

# GHANA COMMUNICATION TECHNOLOGY UNIVERSITY



## **FACULTY OF ENGINEERING DEPARTMENT OF COMPUTER ENGINEERING GROUP ONE CONTROL THEORY PROJECT REPORT ON A SIMPLE RC CIRCUIT**

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## INTRODUCTION:

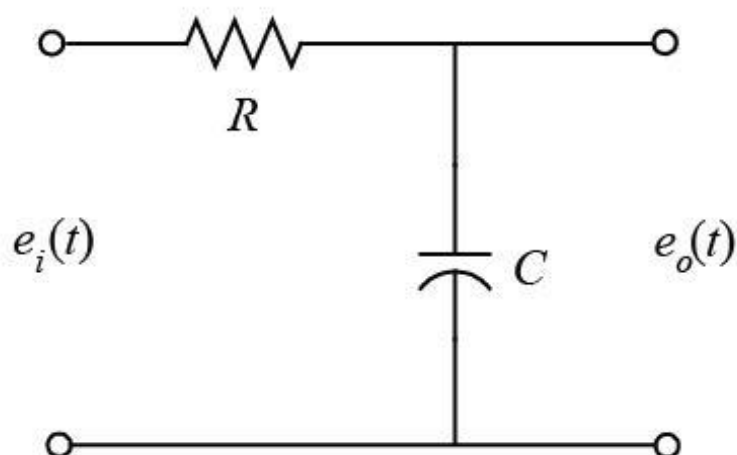
We'll look at how to accurately model an electrical circuit (RC circuit) in order to study its responses at various phases in this project. The time response, frequency response, and establishing control. Elements of this project report will be divided into three stages. In the rest of this report, we'll go over the various stages further.

### A- TIME RESPONSE:

#### Components:

- Arduino board.(Arduino Uno)
- USB cable.
- Breadboard.
- Resistor–10K
- Capacitor–100uF
- Jumper wires

### DIAGRAM OF CIRCUIT.



### **Component Description:**

The Arduino board will be used to generate the RC circuit's input and to measure the circuit's output. The breadboard will act as a ground for the circuit's layout and design, while the jumper wires will serve as links between circuit locations. The resistor will provide the circuit with initial resistance. When current is applied to the circuit, the capacitor in the circuit is constantly charging and discharging. This will allow us to determine some aspects in our response by plotting an efficient graph from the capacitor's output in terms of voltage.

### **SYSTEM MODELLING:**

We must first model our system before determining the time response. This enables us to precisely estimate our system's numerous responses from a theoretical standpoint when values are provided, as well as to develop a transfer function. To model this system, we must first develop an equation in voltage terms. (This is based on Kirchhoff's voltage law.) The variables are R for the resistor's resistance, C for the capacitor's capacitance, ( $e_i$ ) for the input voltage, and ( $e_o$ ) for the output voltage. In order to accomplish so, we examine a loop in one direction. We later devise a formula for the entire process.

$$e_i - iR - \frac{1}{C} \int i \, dt = 0$$

Converting the following from variables in time domain to Laplace, we get:

$$E_i(s) - I(s)R - \frac{1}{Cs} I(s) = 0$$

$$I(s) = \frac{E_i(s)}{R + \frac{1}{Cs}}$$

$$I(s) = \frac{E_o(s)}{\frac{1}{Cs}}$$

Expressing this in terms of output over input we get:

$$\frac{E_i(s)}{R + \frac{1}{Cs}} = \frac{E_o(s)}{\frac{1}{Cs}}$$

$$G(s) = \frac{E_o(s)}{E_i(s)} = \frac{\frac{1}{Cs}}{R + \frac{1}{Cs}}$$

In the end, we arrive at the transfer function:

$$G(s) = \frac{1}{RCs + 1}$$

Considering our transfer function shows us that our system is a first order system, we can manipulate the transfer function so that it has the standard form shown below:

$$\frac{K}{\tau s + 1}$$

Because the resistor and capacitor values affect the circuit's response time, we can calculate our time constant by multiplying the resistor and capacitor values. We chose a resistance value of 10k Ohms and a current of 100 micro-Farad because the circuit's time response is slow enough for the Arduino/Simulink combination to sample the circuit at a fast enough rate to provide a good image of the circuit's output. This provides us a 1 second sampling time.

We'll record the output voltage of the RC circuit for a change in input voltage in this experiment. We'll fit a model to the data based on the output voltage's resulting time response. After we've found a model, we'll compare it to the one we already have obtained from our transfer function.

### **PHYSICAL SETUP:**

Our circuit was built on a breadboard, and the Arduino board was attached to it. We then used digital pin 8 as the board's input and connected the circuit's negative side to ground. The output was likewise connected to the analog pin (A0). This enables the Arduino board to receive Simulink's input command and apply the input voltage to the circuit (via a Digital Output). The board also receives data from the circuit's output voltage (through an Analog Input) and sends it to Simulink.

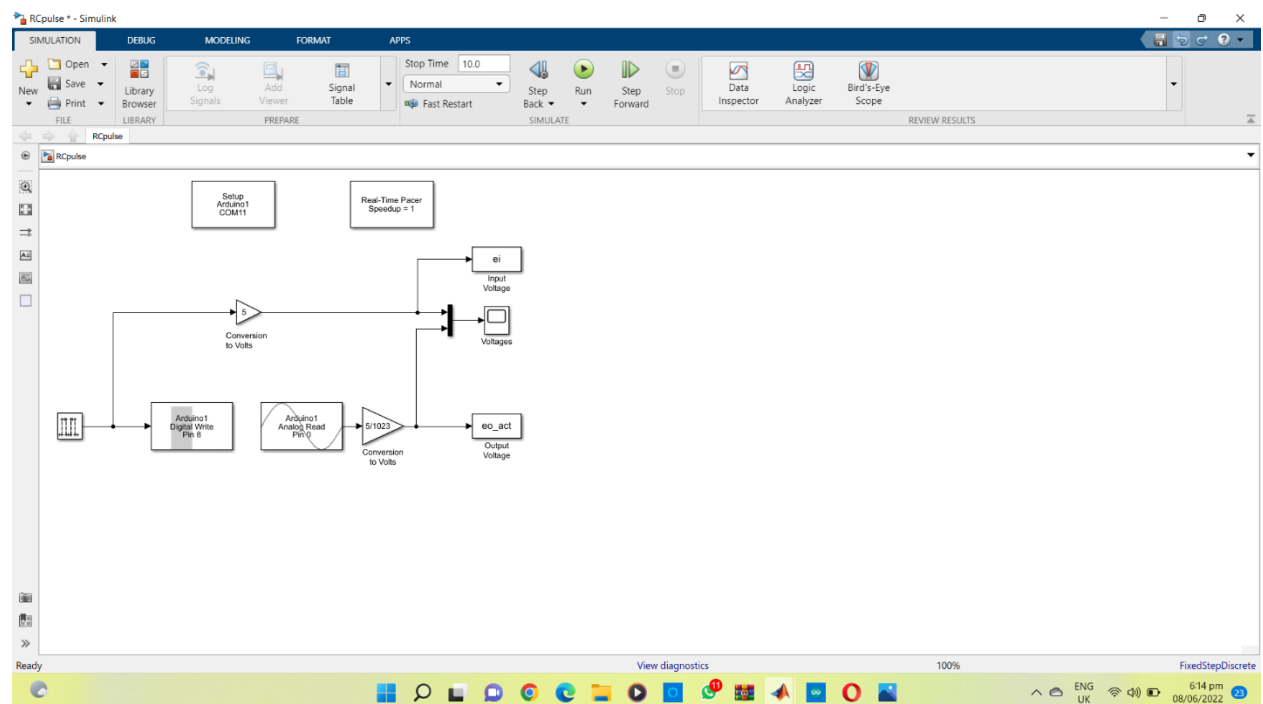
## SOFTWARE SETUP:

Simulink was connected to an Arduino and used to read data from the board and plot it in real time. To enable serial connection between the Arduino board and the Simulink, we had to incorporate the Arduino IO library and a few more packages.

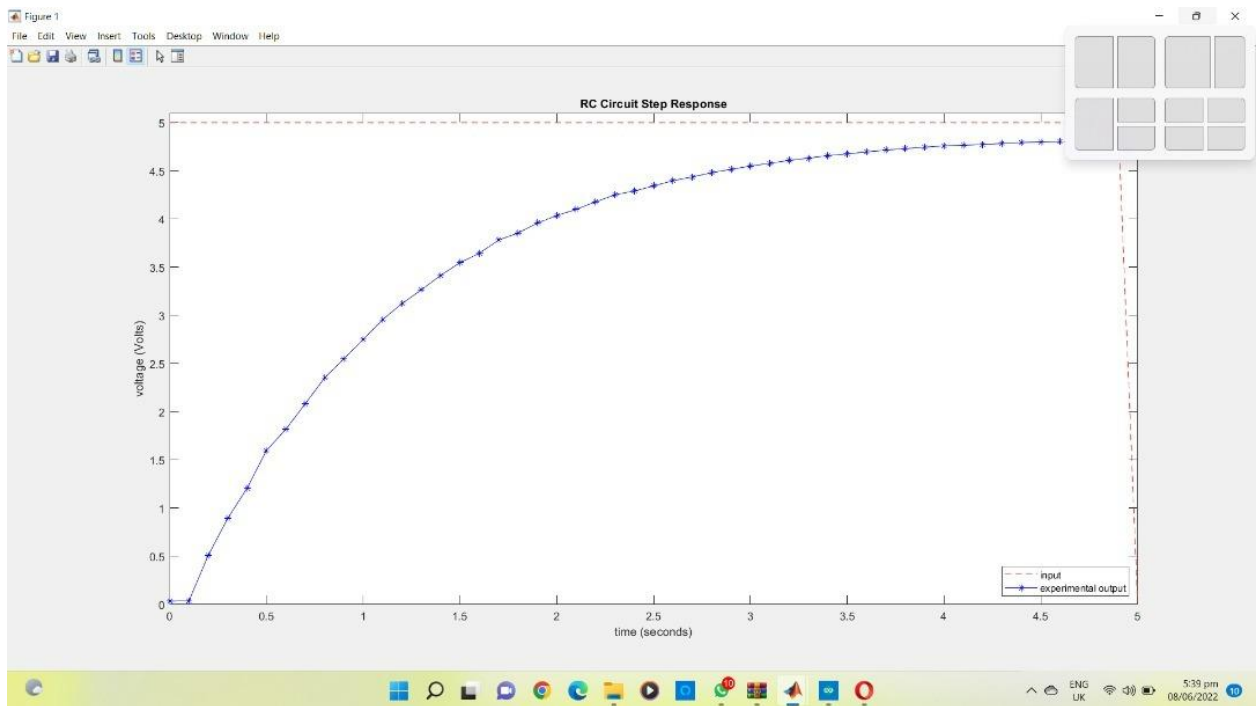
We plotted our graph after working on the physical setup, software setup, and establishing a good connection between the board and Simulink. We execute the Simulink model after it has been generated to receive the input and output voltage data. The capacitor discharging graph was omitted because the plot would only last 5 seconds.

Following the execution of our code, we received the following graph:

## SOFTWARE SETUP ONE



## OBSERVATION



## PARAMETER IDENTIFICATION:

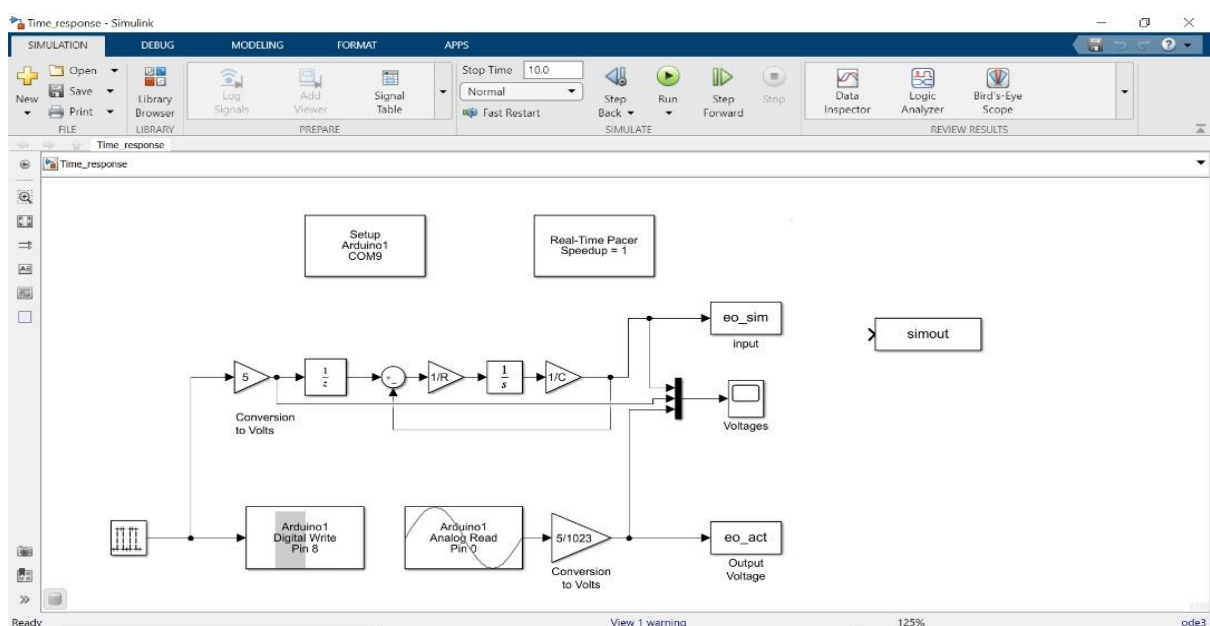
After plotting the graph, we observed that the shape of our plot is that of a first order response. This means we can write an equation in the first order standard form. Given as:

We can then find the values of  $K$  and our time constant ( $\tau$ ) from the recorded response data. Specifically, the steady-state value of the response indicates that the DC gain is  $K$ . Note, we verified with a Voltmeter that the output voltage generated via the Digital Output was very close to 5 Volts. Recalling that by definition the time constant ( $\tau$ ) represents the time it takes the system's response to reach approximately 63 percent of its total change, can be calculated from the following where 63.2 percent of 5 is approximately 3.16.

Since the step input occurs at approximately 0.10 seconds and the output reaches 3.16volts at approximately 1.16seconds, the time

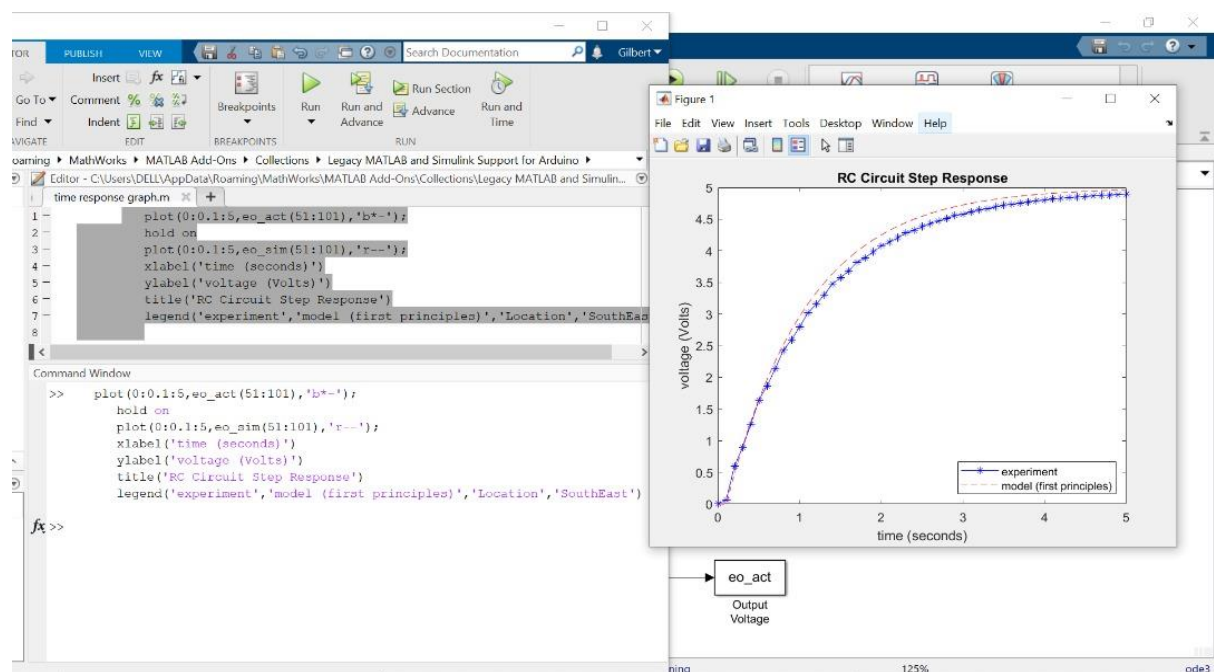
Writing the equation now as:

Next, we compared the two circuit models we derived, the first principles model and the black box model (Black box or experimental modeling is a method for the development of models based on process data). We used the equation from the model generated earlier in the first principles method and generated a Simulink equation for its highest order derivative. We then inserted the blocks into the already existing Simulink model so that we can directly compare the simulated output voltage to the actual recorded output voltage. Also, we added delay so that the timing of the simulated input voltage better matches the timing of the input voltage generated by the arduino board. Diagram shown below.





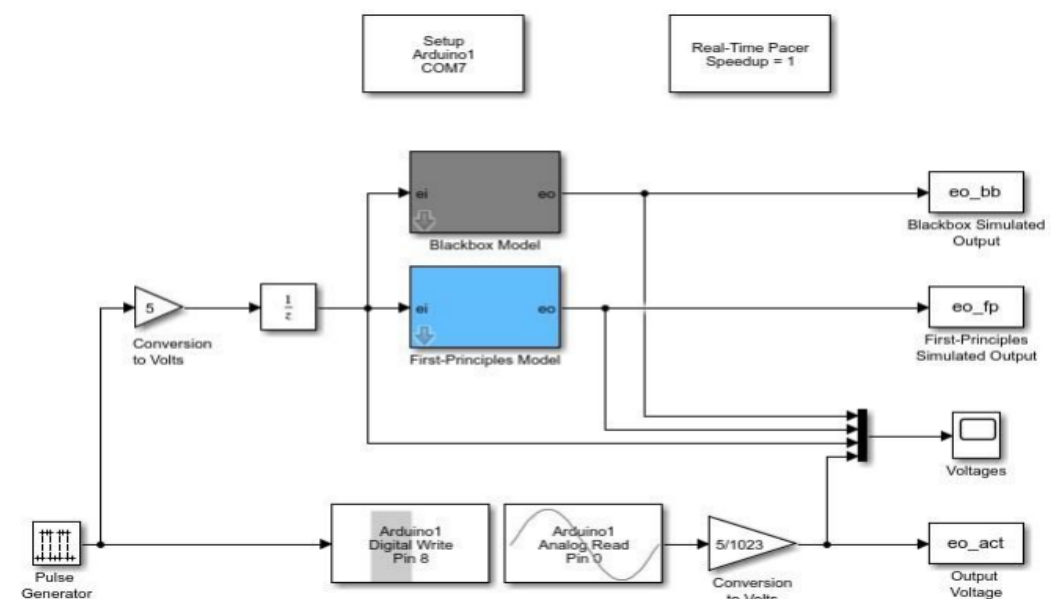
Running this model, we can then compare the simulated output voltage `eo_sim` to the actual experimental output voltage `eo_act`. Recall that the Pulse Generator block is set to output 0 for the first 5 seconds of the run in order to discharge the capacitor completely before stepping to 1.



Here, the above figure shows pretty good, though not exact, agreement between the output voltages predicted by our first-principles model and the physical circuit. In light of the above, the black box model closely matches the results, while the inclusion of more precise R and C estimates did not substantially improve the agreement with the first-principles model. It is hypothesized that, unmodeled resistance and capacitance of the input and output channels connected to the circuit are thought to be the cause of the error in the first principles model. It is generally the case that a black box model will be quite accurate when it is applied to the same

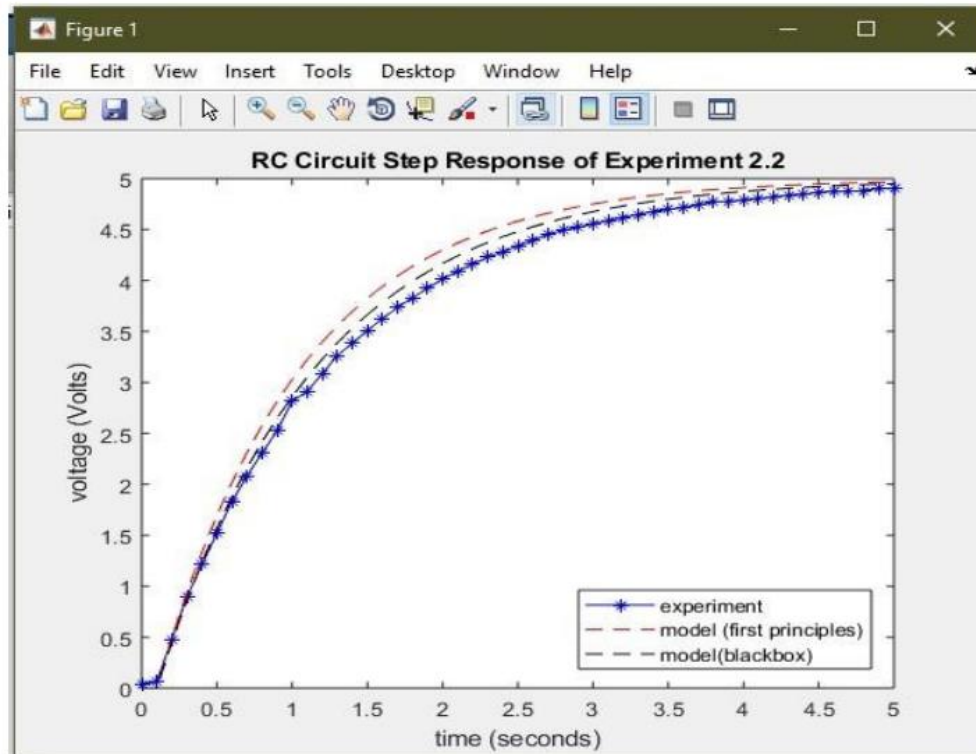
situation (input, environmental conditions, etc.) from which it was derived. A black box model can, however, break down if it is applied under different conditions. Our blackbox model should still work well in this case because we know that the structure of our first-principles model fits the structure of our blackbox model (both first order) and because the resistance and capacitance are unlikely to change much (at least over a short time period). A primary advantage of employing a black box model, aside from its accuracy, is that one does not need to understand the underlying system well in order to derive a model for it.

## Software setup



Observation

## OBSERVATION



## **B- FREQUENCY RESPONSE**

For the second stage, we will employ the same resistor capacitor (RC) circuit. Also, the hardware and software would be the same as used in the previous example, therefore there would be no need to touch on those parts again since we have covered them in the previous stage.

The aim of frequency response analysis is to see how a system reacts to different frequencies of sinusoidally varying inputs.

### **PURPOSE:**

In the previous activity we examined the time response of an RC circuit. The purpose of this activity is rather to understand the frequency response of the same circuit. Specifically, we are going to experimentally construct the magnitude plot portion of the Bode plot for the RC circuit. In this activity we will sweep through a range of frequencies, but we will employ square wave inputs rather than sine wave inputs.

When we consider a system's frequency response, we are specifically interested in how the amplitude and phase of the steady-state output compare to the sinusoidal input. One way to represent this amplitude (magnitude) data and this phase data is as a Bode plot. A Bode plot consists of two graphs, one being the magnitude of the response (the ratio of the output amplitude to the input amplitude,) versus frequency, and the other being the phase of the response versus frequency.

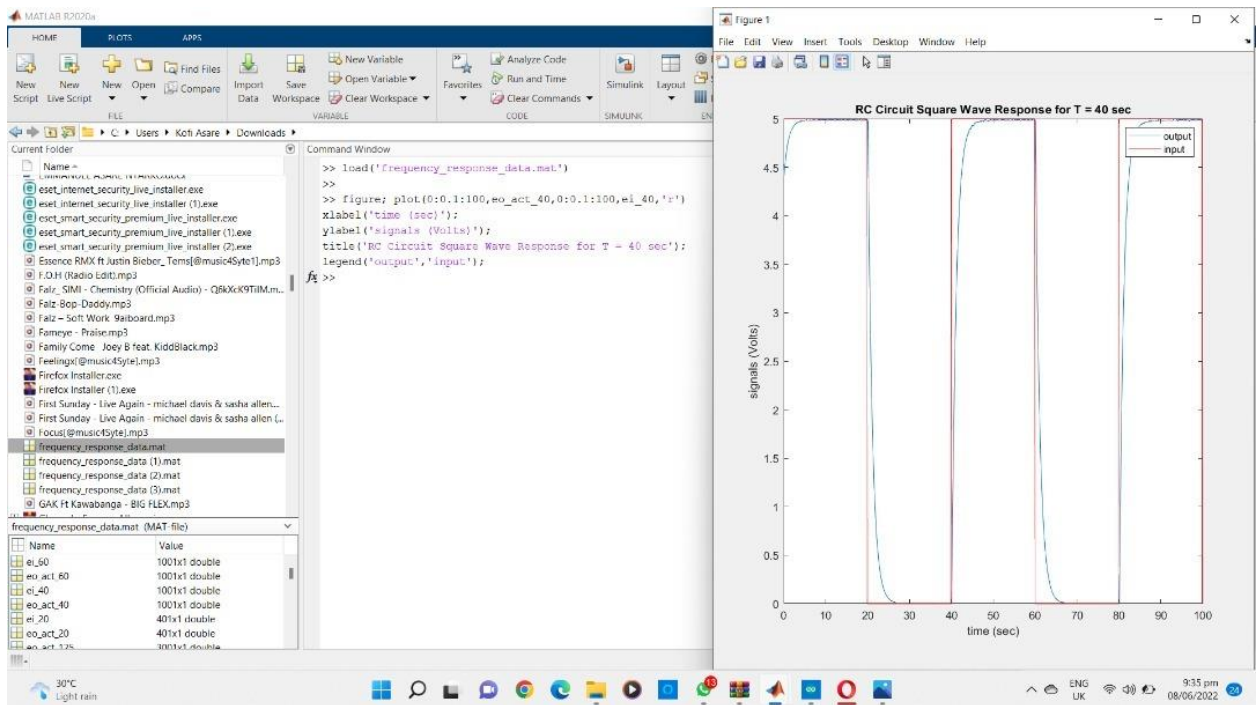
As the frequency of the input increases, the system has a harder time "keeping up" with the input. Through the course of the hardware experiment in the next section, we will try to build some intuition for this phenomenon. In the case of the system we are examining here, the frequency at which the input begins to be attenuated is

determined by the size of the electronic components R and C. Specifically, recall the transfer function for the RC circuit:

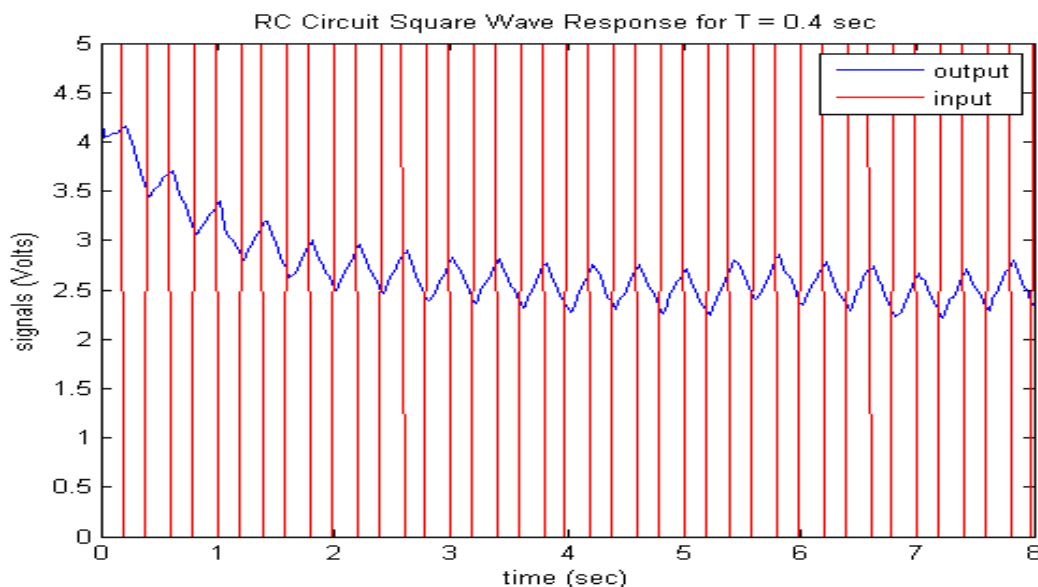
$$G(s) = \frac{1}{RCs + 1} = \frac{K}{\tau s + 1}$$

The break frequency for this circuit is determined by the location of its pole, that is, the break frequency equals  $1/\tau = 1/RC$ . In the case of this circuit,  $RC = 1$  and the break frequency is in the neighborhood of 1 rad/sec. Recalling the form of the RC circuit's step response, we can anticipate how the circuit will respond to a square wave input of varying frequencies. After we have found the break frequency for this circuit, we will sweep through a series of square wave inputs of varying frequency and record the amplitude of the output response. In order to get a complete picture of the RC circuit's frequency response, we need to capture frequencies ranging from at least 1 decade below the break frequency to at least 1 decade above the break frequency. Specifically, we will look at frequencies ranging from approximately 0.05 rad/sec (20 times smaller than 1 rad/sec) all the way up to about 30 rad/sec (30 times larger than 1 rad/sec).

We will examine the system's response for a slower frequency, specifically for T at 40 seconds which corresponds which corresponds to a frequency of approximately 0.157 rad/sec.



Finding the wave response for  $T=0.4\text{sec}$ .



As we expected, the output is much attenuated as compared to the input. Another thing that you will notice is that the output becomes less stable at these higher frequencies. This arises for a couple of reasons. For one, since the output amplitude is much smaller, any errors or disturbances become a larger percentage of the amplitude.

Second, as the frequency of the signals increase, a smaller sample time is required to get a sufficient number of points per period in order to reconstruct the output response. If the signals are sufficiently fast, we will run into the speed limitations of the experimental hardware and software. For the experimental set-up we are employing, we cannot achieve sample times faster than about 0.01 seconds. Therefore, at higher frequencies we may sometimes miss the true extremes of the output signal (the peaks and valleys could occur between samples).

