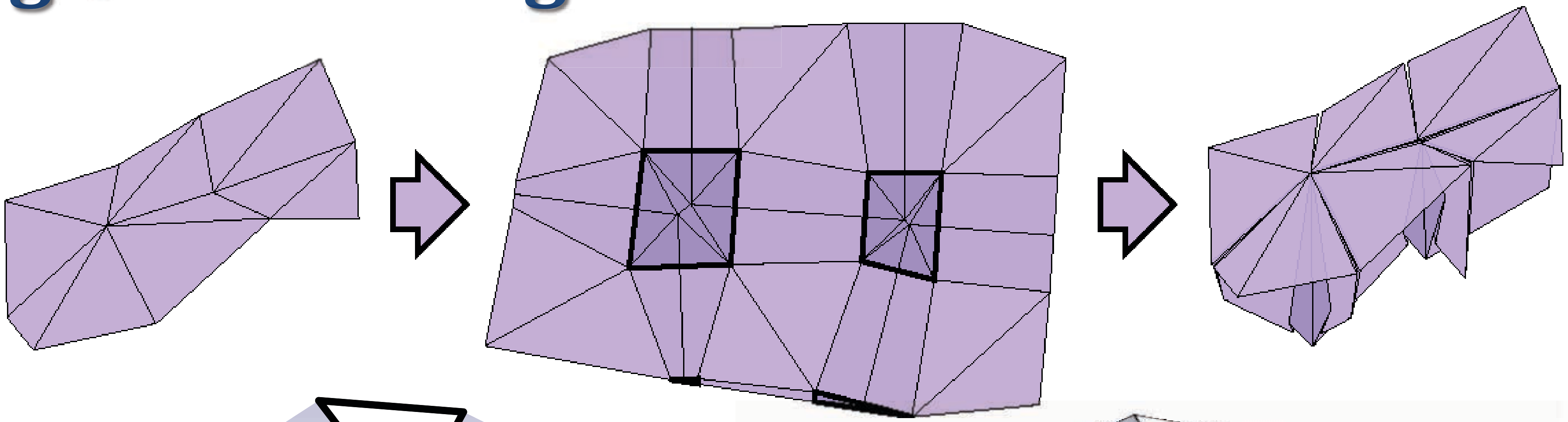
Watertight Folding

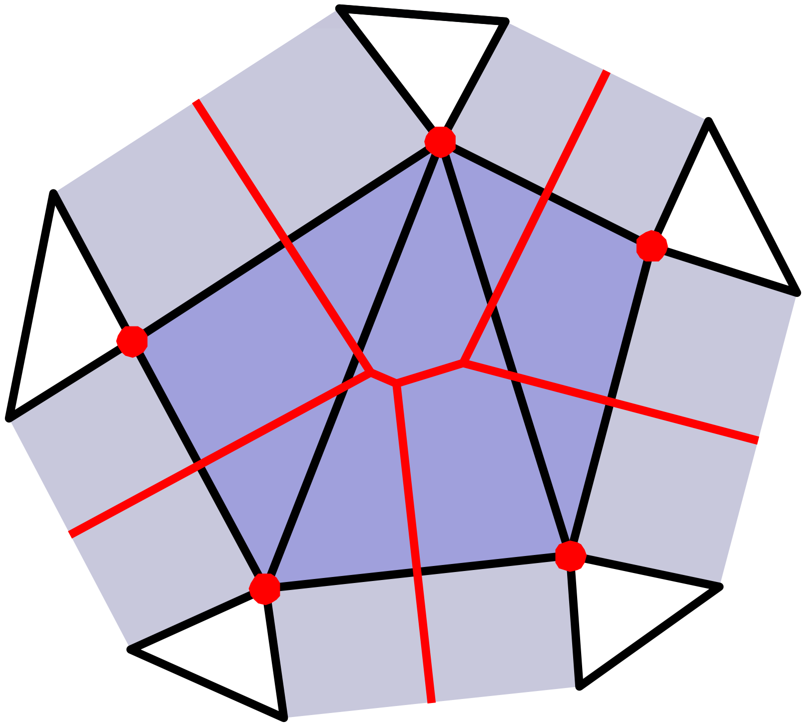


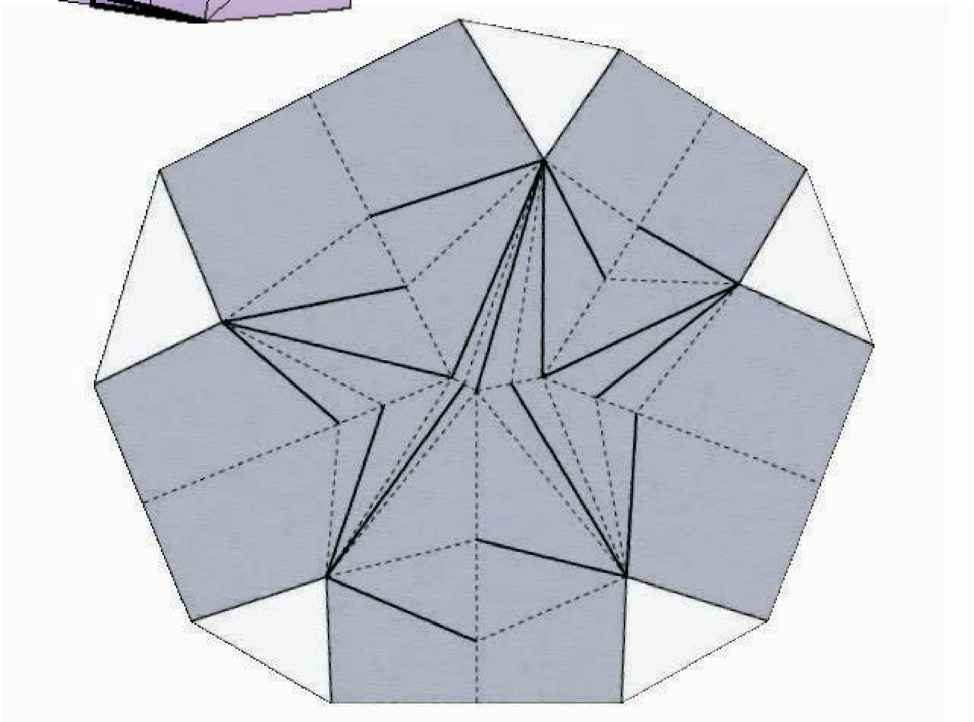


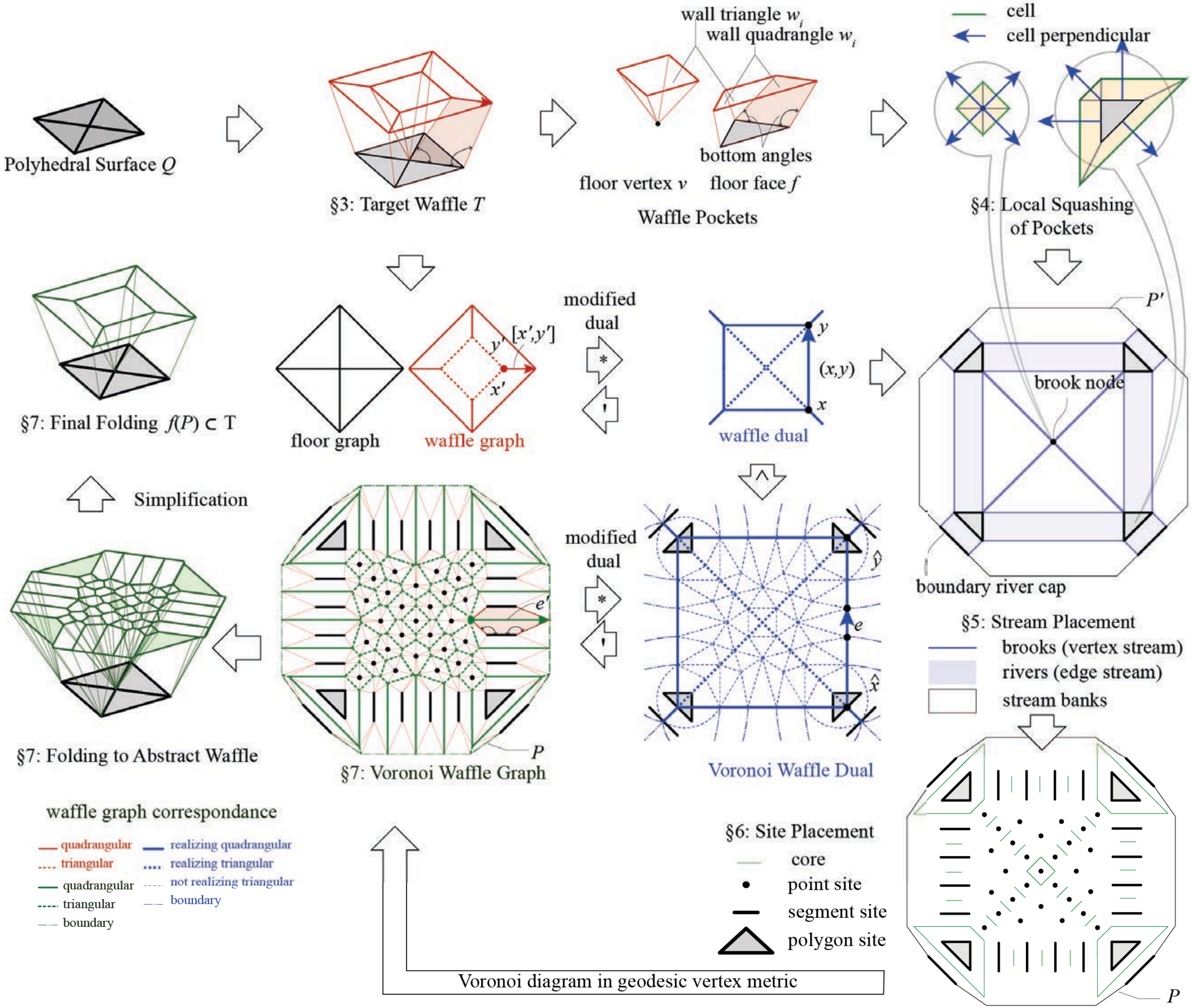
- **Strip algorithm** [1999]
  - Lots of extra boundaries
  - Wrong topology
  - Exact surface
- **Origamizer** [2017]
  - No extra boundary
  - Correct topology
  - Extra tabs

High-Level Approach: Tuck Away Extra Material by Folding Voronoi Diagram



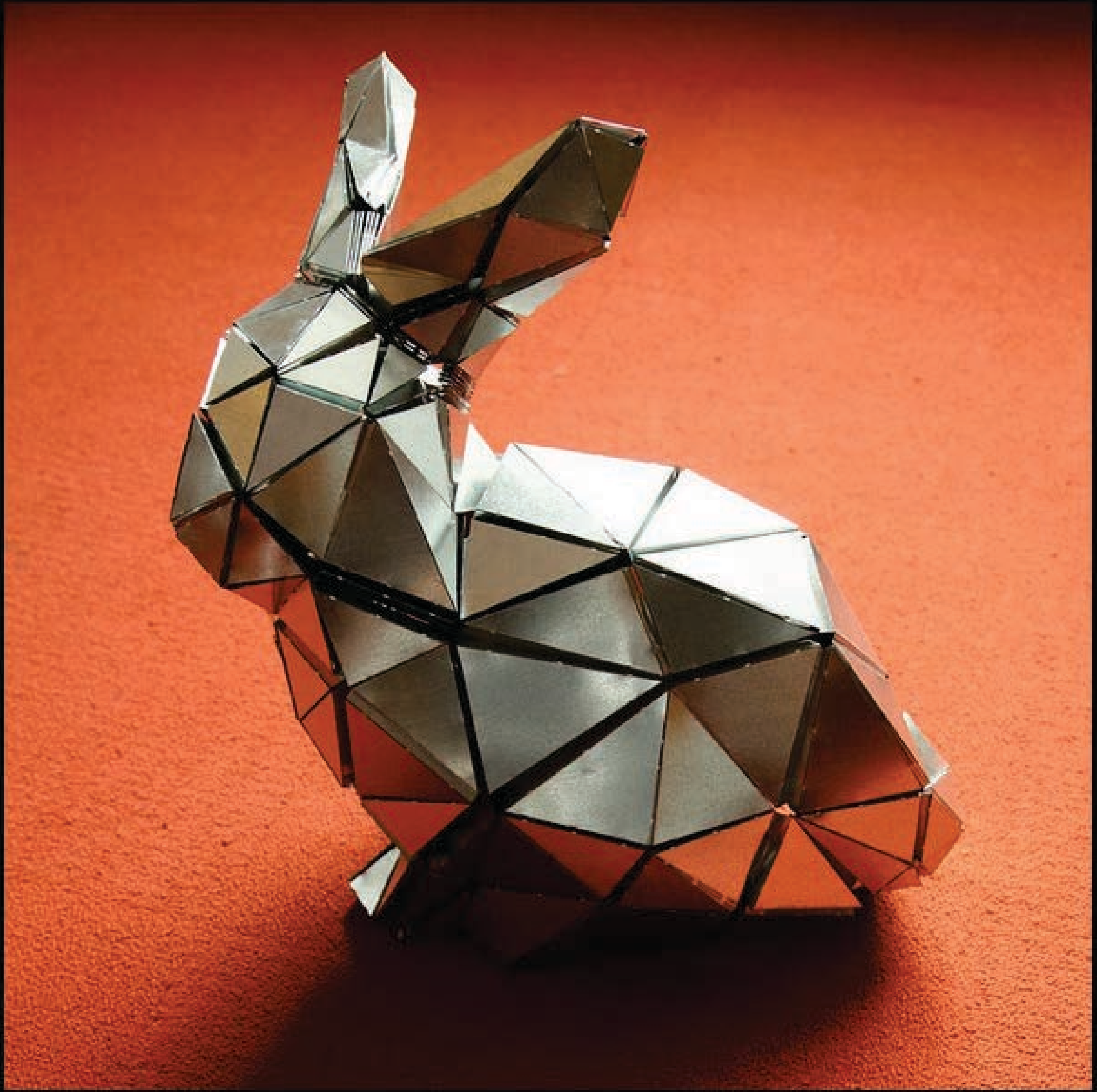






Polyhedral Surface Q
§3: Target Waffle T
wall triangle w_i
wall quadrangle w_i
bottom angles
floor vertex v
floor face f
Waffle Pockets
cell
cell perpendicular
§4: Local Squashing of Pockets
§7: Final Folding f(P) ⊂ T
floor graph
waffle graph
[x′,y′]
y′
x′
modified dual
*
'
waffle dual
(x,y)
y
x
P′
brook node
boundary river cap
§5: Stream Placement
brooks (vertex stream)
rivers (edge stream)
stream banks
Simplification
§7: Folding to Abstract Waffle
§7: Voronoi Waffle Graph
e′
P
modified dual
Voronoi Waffle Dual
ŷ
x̂
e
waffle graph correspondance
quadrangular
triangular
quadrangular
triangular
boundary
realizing quadrangular
realizing triangular
not realizing triangular
boundary
§6: Site Placement
core
point site
segment site
polygon site
Voronoi diagram in geodesic vertex metric
P

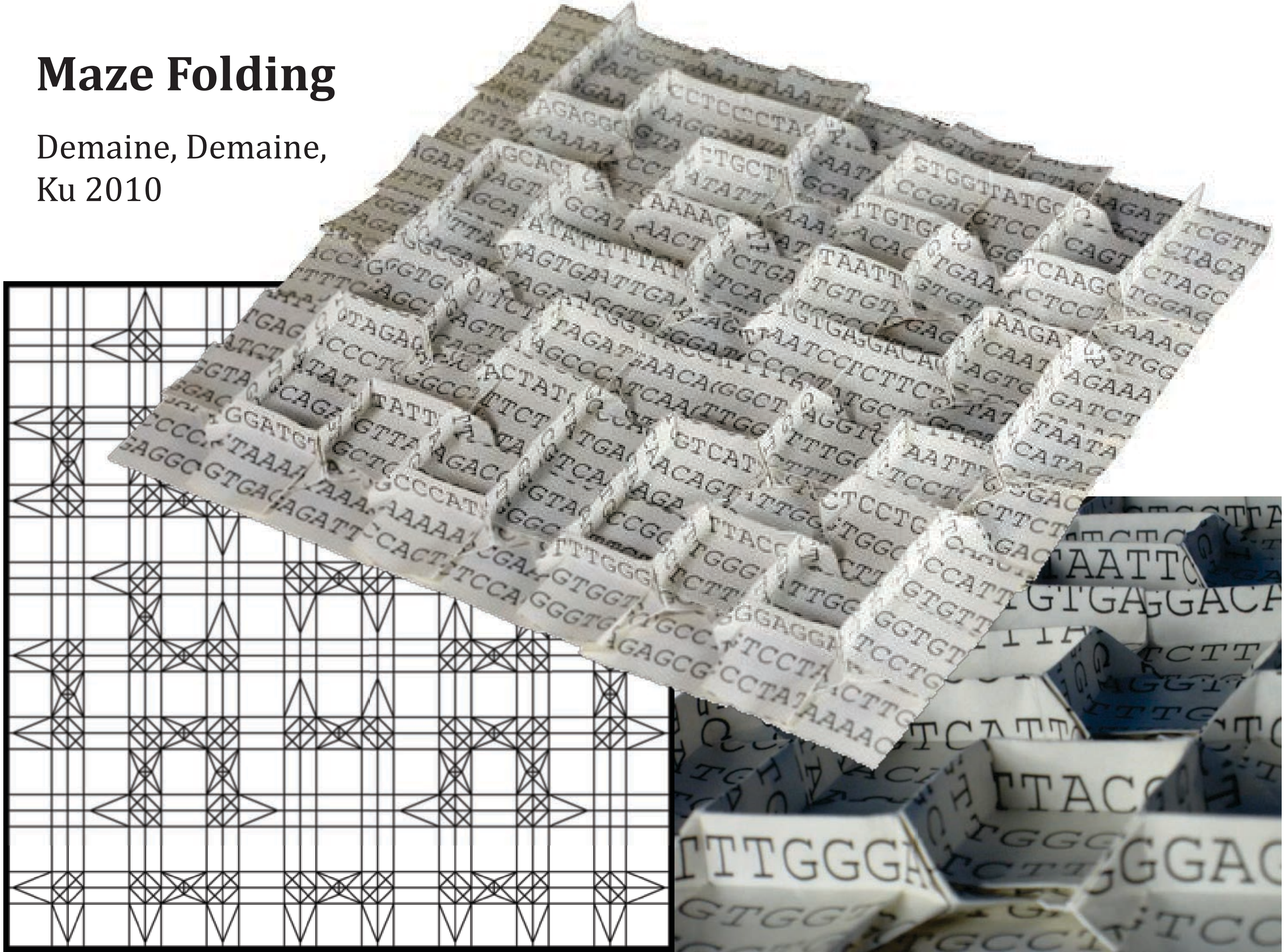
[Cheung, Demaine, Demaine, Tachi 2011]



[Cheung, Demaine,
Demaine, Tachi 2011]

Maze Folding

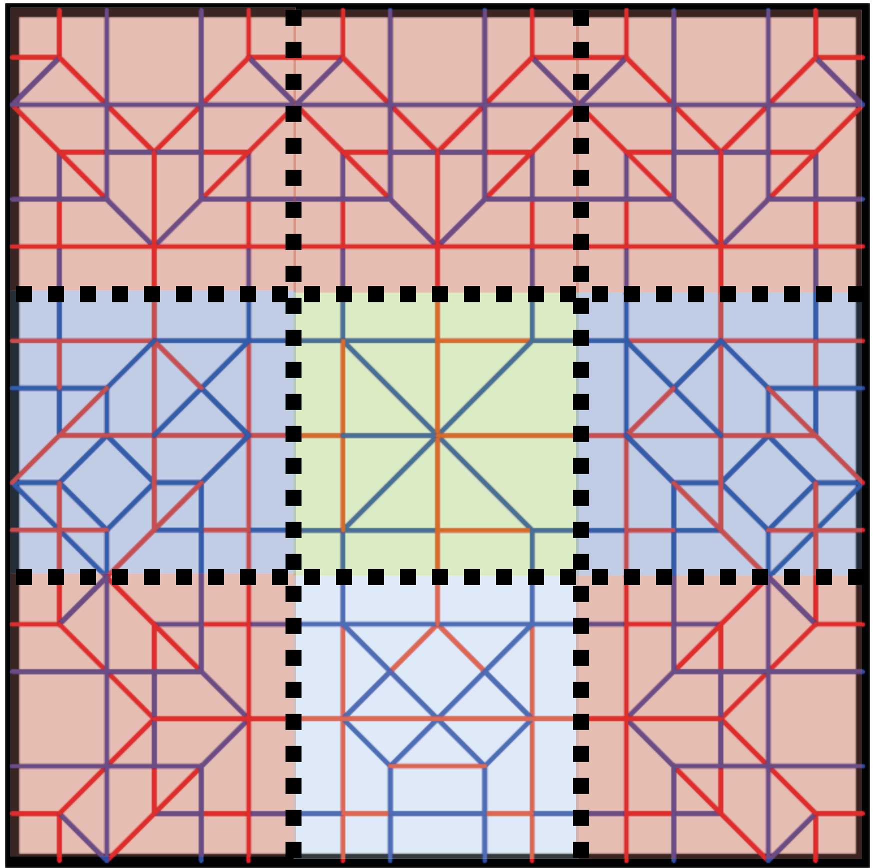
Demaine, Demaine,
Ku 2010

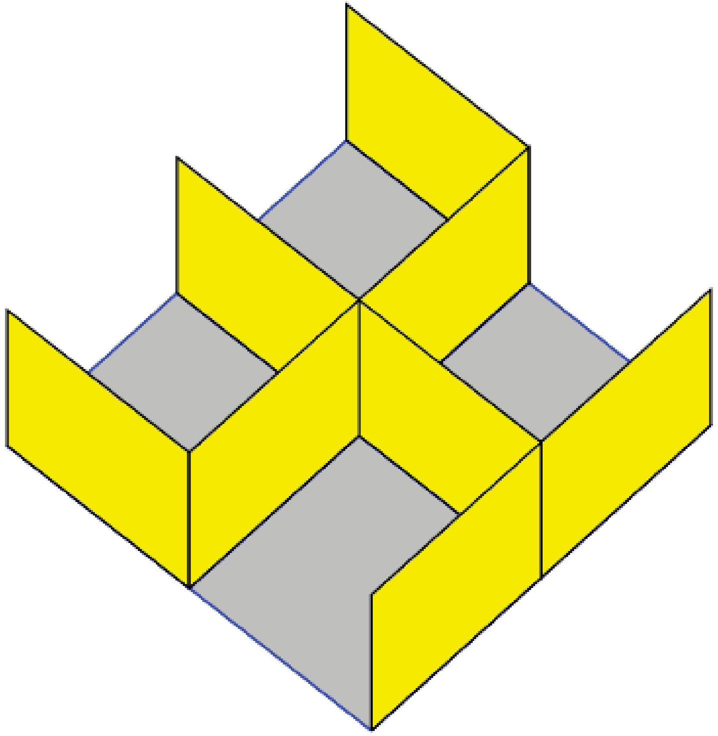


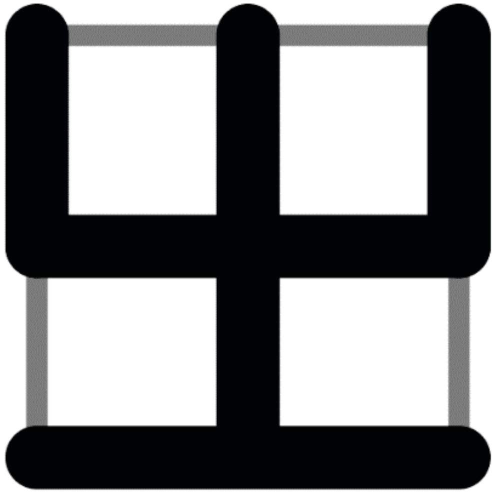
Maze Folding Algorithm

[Demaine, Demaine, Ku 2010]

- Algorithm to fold any $m \times n$ orthogonal graph extruded to height h from a rectangle of dimensions $(1 + 2h)m \times (1 + 2h)n$



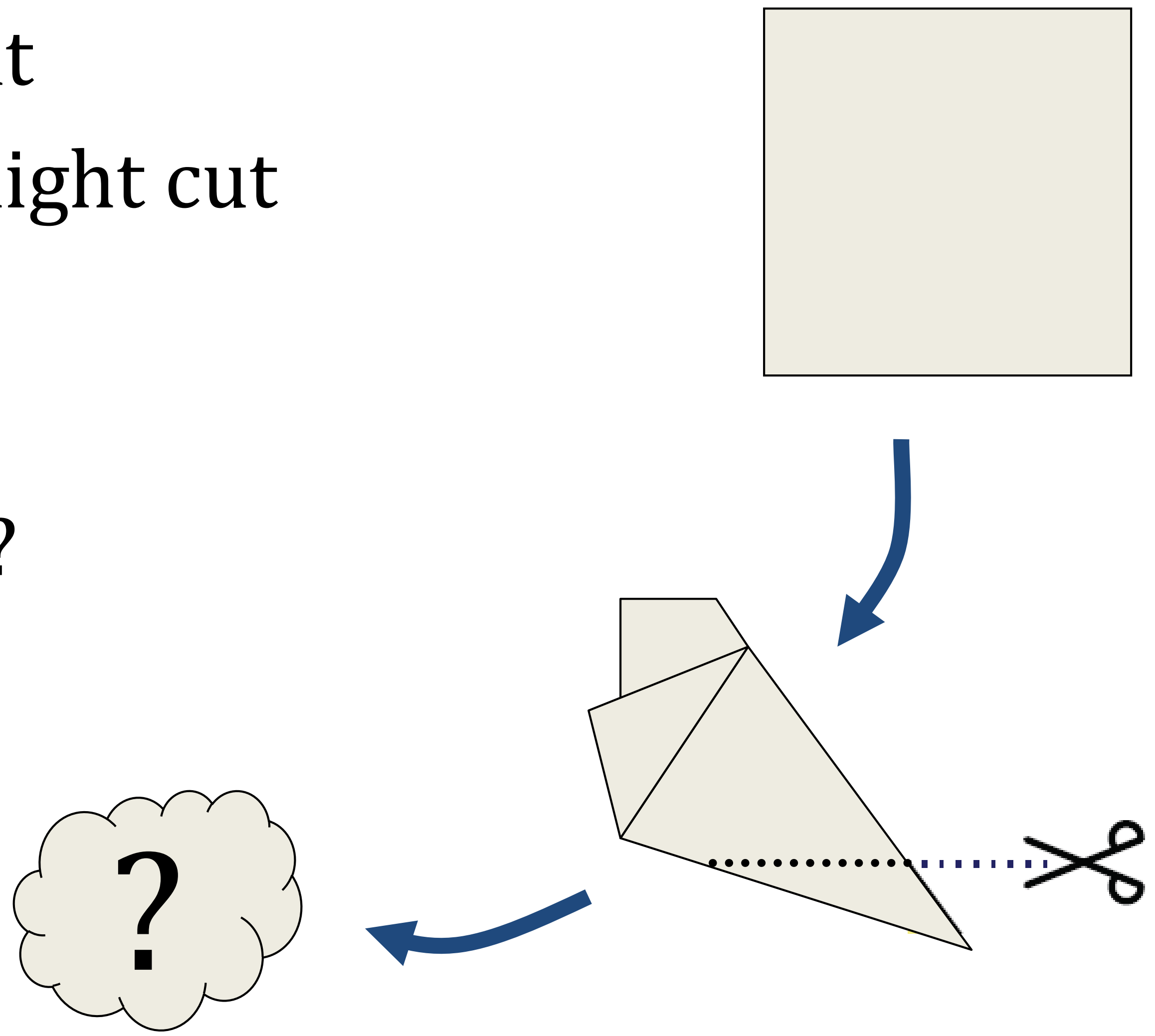




Fold-and-Cut Problem

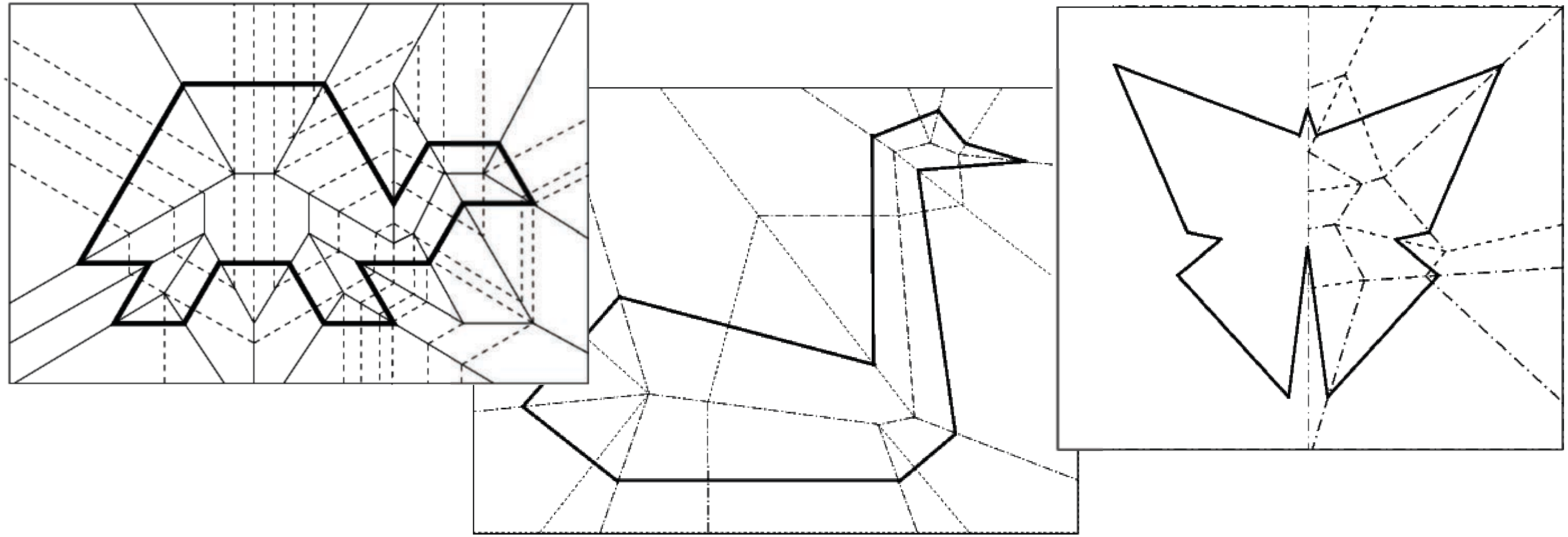
- Fold a sheet of paper flat
- Make one complete straight cut
- Unfold the pieces

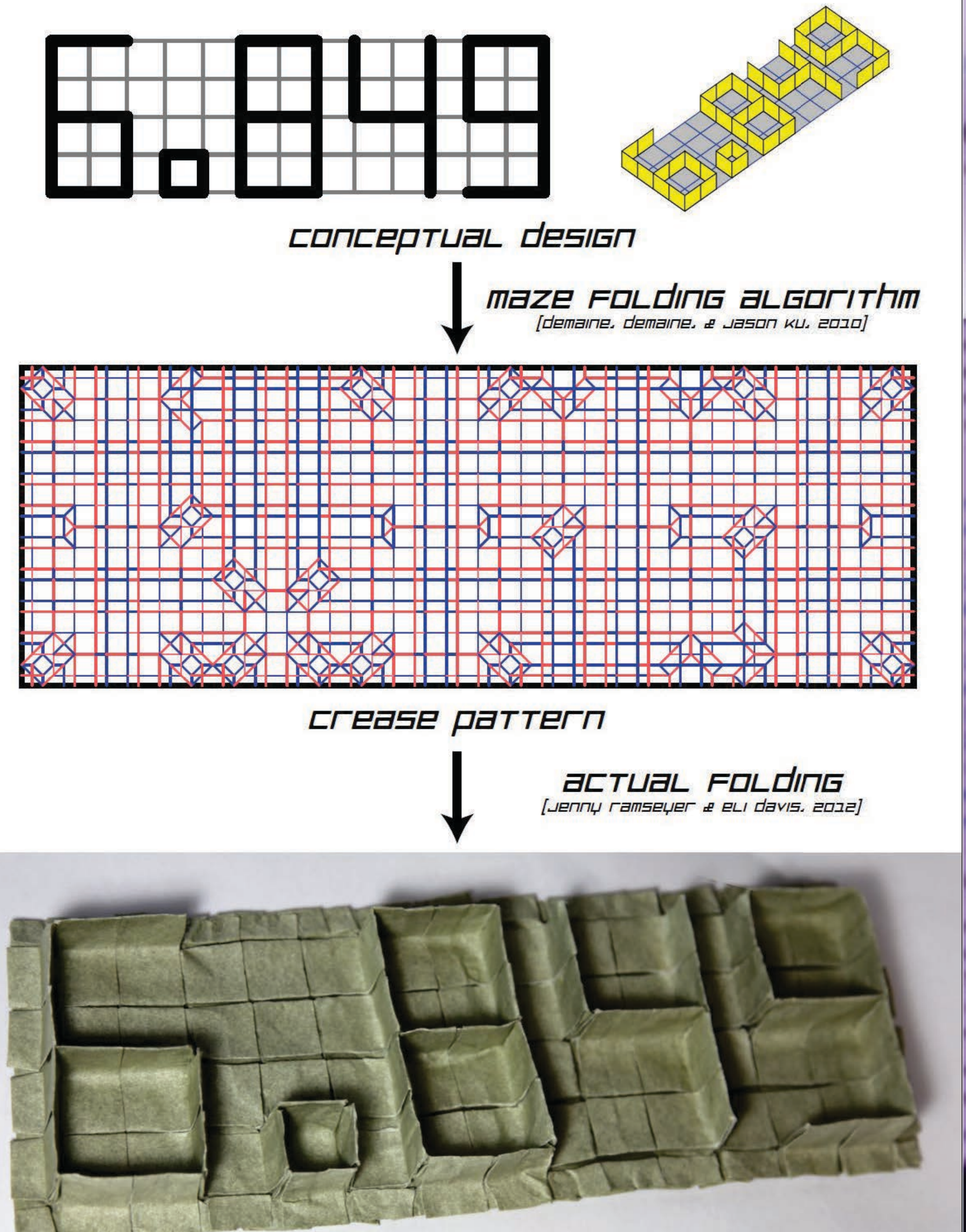
- What shapes can result?

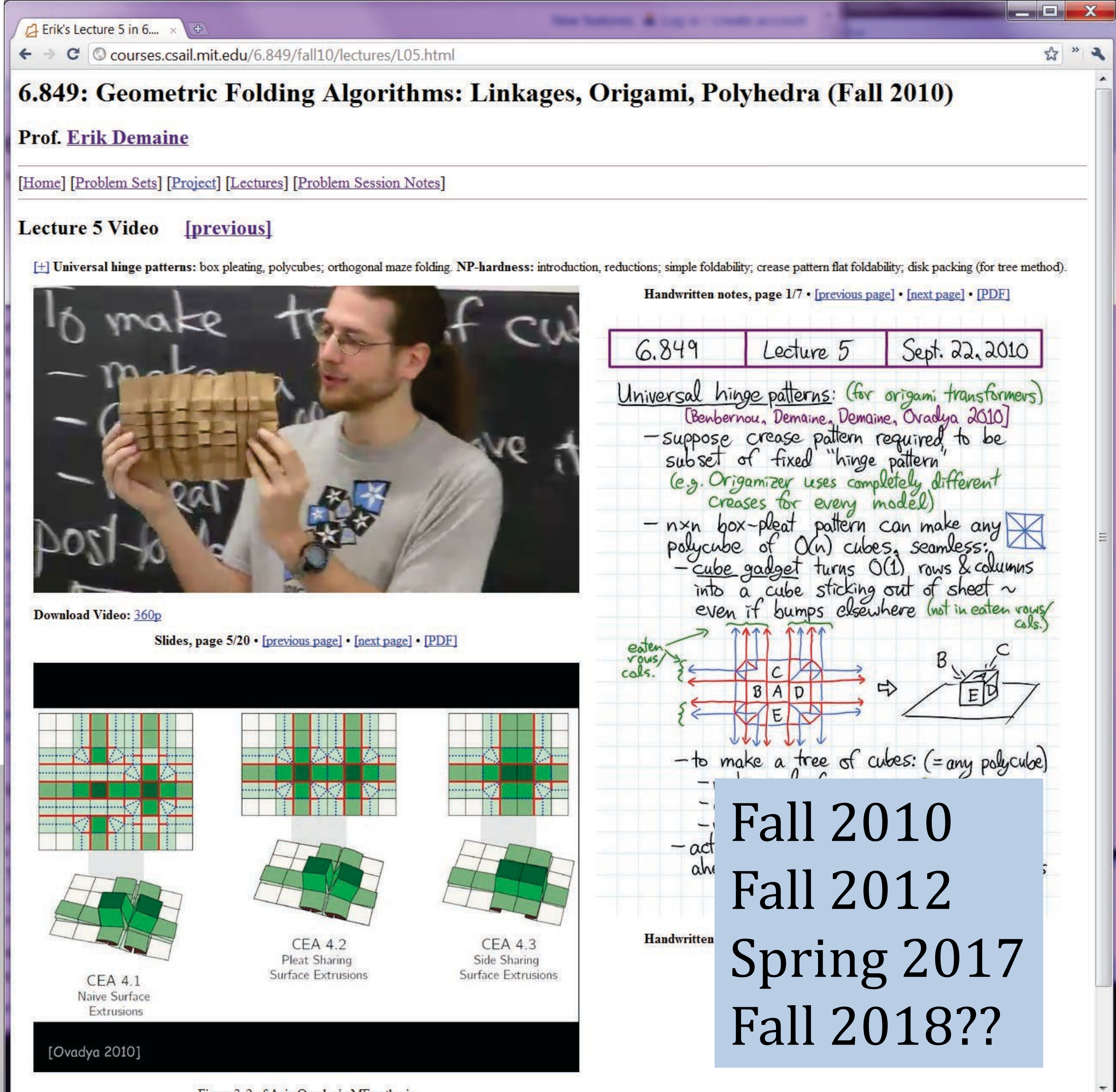


Fold & One Cut Theorem

- Any collection of straight cuts can be made by folding flat & one straight cut
  [Demaine, Demaine, Lubiw 1998] [Bern, Demaine, Eppstein, Hayes 1999]



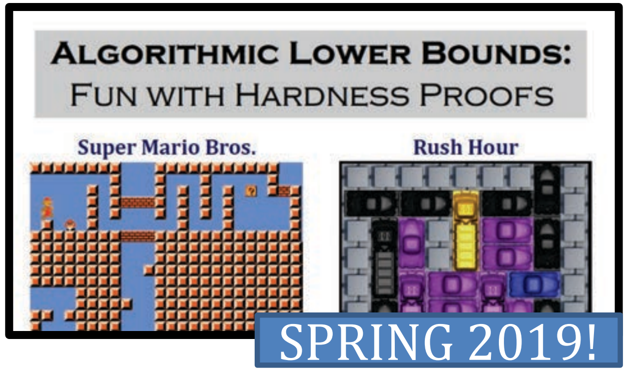




Fall 2010
Fall 2012
Spring 2017
Fall 2018??

Other Fun Research Topics We've Explored

- Super Mario Bros., Mario Kart
- Pokémon
- The Witness
- Portal
- Cookie Clicker
- Bust-a-Move/Puzzle Bobble
- Rubik's Cube
- Symmetric assembly puzzles
- Sliding block/coin puzzles
- UNO
- Losing at Checkers

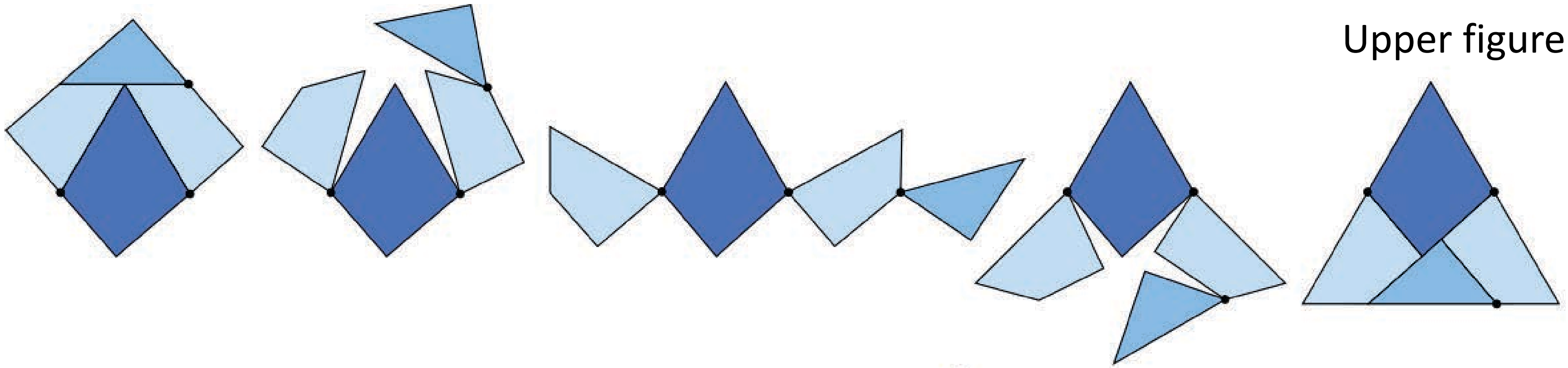


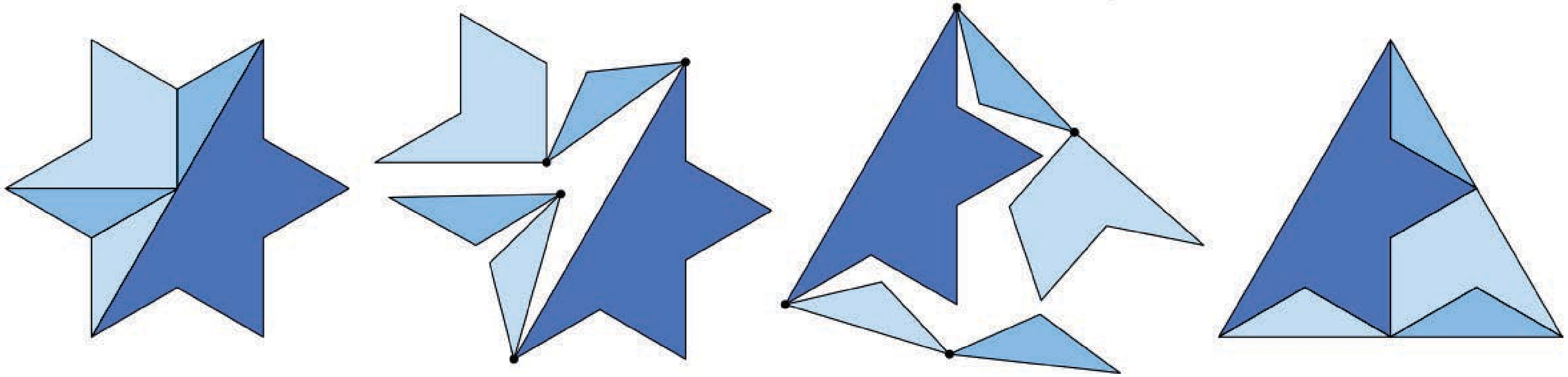
- Solving NP with origami
- Pop-up card design
- Unfoldable polyhedra
- Proving sculptures don't exist
- Folding paper shopping bags
- "Deflating The Pentagon"
- Replicators
- Transformers
- Lego
- Zombies
- Font design

Hinged Dissections

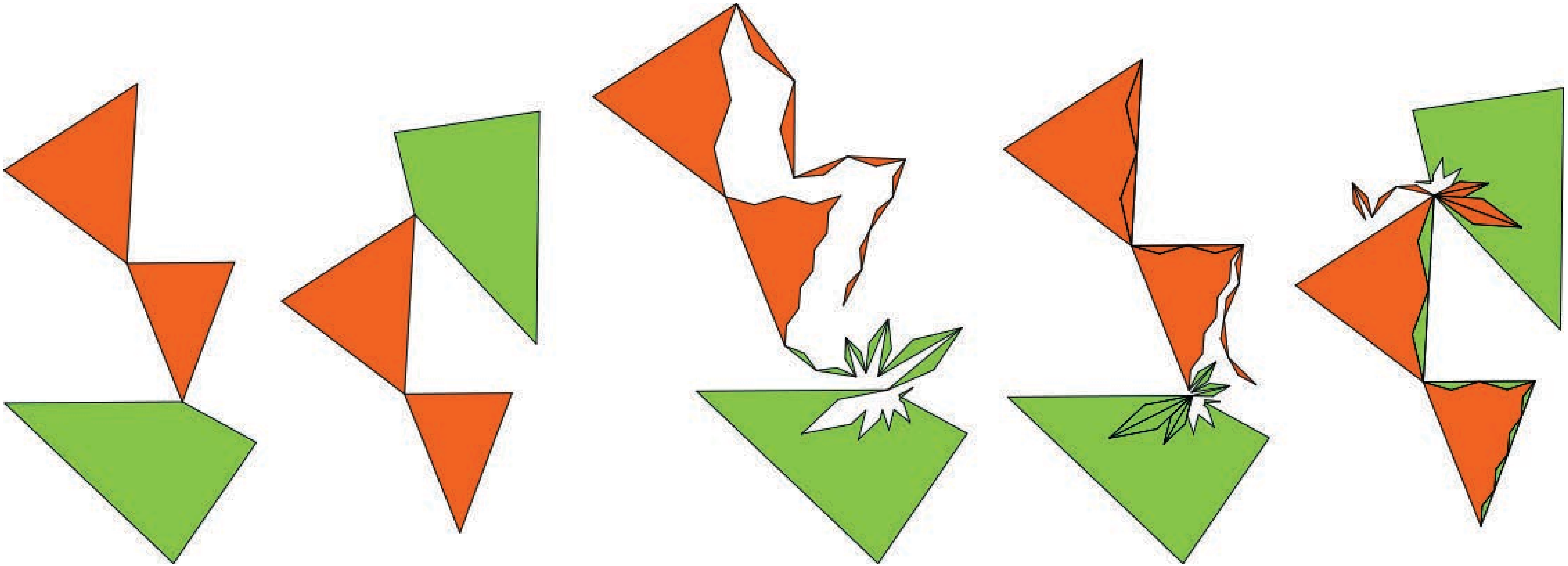
- **Hinged dissection:** polygons attached at rotatable hinges, *all connected*
- Can we always hinge dissect A into B?
- Yes! [Abbott, Abel, Charlton, Demaine, Demaine, Kominers, 2011]

Upper figure from [AACDDK11]

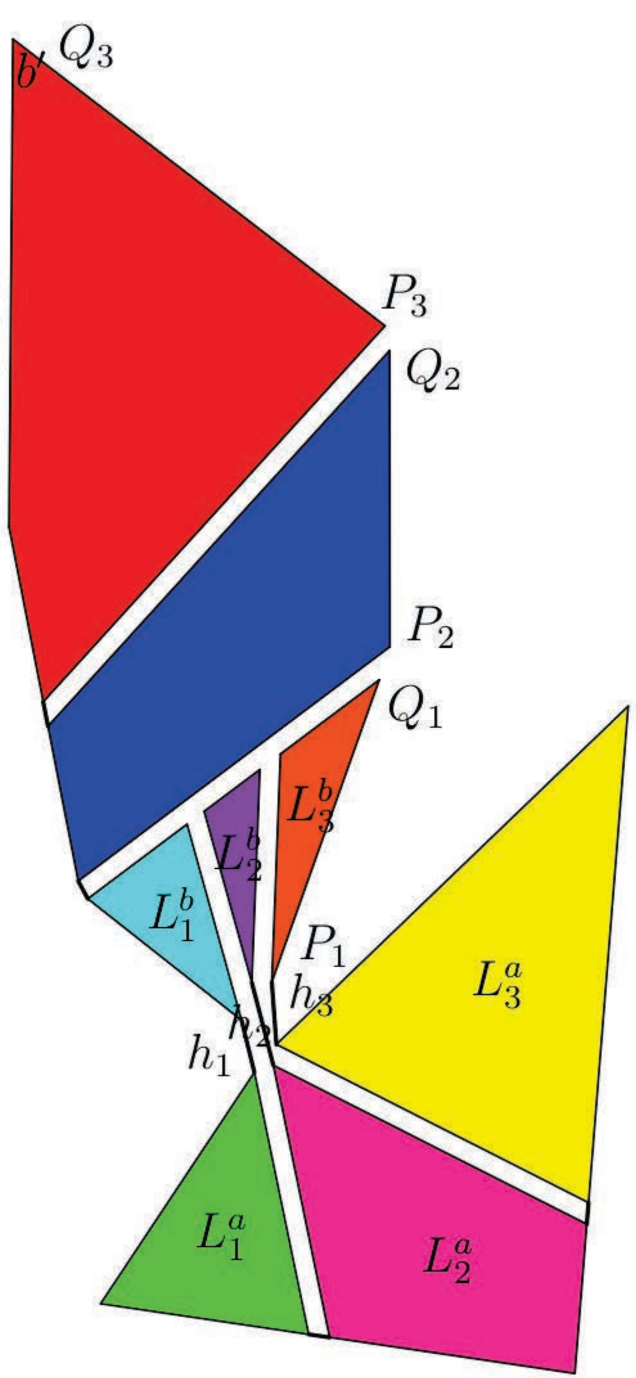


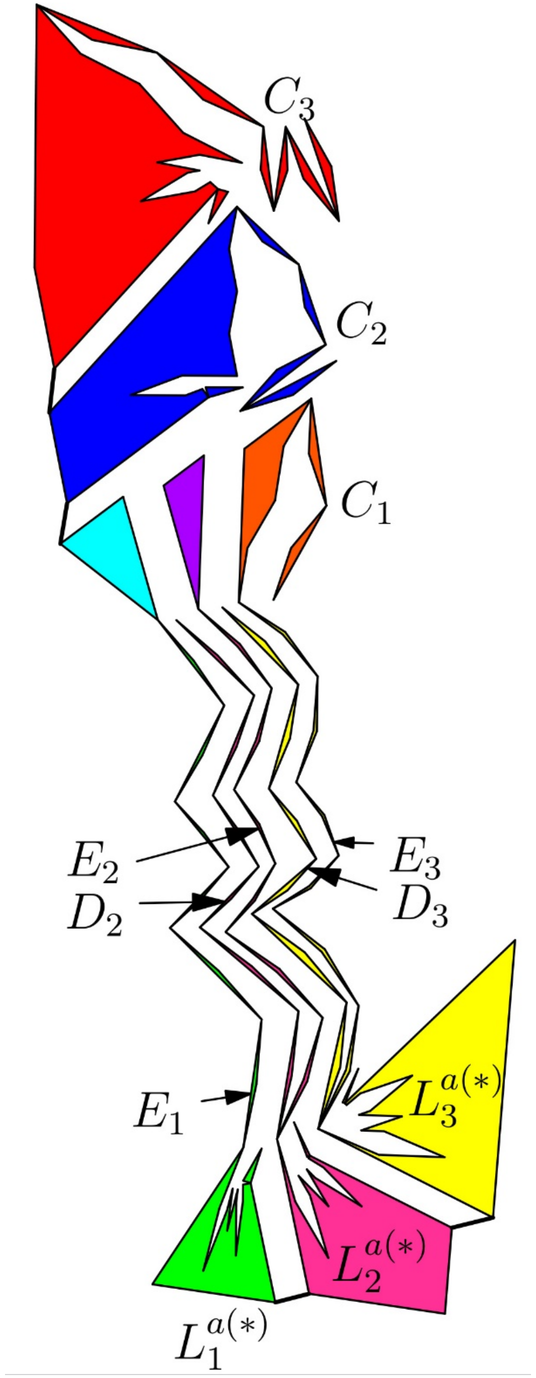


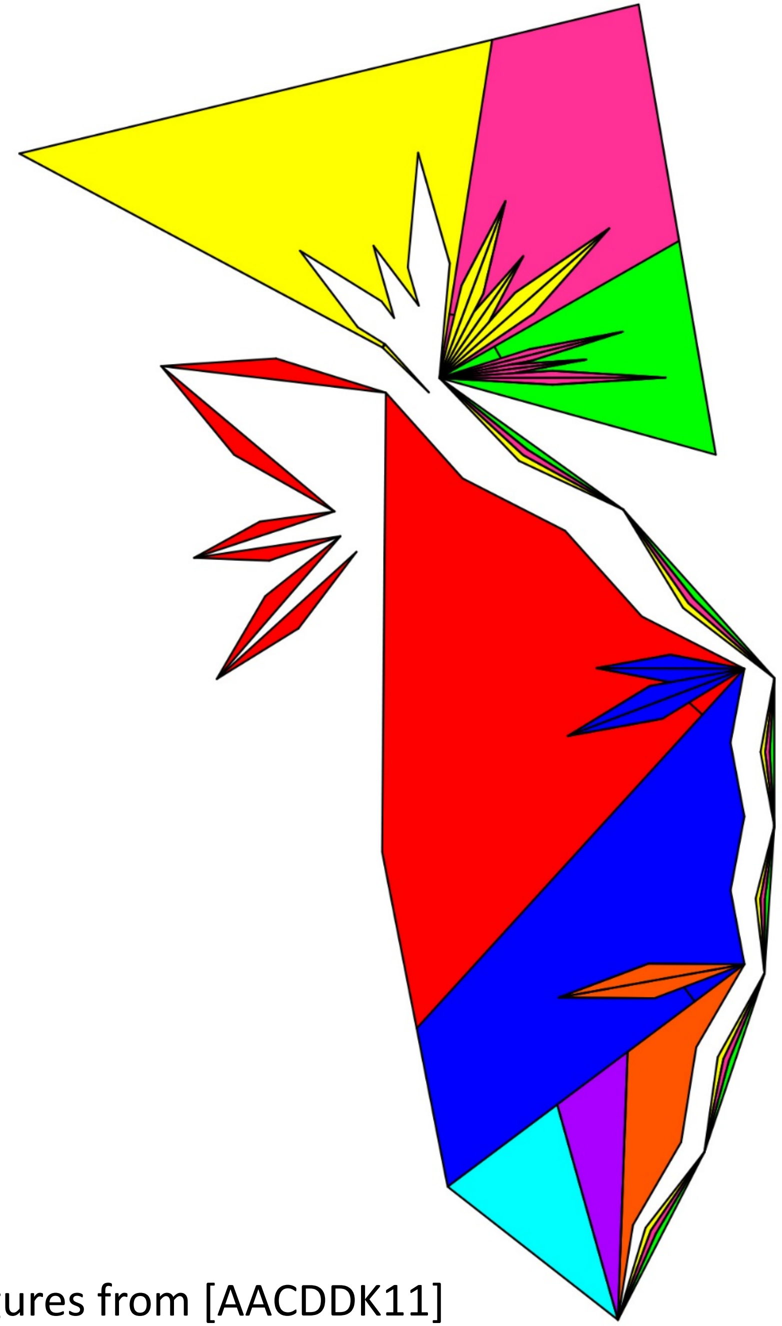
Hinged Dissections Proof Idea



Figures from [AACDDK11]



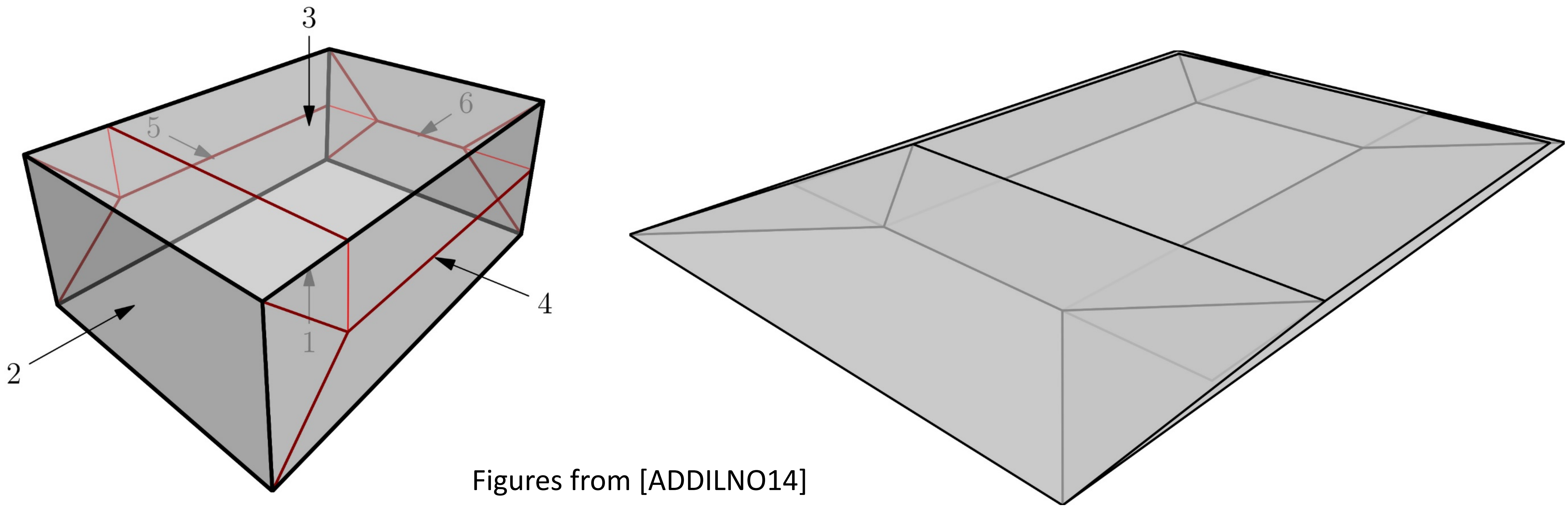




Figures from [AACDDK11]

Polyhedron Flattening

- Flatten a polyhedral surface with an **origami motion**. Always possible?
- Yes for convex! [Itoh, Nara, Vilcu, 2012]
- Yes for convex, but really easy to describe! **“Orderly Squashing”** [Abel, Demaine, Demaine, Itoh, Lubiw, Nara, O'Rourke, 2014]



Figures from [ADDILNO14]

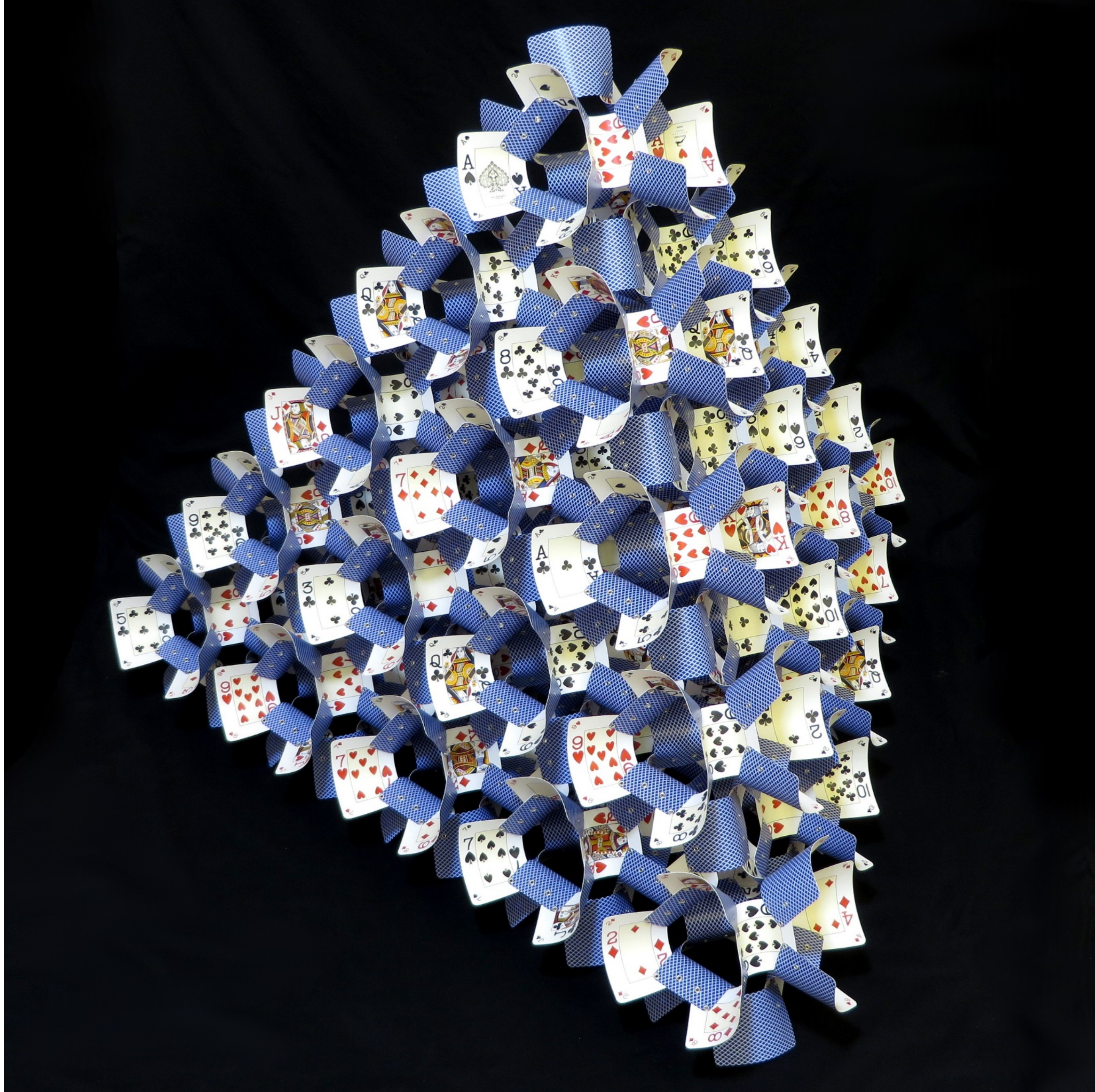
Puzzles!

Snake Cube is NP Complete

[Abel, Demaine, Demaine, Eisenstat, Lynch, Schardl, 2012]

Figure from [ADDELS12]

Sculptures!



Sculptures!