Money Search Model of International Currency with uninsurable risks*

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Preliminary and incomplete

Abstract

I study a two-country monetary model with bilateral matchings in a decentralized market, in which an endogenous ranking of currencies as media of exchange arises from their relative quality as hedging devices. When currencies serves as intertemporal media of exchange and hedging devices, buyers have generally a strict preference for paying sellers with the domestic currency which is the relative better hedging device for sellers. Consequently, when the domestic currency is a better hedge device for domestic sellers, floating exchange rate regime provides a better terms of trade, leading to higher welfare than fixed exchange rate regime. As a by-product, the model endogenously generates a cash-in advance constraint by which a local currency exclusively circulates. It breaks the nominal exchange rate indeterminacy of Kareken and Wallace.

Keywords: Multiple Currencies, Liquidity, Exchange Rates, Nominal Exchange Rate Determinacy, Exchange-Rate Regime

JEL Codes: E42, F31

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1 Introduction

A classic question in international finance is the welfare comparison across floating and fixed exchange rate regimes. With complete financial market and neutrality of money, two exchange-rates arrangements yield the same allocation and welfare as highlighted by Lucas (1982). Breaking this welfare equivalence, Neumeyer (1998) introduces the uninsurable risks in two-country monetary models with incomplete financial markets, following Helpman and Razin (1982). It leads to the conclusion that the flexible exchange rates may have a desirable welfare property over a fixed exchange-rates regime, which helps the reallocation of incomes across states of nature. This occurs by enriching the span of payoffs from asset trades in financial market. The flexible exchange rates allow nominal bonds to be denominated in terms of different currencies and deliver different state-contingent real payoffs. The fixed exchange rate regime, on the other hand, restricts nominal bonds to be denominated in a single common currency, worsening the market incompleteness.

Though the risk sharing may perform well in financial markets under a flexible exchange-rates regime as documented in these previous literature, in the real world, people do not necessarily have equal access to these hedging opportunities. This restricts the welfare benefit to be delivered to a subset of agents in the real economy. In the presence of uninsurable risks and incomplete markets, the literature has not yet come up with clear welfare implications from different exchange-rates regimes beyond those for financial markets. The aim of this paper is to provide one plausible channel.

I present a new channel through which the exchange-rates regimes affect welfare in the presence of uninsurable risks in a two-country monetary model. The new channel comes from terms of trades specifying quantity of goods and currency payments in a bilateral meeting of a decentralized market (DM), rather than financial markets as in previous literature. Through this new channel, I show the flexible exchange-rates regime can Pareto dominate the allocation under a fixed exchange rates regime. In the model, a buyer and seller in a bilateral meeting are exposed to different uninsurable productivity shocks in the future.

Especially, the seller values more the currency which is a better hedge on their future risk. When the domestic currency is a better hedge for the seller but not so relatively so much for the buyer, the buyer is willing to choose it over a foreign currency as a medium of exchange in order to to save the cost of acquiring the DM goods. This leads to a better terms of trade under the flexible exchange-rates regime and enhances the trade surplus in the DM.

This welfare consequence implies that exchange-rates regimes have a welfare implication beyond the incompleteness of financial markets. According to the result here, the flexible exchange-rates can still enhance the surplus from the daily trades in which people exchange goods for monies. Since this type of trades is very ubiquitous in the real world, the benefit from adopting a floating exchange-rates regime turns out to be bigger and broader than previously thought to be.

More specifically, the model builds on and extend the theory developed by Jacquet and Tan (2012) to the setting with multiple currencies. The basic model environment is similar to the monetary model of Lagos and Wright (2005). The period is divided into two subperiods with two currencies. The first subperiod is a DM in which a buyer makes a take-it-or-leave-it offer (TIOLI) to a seller. The second subperiod is a centralized market (CM). Both sellers and buyers live for two periods and the population is overlapping generations. The currencies play dual roles as an intertemporal medium of exchange in a DM of the first subperiod and as a hedging device on productivity shocks in the centralized market (CM) of the second subperiod. Buyers can trade currencies without any restrictions in the CM when young, while young sellers cannot. When a buyer and seller have different exposures to the future CM uninsurable risks, these two parties face different hedging needs. As a result, they differ in their relative valuations of currencies which holds different hedging properties. Then the buyer chooses the currency that is a relatively better hedge for the seller as a payment instrument in the DM. Since the domestic currency then saves the cost of acquiring the DM goods, the buyer obtains a better terms of trade.

The equilibrium consequence of these dual roles of currencies leads to the endogenous adoption of domestic currencies across countries under a floating exchange rate regime, if the domestic currencies provide a better hedge for the domestic sellers. This is the case when the domestic sellers observe the negative shock on their productivity in the CM, the inflation rate of the domestic currency goes down, leading to the higher rate of return for the domestic currency than foreign currency in their bad times. Under a fixed exchange rate, on the other hand, one country must peg their currency to the foreign currency. Then buyers in this country lose the option to use the domestic currency which would provide a better terms of trade. Consequently, the bilateral meetings in this country yield the terms of trade worse than under the flexible exchange rates regime. Therefore, the floating exchange rates provide higher welfare than the fixed exchange rates.

The rest of the paper is organized as follows. The following reviews the related literature. Section 2 describes the model environment. Section 3 formulates individual problems and characterizes the solutions. Section 4 solves the model in terms of the structural primitives and derive the welfare consequence of exchange rate regimes. Section 5 concludes the paper.

Literature review

In addition to the papers mentioned earlier which analyze the welfare effect of exchange rates regimes on risk sharing bu nominal asset tradings in financial markets, Zhu and Wallace (2020) also study the nonequivalence between floating and fixed exchange rates regimes in the presence of uninsurable risks in two-country monetary matching models. Their nonequivalence of allocation comes from the role of flexible exchange rates as an absorber for the demand shock to the specific-country's DM goods. When the buyers demand goods produced in this particular country more, they increase the demand for the producer currencies, which is the medium of exchange in the bilateral meeting. Since the producer currency appreciates, the buyers see the cost of acquiring the additional goods from this producer be higher. This movement of the exchange rate under flexible exchange rate regime suppresses the increase in the DM goods hit by a positive demand shock. Under a fixed exchange rate regime, the exchange rate does not adjust, so the DM goods consumption becomes more volatile in response to the demand shocks. They endogenously breaks the nominal exchange rate indeterminacy by employing the trading protocol developed by Zhu and Wallace (2008), in which the real balance of seller's domestic currency held by buyers determines the surplus of buyers and leads to the cash-in-advance constraints. However, they do not explicitly derive the welfare results under the two regimes.]

Drenik et al. (2021) study currencies as the unit of accounts in financial contracts and the interactions with optimal monetary policies in the presence of political risks. Currency choices matter in their framework for the hedging purpose as in this paper. I focus on the role of monies as media of exchange and hedging devices rather than units of accounts. Moreover, I explicitly describe the economic environment from the grounds up building on the recent advances in monetary theories, where monies are essential because of the information frictions, lack of commitment, and imperfect record keeping technology.

The model in this paper builds on the work by Jacquet and Tan (2012), in which money and a Lucas tree can serve as an intertemporal medium of exchange and hedging device in a one-country monetary model. I extend their framework by introducing two types of sellers with different hedging needs in the DM and two types of currencies with different hedging properties. This leads to the endogenous determinacy of a nominal exchange rate in the model, overcoming the indeterminacy result by Kareken and Wallace.

In international monetary economics literature, Zhang (2014) introduces the information asymmetry of currencies in a two-country monetary model, building on Lester et al. (2012). By restricting government sectors to exclusively use local currencies, the model breaks the indeterminacy of the nominal exchange rate. The model features the multiple equilibria which differ in terms of the international patterns of currency circulations as the result of the strategic complementarities between buyers' currency portfolio choices and sellers' investment decision on a verification technology for foreign currencies. Gomis-Porqueras et al. (2017) breaks the indeterminacy in a two-country monetary model, incorporating a

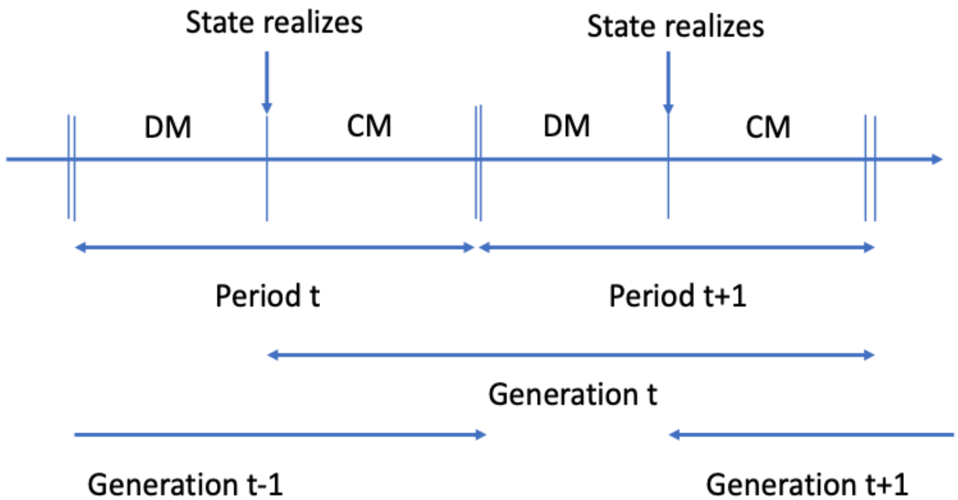

Figure 1: Timing of events

fixed cost of counterfeiting currencies. It generates the endogenous currency-specific limits on the amount of currency transfers in the DM. Nosal and Rocheteau (2016) employs the Zhu and Wallace (2007)'s DM trading protocol to induce the producer currency cash-in-advance constraint, which leads to the nominal exchange rate determinacy. However, these studies do not discuss the welfare implications for the exchange-rates regime.

2 Envirionment

This section describes the model environment by introducing the distribution of the states of nature, private agents, technologies, governments, and the timing of events, which is described in Figure 1.

2.1 Exogenous states of nature

There are two different states $i \in \{1, 2\}$ of nature and each realizes with probability $\pi(i)$ i.i.d. over time. This state of nature controls the currency growth rates and productivity shocks specified later in this section.

2.2 Market structures

Each period is divided into two subperiods. In both subperiods, agents cannot commit to honoring contracts they have agreed to. Moreover, there does not exist a technology keeping track of individual trading histories so that agents are anonymous. These two assumptions jointly restrict the trading in both periods to be *quid pro quo*as in Lagos and Wright (2005). In the first subperiod, there exists a DM where bilateral matching occurs among people and terms of trade is specified in a trading protocol, including the perfectly divisible and perishable DM goods and two types of perfectly divisible and storable currencies $k \in \{1, 2\}$. In the second subperiod, there exists a CM for a numeraire CM consumption goods which is perfectly divisible and perishable, and two types of currencies issued by governments. The state in period t is revealed after the DM but before the CM of the period t.

2.3 People

The initial old with measure 1 has a linear utility with respect to the CM consumption goods. They hold two types of currencies at the beginning to purchase the CM goods.

$$\mathbb{E}_0 C_0^o$$

The subsequent population is overlapping generations of agents living for two periods. There are country j sellers with measure 1, $j \in \{1, 2\}$, denoted by (s, j) and country j buyers with measure 1, $j \in \{1, 2\}$, denoted by (b, j).

The generation t is born after the DM but before the CM period t, knowing their types. Sellers can produce the DM goods at the cost of $\Psi(\cdot)$ and never consume it, while buyers cannot produce the DM goods but consume them. Sellers in this generation cannot participate in the DM in period t, while buyers can. Both sellers and buyers join the DM in period $t+1$, where each type-j buyer is matched with a type-j seller and the terms of trade is determined based on a take-it-or-leave-it (TIOLI) offer by the buyer. Then both sellers and buyers trade currencies and CM consumption goods in the period-$(t+1)$ CM.

Preferences of type-(s,j) agents in generation t is given by

$$\mathbb{E}_t\beta\{-\Psi(c_{t+1}^{s,j}) + \mathbb{E}_t[U(c_{t+1}^{o,s,j}) - \theta_{t+1}^{s,j}N_{t+1}^{o,s,j}]\},$$

where the conditional mathematical expectation is taken for the exogenous state of nature. $c_{t+1}^{s,j}$ is the DM goods produced by this seller; $C_{t+1}^{o,s,j}$ is the CM goods consumed in the period $(t+1)$ CM; $\theta_{t+1}^{s,j}$ is the productivity shock on the labor provided in the period-$(t+1)$ CM; $N_{t+1}^{o,s,t}$ is the labor provided in the period-$(t+1)$ CM.

Preferences of type-(b,j) agents in generation t is given by

$$U(C_t^{y,b,j}) - N_t^{y,b,t} + \beta\{u(c_{t+1}^{b,j})) + \mathbb{E}_t[U(C_{t+1}^{o,b,j}) - \theta_{t+1}^{b,j}N_{t+1}^{o,b,j}]\},$$

where $C_t^{y,b,j}$ is the consumption of CM consumption goods in the period t; $N_t^{y,b,t}$ is the labor provided in the period t CM; $c_{t+1}^{b,j}$ is the DM goods consumed in the period-$(t+1)$ DM; $C_{t+1}^{o,b,j}$ is the CM consumption goods consumed in the period $(t+1)$ DM; $\theta_{t+1}^{b,j}$ is the productivity shock on the labor provided in the period-$(t+1)$ CM; $N_{t+1}^{o,b,t}$ is the labor provided in the period-$(t+1)$ CM. The productivity shock is the function of the state: $\theta_{t+1}^{s,j} = \theta^{s,j}(i)$ and $\theta_{t+1}^{b,j} = \theta^{b,j}(i)$.

These preference specifications imply that (1) the young sellers do not trade and consume in the first CM; (2) Other types of agents can produce the CM goods linearly; (3) Each type of agents are subject to an uninsurable productivity shock in the CM when they become old; (4) Sellers can produce the DM goods linearly but does not consume; (5) Buyers cannot produce the DM goods but want to consume.

As in Jacquet and Tan (2012), I make the following standard assumptions on preferences: (1) $U' > 0$; (2) $u' > 0$; (3) $\Psi' > 0$; (4) $U'' < 0$ (5) $u'' - \Psi'' < 0$; (6) $u - \Psi$ satisfies the Inada conditions; (6) There exists $c^{**} \in (0, \infty)$ such that $u'(c^{**}) = \Psi(c^{**})$; (7) For all θ, there exists $C^{**}(\theta) \in (0, \infty)$ such that $U'(C^{**}(\theta)) = \theta$. For brevity, assume that the linear cost function of the DM goods $\Psi(c) = c$.

I assume that there does not exist any financial contracts that insure each other for productivity shocks across different types of agents. This assumption is extreme but it highlights the crucial difference of this paper from the previous literature in terms of implications of exchange-rates regime for risk sharing. This paper focuses on the implications of the reallocation of hedging devices in the DM for the resulting terms of trade, while previous papers analyze the reallocation through nominal assets trading in centralized financial markets.

2.4 Governments

Under a flexible exchange-rates regime, there exists two country-specific governments $j \in \{1, 2\}$, which control supply of currency j and lump-sum transfer on young country-j buyers in the CM every period. In the initial period, the governments endow the initial olds with the entire stock of monies. The type-j government budget constraint is given by

$$\phi_{j,t}(M_{j,t} - M_{j,t-1}) = T_{j,t},$$

where $M_{j,t}$ is the supply of currency j in period t; $\phi_{j,t}$ is the price of currency j in period t in terms of CM goods. Assume that the currency supply in period t is a function of the state of period t: $M_{j,t}/M_{j,t-1} = \gamma_j(i)$. The right hand side is the seigniorage revenue from supplying additional money.

On the other hand, under a fixed exchange-rates regime, the type 2 government ceases to issue its own currency. As a result, currency 1 becomes a single common currency in this economy.

3 Individual problems

This section formulates individual problems in recursive forms and characterize the solutions. I solve the problems backwardly, starting from the problem faced by old agents in the CM in section 3.1 and then the DM problem in section 3.2 and 3.3, and then the young buyers' problem in the CM in section 3.4.

3.1 The CM for old agents

The maximization problem for type-(x, j) old agents holding real balances (q_1, q_2) is given by

$$V^{x,j}(q_1, q_2) = \max_{C^{o,x,j}, N^{o,x,j}} \{U(C^{o,x,j}) - \theta^{x,j}(i)N^{o,x,j}\}$$

s.t.

$$C^{o,x,j} = N^{o,x,j} + \gamma_1(i)^{-1}q_1 + \gamma_2(i)^{-1}q_2$$

and the nonnegativity constraints of the control variables. Again by assuming interiority of the solution, the quasi-linearity of the utility function implies that $C^{o,x,j} = C^{**}(\theta^{x,j}(i))$ and that $N^{o,x,j} = C^{**}(\theta^{x,j}(i)) - \gamma_1^{-1}(i)q_1 - \gamma_2^{-1}(i)q_2$. Therefore, the expected value to a type-(x, j) agent of entering his last CM with currency portfolio (q_1, q_2) is provided by

$$V^{x,j}(q_1, q_2) = V^{x,j}(0, 0) + v_1^{x,j} q_1 + v_2^{x,j} q_2,$$

where

$$V^{x,j}(0, 0) = \pi(1)[U(C^{**}(\theta^{x,j}(1))) - C^{**}(\theta^{x,j}(1))] + \pi(2)[U(C^{**}(\theta^{x,j}(2))) - C^{**}(\theta^{x,j}(2))]$$

$$v_k^{x,j} = \pi(1)\gamma_k(1)^{-1}\theta^{x,j}(1) + \pi(2)\gamma_k(2)^{-1}\theta^{x,j}(2)$$

Notably, the value function is linear in real balance holdings. This fact greatly simplifies the DM problem described in the following section. The expected valuation of a currency is determined by not only the average rate of return but also its hedging property. The weight on the rate of return is distorted by the realizations of productivity shocks. Therefore, the holding cost and hedge property jointly differentiates the two currencies in terms of agents. They value the currency more which yields higher average return or higher rate of return when his productivity is low.

Now assume that $v_k^{b,j} \leq \beta^{-1}$ for every currency k so that the buyer is never willing to hold real balances beyond the amount used in the DM.

3.2 DM problem

This section derives the partial equilibrium characterizations for the terms of trade given buyers' currency portfolio and the expected valuations for these currencies. The subsequent sections solve the full general equilibrium in terms of structural primitives.

3.2.1 Setup of the DM problem

A country-j buyer is matched with a country-j seller, holding predetermined real balances (q_1, q_2) chosen in the previous CM and making TIOLI offer. The seller does not hold any asset since she does not access to the CM when young. The problem in this matching is to choose the terms of trade (c, d_1, d_2) in order to maximize the buyer's surplus subject to the seller's participation constraint. Using the linearity of the value function of the next CM, the problem is formulated as:

$$\max_{c,d_1,d_2} \{u(c) - [v_1^b d_1 + v_2^b d_2]\}$$

s.t.

$$v_1^s d_1 + v_2^{s,j} d_2 \geq c;$$

$$d_k \leq q_k.$$

Here, v_k^b is the buyer's constant expected valuation of one unit of the real balance of currency k, which is independent of his country profile j; $v_k^{s,j}$ is the type-j seller's constant expected valuation of one unit of the real balance of currency k defined in the previous section; d_k is the real balance transfer of currency k from the buyer to seller. The problem is concave so that the first-order conditions are necessary and sufficient.

$$\epsilon_k^j \equiv \frac{v_k^b}{v_k^{s,j}} \geq u'(c) - \lambda_k^j,$$

with equality if $d_k \geq 0$. λ_k^j is the adjusted multiplier on the upper bound on d_k. Due to the Inada condition, one of the FOCs must hold equality if the seller's total valuation of the buyer's portfolio is positive. Otherwise, the marginal utility of the DM goods consumption becomes infinity, while the marginal cost of production is finite, which is in contradiction.

When the upper bound on d_k is slack, then the buyer equates the marginal utility of the DM consumption goods to the seller's marginal cost of production. When the buyer additionally transfer one unit of real balance of currency k, he loses v_k^b in util, while the seller increases the production by $v_k^{s,j}$. Then the buyer increases his utility by $v_k^{s,j}u'(c)$. At the optimum, this operation must not change the buyer's utility, which leads to the first-order conditions above. If the upper bound is binding, that operation yields a strictly positive utility for the buyer but this is infeasible. In this case, the first-order condition holds with inequality.

3.2.2 Terms of trade with common relative valuations

This section characterizes the terms of trade when both parties have the same relative valuations of assets: $\epsilon_k^j = \epsilon^j$. This nests the special case where both of buyers and sellers have the same valuations of assets $v_k^{b,j} = v_k^{s,j}$ so that $\epsilon_k^j = 1$. This is a common presumption in the monetary theory literature except for Jacquet and Tan (2012).

Let $\omega_j(d_1, d_2)$ be the expected value of a transfer (d_1, d_2) for a seller, $v_1^{s,j}d_1 + v_2^{s,j}d_2$ and

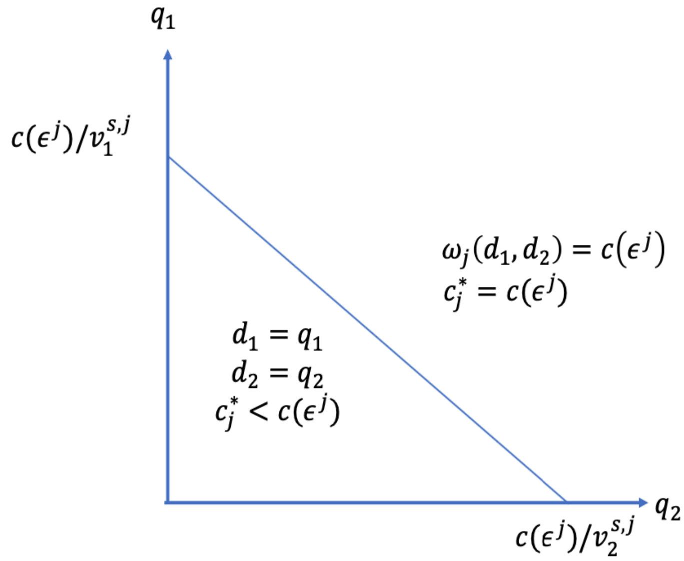

Figure 2: Terms of trade in common valuations

$\overline{c}(\epsilon)$ be the DM consumption of a buyer such that $u'(\overline{c}(\epsilon)) = \epsilon$.

Lemma 1

Assume $\epsilon_k^j = \epsilon^j$ *and the buyer's portfolio* (q_1, q_2)*. The terms of trade in the DM* (c_j^*, d_1^j, d_2^j) *satisfy* $c_j^* = \omega(d_1^j, d_2^j) = \min\{\omega_j(q_1, q_2), \overline{c}(\epsilon^j)\}$*.*

Figure 2 depicts the terms of trade graphically. If the buyer holds enough real balances such that $\omega(q_1, q_2) \geq \overline{c}(\epsilon^j)$, the buyers purchases $\overline{c}(\epsilon^j)$ amounts of goods from the seller by transferring the real balance worth $\overline{c}(\epsilon^j)$ for a seller. In particular, when unconstrained, the buyer is indifferent of which currencies to use as a medium of exchange. In order to obtain an additional unit of goods, the buyer incurs the cost ϵ_j in util, which is independent of the choice of payment instruments. If not, the buyer does not hold real balances to purchase this unconstrained optimal amounts. Then he transfers his entire portfolio to the seller to obtain the maximum level of consumption under his budget.

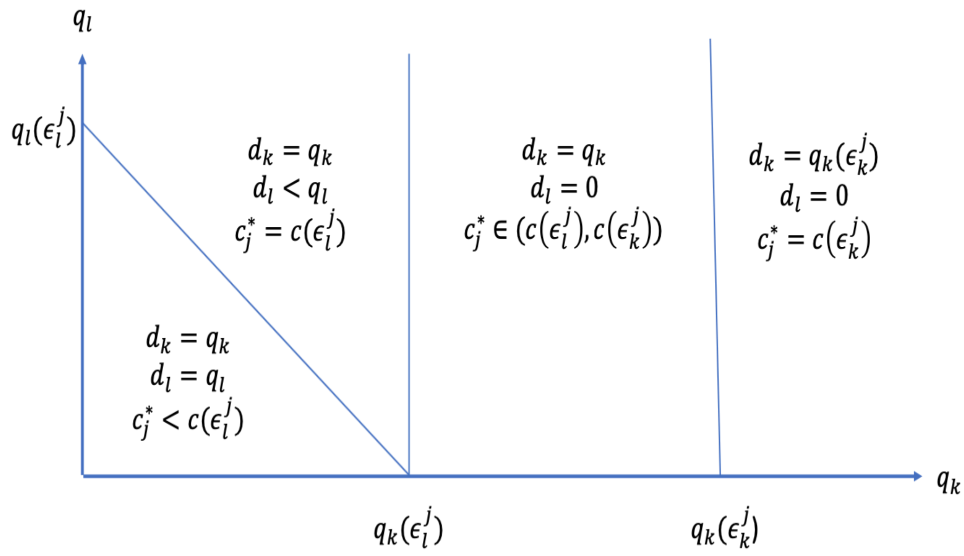

Figure 3: Terms of trade in heterogeneous valuations

3.2.3 Terms of trade with different relative valuations

When two parties have heterogeneous relative valuations of assets, the buyer has a strict preference for using one of the two currencies as a medium of exchange.

Lemma 2

Suppose $\epsilon_l^j > \epsilon_k^j$, $l \neq k$ and buyers' portfolio (q_1, q_2). Then there exist $\overline{q}_k^j(\epsilon_k^j)$ and $\overline{q}_k^j(\epsilon_l^j)$ with $\overline{q}_k(\epsilon_k^j) > \overline{q}_k(\epsilon_l^j)$ such that

$$(c_j^*(q_1, q_2), d_k^j, d_l^j) = \begin{cases} (\overline{c}(\epsilon_k^j), \overline{q}_k(\epsilon_k^j), 0), \ \text{if } q_k > \overline{q}_k^j(\epsilon_k^j) \\ (\overline{c}(0, q_k), q_k, 0), \ \text{if } q_k \in [\overline{q}_k^j(\epsilon_l^j), \overline{q}_k^j(\epsilon_k^j)] \\ (\overline{c}(\epsilon_l^j), q_k, d_l^j(q_k; \epsilon_l^j)), \ \text{if } q_k < \overline{q}_k^j(\epsilon_k^j) \text{ and } \omega(q_k, q_l) \geq \overline{c}(\epsilon_l^j) \\ (c(q_k, q_l), q_k, q_l), \ \text{otherwise,} \end{cases}$$

where $\overline{q}_k^j(\epsilon) \equiv \overline{c}(\epsilon)/v_k^{s,j}$ is the amount of transfer of real balance of currency k to produce $\overline{c}(\epsilon)$ units of consumption; $c(d_k, d_l) \equiv v_k^{s,j} d_k + v_l^{s,j} d_l$ is the consumption level obtained from the transfer (d_k, d_l); $d_l^j(q_k; \epsilon_l^j) \equiv \overline{c}(\epsilon_l^j)/v_l^{s,j} - (v_k^{s,j}/v_l^{s,j})q_k$ is the amount of the less valuable

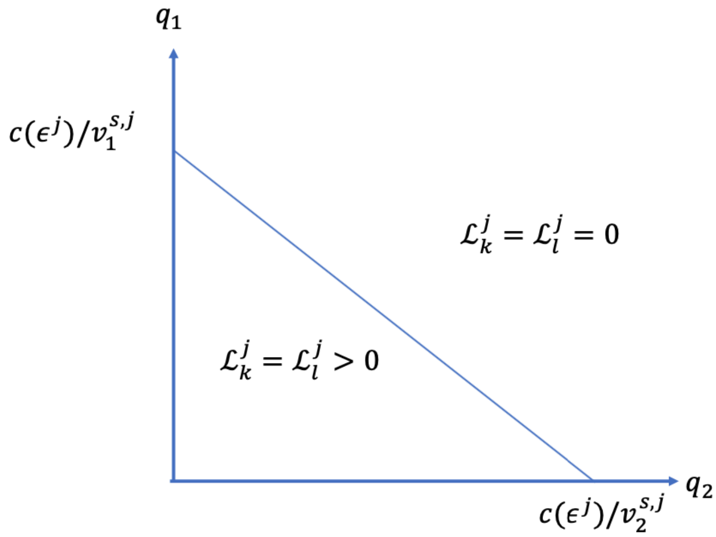

Figure 4: Degrees of liquidity in common valuations

real balance transferred to the seller to compensate for the shortage of the more valuable real balance to produce the consumption at the level of $\bar{c}(\epsilon_l^j)$.

The result of this lemma is described in Figure 3. When the buyer holds enough amounts of the real balance of currency k, $q_k \geq \bar{q}_k^j(\epsilon_k^j)$, he buys the unconstrained optimal amount of goods $\bar{c}(\epsilon_k^j)$. If the real balance falls short of this level, the consumption is constrained and the buyer transfer the entire real balance of currency k but does not use currency l at all when consumption is still higher than $\bar{c}(\epsilon_l^j)$. The reason is that when the buyer uses the currency l as a medium of exchange, the marginal cost of obtaining additional units of consumption ϵ_l^j is still higher than the marginal utility of consumption. As the consumption level becomes below $\bar{c}(\epsilon_l^j)$, the buyer starts to transfer the currency l to the seller.

3.2.4 Relative valuations and liquidity

Let's define the liquidity concept in this model, which plays a crucial role in the currency choice problem for young buyers in the CM. **Definition**

For $k \in 1, 2$, the degree of liquidity of currency k in a type-j meeting is defined to be

$$\mathcal{L}_k^j(q_1, q_2; \epsilon_k^j) \equiv \frac{1}{\epsilon_k^j} u'[c_j^*(q_1, q_2)] - 1$$

By construction, the degree of liquidity is positive if and only if when the currency is additionally given to the buyer, he spends it to obtain additional units of the DM goods. If he prefers rather not to use it in exchange of the DM goods, then the degree of liquidity associated with that currency is negative. If he is indifferent, then the degree becomes zero. In other words, it becomes positive if the DM goods is marginally more valuable than the currency.

Proposition 1

1. *If $\epsilon_k^j = \epsilon^j$, then $\mathcal{L}_1^j = \mathcal{L}_2^j > 0$ if and only if $\omega_j(q_1, q_2) < \overline{c}(\epsilon^j)$, and $\mathcal{L}_1^j = \mathcal{L}_2^j = 0$ otherwise.*

2. *If $\epsilon_l^j = \epsilon_k^j$, $k \neq l$, then:*

   (a) *for currency k, $\mathcal{L}_k^j(q_1, q_2, \epsilon_k^j) > 0$ if and only if $q_k < \overline{q}_k(\epsilon_k^j)$ and $\mathcal{L}_k^j(q_1, q_2, \epsilon_k^j) = 0$ otherwise. For currency l, $\mathcal{L}_l^j(q_1, q_2, \epsilon_k^l) > 0$ if and only if $\omega(q_1, q_2) < \overline{c}(\epsilon_l^j)$; $\mathcal{L}_l^j(q_1, q_2, \epsilon_k^l) = 0$ if and only if $q_k \geq \overline{q}_k(\epsilon_l^j)$ and $\omega_j(q_1, q_2) \geq \overline{c}(\epsilon_l^j)$; and $\mathcal{L}_l^j(q_1, q_2, \epsilon_l^j) < 0$ otherwise.*

   (b) *$\mathcal{L}_k^j(q_1, q_2, \epsilon_l^j) > \mathcal{L}_l^j(q_1, q_2, \epsilon_l^j)$*

Figure 4 depicts this proposition in the case of common valuations, in which two parties are indifferent of which one is used as a medium of exchange so that the degree of liquidities are the same. In case 2, which is represented in Figure 5, since currency k is relatively more valuable for the seller, it is more intensively used as a payment instrument. Therefore, the currency k delivers more liquidity in the DM.

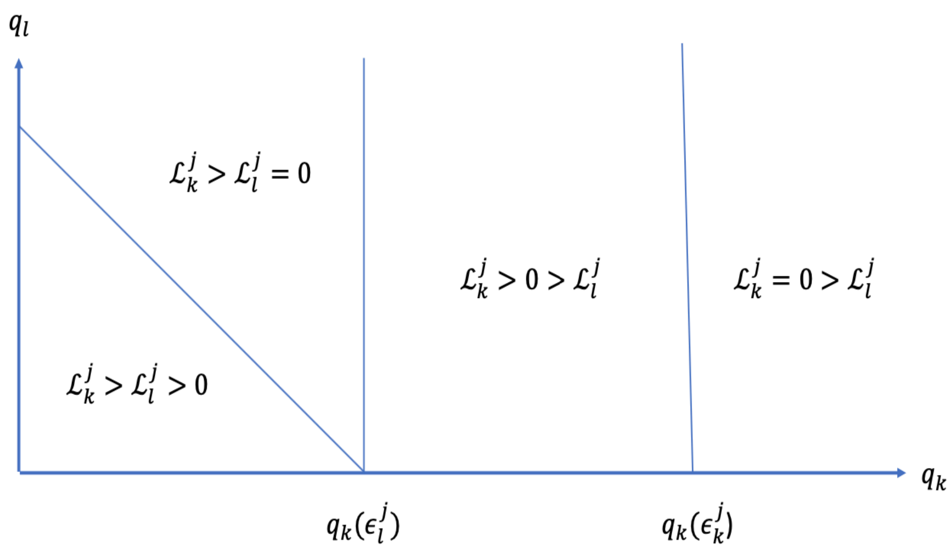

Figure 5: Degrees of liquidity in heterogeneous valuations

3.2.5 Expected values of DM

Given the characterizations of terms of trade, the value of entering the second subperiod with portfolio (q_1, q_2) for a type-j buyer is

$$W^{b,j}(q_1, q_2) = u(c_j^*(q_1, q_2)) + V^{b,j}(0, 0),$$

where $(c_j^*(q_1, q_2), d_1^j(q_1, q_2), d_2^j(q_1, q_2))$ is the terms of trade derived before. The currency holding of buyers are zero since the expected valuation is low enough: $v_k^{b,j} < \beta^{-1}$. Similarly, for a type-j seller, the value of entering the second subperiod with portfolio (q_1, q_2) is

$$W^{s,j} = -c_j^*(q_1, q_2) + [V^{s,j}(0, 0) + v_1^{s,j} \times d_1^j(q_1, q_2) + v_2^{s,j}(q_1, q_2) \times d_2(q_1, q_2)].$$

3.3 The CM for young buyers

Now we characterize the optimal currency holdings for young buyers in the CM. The optimization problem of the type-j young buyer is provided by:

$$\max_{C^{y,b,j},N^{y,b,j},q_1^{y,j},q_2^{y,j}}\{U(C^{y,b,j})-N^{y,b,j}+\beta W^{b,j}(q_1^{y,j},q_2^{y,j})\}$$

s.t.

$$C^{y,b,j}+q_1^{y,j}+q_2^{y,j}=N^{y,b,j}+T_j(i)$$

and nonnegativity constraints on all of the control variables. Here, $W^{b,j}$ is the expected value of continuing in the next DM market; $q_k^{y,b}$ is the real balance of currency k chosen by the type-j young buyers.

Note that the problem is independent of the current realization of the state of the nature because of the i.i.d. assumption on the state evolution. Assuming the interior solution, the quasi linearity of the utility function implies that $C^{y,b,j}=C^{**}(1)$ in every state and that

$$N^{y,b,j}(i)=C^{**}(1)+q_1+q_2-T_j(i).$$

The first order conditions for buyer's currency choice in the CM in both states are given by

$$1\geq\beta W_k^{b,j}(q_1,q_2)$$

with equality if $q_k>0$.

Substituting out the value of the DM, we have the following first-order conditions:

Lemma 4

Let $l_k^j(q_1,q_2;\epsilon_k^j)\equiv\max\{0,\mathcal{L}_k^j(q_1,q_2;\epsilon_k^j)\}$ be the liquidity premium of currency k. The first-order conditions characterizing the currency choice of buyers in both states of the world can *be written as*

$$1 \geq \beta v_k^{b,j} \times [1 + l_k^j(q_1, q_2; \epsilon_k^j)],$$

with equality if $q_k > 0$*.*

This condition features that holding an additional unit of a currency benefits the buyer due to not only the expected currency returns realized in the next CM but also the liquidity in the next DM. Note that the liquidity premia of currencies are always nonnegative. This is because if a currency is more valuable than the consumption goods in the DM, he can rather hold that currency until the next CM instead of spending in the current DM. Due to this option, currencies never incur negative liquidity premia.

4 Equilibrium

This section describes the solution to the equilibrium in terms of structural parameters and derive the welfare implications of exchange-rates regimes.

4.1 Definition of stationary monetary equilibrium

Let $\overline{\gamma}_k^{-1} \equiv \pi(1)\gamma_k(1)^{-1} + \pi(2)\gamma_k(2)^{-1}$ be the one-period expected return for holding currency k. This is negatively associated with the cost of holding currency k. Additionally, let $\sigma_k = \gamma_k(1)/\gamma_k(2)$ denote a volatility measure of the growth rate of currency k. This characterizes the hedging properties of currencies.

Definition

A stationary monetary equilibria (SME) is real balances holdings for households (q_1, q_2) *satisfying the first-order conditions and at least one of the aggregate real balances of two currencies is strictly positive. When* $v_k^{b,j} = \beta^{-1}$*, I consider the SME in the limit as* $v_k^{b,j} \uparrow \beta^{-1}$*.*

4.2 Characterization of SMEs under floating exchange-rates regime

From now on, I make the following assumptions.

Assumption 1: *$\theta^{b,1}(i) = \theta^{b,2}(i) = \theta^{b}(i)$ for all states i.*

Assumption 2: *$\overline{\theta}^{b} = \overline{\theta}^{s,1} = \overline{\theta}^{s,2}$*

Assumption 3.1: *$\theta^{s,1}(2) > \theta^{s,1}(1)$ and $\theta^{s,1}(1) < \theta^{b}(1)$.*

Assumption 3.2: *$\theta^{s,2}(1) = \theta^{s,2}(2)$ and $\theta^{s,2}(2) < \theta^{b}(2)$.*

Assumption 1 is just a normalization of the productivity shocks. Assumption 2 implies that the average productivity in the CM is common across types. The first statements in Assumption 3.1 and 3.2 means that the state j is good for country j sellers since a good productivity shock realizes for them. Moreover, the second statements indicate that the productivity of buyers is less volatile than sellers.

The following proposition characterizes the equilibrium allocations for country 1. A similar result also holds for country 2.

Proposition 2

There exists the following types of SMEs for country 1:

1. *The country-1 meeting prefers the country-1 currency as a medium of exchange when currency 1 provides a relatively better hedge for a seller. More specifically, this is the case when $\epsilon_2^1 > \epsilon_1^1$. This inequality holds for any $\sigma_1 > \sigma_1^*(\sigma_2)$, where $\sigma_1^*(\sigma_2)$ satisfies*

$$\begin{aligned}\epsilon_1^1(\sigma_1) &\equiv \frac{\pi(1)\theta^b(1) + \sigma_1^*(\sigma_2)\pi(2)\theta^b(2)}{\pi(1)\theta^{s,1}(1) + \sigma_1^*(\sigma_2)\pi(2)\theta^{s,1}(2)} \\ &= \frac{\pi(1)\theta^b(1) + \sigma_k\pi(2)\theta^b(2)}{\pi(1)\theta^{s,1}(1) + \sigma_k\pi(2)\theta^{s,1}(2)} \equiv \epsilon_2^1(\sigma_2).\end{aligned}$$

   If such a $\sigma_1^*(\sigma_2)$ does not exist in $[0,\infty)$, then $\epsilon_1^1 < \epsilon_2^1$ for every $\sigma_1 \in [0,\infty)$. If it exists, *then*

$$\frac{d\sigma_1^*(\sigma_2)}{d\sigma_2} > 0.$$

(a) *The DM consumption is high enough such that* $c_1^* \in (\overline{c}(\epsilon_2^1(\sigma_2)), \overline{c}(\epsilon_1^1(\sigma_1))]$. *This is the case when* $\overline{\gamma}_1 \in [\gamma_{1,min}(\sigma_1), \tilde{\gamma}_1(\sigma_1, \sigma_2))$, *where*

$$\gamma_{1,min}(\sigma_1) \equiv \beta \frac{\pi(1)\theta^b(1) + \sigma_1 \pi(2)\theta^b(2)}{\pi(1) + \sigma_1 \pi(2)};$$

$$\tilde{\gamma}_1(\sigma_1, \sigma_2) \equiv \beta \frac{\pi(1)\theta^{s,1}(1) + \sigma_1 \pi(2)\theta^{s,1}(2)}{\pi(1) + \sigma_1 \pi(2)} \epsilon_2^1(\sigma_2).$$

These objects have the following properties:

$$sign\left(\frac{d\gamma_{1,min}(\sigma_1)}{d\sigma_1}\right) = sign(\theta^b(2) - \theta^b(1));$$

$$\frac{\partial \tilde{\gamma}_1}{\partial \sigma_1} > 0;$$

$$\frac{\partial \tilde{\gamma}_1}{\partial \sigma_2} < 0.$$

*The type-*1 *meeting uses only currency* 1 *in this case and the DM consumption is given by* $c_1^* = (u')^{-1}((\beta v_1^{s,1})^{-1})$.

(b) *The DM consumption is low in the sense that* $c_1^* \leq \overline{c}(\epsilon_2^1)$. *This happens when* $\overline{\gamma}_1 \geq \tilde{\gamma}_1(\sigma_1, \sigma_2)$.

i. *Currency* 1 *serves as a medium of exchange, while currency* 2 *does not. More specifically,* $q_1^1 > 0$ *and* $q_2^1 = 0$ *when the inflation rate of currency* 1 *is low enough such that* $\overline{\gamma}_1 < \hat{\gamma}_1(\sigma_1, \sigma_2, \overline{\gamma}_2)$, *where*

$$\hat{\gamma}_1(\sigma_1, \sigma_2, \overline{\gamma}_2) \equiv \overline{\gamma}_2 \frac{\pi(1)\theta^{s,1}(1) + \sigma_j \pi(2)\theta^{s,1}(2)}{\pi(1) + \sigma_1 \pi(2)} \frac{\pi(1) + \sigma_2 \pi(2)}{\pi(1)\theta^{s,1}(1) + \sigma_2 \pi(2)\theta^{s,1}(2)}.$$

It satisfies

$$\frac{\partial \hat{\gamma}_1}{\partial \sigma_1} > 0;$$

$$\frac{\partial \hat{\gamma}_2}{\partial \sigma_1} < 0;$$

$$\frac{\partial \hat{\gamma}_1}{\partial \overline{\gamma}_2} > 0.$$

The DM consumption level is given by $c_1^* = (u')^{-1}((\beta v_1^{s,1})^{-1})$.

ii. *Both currencies are used as media of exchange so that* $q_1^1 > 0$ *and* $q_2^1 > 0$. *This arises when* $\overline{\gamma}_1 = \hat{\gamma}_1(\sigma_1, \sigma_2, \overline{\gamma}_2)$. *In this case, the currency portfolio is indeterminate. The DM consumption level is given by* $c_1^* = (u')^{-1}((\beta v_j^{s,1})^{-1})$.

iii. *Currency* 2 *is used as a medium of exchange* ($q_2^1 > 0$) *while currency* 1 *is not* ($q_1^1 = 0$) *when the inflation rate of currency* 1 *is high enough in the sense that* $\overline{\gamma}_1 > \hat{\gamma}_1(\sigma_1, \sigma_2, \overline{\gamma}_2)$. *The DM consumption level is given by* $c_1^* = (u')^{-1}((\beta v_1^{s,1})^{-1})$.

2. *The country-*1 *meeting prefers the country-*2 *currency as a medium of exchange when currency* 2 *provides a relatively better hedge for a seller. More specifically, this happens if* $\epsilon_2^1 < \epsilon_1^1$. *That is, there exists* $\sigma_1^*(\sigma_2) \in [0, \infty)$ *and* $\sigma_1 < \sigma_1^*(\sigma_2)$.

   (a) *The DM consumption is high enough such that* $c_1^* \in (\overline{c}(\epsilon_1^1(\sigma_1)), \overline{c}(\epsilon_2^1(\sigma_2))]$. *This is the case when* $\overline{\gamma}_2 \in [\gamma_{2,min}(\sigma_2), \tilde{\gamma}_2(\sigma_1, \sigma_2))$. *The country-*1 *meeting uses only currency* 2 *in this case and the DM consumption is given by* $c_1^* = (u')^{-1}((\beta v_2^{s,1})^{-1})$.

   (b) *The DM consumption is low in the sense that* $c_1^* \leq \overline{c}(\epsilon_1^1)$. *This happens when* $\overline{\gamma}_2 \geq \tilde{\gamma}_2(\sigma_1, \sigma_2)$.

      i. *Currency* 2 *serves as a medium of exchange, while currency* 1 *does not. More specifically,* $q_2^1 > 0$ *and* $q_1^1 = 0$ *when the inflation rate of currency* 2 *is low enough such that* $\overline{\gamma}_2 < \hat{\gamma}_2(\sigma_1, \sigma_2, \overline{\gamma}_1)$. *The DM consumption level is given by* $c_1^* = (u')^{-1}((\beta v_2^{s,1})^{-1})$.

ii. *Both currencies are used as media of exchange so that* $q_1^1 > 0$ *and* $q_2^1 > 0$. *This arises when* $\overline{\gamma}_2 = \hat{\gamma}_2(\sigma_1, \sigma_2, \overline{\gamma}_1)$. *In this case, the currency portfolio is indeterminate. The DM consumption level is given by* $c_1^* = (u')^{-1}((\beta v_2^{s,1})^{-1})$.

iii. *Currency* 1 *is used as a medium of exchange* ($q_1^1 > 0$) *while currency* 2 *is not* ($q_2^1 = 0$) *when the inflation rate of currency* 2 *is high enough in the sense that* $\overline{\gamma}_2 > \hat{\gamma}_2(\sigma_1, \sigma_2, \overline{\gamma}_1)$. *The DM consumption level is given by* $c_1^* = (u')^{-1}((\beta v_2^{s,1})^{-1})$.

3. *The country-*1 *meeting is indifferent of two currencies as hedging devices* $\epsilon_2^1 = \epsilon_1^1$. *That is, there exists* $\sigma_2^*(\sigma_2) \in [0, \infty)$ *and* $\sigma_1 = \sigma_1^*(\sigma_2)$.

   (a) *Currency* 1 *is used as a medium of exchange when* $v_1^b > v_2^b$. *Then the DM consumption is given by* $c_1^* = (u')^{-1}(\epsilon/(\beta v_1^b))$.

   (b) *Currency* 2 *is used as a medium of exchange when* $v_2^b < v_1^b$. *Then the DM consumption is given by* $c_1^* = (u')^{-1}(\epsilon/(\beta v_2^b))$.

   (c) *Both currencies can be used as media of exchange when* $v_1^b = v_2^b$. *Then the DM consumption is given by* $c_1^* = (u')^{-1}(\epsilon/(\beta v_1^b))$.

Assumption 3 make the equilibrium in which domestic currencies are exclusively used in each country more likely in the following sense: For σ_1 large enough, currency 1 is less costly for the country 1 buyer in the DM so that $\epsilon_1^1 < \epsilon_2^1$. Similarly, for σ_2 small enough, currency 2 is less costly for the country 2 buyer in the DM so that $\epsilon_2^2 < \epsilon_1^2$. Unless the inflation rate of the domestic currency is extremely high, it is exclusively used as a medium of exchange in the domestic DM.

Now I assume the following under the floating exchange rates regime, which implies the SME in which each domestic currency is the only medium of exchange in the domestic bilateral meeting of each country.

Assumption 4.1*:* $\gamma_2(1)^{-1} > \gamma_2(2)^{-1}$

Assumption 4.2*: $\gamma_1(2)^{-1} > \gamma_1(1)^{-1}$.*

Assumption 5*: $\overline{\gamma}_1^{-1} = \overline{\gamma}_2^{-1} = 1$*

Assumption 4 makes currency j be a relatively good hedge for a country-j seller. In other words, the seller's domestic currency delivers a higher rate of return when the domestic sellers receive a lower productivity shocks in the CM whey they become old. Assumption 5 pins down the average inflation rates, which are common across currencies. Assumption 5 removes the expected transfers to young buyers from governments, which sharpen the welfare consequence of exchange rates regimes.

Proposition 3:

Under Assumption 1, 2, 3, 4, 5, the SME prevails, where the medium of exchange in the DM is the domestic currency in each country, $q_1^1 > 0, q_2^1 = 0$ and $q_2^2 > 0, q_1^2 = 0$. In each country $j \in \{1, 2\}$, $c_j^* = (u')^{-1}((\beta v_j^{s,j})^{-1})$. The nominal exchange rate is determinate:

$$\frac{\phi_{2,t}}{\phi_{1,t}} = \frac{M_{1,t}}{M_{2,t}} \frac{v_1^{s,1}}{v_2^{s,2}} \frac{(u')^{-1}[(\beta v_2^{s,2})^{-1}]}{(u')^{-1}[(\beta v_1^{s,1})^{-1}]}.$$

The nominal exchange rate increases in $v_2^{s,2}$ and decreases in $v_1^{s,1}$. Therefore, as the seller values the domestic currency more, the local currency appreciates.

4.3 Characterization of SMEs under fixed exchange-rates regime

Under the fixed exchange-rates regime, country 2 government removes currency 2 so that currency 1 is the only medium of exchange in the bilateral meetings of both countries.

Lemma 3:

A SME under fixed exchange-rates regime is the allocation such that $c_j^* = (u')^{-1}((\beta v_1^{s,j})^{-1})$ for each country $j \in \{1, 2\}$.

4.4 Welfare comparison across exchange-rates regime

When I compare the welfare under these two different exchange rate arrangements, I can do so only by comparing the expected utilities of buyers and the initial olds since the TIOLI offer make sellers' expected surplus from the DM trade zero and their expected utilities are invariant across regimes. Moreover, under Assumption 5, I can ignore the welfare effects of expected government transfers under two regimes. This does not mean that they are not relevant in reality but rather I make this assumption to sharpen the welfare implications of involving monies that play dual roles as intertemporal media of exchange and hedging devices.

Moreover, the following assumption leads to more sharper welfare implications.

Assumption 6: *The intertemporal elasticity of substitutions is greater than 1,* $xu''(x)/u'(x) \leq 1$.

Proposition 4

The allocation under floating exchange rate regime in Proposition 3 Pareto-dominates that of the fixed exchange rate regime in Lemma 3. More specifically, the DM consumption is higher in country-2 meetings under the flexible exchange rate regime than the fixed exchange rate regime. Moreover, the larger real balance leads to the bigger transfer to the initial old under the flexible exchange rate regime.

Proof of Proposition 3:

Let $\Delta^{b,2}$ *denote the expected utility difference of type-2 young buyers in the SME between flexible and fixed exchange rates regimes. I will show* $\Delta^{b,2} > 0$ *so that the flexible exchange rate yields a higher expected lifetime utilities for young buyers. First notice that when they get old, they do not bring any currencies to the CM due to the assumption that* $\beta^{-1} > v_k^{b,2}$ *for* *any k. Therefore, the expected continuation values of being the CM when old are equalized in both regimes. Moreover, the expected government transfer is zero in both regimes. Therefore,*

$$\Delta^{b,2} = \max_{q_2}\{-q_2 + \beta u(v_2^{s,2} q_2)\} - \max_{q}\{-q + \beta u(v_1^{s,2} q)\},$$

which is strictly positive since $v_2^{s,2} > v_1^{s,2}$ under Assumption 4 and 5. Let q_2^* be the maximand of the first maximization problem under the flexible exchange rates regime and q^* be that of the second problem under the fixed exchange rate regime. Assumption 6 implies $q_2^* > q^*$. This occurs because the intertemporal elasticity of substitutions being higher than 1 implies the substitution effect dominates the wealth effect so that the consumption is tilted forward. Since the type-1 buyers have the same expected utilities and demand the same real balances across regimes, the initial old holds more real balance at the beginning and consumes more in the initial CM. Therefore, the floating exchange rates regime Pareto-dominates the fixed exchange rates regime. Q.E.D.

The intuition of this result is as follows. The country-2 sellers have higher valuations for currency 2 than 1 due to the better hedging property on their productivity shock in the next CM. Thus, the country-2 buyers can save the cost of buying the DM goods by using this medium of exchange under the flexible exchange rates regime. However, this is impossible to practice under the fixed exchange rate regime since only currency 1 is circulated, which is a worse hedge for type-2 sellers. This leads to lower welfare of type-2 buyers under the fixed exchange rate regime because of the poorer terms of trade. Moreover, the larger real balance that type-2 buyers demand in the CM under flexible exchange-rates regime. This leads to a larger transfer to the initial old and higher welfare.

Notice that the Pareto dominance of the floating exchange rates comes from the decentralized market, where different hedging motive of buyers and sellers across determine the ranking of the preferred medium of exchange. This is different from the channel of the previous literature, in which nominal asset trading in a centralized financial market affects the welfare. Since the money-goods trades are ubiquitous in the real economy, the Proposition 4 implies that the floating exchange rate regime enhances the welfare more broadly when the domestic currencies are better hedges on sellers' productivity shocks. This channel does not require the access to financial markets. As a result, the welfare implications of exchange-rates regimes are richer and broader than previously thought to be.

Finally, the Pareto dominance of floating exchange-rates regime obviously depends on the assumptions made above. Therefore, the economic environment in this paper does not always support this regime in terms of welfare. For example, if the IES is below 1, the floating exchange-rates regime hurts the welfare of the initial olds by reducing the real balance transfers. So far, I have assumed that a domestic currency is a better hedge for the domestic sellers. However, if it is actually a worse hedge than the foreign currency, it is beneficial for the country to adopt the fixed exchange-rates, abandoning the domestic currency in order to enhance the welfare of young buyers. Still, the framework here provides a useful perspective on the distribution of the benefit adopting different exchange-rates regimes in the presence of uninsurable risks even with flexible prices.

5 Conclusion

This paper presents a model of multiple currencies with uninsurable risks, in which currencies serve as both an intertemporal medium of exchange and hedging device. Under a flexible exchange rate regimes, these two properties jointly pins down buyers' currency portfolio and the nominal exchange rate. When the returns for domestic currencies move in such a way that it insures sellers against sellers' future productivity shocks, the allocation under the flexible exchange rates regime Pareto-dominates the fixed exchange rates regime. This happens by improving the terms of trade in the DM meetings. This Pareto improvement occurs through the decentralized goods-money market transactions, which are ubiquitous in the real economy. Although this conclusion depends on the simplifying assumptions, the new welfare channel developed in this paper reveals the broader impact of exchange-rates regimes beyond financial markets in the presence of uninsurable risks even under flexible prices.

There are several interesting extensions to the model developed in this paper. Especially, though this paper assumes the exogenous parameter restrictions on inflation rates, it is interesting to investigate what additional structural features pin down the hedging properties of currencies. For example, how policy makers manipulate inflation rates across states to maximize the welfare of domestic agents is going to be an interesting direction. Comparing the resulting policies with the social planner's solution would inform of the optimal international cooperation of monetary policies.

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