Hierarchical-block conditioning approximations for high-dimensional multivariate normal probabilities

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Introduction

Introduction

- The computation of the multivariate normal (MVN) probabilities

$$\Phi_n(\mathbf{a}, \mathbf{b}; 0, \Sigma) = \int_a^b \frac{1}{\sqrt{(2\pi)^n |\Sigma|}} \exp\left(-\frac{1}{2}\mathbf{x}^T \Sigma^{-1} \mathbf{x}\right) d\mathbf{x}, \qquad (1)$$

  where **a** and **b** are integration limits, the mean vector μ is assumed to be 0, Σ is a positive-definite covariance matrix, is required for a variety of applications.
- In high-dimensional settings (large n), it is hard to calculate (1) directly.
- We review new approaches proposed by Cao et al. (2019) to approximate high-dimensional multivariate normal probability (1) using the hierarchical matrix $\mathcal{H}$ (Hackbusch, 2015) for the covariance matrix Σ.

Motivation

The methods are based on

1. the bivariate conditioning method (Trinh and Genz, 2015) and
2. the hierarchical Quasi-Monte Carlo method (Genton et al., 2018).

Elements

Typography

```
The theme provides sensible defaults to
\emph{emphasize} text, \alert{accent} parts
or show \textbf{bold} results.
```

becomes

The theme provides sensible defaults to *emphasize* text, accent parts or show **bold** results.

Font feature test

- Regular
- *Italic*
- SmallCaps
- **Bold**
- ***Bold Italic***
- **Bold SmallCaps**
- `Monospace`
- *`Monospace Italic`*
- **`Monospace Bold`**
- ***`Monospace Bold Italic`***

Lists

Items

- Milk
- Eggs
- Potatos

Enumerations

1. First,
2. Second and
3. Last.

Descriptions

PowerPoint Meeh.

Beamer Yeeeha.

Animation

- This is important

Animation

- This is important
- Now this

Animation

- This is important
- Now this
- And now this

Animation

- This is really important
- Now this
- And now this

Figures

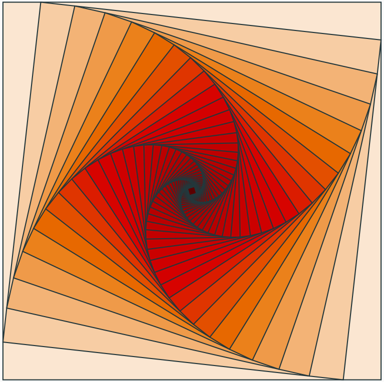

Figure 1: Rotated square from texample.net.

Tables

Table 1: Largest cities in the world (source: Wikipedia)

City	Population
Mexico City	20,116,842
Shanghai	19,210,000
Peking	15,796,450
Istanbul	14,160,467

Blocks

Three different block environments are pre-defined and may be styled with an optional background color.

Default
Block content.

Alert
Block content.

Example
Block content.

Default

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Alert

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Example

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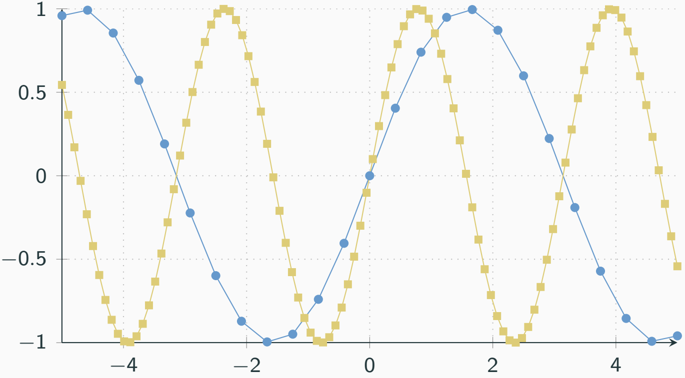
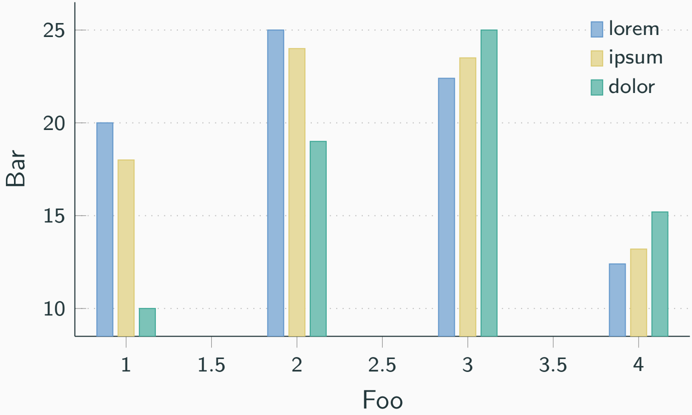
Math

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

Line plots



Bar charts



Quotes

> *Veni, Vidi, Vici*

Frame footer

metropolis defines a custom beamer template to add a text to the footer. It can be set via

```
\setbeamertemplate{frame footer}{My custom footer}
```

References

Some references to showcase [allowframebreaks] Erdős (1995); Graham et al. (1989); Greenwade (1993); Knuth (1992); Simpson (2003)

Conclusion

Summary

Get the source of this theme and the demo presentation from

`github.com/matze/mtheme`

Questions?

Backup slides

Sometimes, it is useful to add slides at the end of your presentation to refer to during audience questions.

The best way to do this is to include the `appendixnumberbeamer` package in your preamble and call `\appendix` before your backup slides.

metropolis will automatically turn off slide numbering and progress bars for slides in the appendix.

References

Cao, J., Genton, M. G., Keyes, D. E., and Turkiyyah, G. M. (2019). Hierarchical-block conditioning approximations for high-dimensional multivariate normal probabilities. *Statistics and Computing*, 29(3):585–598.

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Trinh, G. and Genz, A. (2015). Bivariate conditioning approximations for multivariate normal probabilities. *Statistics and Computing*, 25(5):989–996.