A Model of Capital and Crisis with Structured Ambiguity*

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Abstract

I present a general-equilibrium framework of intermediary asset pricing where agents are uncertain about parameters and states. Using observable information and the understanding of the underlying economic structure, agents develop a set of alternative beliefs about parameters and states, predicting different future asset returns. Intermediaries fear scenarios with low expected excess returns, leading to reduced asset demand due to uncertainty aversion and amplifying the risk premium, especially during crises when capital within the intermediation sector is scarce. I evaluate government policies that restrict the set of beliefs that intermediaries view as plausible, such as deposit insurance, capital requirements, and government guarantees for risky asset payoffs. I demonstrate the efficacy of announcements that eliminate pessimistic prospects for cash-flow growth and restore risk appetite.

Keywords: Asset prices, financial frictions, heterogeneous agent model, risk, robustness, model uncertainty

JEL Codes: C52, C54, D51, D52, D53, D81, D84, E44, E5, E6, G1, G2

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1 Introduction

A growing literature in macroeconomics and asset pricing has developed general equilibrium models with intermediaries who are marginal investors in financial markets. These models are used to study asset valuation and policy impacts during financial crises. However, they involve hard-to-measure parameters and states which agents inside these models can plausibly view as uncertain.

Existing theoretical frameworks overlook this uncertainty by assuming a rational expectations equilibrium, where agents fully trust the true equilibrium data generating process for asset returns. In these settings, government policies are limited to direct interventions such as recapitalization of distressed financial institutions, aimed at mitigation of heightened risk premia during crises. Such policy analysis ignores the possibility that parameter and state uncertainty are crucial contributors to risk premia, and that there are alternative policy prescriptions aimed at resolving the uncertainty, for instance as emphasized in Haddad et al. (2023), by altering the market perception that policymakers might do whatever it takes, in that they will go to much greater lengths to backstop markets if the situation gets worse.

In this paper, I develop a general-equilibrium model featuring financial intermediaries, who are ambiguous about parameters and states that are key return predictors. Those agents find many alternative combinations of predictors consistent with observable information and their understanding of the mapping from observable information to the uncertain predictors. I refer to this form of uncertainty as "structured" ambiguity. In order to guard themselves against the uncertainty, ambiguity-averse agents make cautious investment decisions under a worst-case scenario which has the most adverse utility consequences, leading to a reduction in risky investment.

My findings reveal that compensation for structured ambiguity accounts for a substantial portion of total risk premia, revealing the importance of policy interventions and regulations affecting this component of the risk premia. Using this framework as a laboratory, I demonstrate the effectiveness of conditional policy promises aimed at resolving agents' uncertainty about return predictors. Those promises eliminate the worst-case scenario by shrinking the set of alternative return predictors consistent with observable information and understanding of the underlying economic structure.

The economic structure builds upon He and Krishnamurthy (2013) (thereafter HK). There are risky and risk-free asset markets. The risky asset represents the claim on the aggregate dividend that stochastically evolves and has a fixed unit supply. The net supply of the risk-free asset is zero.

Two types of identical agents exist: intermediaries and households. Households, trusting the true data generating process, can invest in the risk-free asset market without frictions. However, they cannot directly invest in the risky asset. They must invest in the funds of financial intermediaries due to margin constraints. The contribution of households is capped by a fraction of intermediaries' own wealth, reflecting the tightness of the constraint. Intermediaries invest their own wealth and the contribution from households in risky and risk-free assets without frictions to maximize their lifetime utility.

In this economy, the total wealth share of intermediaries, or the financial sector's capitalization and the tightness of the constraint, are key predictors for future risky asset returns. Low capitalization and a tight financial constraint constrain households' contribution to the financial intermediary. Intermediaries, in turn, must borrow larger amounts of the risk-free funds from households to finance the risky asset, leading to a highly leveraged position. To compensate for this, the risk premium must increase in equilibrium to clear the market.

Additionally, the expected cash-flow growth from the risky asset is a predictor for the future risk-free rate. Higher expected cash-flow growth predicts higher economic growth, increasing future risk-free rates.

In contrast to HK, intermediaries under ambiguity aversion are uncertain about these three return predictors and make cautious decisions under the worst-case belief regarding the return predictors. This worst-case belief represents the scenario that minimizes their lifetime utility. In equilibrium, intermediaries fear lower expected returns on the risky asset, which would slow down their wealth accumulation.

Intermediaries discipline the set of combinations of the three return predictors using observable information on return volatility of the risky asset, realized risk-free rates, and their understanding of the mapping from those predictors to observable information. Intermediaries find many combinations of predictors consistent with observable information, leading to a partial identification problem.

I illustrate the presence of multiple combinations of return predictors consistent with observable information with the following example. During a financial crisis, the capitalization of the financial sector is low, and the margin constraint is binding. The economy experiences high return volatility and a lower risk-free rate. High return volatility arises because adverse shocks amplify reductions in intermediaries' wealth, reducing risky asset demand and asset prices. The risk-free rate drops due to the higher precautionary saving motive of the highly-leveraged financial sector.

However, high return volatility and a low risk-free rate can also be consistent with high financial sector capitalization and lower cash flow growth. High capitalization makes the financial sector a more significant player, significantly fluctuating asset prices even with moderate leverage, indicating high return volatility. Lower cash-flow growth prospects lead to a lower risk-free rate, aligning with observable information. This alternative scenario corresponds to a lower expected return perceived by each individual intermediary.

Since individual intermediaries cannot observe aggregate capitalization of the entire financial sector or tightness of margin constraints of other intermediaries, they view both scenarios as possibly plausible, which both rationalize available information. The latter scenario is, however, more adverse from their perspective, since the lower perceived expected returns constitute less advantageous investment opportunities.

To guard against this alternative scenario, intermediaries reduce their demand for the risky asset compared to demand without this form of structured ambiguity. In equilibrium, the difference in expected excess returns under the true data generating process and under the worst-case belief accounts for approximately 40% of the total risk premia in this economy. I term this difference as "the price of partial identification" representing the compensation for uncertainty induced by the partial identification problem.

In policy experiments, conditional policy promises resolving uncertainty about cash flow growth prove highly effective at mitigating heightened risk premia. Under this policy, similar to guaranteeing the cash flow from mortgage-backed securities, agents infer that the lower risk-free rate arises from the high precautionary saving motive of the sector associated with lower capitalization, not the lower cash flow growth, and that the high return volatility is caused by a lower capitalization. Then they conclude that expected excess returns must be high, which increases their appetite for risky assets.

By contrast, policy promises resolving uncertainty about the tightness of financial constraints such as capital requirements and deposit insurance are not as effective. They do not eliminate the possibility of higher capitalization of the financial sector and less profitable opportunities in the market.

These examples provide the main conceptual insight of the paper. Policymakers should be aware of the ways how their policies alleviate uncertainty and shape the beliefs of market participants during financial crises in order to understand their efficacy. In particular, the understanding of how policy actions mitigate the worst-case scenario is crucial, which requires the explicit modelling of the worst-case scenario in this paper.

This paper contributes to two strands of the literature. First, I make an applied contribution to the literature that develops general-equilibrium models with financial frictions in macroeconomics and asset pricing initiated by Kiyotaki and Moore (1997) and Bernanke et al. (1999). I incorporate ambiguity over hard-to-measure parameters and states into a canonical model of this class and explore policy implications. Consistent with the theoretical prediction in this paper, Bachmann et al. (2020) provides empirical findings documenting joint movements in perceived ambiguity and measured risk premia, such as those present in credit spreads.

The novel aspect of my approach lies in the incorporation of this uncertainty following the ambiguity literature. This methodology offers a practical and manageable way to address multi-dimensional uncertainty without requiring the tracking of an extensive array of state variables encompassing time-varying parameters and states. Maintaining a small state space within the model is crucial for a computational characterization of the equilibrium globally. This is particularly crucial when characterizing the essential state dependence observed in this class of models, as underscored by recent contributions Adrian and Boyachenko (2012), He and Krishnamurthy (2013),Brunnermeier and Sannikov (2014), Di Tella (2017), He and Krishnamurthy (2019).

The focus on the endogenous subjective beliefs during financial crises in the paper is close to the work examining the presence of multiple equilibria under rational expectations equilibrium in macro-finance models and implications of jumps across multiple equilibria for equilibrium dynamics such as Gertler et al. (2020) and Khorrami and Mendo (2023). Similar to the classic study of bank runs by Diamond and Dybvig (1983), those models do not predict any subjective and objective probability over multiple equilibria.

In contrast, the set of the beliefs over alternative scenario and the worst-case scenario are all endogenous in this paper, depending on the economic fundamental such as the capitalization of the financial sector. This paper provides a framework to analyze how policy interventions alter the endogenous beliefs and implications for equilibrium dynamics.

More broadly, some recent work integrates the deviation from a rational expectations equilibrium in the form of behavioral expectations biases into macro-finance models including Krishnamurthy and Li (2021) and Maxed (2023), which attempt to replicate the empirical boom-bust credit cycles surrounding financial crises. Fontanier (2022) studies the optimal policies in the presence of these expectations biases with financial frictions. My work differs by explicit modeling of the endogenous formation of the subjective beliefs due to ambiguity concerns, predicting the joint movements in perceived ambiguity and measured risk premia, empirically documented in Bachmann et al. (2020).

Second, this paper provides a theoretical contribution to the ambiguity literature. Recent work by Hansen and Sargent (2022) refines the concept of ambiguity and distinguishes between model misspecification concerns and structured ambiguity. In the former, agents fear that all parametric economic theories are misspecified, leading to cautious decisions based on a worst-case model that is statistically close to alternative models but not necessarily a well-specified parametric model itself[1].

In contrast, structured ambiguity involves concerns about identifying which parameterized economic models represent the true data generating process, building on the static setting of Gilboa and Schmeidler (1989) and the dynamic extension by Chen and Epstein (2002). A challenge associated with incorporating the structured ambiguity is how agents inside the models find alternative parametric models.

The existing applications of structured ambiguity are limited to simple environments involving representative agents' decision making, which is reduced to solving a planner's problem[2], where agents or planners are uncertain about parameters characterizing exogenous processes in the economy and discipline the set of alternative parameters statistically, independent of equilibrium allocation and prices.

In contrast, I introduce structured ambiguity into a general environment where prices, allocations, and the worst-case belief are jointly determined. This approach necessitates that belief formation is consistent with observed endogenous prices. In this paper, agents contemplate alternative parametric economic theories of financial intermediation with different values of parameters and states.

The rest of this paper is structured as follows: Section 2 describes the model ingredients and the equilibrium in the sequential formulation. Section 3 and Section 4 characterize the set of alternative models and the worst-case beliefs in a Markovian setting. In Section 5, I calibrate the model and verify the worst-case model is statistically hard to distinguish from

[1]See Pouzo and Presno (2016) and Bhandari et al. (2023) for applications of model misspecification concerns to general economic environments such as sovereign debt markets and labor markets.

[2]For example, see Hansen and Sargent (2010), Hansen and Sargent (2021), Balter et al. (2023).

the true date generating process. Section 6 shows the impact of the partial identification challenge faced by investors on the risk premia, and Section 7 illustrates how government announcements can mitigate heightened risk premia in crisis episodes. Finally, Section 8 concludes with a discussion of future research.

2 Model

I introduce ambiguity regarding alternative price processes, or asset return processes, into agents' preferences due to uncertainty regarding certain parameters and states in the intermediary asset pricing model of He and Krishnamurthy (2013) (henceforce, HK). The HK model stands as one of the pioneering quantitative papers in the continuous-time intermediary asset pricing literature.

2.1 Model Set-Up

Time is continuous, denoted by $t \in [0, \infty)$ representing the current period. There are two distinct groups of agents: households and intermediaries. While households lack the expertise to directly invest in the risky asset market, intermediaries possess the required knowledge. Intermediaries can act on behalf of households in risky asset investments. This intermediary role is central to the model's structure — households demand intermediation services, and intermediaries supply these services.

Consequently, households face a portfolio decision, involving the allocation of their wealth between acquiring equity in the intermediaries and investing in risk-free bonds. Intermediaries, in turn, receive the equity contribution from households, combine them with their own wealth, and allocate the entire pool of managed funds between the risky asset and risk-free bonds. I will delve into a detailed examination of each component of this model in the forthcoming sections. Firstly, I will review the common elements shared with the HK model and then describe the novel element: intermediaries' preferences incorporating structured ambiguity about the true equilibrium data generating process (DGP) due to the uncertainty about parameters and states.

2.2 Assets

The assets in this model adhere to the structure outlined in Lucas (1978) tree economy. There is a single perishable consumption good serving as the numeraire. I normalize the total supply of intermediated risky assets to one unit. Meanwhile, the riskless bond has zero net supply and is open for investment by both households and intermediaries.

The risky asset in this model yields a dividend flow D_t that follows a geometric Brownian motion described by the following stochastic differential equation:

$$\frac{dD_t}{D_t} = gdt + \sigma dZ_t,$$

with D_0 as the initial condition. Here, g and σ represent the mean dividend growth and volatility.

In this paper, we work within the framework of a probability space denoted as $(\Omega, \mathcal{F}, \mathcal{P})$, where $\mathcal{P}$ represents the true DGP. The stochastic process Z_t is established as a standard Brownian motion on this complete probability space.

Additionally, we define two key processes, namely P_t and r_t, which correspond to the risky asset price and interest rate processes, respectively. Furthermore, we define the total return on the risky asset, denoted as dR_t, which follows the equation:

$$dR_t = \frac{D_t dt + dP_t}{P_t} = (\pi_{R,t} + r_t)dt + \sigma_{R,t} dZ_t,$$

Here, $\pi_{R,t}$ and $\sigma_{R,t}$ represent the expected excess return and return volatility, respectively, under the true DGP determined in equilibrium.

2.3 Intermediary and Margin Constraint

At any given time t, each intermediary is randomly matched with a household. These interactions happen instantly, resulting in a continuum of identical bilateral relationships. Household j allocates a part of its wealth, $H_{j,t}$ to purchase equity issued by the intermediary. The wealth of intermediary i at time t, denoted as $w_{i,t}$. Intermediaries execute trades in a Walrasian risky asset and bond market, while households trade solely in the bond market. At $t + dt$, the match concludes, and the intermediation market repeats the process.

Considering a relationship between an intermediary i and household j, the intermediary's total funds comprise the intermediary's own wealth, $w_{i,t}$, and the wealth allocated to the intermediary by the household, $H_{j,t}$. The intermediary makes all investment decisions for these total funds and faces no portfolio restrictions regarding buying or short-selling either the risky asset or the risk-free bond. Let $\alpha^I_{i,t}$ denote the ratio of the intermediary's risky asset holdings to its total funds, $w_{i,t} + H_{j,t}$. This ratio, capturing leverage, is typically larger than one. Therefore, the return on funds delivered by the intermediary is described by the equation:

$$dR^I_t = r_t dt + \alpha^I_t (dR_t - r_t), \tag{1}$$

where dR_t represents the total return on the risky asset. When $\alpha_{i,t} > 1$, the intermediary invests more than 100 percent of the total funds in risky assets and borrows $(\alpha^I_{i,t} - 1)(w_{i,t} + H_{j,t})$ through the risk-free short-term bond market, thus making a leveraged investment in the risky asset.

The household is unwilling to invest more than $mw_{i,t}$ in the equity of a matched intermediary, where $m > 0$ is a constant parametrizing the financial constraint and $w_{i,t}$ is the wealth held by an intermediary i managing the intermediary. If an intermediary invests one dollar in the intermediary's entire pool of managed funds, the household will invest at most m dollars of its own wealth. The margin constraint implies that the intermediary's demand

$H_{j,t}$ facing a household is at most:

$$H_{i,t} \leq m w_{i,t}. \quad (2)$$

A small m or $w_{i,t}$ restricts the household's ability to participate indirectly in the risky asset market. This constraint influences risk premia and asset prices in equilibrium[3].

2.4 Households: The Demand for Intermediation

In the HK model, the household sector is represented as an overlapping generation (OG) of agents, simplifying the household's decision problem. To explain the OG environment in a continuous-time model, time is indexed as t, $t+\delta$, $t+2\delta$, ... considering the continuous time limit when δ is of the order dt. A unit mass of generation t agents is born with wealth w_t^h and lives during periods t and $t+\delta$. Unlike intermediaries, households fully trust the true DGP and aim to maximize utility:

$$\rho\delta \log c_t^h + (1-\rho\delta) E_t[\log w_{t+\delta}^h],$$

where c_t^h is the household's consumption rate in period t and w_t^h represents a bequest for generation $t+\delta$. Importantly, E_t is the expectations operator under the true DGP. Additionally, generation t households are assumed to receive labor income at date t of $lD_t\delta$. Here, $l > 0$ is a constant, and recall that D_t is the dividend rate on the risky asset at time t. Thus, labor income is proportional to the aggregate output of the economy. This inclusion of labor income is crucial because it prevents scenarios where the household sector vanishes from the economy due to lack of income.

Households invest their wealth w_t^h from t to $t+\delta$ in financial assets. They are required

[3] This constraint, linking "net worth" and external financing, is standard in financial friction literature and can be justified by various agency or informational frictions. For instance, in He and Krishnamurthy (2012), hedge fund managers often have a significant portion of their wealth tied up in the fund. External investors require managers to have a substantial stake ("skin in the game") to align incentives. If the hedge fund sustains losses, depleting managers' stakes, investors are reluctant to contribute further capital due to concerns about mismanagement or more losses. This scenario, known as a "hedge fund capital shock," is captured in the model.

to keep a minimum of λw_t^h ($\lambda < 1$) in short-term debt issued by the intermediary sector as debt households, representing a baseline demand for holding a portion of household wealth in a risk-free asset. This demand for liquid balances by the household sector is satisfied by issuing bank deposits. This feature is important as it generates leverage in the intermediary sector, even when the margin constraint doesn't bind, allowing for stochastic dynamics of the key state variable, the aggregate wealth share of intermediaries, in unconstrained states. The remaining fraction of wealth $(1-\lambda)w_t^h$ is managed by risky households who can invest in matched intermediaries and risk-free bonds.

To summarize, a debt and risky asset household are born at generation t with wealth of w_t^h . The households receive labor income, choose consumption, and make savings decisions, respecting the restriction on their investment options. It is easy to verify that in the continuous time limit, i.e., when $\delta \to dt$, the households' consumption rule is

$$c_t^h = \rho w_t^h.$$

Debt households invests λw_t^h invests in the bond market at the interest rate r_t. A risky asset household with wealth $(1-\lambda)w_t^h$ decides decides the fraction $\alpha_t^h \in [0,1]$ to invest in intermediaries' equity. The remaining $1-\alpha_t^h$ of the risky asset household's wealth is allocated to the risk-free bond. Given the log utility, the risky asset households chooses α_t^h to solve the optimization problem:

$$\max_{\alpha_t^h \in [0,1]} \alpha_t^h E_t[dR_t^I - r_t dt] - \frac{1}{2}(\alpha_t^h)^2 Var_t[dR_t^I - r_t dt]$$

subject to the margin constraint (2)

$$H_t = \alpha_t^h (1-\lambda) w_t^h \leq m w_t,$$

where I omitted the dependence on identity i, j since households and intermediaries are identical within their classes ($w^h_{j,t} = w^h_t$) and ($w_{i,t} = w_t$). The evolution of w^h_t across generations is described by

$$dw^h_t = (lD_t - \rho w^h_t)dt + w^h_t r_t dt + \alpha^h_t (1-\lambda) w^h_t (dR^I_t - r_t dt).$$

2.5 Intermediaries

There exists a unit mass of identical and infinitely-lived intermediaries where households invest their resources. Intermediaries contemplate alternative stochastic processes of returns on the risky asset. Since the absolute continuity implies the return volatility $\sigma_{R,t}$ is observable in the continuous-time Brownian motion environment, intermediaries are solely concern of alternative specifications of expected excess return on the risky asset. Since they distrust the baseline true parameters and current state realization, they contemplate alternative expected excess returns which prevail in equilibria with alternative true parameter and current state realizations.

I assume intermediaries doubt the baseline, or true values of margin constraint parameter m, the mean dividend growth g, and the aggregate wealth share of intermediaries $x_t \equiv w_t/P_t$, which turn out to be important return predictors in this model as discussed in section 4. From now on, I distinguish the true (baseline) parameter and state values with hats ($\hat{m}, \hat{g}, \hat{x}_t$) from the general notation of these variables without hats (m, g, x_t).

2.5.1 Formulation of Alternative Models

Following Hansen and Sargent (2022), I describe a set of alternative models for the expected excess returns using convenient mathematical representations of positive martingales that modify a baseline probability model. For intermediaries, these martingales are likelihood ratios between alternative and the baseline model. Starting from the intermediaries' baseline probability measure, I use martingales to represent probabilities that intermediaries consider as plausible alternative models.

For clarity, I use the following baseline model, or the true DGP of the stochastic process governing the dynamics of excess returns:

$$dR_t - r_t dt = \hat{\pi}_{R,t} dt + \sigma_{R,t} dZ_t,$$

where $\hat{\pi}_{R,t}$ represents the expected excess return under the baseline model and is a measurable function with respect to the filtration $\mathcal{F}$.

Intermediaries contemplate alternative models for the excess return represented as likelihood ratios, which, in my setting, are positive martingales with unit expectations. In the continuous-time Brownian information environment, owing to the Girsanov Theorem and related results, we can describe the evolution of a likelihood ratio denoted as M^S of an alternative process relative to the baseline specification as follows:

$$dM_t^S = M_t^S S_t dZ_t,$$

where S_t is progressively measurable with respect to the filtration $\mathcal{F}$. If

$$\int_0^t |S_\tau|^2 d\tau < \infty \tag{3}$$

with probability one, the stochastic integral $\int_0^t S_\tau dZ_t$ is well-defined. Imposing the initial condition $M_0^S = 1$, we express the solution of the stochastic differential equation (2) as a so-called stochastic exponential:

$$M_t^S = \exp\left(\int_0^t S_\tau dZ_\tau - \frac{1}{2}\int_0^t |S_\tau|^2 d\tau\right). \tag{4}$$

Definition 1 $\mathcal{M}$ *denotes the set of all martingales* M^S*, constructed as stochastic exponentials via representation* (4) *with an* S_t *that satisfies* (3) *and is progressively measurable with respect to* $\mathcal{F}$*.*

In the subsequent discussion, I use the process S to represent alternative martingales of interest. I describe probabilities implicitly by delineating the family of conditional expectations associated with each such S process, namely,

$$E^S(B_t|\mathcal{F}_0) = E(M_t^S B_t|\mathcal{F}_0)$$

for any $t \geq 0$ and any bounded $\mathcal{F}_t$-measurable random variable B_t. This representation uses the positive random variable M_t^S as a Radon-Nikodym derivative for the date t conditional expectation operator $E^S(\cdot|\mathcal{F}_0)$. The martingale property for M^S ensures that the Law of Iterated Expectations applies to the constructed probability measures. In what follows, I will refer to this probability measure as being affiliated with the martingale M^S.

Under the alternative model, the evolution of expected excess returns follows:

$$dR_t - r_t = \pi_R^S dt + \sigma_{R,t} dZ_t^S,$$

where π_R^S represents the expected excess return under the alternative model, and

$$dZ_t = S_t dt + dZ_t^S,$$

where dZ_t^S is now a standard Brownian motion under the alternative model.

Importantly, intermediaries restrict the set of alternative specifications of the excess return processes in a structured way so that the alternative expected excess return must correspond to the equilibrium outcome in an alternative economy with alternative parameters (m, g) and aggregate state x_t, which is the aggregate wealth share of intermediaries, or the capitalization of the financial sector. This restriction disciplines the set of alternative models, S and M^S.

Definition 2 *$\mathcal{M}^S$ denotes the set of all martingales M^S in $\mathcal{M}$ that satisfies $S_t = \hat{\pi}_{R,t} - \pi_R(x_t, m, g)$, where $\pi_R(x_t, m, g)$ is the equilibrium expected excess return in an alternative* *economy with a wealth share of intermediaries x_t and parameters $m \in (0,\infty)$ and $g \in (-\infty,\infty)$.*

Under an alternative model in $\mathcal{M}^S$, the expected excess return evolves as

$$dR_t - r_t = \pi_R^S(x_t, m, g)dt + \sigma_{R,t} dZ_t^S.$$

2.5.2 Restricting the Set of Structured Alternative Models

Without any further restrictions on alternative combinations of parameters and state, the set of alternative models $\mathcal{M}$ is too large. In fact, some of those alternative combinations are not consistent with the observable information from the realized excess returns, or return volatility and risk-free rates[4], along with the understanding of equilibrium relationships implied by the structure of the alternative economy, i.e. cross-equation restrictions. In what follows, I elaborate on how intermediaries discipline the set of alternative models for expected excess returns by exploiting these restrictions.

In particular, in each period t, intermediaries form a set of (x_t, m, g) consistent with the observable return volatility $\sigma_{R,t}$ and risk-free rate r_t by using the understanding of the equilibrium relationships between (x_t, m, g) and the observable σ_R and r, so called cross-equation restriction such that

$$\underbrace{\sigma_{R,t}}_{Observable\ information} \underbrace{=}_{Cross-equation\ restriction} \underbrace{\sigma_R(x_t, m, g)}_{Model}.$$

$$\underbrace{r_t}_{Observable\ information} \underbrace{=}_{Cross-equation\ restriction} \underbrace{r(x_t, m, g)}_{Model}.$$

$\sigma_R(x_t, m, g)$ and $r(x_t, m, g)$ are the equilibrium return volatility and risk-free rate, respectively, in an alternative economy with true value of margin constraint parameter m, current wealth share x_t, and the mean dividend growth rate g.

[4]In continuous time with Brownian motion, observing returns on the risky asset allows for observing return volatility due to the continuity.

Those collection of (x_t, m, g) form the partially-identified set, with each element characterizing the alternative infinitesimal expected excess return $\pi^S_{R,t}$ and the associated local drift distortion S_t in Ξ_t. Because of the one-to-one correspondence between (x_t, m, g) and S_t, I denote the elements in Ξ_t as either (x_t, m, g) and S_t, Following Chen and Epstein (2002), I formulate the restricted set of $\mathcal{M}^O$ in terms of Ξ_t

$$\mathcal{M}^o \equiv \{M^S \in \mathcal{M}^{\mathcal{S}} : S_t \in \Xi_t \text{ for all t} \geq 0\}. \tag{5}$$

Discussion on dynamic consistency and admissibility of the worst-case belief: *In general, the set Ξ_t is neither convex nor compact, which are both sufficient as discussed in Epstein and Schneider (2003) to ensure the dynamic consistency of dynamic max-min preferences introduced later. To overcome this issue, I expand the original set Ξ_t to make it convex and compact such that the expanded set $\tilde{\Xi}_t$ includes all the alternative models S of expected excess returns that reside in the interval between the upper and lower bound of the original set Ξ_t. Even in this expanded set, since the minimization problem turns out to have linear objective function with respect to S, the minimization problem will choose either maximum or minimum S in the original set. Thus, the minimization problem under the expanded set still yields the admissible belief. Therefore, assuming the formation* (5) *leads to the dynamic consistency of max-min preferences is not an issue in this model.*

To cope with uncertainty about alternative specifications, for a given consumption stream $\{c_{i,t} : t \geq 0\}$, intermediaries' alter ego chooses the alternative model to minimize the expected lifetime utility. More formally, the continuation value process $\{V_{i,t} : t \geq 0\}$ of an intermediary i is

$$\begin{aligned} V_{i,t} &= \min_{S_\tau \in \Xi_\tau : t \leq \tau < \infty} E\bigg(\int_0^\infty \exp(-\rho\tau) \bigg(\frac{M^S_{t+\tau}}{M^S_t} [u(c_{i,t+\tau}) d\tau | \mathcal{F}_\tau] \bigg) \bigg) \\ &= \min_{\mathcal{M}^o} E^S_t \bigg[\int_0^\infty \exp(-\rho\tau) u(c_{i,t+\tau}) d\tau \bigg], \end{aligned}$$

where $u(c_{i,t}) = \frac{c_{i,t}^{1-\gamma}}{1-\gamma}$.

Given the worst-case belief, the intermediary chooses his consumption rate and the portfolio decision of the intermediary to solve

$$\max_{c_{i,t+\tau},\alpha^I_{i,t+\tau}} \min_{\mathcal{M}^o} E^S\left[\int_0^\infty \exp(-\rho\tau)u(c_{i,t+\tau})d\tau\right]$$

s.t.

$$dw_{i,t} = -c_{i,t}dt + w_{i,t}r_t dt + w_{i,t}(dR^I_t(\alpha^I_{i,t}) - r_t dt).$$

where the intermediary return $dR_t(\alpha^I_{i,t})$ is given by (1). We can also rewrite the budget constraint in terms of the underlying return:

$$dw_{i,t} = -c_{i,t}dt + w_{i,t}r_t dt + \alpha^I_{i,t}w_{i,t}(\pi^S_{R,t}dt + \sigma_{R,t}dZ^S_t).$$

Note that the intermediary's portfolio choice of $\alpha^I_{i,t}$ effectively maximizes his lifetime utility.

Discussion on parameter and state uncertainty: *As an illustration of uncertainty about capitalization of the financial institutions in the intermediated asset markets, Adrian et al. (2014) and He et al. (2017) differ in their definitions of capital. He et al. (2017) include the capital of the bank holding companies to that of their US broker-dealers, whereas Adrian et al. (2014) do not. There is a controversy regarding whether the internal market within those banks allows for transferring the capital in one subsidiary to another.*

Regarding the uncertainty about the tightness of constraint m, footnote 10 in He and Krishnamurthy (2013) documents the concern about the correct value of this parameter in their calibration[5].

[5] *"The m in our calibration applies to the entire intermediation sector, and as is evident in Table 1, there is functional heterogeneity across the modes of intermediation. In particular, it is not obvious what the m for the mutual fund or pension fund sector should be, which may lead one to worry about our choice of m based solely on considering the leveraged sector".*

2.6 Equilibrium

Definition 3 *The equilibrium parameterized by a baseline value of* $(\hat{m}, \hat{g})$ *must satisfy the following conditions: it comprises price processes* $\{P_t\}$ *and* $\{r_t\}$*, decisions* $\{c_t, c_t^h, \alpha_t^I, \alpha_t^h\}$*, and the set of alternative beliefs* $\{\Xi_t\}$ *such that:*

1. *Given the price processes and beliefs, decisions solve the consumption-savings problems of the debt household, the risky asset households and the intermediaries;*

2. *Decisions satisfy the intermediation constraint;*

3. *The risky asset market clears*

$$\frac{\alpha_t^I(w_t + \alpha_t^h(1-\lambda)w_t^h}{P_t} = 1;$$

4. *The goods market clears;*

$$c_t + c_t^h = D_t(1+l);$$

5. *The set of alternative models* Ξ_t *must be consistent with the observed return volatility and risk-free rate, where* $\sigma_R(x_t, m, g)$ *and* $r(x_t, m, g)$ *must be implied by an equilibrium in the set of alternative economies parameterized by some* (m, g)*.*

In the following, I will focus on a stationary Markov equilibrium, where the aggregate state variables consist of the aggregate dividend D_t and the aggregate wealth share of intermediaries $\hat{x}_t$. As standard in any economy with CRRA agents where endowments follow a Geometric Brownian Motion, I conjecture that the equilibrium risky asset price is

$$P_t = D_t p(\hat{x}_t),$$

where $p(\hat{x}_t)$ is the price-dividend ratio of the risky asset. Additionally, I conjecture that the equilibrium expected excess return $\pi_{R,t}$, return volatility $\sigma_{R,t}$, and risk-free rate r_t are all functions of only $\hat{x}_t$. Moreover, the aggregate intermediaries' wealth share evolves as

$$dx_t = \mu_x(\hat{x}_t)dt + \sigma_x(\hat{x}_t)dZ_t.$$

3 Recursive Representation of Preferences and Decisions

This section outlines the Markovian decision problem, corresponding to the sequential formulation in subsection 2.5, by characterizing a set of structured models and a continuation value process over consumption streams. The set of structured models, denoted as Ξ_t, is defined in terms of alternative parameters and the current realizations of the Markov state variable, x_t. In the context of Markovian decision problems, the evolution of continuation values is described by a Hamilton-Jacobi-Bellman (HJB) equation.

3.1 Hamilton-Jacobi-Bellman Equation

The homothetic property of intermediaries' preferences implies their value function in the form:

$$J(w_{i,t}, Y(x_t)) = \frac{[w_{i,t}Y(x_t)]^{1-\gamma}}{1-\gamma},$$

where $Y(x_t)$ represents the future investment opportunity, evolving as

$$\frac{dY}{Y} = \mu_Y(x)dt + \sigma_Y(x)dZ.$$

Then the Hamilton-Jacobi-Bellman (HJB) equation of an intermediary i is formulated as:

$$\rho J_i = \max_{\alpha_i, c_i} \min_{S\in\Xi} \frac{c_i^{1-\gamma}}{1-\gamma} + (\partial_Y J_i)\mu_Y^S Y + (\partial_W J_i)(rw_i + \alpha_i \pi_R^S w_i - c_i) + \frac{1}{2}(Y\sigma_Y)^2(\partial_{YY} J_i)$$
$$+ \frac{1}{2}(\partial_{WW} J_i) w_i^2 (\alpha_i \sigma_R)^2 + w_i(\alpha_i\sigma_R)Y\sigma_Y(\partial_{WY} J_i),$$

where

$$\pi_R^S = \pi_R - \sigma_R S;$$

$$\mu_Y^S = \mu_Y - \sigma_Y S.$$

The differences between the subjective and true local drift are captured by S. In particular, it turns out that

$$\mu_Y - \mu_Y^S = (\partial_x Y)\hat{x}(\mu_x - \mu_x^S) = (\partial_x Y)\hat{x}(\hat{\alpha}^I - 1)(\pi_R - \pi_R^S),$$

where $\hat{\alpha}^I$ is the portfolio weight on the risky asset of the aggregate financial sector, which is greater than 1 in equilibrium. Therefore, alternative expected excess returns on the risky asset affect the individual utility not only through the accumulation of individual intermediaries' wealth $w_{i,t}$ but also the accumulation of aggregate intermediaries' wealth x_t. Further elaboration on the tradeoff of alternative higher expected excess returns is provided in subsection 4.3, detailing how it determines the worst-case parameter and state $(x^{worst}, m^{worst}, g^{worst})$.

3.2 Markovian Characterization of Set of Alternative Models

The Markov property of the equilibrium objects allow for convenient characterizations of the set of alternative models Ξ_t in terms of (x, m, g). I illustrate how individual intermediaries identify (m, x) by focusing a lower aggregate state where financial constraints are binding. Figure 1 plots return volatility as a function of the aggregate state x given a fixed m.

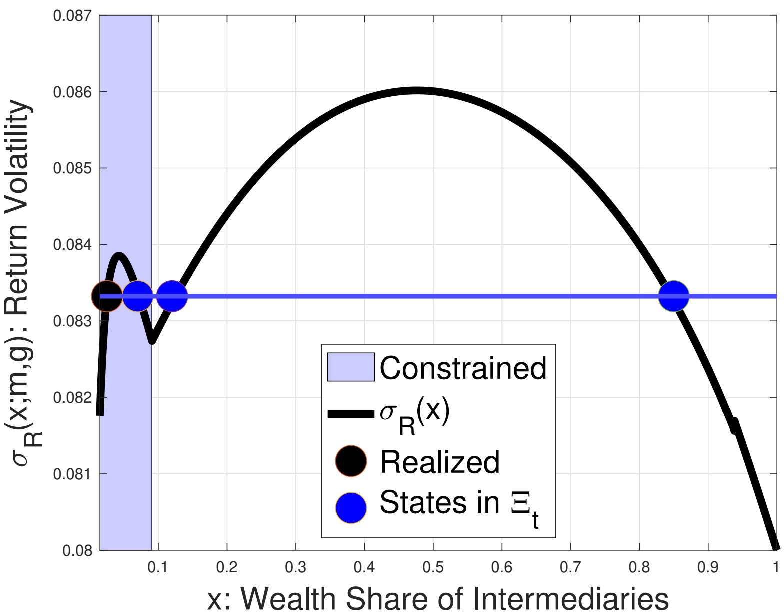

Figure 1: Return Volatility

The return volatility is graphed against $x = w/P$, intermediaries' aggregate wealth as a percentage of the assets in the economy. Parameters are from Table 1 and Table 2. Shaded blue area corresponds to the constrained region.

To interpret the shape of the return volatility, note that

$$\sigma_x = x \times vol_t\left(\frac{dw}{w} - \frac{dp}{p}\right) = \hat{x}(\hat{\alpha}^I - 1)\sigma_R,$$

where vol_t denotes the volatility component. On one hand, as the wealth share of intermediaries x declines, the intermediation sector starts to increase the leverage $\hat{\alpha}^I >> 1$, which increases the relative volatility of the sector's wealth growth to the aggregate wealth growth of the entire economy and therefore, the volatility of the wealth share. At the same time, as the wealth share of intermediaries decrease, the change of entire intermediaries' wealth level becomes relatively small compared to the entire economy, which decreases the volatility of the wealth share. More explicitly solving for σ_x, since $\sigma_R = vol_t(dp/p) = \frac{dp}{p}\sigma_x + \sigma$,

$$\sigma_x = \frac{\hat{x}(\hat{\alpha}^I - 1)\sigma}{1 - \hat{x}(\hat{\alpha}^I - 1)\frac{p'}{p}}.$$

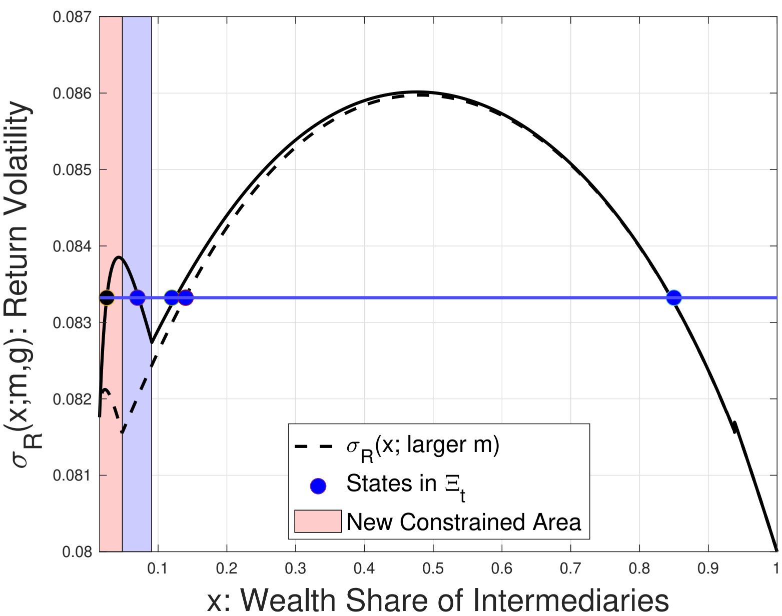

Figure 2: Return Volatility

The dashed curve represents the return volatility against $x = w/P$, the intermediaries' aggregate wealth as a percentage of the assets in the economy, when the underlying baseline value of $m = 2$. Other Parameters are from Table 1 and Table 2. Shaded blue area corresponds to the constrained region. See the note below Figure 1 for the actual return volatility in black.

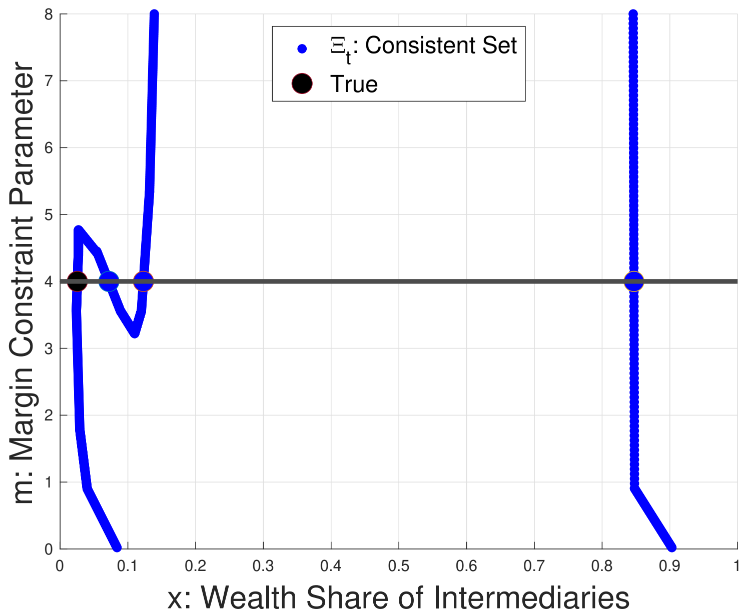

Figure 3: Set of Consistent (m, x)

The set of combinations (m, x) consistent with observable information is graphed in blue. The horizontal line is the aggregate wealth share of intermediaries and the vertical line is the margin constraint parameter. The black dot corresponds to the baseline. Each point in blue has the corresponding value of g in Figure 4.

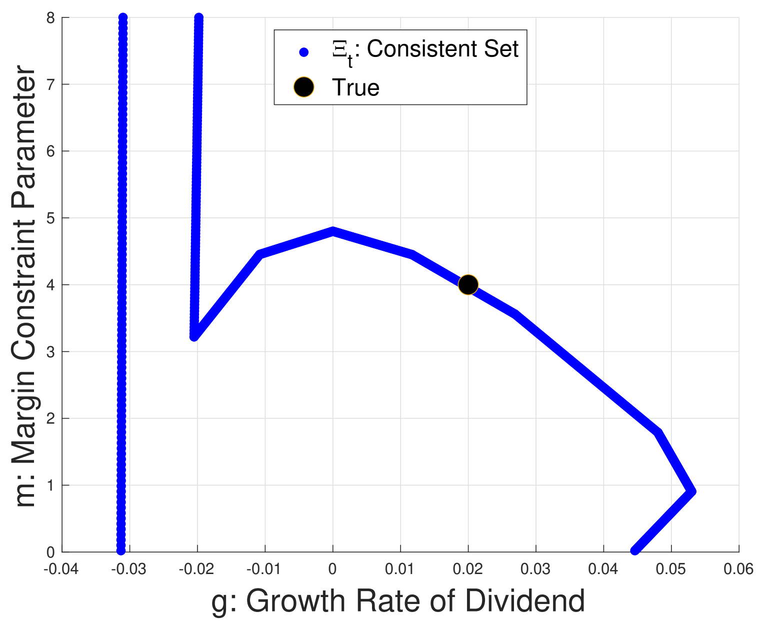

Figure 4: Set of Consistent (m, g)

The set of combinations (m, g) consistent with observable information is graphed in blue. The horizontal line is the dividend growth and the vertical line is the margin constraint parameter. The black dot corresponds to the baseline. Each point in blue has the corresponding value of g in Figure 3

.

The denominator of the right-hand side reflects the multiplier effects as in Brunnermeier and Sannikov (2014) and Di Tella (2017): the decline in the wealth share of intermediaries decrease the asset price due to their higher effective risk aversion and it further decreases the wealth share. Starting from $\hat{x} = 1$, as the wealth share of intermediaries declines, the first effect dominates the second effect until $\hat{x} \approx 0.5$ and return volatility increases, while the second effect starts to dominate below $\hat{x} \approx 0.5$ and return volatility decreases. Without financial constraints, the volatility of wealth share $\hat{x}$ would converge to zero and return volatility to the fundamental volatility σ as $\hat{x} \downarrow 0$.

When the economy starts to be constrained, the first effect starts to dominate the second effect again since the leverage of the intermediation sector is forced to rapidly increase. As further x goes down to 0, the second effect starts to dominate eventually.

Intermediaries find alternative three additional aggregate state x's in blue circles consistent with the observed return volatility, in addition to the true $\hat{x}_t$ in the black circle. Without observing the current aggregate capitalization of the financial sector $\hat{x}_t$, individual intermediaries view those alternative capitalizations as possibly plausible since they all rationalize available information.

Now intermediaries also contemplate an alternative value of the margin constraint parameter m. In Figure 2, the dashed line is the equilibrium return volatility in an alternative economy with the different m. Intermediaries find six combinations of (m, x) consistent with the observed return volatility. Contemplating all combinations of (m, x, g) consistent with observed return volatility and risk-free rate, Figure 3 and Figure 4 plot all such combinations of (m, x, g). Since all of these combinations are consistent with observable information, they constitute the set of alternative models Ξ_t. Each individual intermediary regards all the elements in this set as equally plausible.

4 Characterization of Beliefs and Decisions

4.1 Equilibrium Risk Premium

In this section, the dynamics of the risk premium ($\pi_{R,t}$) are analyzed first under the assumption of logarithmic utility for intermediaries and in the absence of ambiguity. The risk premium represents the additional return required by investors for holding risky assets, and it is determined by the Euler equation of intermediaries:

$$\pi_{R,t} = Cov_t\left[\frac{dw_t}{w_t}, dR_t\right].$$

This equation states that the risk premium is the covariance between the growth rate of intermediation sector's capital (dw_t/w_t)and the risky asset returns (dR_t). Equivalently, it can be expressed as the product of the intermediary's exposure to the risky asset (α_t^I) and the variance of returns ($\sigma_{R,t}^2$):

$$\pi_{R,t} = \alpha_t^I \sigma_{R,t}^2.$$

The risk premium is influenced by two main factors: the intermediary's exposure to the risky asset (α_t^I) and the variance of returns ($\sigma_{R,t}$). The analysis primarily focuses on the exposure term (α_t^I) since it is the main determinant of the risk premia in the calibration below.

Consider a scenario where the margin constraint binds, leading to entire intermediaries raising total funds of $(1 + \hat{m})w_t$. This situation occurs when either the aggregate wealth share of intermediaries is low or the margin constraint is tight, represented by the condition $\hat{x}_t \leq x^c(\hat{m})$. Here, $x^c(\hat{m})$ denotes the critical wealth share below which the constraint binds and is given by:

$$x^c(\hat{m}) = \frac{1-\lambda}{1-\lambda+\hat{m}}$$

In this constrained region, all risky assets are held through the intermediary. The equilibrium market clearing condition in this region implies:

$$\hat{\alpha}_t^{I,const}(w_t + mw_t) = P_t.$$

which, rearranged, gives the intermediary's exposure α_t^I in the constrained region as:

$$\hat{\alpha}_t^I = \frac{1}{\hat{x}_t}\frac{1}{1+\hat{m}},$$

where $\hat{\alpha}_t^I$ emphasizes the dependence on the baseline value of m. As intermediaries' total capitalization $\hat{x}_t$ decreases within the constrained region, the risk premium rises. Moreover, when the parameter m is larger (indicating that the intermediaries can raise more equity capital from households for a given amount of their own equity stake), the effect of decreasing intermediaries' capitalization on the risk premium is dampened.

This analysis highlights the intricate relationship between margin constraints, the intermediary's exposure to risky assets, and the resulting risk premium. It provides insights into how changes in the intermediaries' capitalization and margin constraint parameter m predicts the future asset returns in the markets.

In the unconstrained region, where the margin constraint does not bind, the total funds of the intermediary sector is the sum of the intermediaries' wealth and the risky asset household's equity contribution to the intermediary sector. The market clearing condition for the risky asset in this region is given by:

$$\alpha_t^{I,unconst}(w_t + (1-\lambda)w_t^h\alpha_t^h) = P_t.$$

Here, α_t^h represents the risky asset household's share of wealth invested in the intermediaries, and $\alpha_t^{I,unconst}$ is the intermediary's exposure to the risky asset in the unconstrained region. In the calibration, it is assumed that the risky asset household chooses to invest 100% of their wealth in the intermediaries when there are no binding margin constraints ($\alpha_t^h =$

1). Consequently, the intermediary's exposure in the unconstrained region ($\alpha_t^{I,unconst}$) is calculated as:

$$\alpha_t^{I,unconst} = \frac{1}{1 - \lambda(1 - \hat{x}_t)}.$$

In the scenario where $\lambda = 0$ (indicating the absence of debt households in the economy), $\alpha_t^{I,unconst}$ is constant and equal to one. This implies that the risk premium in the unconstrained region remains constant over time.

In contrast to the unconstrained region, as demonstrated earlier, the risk premium increases in the constrained region when the intermediaries' total funds fall due to binding margin constraints. This asymmetry in the response of risk premia to changes in intermediaries' capitalization is a key characteristic of the model and is a central feature of the analysis, reflecting the intricate interplay between margin constraints, leverage effects, and risk premia in the intermediated asset markets.

For the case where $\lambda > 0$, which we consider in the calibration, the risk premium also rises in the unconstrained region because of a leverage effect. In the calibration, however, this effect in the unconstrained region is small compared to the constrained region.

Now the concept of intermediaries' uncertainty regarding the baseline values of (x, m, g) is introduced, along with an exploration of how they evaluate alternative models of expected excess returns in light of this uncertainty. Intermediaries consider alternative values of (x, m, g), denoted as (x_t, m_t, g_t), within the set of possible models Ξ_t. Based on these alternative values, intermediaries calculate the expected excess return, denoted as $\pi_{R,t}^S(x_t, m_t, g_t)$, taking into account both their exposure to the risky asset and the compensation for parameter and state uncertainty. In equilibrium, the expected excess return under the true DGP is formulated as

$$\pi_{R,t}(\hat{x}_t, \hat{m}, \hat{g}) = \hat{\alpha}_t^I \sigma_{R,t}^2 + UP_t,$$

where $\alpha_t^I = \alpha_t^{I,unconst}$ in the unconstrained region and $\alpha_t^I = \alpha_t^{I,const}$ in the constrained region. UP_t is the equilibrium compensation for parameter and state uncertainty known to intermediaries, whose definition I will provide in (6). Each individual intermediary contemplates alternative models of expected excess return by exploring alternative values of $(x, m, g) \in \Xi_t$:

$$\pi^S_{R,t}(x_t, m_t, g_t) = \alpha^I(x_t, m_t)\sigma^2_{R,t} + UP_t,$$

where if $x \le x^c(m)$

$$\alpha^I_t = \frac{1}{x}\frac{1}{1+m},$$

otherwise

$$\alpha^I_t = \frac{1}{1-\hat{\lambda}(1-x)}.$$

Considering a general CRRA utility case, the expected excess return under the true DGP is formulated as

$$\hat{\pi}_{R,t} = \gamma\alpha^I_t(\hat{x}_t, \hat{m})\sigma^2_R + (\gamma-1)\sigma_R\sigma_Y(\hat{x}_t, \hat{m}, \hat{g}) + UP_t.$$

Here, the first term on the right-hand side represents the compensation for the exposure to the underlying aggregate shock. It quantifies the additional exposure to risk (quantity of risk) denoted by $\sigma_{R,t}$, multiplied by the price of risk, which is proportional to the volatility of the aggregate wealth of intermediaries. The second term signifies compensation for the intertemporal hedging motive, where σ_Y denotes the volatility of investment opportunities of the aggregate intermediary sector. The third term reflects compensation for the uncertainty associated with both the state and the parameters.

Then each individual intermediary confronts alternative models for the expected excess return by contemplate alternative values of $(m, x, g) \in \Xi_t$:

$$\pi^S_{R,t} = \gamma \cdot \alpha^I_t(x, m)\sigma^2_R + (\gamma-1)\sigma_R\sigma_Y(x, m, g) + UP_t,$$

In this equation, the parameter and state uncertainty primarily affects the first term on the right-hand side in the context of the calibration described below.

The compensation for the parameter and state uncertainty due to the partial identification problem, UP_t is defined as the difference of the expected excess returns under the true and the equilibrium worst-case DGP:

$$UP_t = \hat{\pi}_{R,t} - \pi^S_{R,t}(x_t^{worst}, m_t^{worst}, g_t^{worst}), \tag{6}$$

where $(x^{worst}, m^{worst}, g^{worst})$ is the equilibrium worst-case parameter which is the solution to the minimization problem in equilibrium. Henceforce, UP_t is referred to as the "price of partial identification".

4.2 Equilibrium Computation

The iterative procedure for solving equilibria involves solving a sequence of ordinary differential equations for the equilibrium price-dividend ratios $p(x)$ in various economies with alternative baseline parameter values $(\hat{m}, \hat{g})$. In these economies, intermediaries face uncertainty regarding parameter and state realizations, leading them to contemplate diverse stochastic processes for asset returns. Algorithm 1 displays the algorithm to find the equilibria. The computational details of this procedure are outlined in the Appendix.

The challenge lies in simultaneously solving for the price-dividend ratio and the local mean belief distortion S, ensuring the following conditions:

- In an economy with a baseline parameter value $(\hat{m}, \hat{g})$, intermediaries construct the set of alternative beliefs Ξ_t using equilibrium return volatilities and risk-free rates from alternative economies with alternative baseline $(\hat{m}, \hat{g})$ values.

- Given the worst-case beliefs, the economy's equilibrium must determine the set of prices and allocations.

The iterative solution process begins by gradually expanding the set of alternative models. Initially, the equilibrium price-dividend ratios are assumed to follow rational expectations

Algorithm 1: Fixed-Point Algorithm

Data: Guess for $\sigma_R(x,m,g)$ and $r(x,m,g)$, $x \in [0,1]$, $m \in (0,\overline{m})$, $g \in (\underline{g},\overline{g})$
Result: Equilibrium $\sigma_R(x,m,g)$ and $r(x,m,g)$
Initialization;
Set $n=1$ and $\sigma_R^{(0)}(x,m,g) = \sigma_R^{REE}(x;m,g)$ and $r^{(0)}(x,m,g) = r^{REE}(x;m,g)$
while do
  for $(g_i,m_i) \in (\underline{g},\overline{g}) \times (0,\overline{m})$ **do**
    Compute a competitive equilibrium where

- intermediaries form a set of beliefs $\{\Xi_t\}$ using $\sigma_R^{(n-1)}(x,m,g)$ and $r^{(n-1)}(x,m,g)$.
- (g_i,m_i) is true (baseline) parameter value in this equilbrium.

  end
  $\Rightarrow \{\sigma_R^{(n+1)}(x,m,g)\},$, $x \in [0,1]$, $m \in (0,\overline{m})$, $g \in (\underline{g},\overline{g})$
  if $\max_{(x,m,g)\in[0,1]\times(0,\overline{m})\times(\underline{g},\overline{g})} |\sigma_R^{(n+1)}(x,m,g) - \sigma_R^{(n)}(x,m,g)| + |r^{(n+1)}(x,m,g) - r^{(n)}(x,m,g)| < \epsilon$ **then**
    break;
  end
  Set $n \Rightarrow n+1$
end

where the set of alternative models contains only a baseline model, restricting $S=0$. Utilizing equilibrium solutions of price-dividend ratios from previous iterations, intermediaries form the set of alternative models Ξ_t for the excess return process. In the current iteration, the set Ξ_t is expanded in terms of the magnitude of S, and the worst-case belief and price-dividend ratios are computed. This iterative process continues until the equilibrium price-dividends converge.

In equilibrium, the fraction of intermediaries' capitalization $\hat{x}_t$ ranges between 0 and 1. As $\hat{x}_t$ approaches zero, the economy is predominantly comprised of households. In this case, the boundary condition is as follows. Considering the intermediary's consumption from the goods market clearing condition:

$$c_t = D_t(1+l) - c_t^h = D_t(1+l) - \rho(1-\hat{x}_t)P_t = D_t[(1+l) - \rho(1-\hat{x}_t)p(\hat{x}_t)],$$

where the second equality arises from the myopic decision rule of households $c_t^h = \rho w_t^h$.

Consequently, as $\hat{x}_t \to 0$:

$$p(0) = \frac{1+l}{\rho}.$$

Conversely, when $\hat{x}_t \to 1$, the economy behaves as if solely comprised of intermediaries. In this scenario, the ordinary differential equation in the limit $\hat{x}_t \to 1$ determines $p'(1)$ given the worst-case belief distortion at $x_t \to 1$, denoted as $S(1)$. The boundary condition for $S(1)$ is enforced by solving the minimization problem for intermediaries using equilibrium parameters at $x_t \to 1$ from the preceding iteration.

4.3 Characterizations of Worst-Case Model

In the minimization problem, the objective is to choose the expected excess return consistent with the parameterization in Ξ_t. Specifically[6],

$$\min_{\pi_R^S} \underbrace{(\partial_Y J_i)(\partial_x Y)\hat{x}(\hat{\alpha}^I - 1)}_{<0} \pi_R^S + \underbrace{(\partial_W J_i)\alpha_i^I}_{>0} \pi_R^S,$$

This minimization is subject to

$$\pi_R^S \in \{\pi_R - \sigma_R S : S(m, x, g) \in \Xi_t\}.$$

A higher expected return has two opposing effects on individual lifetime utility. On one hand, today's higher expected return improves the individual utility by speeding up the accumulation of the individual wealth. On the other hand, it also accelerates the accumulation of the entire financial sector, x_t. Then the excess returns tomorrow and thereafter are expected to be low since the financial sector accumulates wealth and reduce the leverage, which reduce future risk premia.

[6]The dependence of α_i^I and c_i on S is eliminated due to the envelope condition.

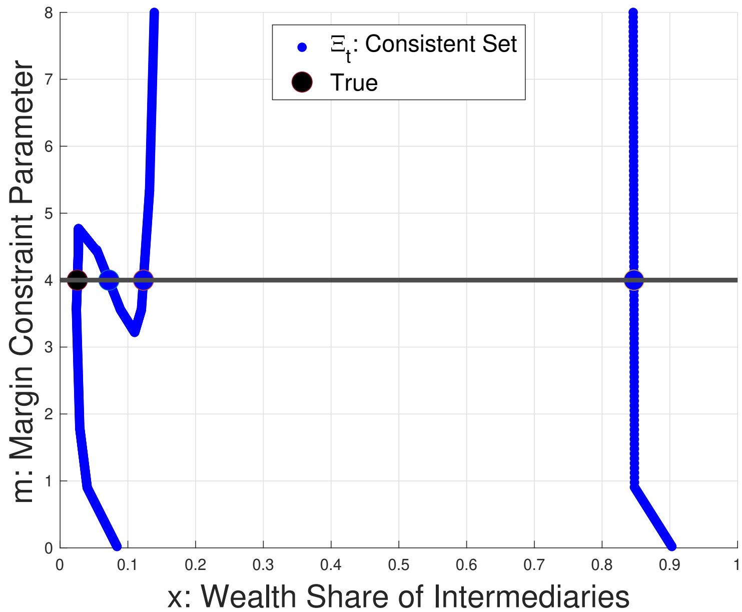

Figure 5: The worst-case combination and set of consistent (m, x)

The worst-case combination of (m, x) is in red. The set of combinations (m, x) consistent with observable information is graphed in blue. The horizontal line represents the aggregate wealth share of aggregate intermediaries and the vertical line is the margin constraint parameter. The black dot corresponds to the baseline. Each point in blue has the corresponding value of g in Figure 6

.

In equilibrium, intermediaries fear scenarios where the expected excess return is low, and their individual wealth growth is slow. In Figure 5, the worst-case combination of (m, x) in red is characterized by a higher aggregate wealth share than the actual, which reduces the leverage of the whole intermediation sector and hence risk premium, predicting a lower future excess return.

The higher capitalization of the unconstrained sector is still consistent with higher observed return volatility since a higher capitalization makes the financial sector a more significant player, significantly fluctuating asset prices even with moderate leverage, indicating high return volatility.

Figure 6 plots the implied g consistent with observed risk-free rate. The worst-case expected growth rate of dividend is the lowest among alternatives. The lowest leverage

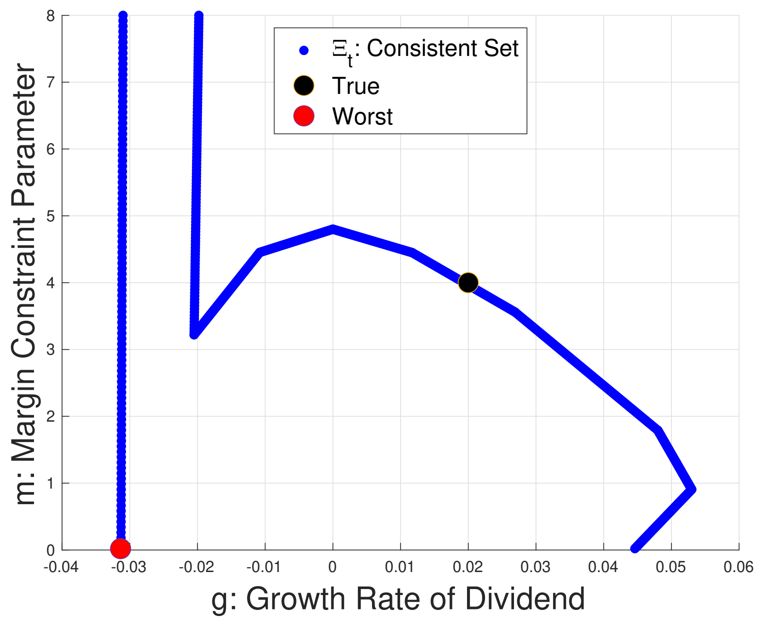

Figure 6: The worst-case combination and set of consistent (m, g)

The worst-case combination of (m, g) is in red. The set of combinations (m, g) consistent with observable information is graphed in blue. The horizontal line is the dividend growth and the vertical line is the margin constraint parameter. The black dot corresponds to the baseline. Each point in blue has the corresponding value of g in Figure 5

implied by the worst-case (m, x) implies the lowest precautionary saving motive by the aggregate financial sector, which increases the implied risk-free rate. This is inconsistent with the observed lower risk-free rate driven by the actually higher saving motive due to the lower worst-case expected return on the risky asset. To be consistent, the expected growth rate of the aggregate dividend must be the lowest, which induces the economy to grow slowly and reduce the implied risk-free rate.

4.4 Characterizations of Consumption-Saving and Portfolio Choice

Given the subjective expected return $\pi_R^S(x_t^{worst}, m_t^{worst}, g_t^{worst})$, which is the expected excess return under the worst-case alternative model, the maximization problem yields a standard

Parameter	Description	Value	Target	Target value	Model
γ	Relative Risk Aversion	1.8	Average expected excess return of MBS	3.4	3.8
ρ	Discount Rate	0.08	Average risk-free rate	1	0.78
σ	Dividend Volatility	0.08	Return volatility of MBS	0.81	0.83
λ	Debt Household Share	0.6	Average Debt-to-Asset ratio in 2007	0.52	0.55
l	Labor Income Ratio	1.84	Share of Labor Income in Total Income	0.66	0.64

Table 1: Matched Moments and Internally Calibrated Parameters

Parameter	Description	Value	Source
m	Intermediation multiplier	4	HK (Share of managers compensation in intermediarys' profit)
g	Dividend Growth	2%	HK (Average real output growth in the U.S.)

Table 2: Fixed Parameters

consumption and portfolio choice rule:

$$c_{i,t} = \rho^{\frac{1}{\gamma}} Y^{\frac{\gamma-1}{\gamma}} \times w_{i,t};$$

$$\alpha^I_{i,t} = \underbrace{\frac{1}{\gamma} \frac{\pi^S_R(x^{worst}_t, m^{worst}_t, g^{worst}_t)}{\sigma^2_{R,t}}}_{Myopic} + \underbrace{\frac{1-\gamma}{\gamma} \sigma_{R,t}\sigma_{Y,t}}_{Hedging\ motive}.$$

5 Calibration and Verification of Amount of Uncertainty

In the calibration shown in Table 1, the model targets the same moments as in HK. The parameters of relative risk aversion and dividend growth volatility $(\gamma, \sigma) = (1.8, 0.08)$ are calibrated to match the model-implied average excess return (3.8%) and return volatility (0.78%) of mortgage-backed securities with the empirical moments (3.4% and 0.81% respectively). The discount rate $\rho = 0.08$ is set to match the average risk-free rate (1%), yielding the model-implied value of 0.78%. The fraction of households $\lambda = 0.6$ is chosen to align with the model-implied average debt-assets ratio of the intermediary sector (0.55) with that of the financial sector in 2007 (0.52). The value $l = 1.84$ is based on the share of labor income to total income for the United States (66%). The model generates an average labor income share of 64%.

For fixed parameters in Table 2, the values for the intermediation multiplier $m = 4$ and dividend growth rate $g = 0.02$ are taken from HK. The choice of $m = 4$ aligns with the compensation of financial managers in the intermediary sector, and $g = 0.02$ corresponds to the average per-capita growth rate of the U.S. GDP.

In order to quantify the plausibility of the worst-case parametric alternative, I employ detection error probabilities as proposed by Anderson et al. (2003). Consider a random sample of independent draws indexed by i of time-series data of excess returns on the risky asset $\{dR_t^B - r_t^B dt\}_{t,i}$ drawn from the baseline distribution parameterized by the baseline values $f(\{dR_t - r_t dt\}_{t,i}; \hat{m}, \hat{g}, \{\hat{x}_t\}_t)$, and a sample $\{dR_t^A - r_t^A dt\}_{t,j}$ indexed by j drawn from the alternative distribution $\tilde{f}(\{dR_t - r_t\}_{t,j}; \{m_t^{worst}\}, \{g_t^{worst}\}, \{x_t^{worst}\}_t)$ determined as the worst-case distribution under worst-case parameters. I evaluate the probability that the random sample is assigned a higher likelihood under the alternative distribution than under the correct benchmark distribution:

$$P\left(\sum_{i=1}^{I} \log \tilde{f}(\{dR_t^B - r_t^B dt\}_{t,i}) > \sum_{i=1}^{I} \log f(\{dR_t^B - r_t^B dt\}_{t,i})\right) = P\left(\sum_{i=1}^{I} \log m(\{dR_t^B - r_t^B dt\}_{t,i}) > 0\right). \tag{7}$$

Here, I is the number of samples. Conversely, the probability that the random sample $\{dR_t^A - r_t^A dt\}_{t,j}$ is assigned a higher likelihood under the benchmark distribution than under the correct alternative distribution is given by,

$$P\left(\sum_{i=j}^{I} \log f(\{dR_t^A - r_t^A dt\}_{t,j}) > \sum_{j=1}^{I} \log \tilde{f}(\{dR_t^A - r_t^A dt\}_{t,j})\right) = P\left(\sum_{i=1}^{I} \log m(\{dR_t^A - r_t^A dt\}_{t,j}) < 0\right). \tag{8}$$

The detection error probability is then defined as the average of the two probabilities above,

$$d(I) = \frac{1}{2}\left(P\left(\sum_{j=1}^{I} \log m(\{dR_t^A - r_t^A dt\}_{t,j}) < 0\right) + P\left(\sum_{i=1}^{I} \log m(\{dR_t^B - r_t^B dt\}_{t,i}) > 0\right)\right) \tag{9}$$

The detection error probability expresses the chance that the likelihood ratio leads to the erroneous conclusion about which of the two distributions generated the random sample. The construction implies that $0 \leq d(I) \leq 1/2$, achieving the upper bound when $f(\{dR_t - r_t dt\}_t)$ and $\tilde{f}(\{dR_t - r_t dt\})$ are identical. The detection error probability is also decreasing in the sample size I, as long as f and $\tilde{f}$ are statistically distinguishable.

When $I = 600$ months, which implies 50 years, the detection error probability is 32%, exceeding the commonly accepted threshold of 20% in the literature. Therefore, the worst-case model is statistically challenging for intermediaries to distinguish from the true DGP and the worst-case concern held by intermediaries is admissible.

6 Results

Figure 7 illustrates the worst-case scenarios for mean dividend growth, margin constraint parameters, and intermediaries' capitalization (g, m, x) in relation to the actual states of the economy. In the worst-case scenarios, the parameter m consistently approaches zero, suggesting minimal intermediation perceived by each individual intermediary. Furthermore, the worst-case capitalization x exceeds the actual values, with a notable discrepancy, especially in low capitalization situations when margin constraints are active. Despite the high leverage in the financial sector during constrained periods, individual intermediaries perceive the aggregate leverage as low in the worst-case scenario. Consequently, the sector's precautionary saving motive remains low in their perception, leading to a higher risk-free rate implied by the worst-case model compared to the actual values. To align with these observed tendencies, the worst-case value for g needs to be significantly lower than the actual value, especially in constrained regions.

In Figure 8, equilibrium returns are plotted as functions of actual state realizations. The objective risk premium, representing the expected excess return under the true DGP, is

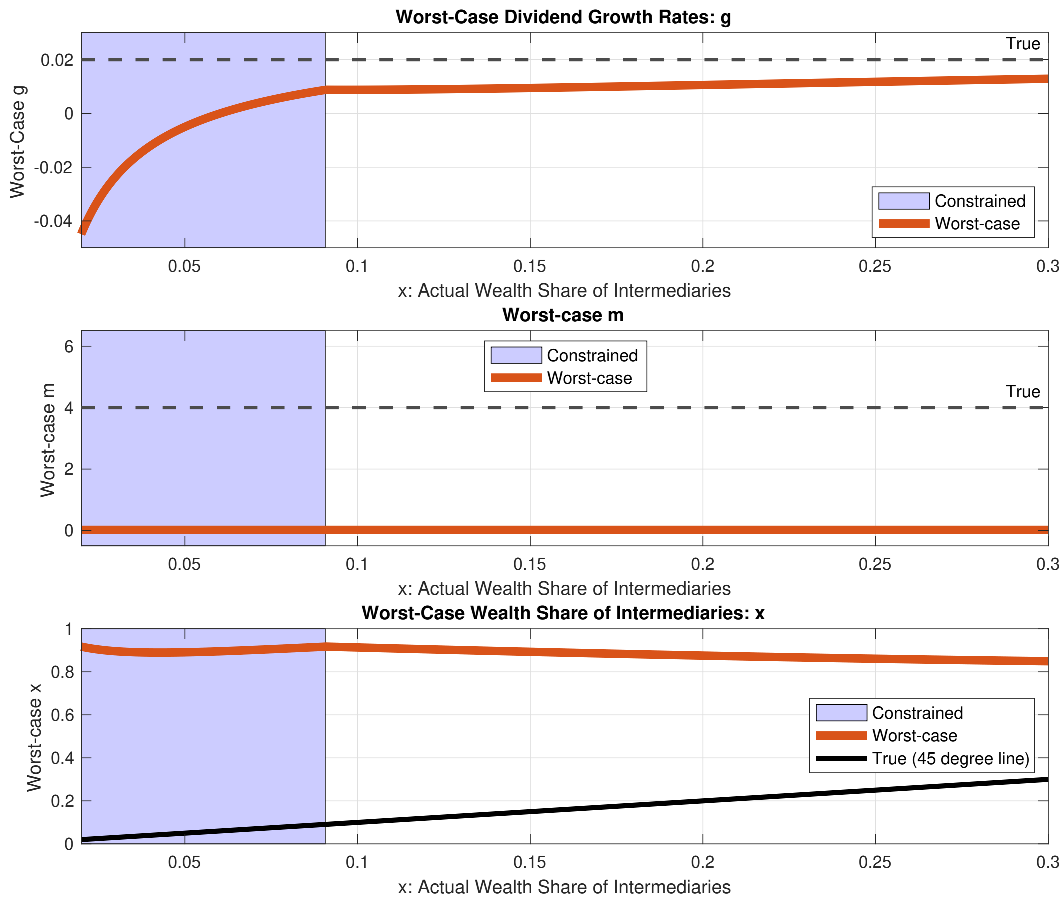

Figure 7: Worst-case parameters and state

The worst-case dividend growth rate, margin constraint parameter, and the aggregate wealth share of intermediaries are graphed in red against the actual realizations of the aggregate state. The flat dashed lines in the upper and middle panel corresponds to the baseline values. The solid black line is a 45-degree line, corresponding to the baseline values of the aggregate state.

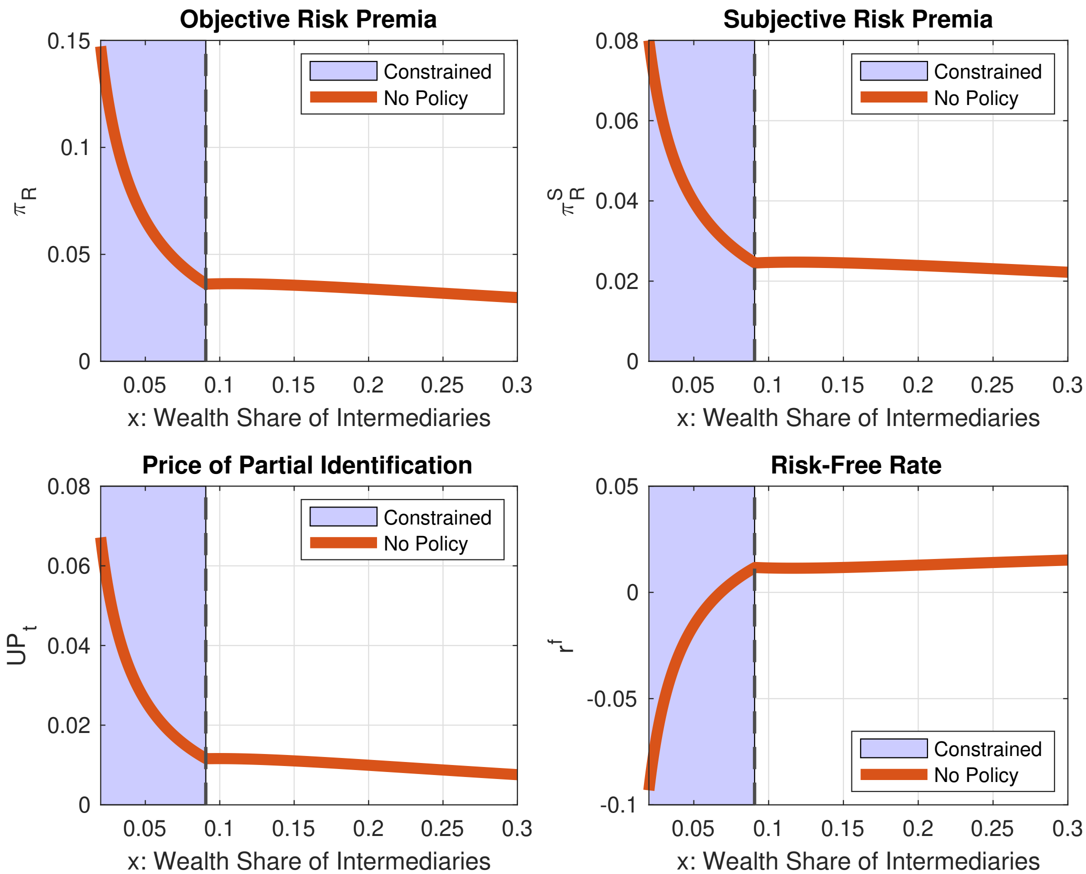

Figure 8: Equilibrium Returns as functions of actual state realization

Components of risk premium and risk-free rate are graphed against actual realizations of $x = w/P$. The objective risk premium is the expected excess return under the true DGP, while the subjective risk premium is that under the worst-case model. The price of partial identification is the difference of the compensation for the actual and worst-case leverage of the intermediation sector.

amplified in the constrained region. This objective risk premium comprises the subjective risk premium and compensation for parameter and state uncertainty, known as the price of partial identification. Both components contribute to amplifying the objective risk premium in the constrained region.

The subjective risk premium represents the expected excess return under the intermediaries' worst-case model. When each individual intermediary faces higher leverage, it reduces its demand for the risky asset. Consequently, the current asset price decreases, causing the subjective risk premium to rise in equilibrium. This adjustment in the risk premium restores market clearing by stimulating demand for the risky asset. This concept of risk compensation parallels the risk premium discussed in the rational expectations equilibrium presented in He and Krishnamurthy (2013).

The price of partial identification amplifies in the constrained region. Intermediaries perceive the aggregate leverage to be lower than it actually is. Consequently, the actual compensation for this leverage is significantly higher than what they perceive. This disparity is reflected in the elevated price of partial identification in the constrained region.

The risk-free rate becomes lower in the constrained region due to two reasons. First, individual intermediaries are exposed to higher leverage, which causes the higher precautionary saving motive. Second, the lower subjective expected excess return under the worst-case model induces intermediaries to save more to smooth consumption intertemporally. This is the precautionary saving motive due to the uncertainty from the partial identification problem, or structured ambiguity.

Notably, the price of partial identification accounts for approximately 40% of the objective risk premia under the plausible value of the detection error probability. Moreover, the structured ambiguity amplifies the precautionary saving motive in the constrained region. This finding calls for government policies during financial crises to mitigate the structured ambiguity. While He and Krishnamurthy (2013) explored several government interventions in the rational expectations equilibrium, those policies aimed to increase the financial sec-

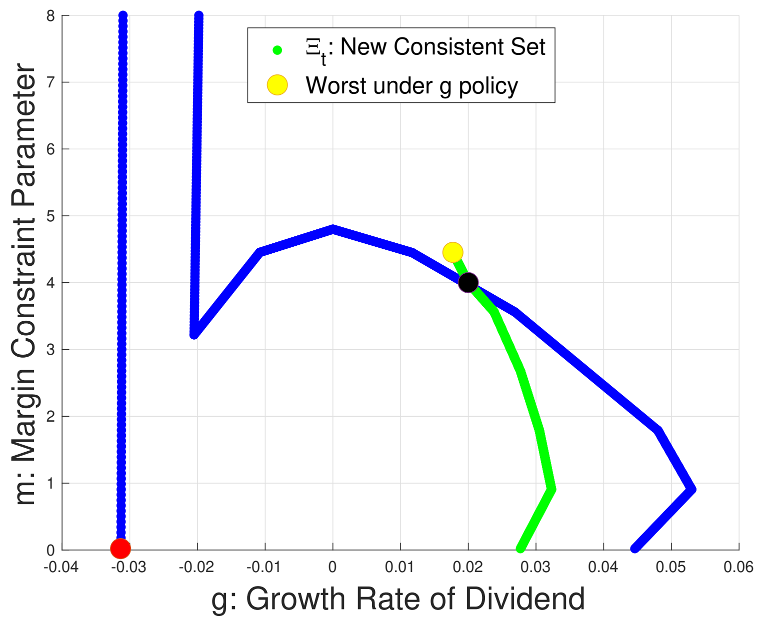

Figure 9: Set of (m, g) after g policy intervention

The set of combinations (m, g) under g policy in green and without policies in blue are graphed. The worst-case combinations are plotted in yellow under g policy and in red without policies. The baseline combination is graphed as a black dot.

tor's capitalization and do not directly speak to how policies can resolve the heightened uncertainty embedded in the price of partial identification and high objective risk premia. My framework provides a laboratory to examine the effects of government announcements on the intermediated asset markets.

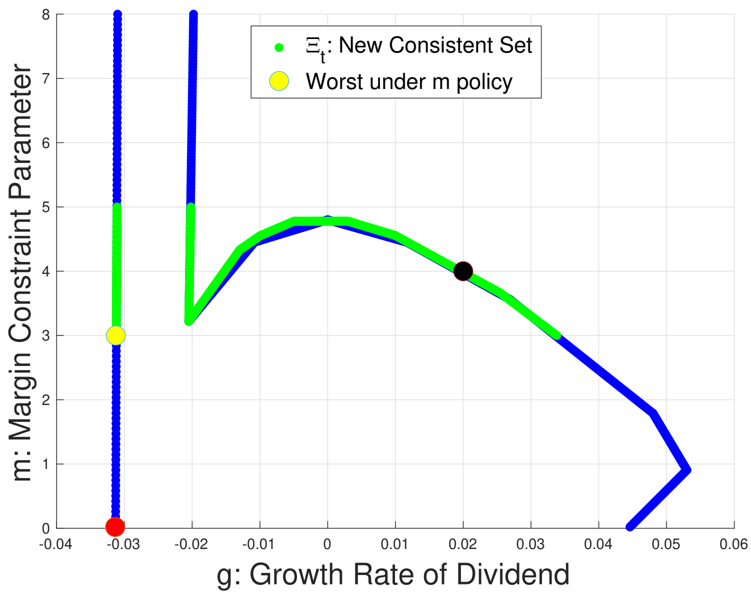

Figure 10: Set of (m, g) after m policy intervention

The set of combinations (m, g) under m policy in green and without policies in blue are graphed. The worst-case combinations are plotted in yellow under m policy and in red without policies. The baseline combination is graphed as a black dot.

7 Belief Management Policies

7.1 Policy Experiments during Crises

I examine the efficacy of the two policies aimed at resolving uncertainty about the mean dividend growth rate g (referred to as the g policy) and the margin constraint parameter m (referred to as the m policy). These policies narrow down the set of parameters and states. g policy restricts g in Ξ_t to be greater than 1%, similar to guaranteeing the cash flow from the intermediated risky asset. m policy corresponds to the deposit insurance and capital requirement, which impose constraints that $3 < m$ and $m < 5$ in Ξ_t, respectively, narrowing down the potential perceived range of tightness of constraints m.

In line with policy experiments in HK, the success of these policies are measured in terms of how they mitigate the heighted risk premia during financial crises when the financial sector is less capitalized and the constraint is binding.

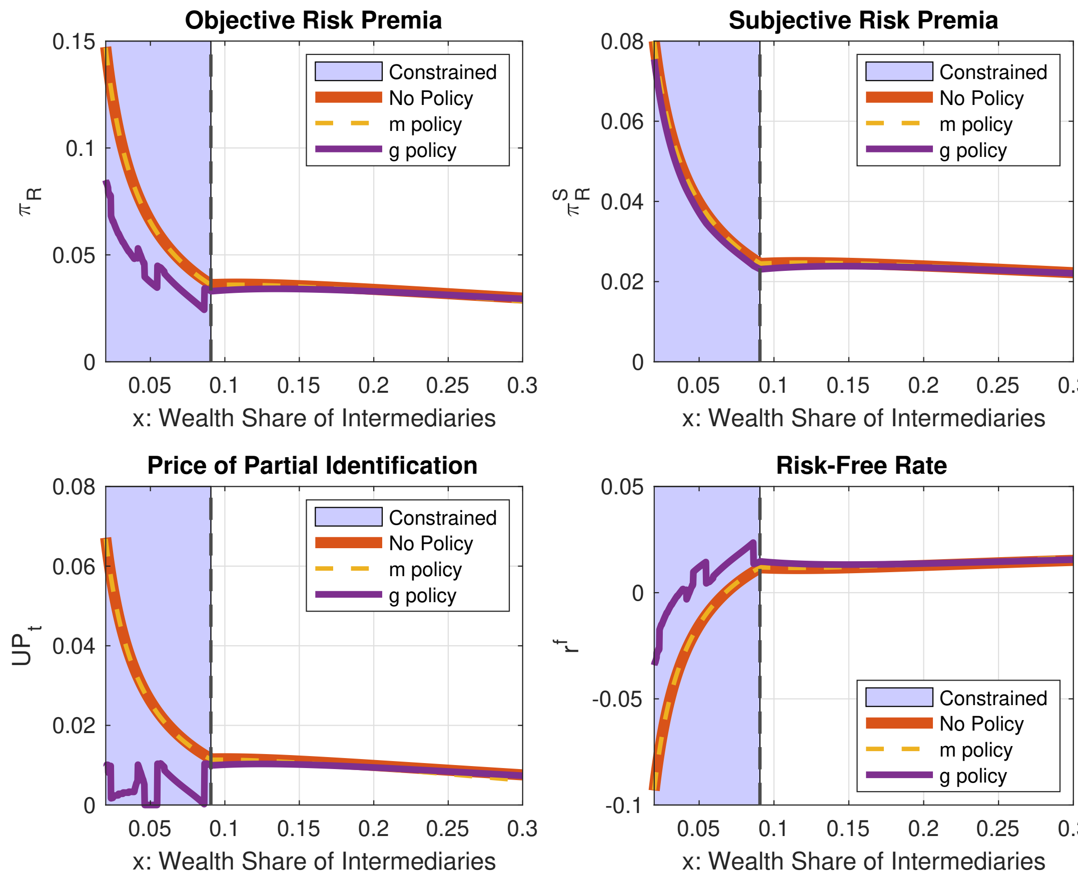

Figure 11: Equilibrium Returns under Policy Announcements

See the note of Figure 8 for the detailed description of the graph. The equilibrium objects under m policy (dashed yellow lines) overlap those without policy announcements (solid orange lines). The purple lines plot the equilibrium returns under g policy announcement.

At the beginning of period 0, the state is characterized by a risk premium of 12% before any announcements are made. Agents make their optimal decisions without prior knowledge of these announcements. Then the government announcements are unexpectedly made in period 0. The asset prices and the wealth share of intermediaries experience an immediate jump. These announcements alter the entire equilibrium dynamics.

The g policy is effective at mitigating the heightened risk premia. Figure 9 illustrates the responses of the (m, g) set to the government's announcement of the g policy. The new worst-case belief, depicted in yellow, aligns much more closely with the actual scenario shown

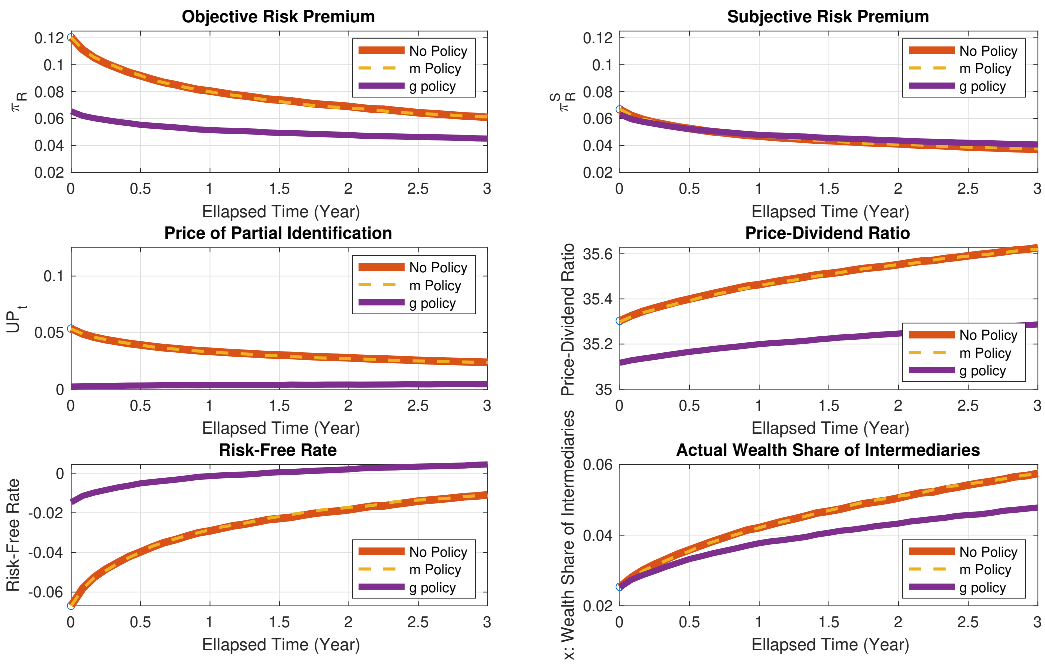

Figure 12: Mean Transition Dynamics

The mean transition dynamics with and without policy announcements are graphed against elapsed time (year). See the note of Figure 8 for the description of plotted objects. The mean transition paths are computed as the average of 5000 stochastic simulations starting the time 0 from the state with objective risk premium 12% in equilibrium without policy announcements. Once announcements are made at time 0, the entire equilibrium dynamics immediately jump to the new equilibrium. The asset prices and wealth share jump but the portfolio holdings made in equilibrium without announcements are fixed at time 0 after announcements.

Transit to	10 (%)	7.5	6.0	5.0
No policy	0.45	1.38	2.68	4.7
m Policy	0.44	1.4	2.76	4.62
g Policy	0	0	0.52	1.11

Table 3: Mean Transit Time (Year)

Table 4: See Figure 12 for the construction of the mean transit time. This table shows how many years it take the economy to hit the states with specific levels of the objective risk premia in the first row.

in black. This adjustment occurs because the g policy eliminates the possibility of the worst-case belief characterized by overly pessimistic cash-flow growth consistent with observable information and the structure of an alternative economy. Each individual intermediary conjectures the observed lower risk-free rate arises not due to the lower cash-flow growth but higher precautionary saving motive by the entire financial sector exposed to a higher leverage.

Moreover, individual intermediaries then conclude that expected excess returns must be high, which increases their demand for the risky asset. In the new equilibrium, this force reduces the price of partial identification and the risk premium under the true DGP.

On the other hand, the m policy is not so effective in this model. Figure 10 displays the response of the set of beliefs concerning (m, g) to the m policy announcement. In this scenario, the new worst-case g remains close to the previous value and significantly lower than the actual value. This suggests that the financial sector's perceived precautionary saving motive by each individual intermediary remains relatively low, and the aggregate sector leverage is perceived as low in the worst case. This policy prediction arises because as illustrated in section 6, even without uncertainty about m, a higher capitalization of the unconstrained financial sector is still consistent with the observed higher return volatility. Individual intermediaries are concerned about the possibility of the lower aggregate leverage and lower profitable opportunity in the market, which depresses their appetite for the risky asset.

Figure 11 displays the equilibrium returns as functions of the actual realizations of the aggregate state, similar to Figure 8. While the m policy does not succeed in reducing the risk premia, the g policy effectively mitigates the heightened risk premia in the constrained region by reducing the price of partial identification. The resolution of uncertainty about g concurrently addresses the uncertainty associated with the partial-identification problem of (m, x). Importantly, the subjective risk premia remain unchanged in response to the policy announcement across different states. Although the announcements resolve uncertainty related to partial identification problems, they do not alleviate the scarcity of capitalization in the financial sector in the constrained region, in contrast to the actual implementation of strategies such as capital injection, discount rate lending, or direct asset purchases analyzed in the rational expectations equilibrium by He and Krishnamurthy (2013).

Figure 12 illustrates the mean transition dynamics over elapsed time (in years) under different policy announcements. These paths are computed as the average of 5000 stochastic simulations, commencing at time 0 from the state characterized by an objective risk premium of 12% in equilibrium without policy announcements. When announcements are made at time 0, the entire equilibrium dynamics promptly shift to the new equilibrium. While asset prices and wealth shares experience a jump, the portfolio holdings established in the equilibrium without announcements remain fixed at time 0 after the announcements.

The g policy reduces the risk premium through a reduction in the price of partial identification, not the subjective risk premium. Although the announcement resolves uncertainty, it does not directly change the compensation for the high leverage faced by individual intermediaries.

In response to the g policy, the risk-free rate jumps up as it resolves the precautionary saving motive arising from parameter and state uncertainty. Due to this higher risk-free rate, the price-dividend ratio slightly decreases. The wealth share of intermediaries does not experience a significant jump following the announcements, but it grows at a slower rate due to the reduction in objective risk premia caused by the decreased price of partial identification. Table 4 displays the average transition time of the economy reaching specific levels of the objective risk premium. The g policy significantly accelerates the convergence of risk premia to lower levels.

A novel insight from this exercise is that policymakers should be cognizant of the endogenous linkages among different aspects of uncertainty. The exercise demonstrates that resolving one type of uncertainty about g simultaneously mitigates uncertainty concerning other dimensions (m, x). Conversely, resolving uncertainty about m alone does not have the same effect. Policymakers should be thoughtful of what the worst-case scenario is and what aspects of uncertainty are crucial to sustain the worst-case scenario for market participants.

7.2 Long-Run Effects of Announcement Policies

Now I examine the long-run implications of the aforementioned belief-management policies. Figure 13 compares the stationary distributions with and without g policy announcements. Under the g policy, the economy frequently encounters constrained states. Table 5 presents measurements from simulations under various policy interventions. The g policy reduces the objective risk premia in the constrained region but not in the unconstrained region relative to the equilibrium without policy announcements.

The worst-case g becomes significantly lower than the baseline value endogenously only in the constrained region. Therefore, the announcement policy does not alter asset demand in non-crisis states, preserving the risk premia comparable to those without government interventions and capital accumulation within the financial sector.

The higher probability of remaining in constrained states under the g policy arises from the lower objective risk premia in the constrained region, although to a lesser extent in the unconstrained region. Once the economy enters a constrained state, these periods persist longer under the g policy, accompanied by moderately elevated risk premia. Additionally, the resolution of uncertainty under the g policy leads to an increase in the risk-free rate due to the mitigated precautionary saving motive.

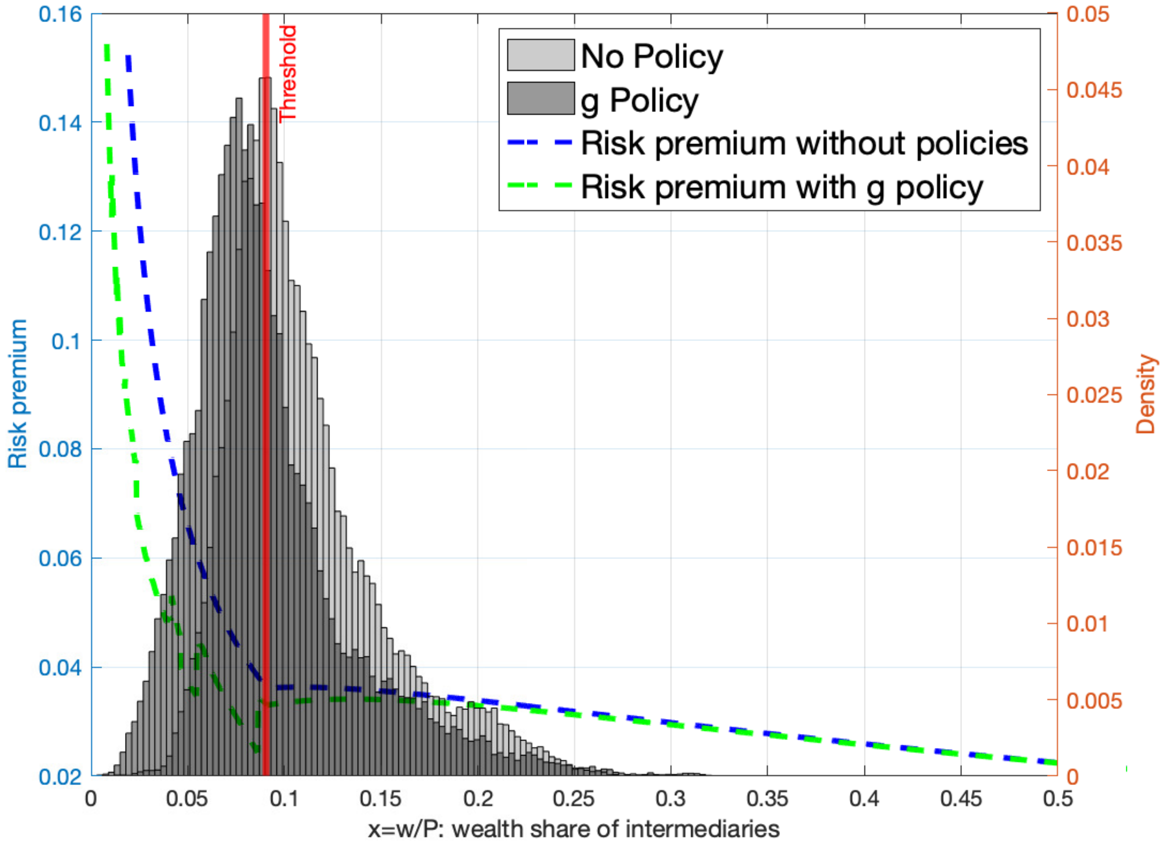

Figure 13: Stationary Distributions

The stationary distribution and objective risk premium against the actual realization of the aggregate state $x = w/P$ without and with policy announcements. The vertical red line is the constrained threshold.The threshold is not quatitatively different with and without policies. The darker histogram is the stationary distribution under the g policy, whereas, the lighter one is that without policies. The dashed blue line is the risk premia without policies, while the green is those under the g policy.

	No Policy	m Policy	g Policy
Average risk premium (%)	3.84	3.88	3.6
Risk premium in unconstrained region	3.58	3.51	3.36
Risk premium in constrained region	4.61	4.53	3.73
Average Sharpe ratio	47.89	46.54	43.32
Average return volatility (%)	8.32	8.33	8.32
Average interest rate (%)	0.8	0.9	1.22
Average price-dividend ratio	36.16	36.24	35.68
Prob(unconstrained) (%)	60.36	64.33	36.09
Labor income ratio	0.64	0.64	0.64
Debt-to-asset ratio	0.56	0.55	0.62

Table 5: Measurements

The table presents a number of key moments from stochastic simulations in the models with and without policy announcements. I report the unconditional average risk premium, average risk premium in the unconstrained region, average risk premium in the constrained region, average Sharpe ratio, average return volatility, average interest rate, average price-dividend ratio, average labor income ratio, and debt-to-asset ratio. I also report the unconditional probability of the margin constraint not binding.

8 Conclusion

In this study, I introduced a model designed to investigate risk premia dynamics during crisis events, where intermediaries grapple with capital scarcity and fear low-profitable investment opportunities in asset markets. The worst-case parameterized model for asset returns proves challenging to distinguish statistically from the baseline model. Notably, compensation for parameter and state uncertainty contributes to approximately 40% of the total risk premium during crisis episodes. Evaluating the effectiveness of government announcements using the model reveals that guaranteeing the cash flow of risky assets is the most impactful policy to mitigate the heightened risk premium during crises.

A promising avenue for future research lies in integrating default risk for financial institutions exposed to Poisson arrival shocks. This element, pivotal during the 2007-2009 financial crises, represents a source of uncertainty that was challenging to accurately measure. The modeling methodology presented in this paper stands as a useful starting point, providing a foundation to capture structured ambiguity in a broader and more general environment.

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Appendix A. Technical Appendix

In this Appendix, I detail the derivations of the ODE that characterizes the equilibrium and boundary conditions. I the main text, I use x, which is the ratio of the aggregate intermediaries' wealth w to the price of risk asset price P, as the state variable. As in He and Krishnamurthy (2013), in writing the ODE, it turns out the expressions are simpler and it leads to a computationally tractable boundary condition by changing variables to an alternative state variable $y \equiv w^h/D$, which is households' wealth w^h scaled by the current dividend level D. Denote the price-dividend ratio with the alternative state variable $F(y)$. Once we solve for the equilibrium price-dividend ratio $F(y)$, we can convert back to the original state variable x by

$$x = 1 - y/F(y), \tag{10}$$

and the price-dividend ratio $p(x)$ as a function of the fraction of intermediaries' wealth x satisfies

$$F(y) = p(1 - y/F(y)).$$

When x ranges from 0 to 1, y takes value from $F(y_b)$ to 0, where the maximum households wealth $y_b \equiv (1+l)/\rho$.

A.1. Derivation of the ODE for price-dividend ratio

Denote the dynamics of y_t as

$$dy_t = \mu_y dt + \sigma_y dZ_t, \tag{11}$$

for unknown function μ_y and σ_y. We write dc_t/c_t and dR_t as functions of mu_y, σ_y and the derivatives of $F(y)$. Due to the market clearing, since $c_t = D_t(1 + l\rho y_t)$, I have

$$\begin{aligned}
\frac{dc_t}{c_t} &= \frac{dD_t}{D_t} - \frac{\rho dy}{1+l-\rho y} - \frac{\rho}{1+l-\rho y} Cov_t\left[dy, \frac{dD}{D}\right] \\
&= \left(g - \frac{\rho}{1+l-\rho y}(\mu_y + \sigma_y \sigma)\right)dt + \left(\sigma - \frac{\rho \sigma_y}{1+l-\rho y}\right)dZ_t \\
&= \mu_c dt + \sigma_c dZ.
\end{aligned}$$

Also,

$$dR_t = \frac{dP_t + D_t dt}{P_t} = \left[g + \frac{F'}{F}\mu_y + \frac{1}{2}\frac{F''}{F}\sigma_y^2 + \frac{1}{F} + \frac{F'}{F}\sigma_y\sigma\right]dt + \left(\sigma + \frac{F'}{F}\sigma_y\right)dZ_t.$$

Given the solution S to the minimization problem in subsection 3.1, the Euler equation of intermediaries for the risky asset is given by

$$-\rho dt - \gamma E_t^S\left[\frac{dc_{i,t}}{c_{i,t}}\right] + \frac{1}{2}\gamma(\gamma+1)Var_t\left[\frac{dc_{i,t}}{c_{i,t}}\right] + E_t^S[dR_t] = \gamma Cov_t\left[\frac{dc_{i,t}}{c_{i,t}}, dR_t\right]. \tag{12}$$

Note that the worst-case alternative model differs from the true DGP only in terms of the first moment but not second moment ($Var_t^S = Var_t$ and $Cov_t^S = Cov_t$).

The intermediary i's consumption evolves under the worst-case model as

$$\frac{dc_{i,t}}{c_{i,t}} = (\mu_{c,i} - \sigma_{c,i}S)dt + \sigma_{c,i}dZ^S.$$

Substituting this into the Euler equation above,

$$-\rho dt - \gamma(\mu_{c,i} - \sigma_{c,i}S) + \frac{1}{2}\gamma(\gamma+1)\sigma_{c,i}^2 + E_t[dR_t] - \sigma_R S = \gamma\sigma_{c,i}\sigma_R \tag{13}$$

Since intermediaries are identical in equilibrium and $\mu_{c,i} = \mu_c$ and $\sigma_{c,i} = \sigma_c$,

$$-\rho dt - \gamma(\mu_c - \sigma_{c,i}S) + \frac{1}{2}\gamma(\gamma+1)\sigma_c^2 + E_t[dR_t] - \sigma_R S = \gamma\sigma_c\sigma_R. \tag{14}$$

Substituting the expressions of dR_t and dc_t/c_t above, I obtain the ODE for F:

$$
\begin{aligned}
&g - \sigma S + \frac{F'}{F}(\mu_y - \sigma_y S + \sigma_y \sigma) + \frac{1}{2}\frac{F''}{F}\sigma_y^2 + \frac{1}{F} = \rho + \gamma(g - \sigma S) - \\
&- \frac{\gamma\rho}{1 + l + \rho y}(\mu_y - \sigma_y S + \sigma_y \sigma) + \gamma\left(\sigma - \frac{\rho\sigma_y}{1 + l + \rho y}\right)\left(\sigma + \frac{F'\sigma_y}{F}\right) - \frac{\gamma(\gamma+1)}{2}\left(\sigma - \frac{\rho\sigma_y}{1 + l + \rho y}\right)^2.
\end{aligned}
\tag{15}
$$

Similarly, the Euler equation of intermediaries for the risk-free asset is given by

$$
r_t dt = \rho dt + \gamma E_t\left[\frac{dc_{i,t}}{c_{i,t}}\right] - \frac{\gamma(\gamma+1)}{2} Var_t\left[\frac{dc_{i,t}}{c_{i,t}}\right].
$$

After rearranging,

$$
r = \rho + \gamma(g - \sigma S) - \frac{\gamma\rho}{1 + l - \rho\gamma}(\mu_y - \sigma_y S + \sigma\sigma_y) - \frac{\gamma(\gamma+1)\sigma^2}{2}\left(\sigma - \frac{\rho\sigma_y}{1 + l - \rho y}\right).
$$

Regarding the derivation of (μ_y, σ_y), refer to the Appendix of HK since the derivation is exactly the same. The result is

$$
\sigma_y = -\frac{\theta_b}{1 - \theta_s F'}\sigma
$$

and

$$
\mu_y + \sigma\sigma_y = \frac{1}{1 - \theta_s F'}\left(\theta_s + l + (r - g)\theta_b - \rho y + \frac{1}{2}\theta_s F''\sigma_y^2\right)
$$

where θ_s are the number of shares that the risky asset household owns, and $\theta_b D = w^h - \theta_s P$ is the amount of funds that the households have invested in the risk-free bond. They depend on whether the economy is constrained or not. In the unconstrained region, $\theta_s = \frac{(1-\lambda)y}{F - \lambda y}$ and $\theta_b = \lambda y \frac{F-y}{F-\lambda y}$. In the constrained region, $\theta_s = \frac{m}{1+m}$ and $\theta_b = y - \frac{m}{1+m}F$. The economy is unconstrained if $0 < y \leq y^c$, where $y^c = \frac{m}{1-\lambda+m}F(y^c)$.

The ODE for F can be derived by substituting these expressions into the Euler equations and then combining them to substitute out the risk-free rate r.

A.2. Boundary Conditions

The upper boundary condition is given by $F(y_b) = y_b$ since the households hold the entire wealth in the economy. As in HK, I derive the lower boundary condition by taking the limit $y \to 0$ of the ODE:

$$F(0) = \frac{1 + F'(0)l}{\rho + g(\gamma - 1) + \frac{1}{2}\gamma(1-\gamma)\sigma^2 - \frac{l\gamma\rho}{1+l} - (\gamma + 1)\sigma S(0)},$$

where I require $F(0) > 0$ so that

$$\rho + g(\gamma - 1) + \frac{1}{2}\gamma(1-\gamma)\sigma^2 - \frac{l\gamma\rho}{1+l} - (\gamma + 1)\sigma S(0) > 0.$$

As documented in subsection 4.2, I approximate $S(0)$ by solving the minimization problem of intermediaries at $y = 0$ using the objective function from the previous iterations.

A.2. Numerical methods

I start the iterations with rational expectations equilibria with different values of (m, g), where $m \in (1e - 1, 20)$ and $g \in (-0.2, 0.2)$. The space of (m, g) is discretized, where $m \in (1e - 1, 20)$ and $g \in (-0.2, 0.2)$ are approximated by the equal-distant grids with 40 grid points.

Given the equilbrium objects from the previous iteration, I solve the new iteration as follows. Using the previous equilibrium objects, I compute the belief distortion $S(0)$. Then I implement the shooting algorithm that search for $F'(0)$ that jointly satisfies the lower and upper boundary conditions, and the ODE. Since both boudnaries are singular, I truncate the state space as in HK and extrapolate the truncated solution of F following the same procedure of HK. Notice that each equilibrium can be computed in parallel during iterations.

To obtain the stability of the algorithm, I gradually expand the set of alternative models by increasing the limit on the magnitude of the belief distortion S, starting $S = 0$ from the rational expectations equilibrium. The solution is regarded as converged if the solutions from the current iteration does not change from the previous iteration.