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Hierarchical-block conditioning approximations for high-dimensional multivariate normal probabilities

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Remerciements

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Liste des sigles et acronymes

ASK	*Amplitude Shift Keying*
AWGN	*Additive White Gaussian Noise*
BABG	Bruit Additif Blanc Gaussien
BCJR	*Bahl, Cocke, Jelinek, Raviv*
BER	*Binary Error Rate*
BFDM	*Biorthogonal Frequency Division Multiplexing*

Chapter 1

Introduction

The computation of multivariate normal probability appears various fields. For instance, the inferences based on the central limit theorem, which holds when the sample size is large enough, is widely used in the social sciences and engineering as well as in the natural sciences. Recently, the dimensionality of data and models has been grown significantly, and in this respect, so does a need for the methodology to efficiently calculate high-dimensional multivariate normal probability.

Cao et al. (2019) proposes new approaches to approximate high-dimensional multivariate normal probability

$$\Phi_n(a, b; 0, \Sigma) = \int_a^b \frac{1}{\sqrt{(2\pi)^n |\Sigma|}} \exp\left(-\frac{1}{2}\mathbf{x}^T \Sigma^{-1} \mathbf{x}\right) d\mathbf{x}$$

using the hierarchical matrix $\mathcal{H}$ (Hackbusch, 2015) for the covariance matrix Σ. The methods are based on two state-of-arts methods, among others, are the bivariate conditioning method (Trinh and Genz, 2015) and the hierarchical Quasi-Monte Carlo method (Genton et al., 2018). Specifically, Cao et al. (2019) generalize the bivariate conditioning method to a d-dimension and combine it with the hierarchical representation of the covariance matrix.

Chapter 2

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2.2 d-Dimensional Conditioning Approximation

2.2.1 CMVN

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