
Literature Review

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1 Parameters

- p_i : Number of Parameters
- m : Number of Samples

2 Naming

- HDP: High Dimensional Probability
- NTK: Neural Tangent Kernel
- MLP: Multi-Layer Perceptron, *a.k.a.* Fully Connected (FC) Neural Network
- ResNet: Residual Neural Network, MLP with skip connections

3 Convergence and Generalization of Wide Neural Network

Paper	Key Result	Condition Setup
[1]	Convergence in Polynomial Time with Polynomial Size of Data	Two or Three Layer MLP with SGD
[2]	ReLU networks is not Lipschitz but holds weaker Hölder smoothness	CNN or MLP with ReLU activation
[3]	Tighter and less conditioned bound.	2 layer MLP under less over-parameterized condition, ReLU, SGD
[4]	Analyze CNN with NTK and an algorithm designed for CNN inspired by NTK.	MLP, CNN, ReLU, SGD

4 Convergence and Generalization of Deep Neural Network

Paper	Key Result	Condition Setup
[5, 6, 7, 8]	NTK does not work under finite width infinity depth NN architecture. However, NTK does work under ResNet architecture. Initialization matters more for deep network.	MLP, ResNet, ReLU, SGD
[9]	An ordinary differential equations point of view	CNN, ReLU, SGD
[10]		MLP, CNN, ReLU, SGD

5 Convergence and Generalization of Wide Deep Neural Network

Paper	Key Result	Condition Setup
[11]	Triple Descent Phenomenon	Single Layer MLP, $p_1 \ll m \ll p_2 \ll m^2 \ll p_3$
[12, 13]	When both depth and width are finite, the convergence and generalization is determined by the ratio of p/m	

6 Spectral Analysis of NTK

Paper	Key Result	Condition Setup
[14]	The Spectrum of ResNTK shows stable frequencies while FC-NTK has spike frequencies	The eigenfunctions of ResNTK are (scaled) spherical harmonics and that its eigenvalues decay with frequency k at the rate of k^{-p} .
[15]	NTK fail to learn high spectral modes unless the sample size p is sufficiently large	
[16]	NTK fail to filter out high frequency noise and thus fail to reconstruct the signal, while CNN shows robust performance under such a case.	MLP, CNN, ReLU, SGD
[17, 18]	RFF, NTK and Neural Value Approximation	one line code to help RFF and NTK capture high frequency feature
[19]	FIM, eigenspace, etc	

A common agreement is that the NTK fail to capture high frequency feature at the initial setup.

7 Researches inspired by NTK

Paper	Key Result	Condition Setup
[20, 21]	A second order optimization method to train NN inspired by NTK	
[22]	In the transfer learning setup, the model convergence does not depends on input dimension but the feature dimension.	
[23]	Using NTK analyzes the generalization bound of robust optimization	
[24, 25, 26, 27]	NTK enables Gaussian Process and Bayesian Inference in designing Neural Network based bandit algorithm.	contextual bandit algorithm, reinforcement learning
[28]	How the intermediate parameter looks like after train the Neural Network with NTK.	
[29]	Explains the feature representation capability with NTK.	MLP, ReLU, SGD
[30]	Explain the superior performance of ResNet over MLP by analyzing NTK.	MLP, ResNet, ReLU, SGD
[31]	NTK under L_2 regularizer.	MLP, ReLU, SGD
[32]	NTK for Domain Adaptation.	
[33]	Reinforcement with NTK and Gaussian Process	
[34, 35]	NTK is similar to the Laplace Kernel	

8 Other Kernel

Paper	Key Result	Condition Setup
[36]	A new type of random features as a kernel function	
[37]	Composition Kernel	
[38]	Another sampling based kernel function	

9 Proofs of NTK

Paper	Key Result	Condition Setup
[39]	The first proof, a function space perspective	
[40]	Reverse Proof of NTK	
[41]	Parameter Space Perspective	
[42]	Random Kernel Function converges to NTK in expectation with high Probability.	MLP, ReLU, SGD
[43, 44, 45]	Under infinity width architecture, NN's parameters almost remaining unchanged during training. This is called lazy training.	

References

- [1] Zeyuan Allen-Zhu, Yuanzhi Li, and Yingyu Liang. “Learning and Generalization in Overparameterized Neural Networks, Going Beyond Two Layers”. In: ().
- [2] Alberto Bietti and Julien Mairal. “On the Inductive Bias of Neural Tangent Kernels”. In: ().
- [3] Atsushi Nitanda, Geoffrey Chinot, and Taiji Suzuki. “Gradient Descent Can Learn Less Over-parameterized Two-layer Neural Networks on Classification Problems”. In: ().
- [4] Sanjeev Arora et al. *On Exact Computation with an Infinitely Wide Neural Net*. Nov. 4, 2019. arXiv: 1904.11955 [cs, stat]. URL: <http://arxiv.org/abs/1904.11955> (visited on 08/14/2023). preprint.
- [5] Soufiane Hayou, Arnaud Doucet, and Judith Rousseau. “Exact Convergence Rates of the Neural Tangent Kernel in the Large Depth Limit”. In: ().
- [6] Soufiane Hayou, Arnaud Doucet, and Judith Rousseau. “The Curse of Depth in Kernel Regime”. In: ().
- [7] Etai Littwin, Tomer Galanti, and Lior Wolf. *On Random Kernels of Residual Architectures*. June 17, 2020. arXiv: 2001.10460 [cs, stat]. URL: <http://arxiv.org/abs/2001.10460> (visited on 08/17/2023). preprint.
- [8] Kaixuan Huang et al. *Why Do Deep Residual Networks Generalize Better than Deep Feedforward Networks? – A Neural Tangent Kernel Perspective*. Dec. 22, 2020. arXiv: 2002.06262 [cs, stat]. URL: <http://arxiv.org/abs/2002.06262> (visited on 08/17/2023). preprint.
- [9] Jiaoyang Huang and Horng-Tzer Yau. “Dynamics of Deep Neural Networks and Neural Tangent Hierarchy”. In: ().
- [10] Jongmin Lee et al. “Neural Tangent Kernel Analysis of Deep Narrow Neural Networks”. In: ().
- [11] Ben Adlam and Jeffrey Pennington. “The Neural Tangent Kernel in High Dimensions: Triple Descent and a Multi-Scale Theory of Generalization”. In: ().
- [12] Boris Hanin and Mihai Nica. “FINITE DEPTH AND WIDTH CORRECTIONS TO THE NEURAL TANGENT KERNEL”. In: (2020).
- [13] Yuan Cao and Quanquan Gu. “Generalization Bounds of Stochastic Gradient Descent for Wide and Deep Neural Networks”. In: ().
- [14] Yuval Belfer et al. “Spectral Analysis of the Neural Tangent Kernel for Deep Residual Networks”. In: ().
- [15] Blake Bordelon, Abdulkadir Canatar, and Cengiz Pehlevan. “Spectrum Dependent Learning Curves in Kernel Regression and Wide Neural Networks”. In: ().
- [16] Stefani Karp et al. “Local Signal Adaptivity: Provable Feature Learning in Neural Networks Beyond Kernels”. In: ().
- [17] Matthew Tancik et al. *Fourier Features Let Networks Learn High Frequency Functions in Low Dimensional Domains*. June 18, 2020. arXiv: 2006.10739 [cs]. URL: <http://arxiv.org/abs/2006.10739> (visited on 11/11/2022). preprint.
- [18] Ge Yang, Anurag Ajay, and Pulkit Agrawal. *Overcoming the Spectral Bias of Neural Value Approximation*. June 9, 2022. arXiv: 2206.04672 [cs, stat]. URL: <http://arxiv.org/abs/2206.04672> (visited on 08/14/2023). preprint.
- [19] Ryo Karakida, Shotaro Akaho, and Shun-ichi Amari. *Pathological Spectra of the Fisher Information Metric and Its Variants in Deep Neural Networks*. Sept. 27, 2020. arXiv: 1910.05992 [cond-mat, stat]. URL: <http://arxiv.org/abs/1910.05992> (visited on 07/08/2023). preprint.
- [20] Tianle Cai et al. “Gram-Gauss-Newton Method: Learning Overparameterized Neural Networks for Regression Problems”. In: ().
- [21] Arthur Jacot, Franck Gabriel, and Clement Hongler. “The Asymptotic Spectrum of the Hessian of DNN throughout Training”. In: (2020).
- [22] Alex Damian, Jason D Lee, and Mahdi Soltanolkotabi. “Neural Networks Can Learn Representations with Gradient Descent”. In: ().

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- [23] Zhun Deng, Kenji Kawaguchi, and Jiaoyang Huang. “How Shrinking Gradient Noise Helps the Performance of Neural Networks”. In: ().
 - [24] Michal Lisicki, Arash Afkanpour, and Graham W Taylor. “EMPIRICAL ANALYSIS OF REPRESENTATION LEARNING AND EXPLORATION IN NEURAL KERNEL BANDITS”. In: ().
 - [25] Dongruo Zhou, Lihong Li, and Quanquan Gu. “Neural Contextual Bandits with UCB-based Exploration”. In: ().
 - [26] Parnian Kassraie and Andreas Krause. *Neural Contextual Bandits without Regret*. Feb. 28, 2022. arXiv: 2107.03144 [cs, stat]. URL: <http://arxiv.org/abs/2107.03144> (visited on 08/17/2023). preprint.
 - [27] Yiling Jia et al. *Learning Neural Contextual Bandits Through Perturbed Rewards*. Mar. 18, 2022. arXiv: 2201.09910 [cs]. URL: <http://arxiv.org/abs/2201.09910> (visited on 08/17/2023). preprint.
 - [28] Philip M Long. “Properties of the After Kernel”. In: ().
 - [29] Yizhang Lou et al. “Feature Learning and Signal Propagation in Deep Neural Networks”. In: ().
 - [30] Tom Tirer. “Kernel-Based Smoothness Analysis of Residual Networks”. In: ().
 - [31] Colin Wei et al. “Regularization Matters: Generalization and Optimization of Neural Nets v.s. Their Induced Kernel”. In: ().
 - [32] Jun Wu et al. “Distribution-Informed Neural Networks for Domain Adaptation Regression”. In: ().
 - [33] Imene R. Goumri, Benjamin W. Priest, and Michael D. Schneider. “Reinforcement Learning via Gaussian Processes with Neural Network Dual Kernels”. In: *2020 IEEE Conference on Games (CoG)*. 2020 IEEE Conference on Games (CoG). Osaka, Japan: IEEE, Aug. 2020, pp. 1–8. ISBN: 978-1-72814-533-4. DOI: 10.1109/CoG47356.2020.9231744. URL: <https://ieeexplore.ieee.org/document/9231744/> (visited on 08/17/2023).
 - [34] Lin Chen and Sheng Xu. “DEEP NEURAL TANGENT KERNEL AND LAPLACE KERNEL HAVE THE SAME RKHS”. In: (2021).
 - [35] Amnon Geifman et al. *On the Similarity between the Laplace and Neural Tangent Kernels*. Nov. 14, 2020. arXiv: 2007.01580 [cs, stat]. URL: <http://arxiv.org/abs/2007.01580> (visited on 08/17/2023). preprint.
 - [36] Insu Han, Amir Zandieh, and Haim Avron. “Random Gegenbauer Features for Scalable Kernel Methods”. In: ().
 - [37] Vaishaal Shankar et al. “Neural Kernels Without Tangents”. In: ().
 - [38] David P Woodruff and Amir Zandieh. “Leverage Score Sampling for Tensor Product Matrices in Input Sparsity Time”. In: ().
 - [39] Arthur Jacot, Franck Gabriel, and Clément Hongler. *Neural Tangent Kernel: Convergence and Generalization in Neural Networks*. Feb. 10, 2020. arXiv: 1806.07572 [cs, math, stat]. URL: <http://arxiv.org/abs/1806.07572> (visited on 08/13/2023). preprint.
 - [40] James B Simon, Sajant Anand, and Michael R DeWeese. “Reverse Engineering the Neural Tangent Kernel”. In: ().
 - [41] Jaehoon Lee et al. “Wide Neural Networks of Any Depth Evolve as Linear Models Under Gradient Descent”. In: *Journal of Statistical Mechanics: Theory and Experiment* 2020.12 (Dec. 1, 2020), p. 124002. ISSN: 1742-5468. DOI: 10.1088/1742-5468/abc62b. arXiv: 1902.06720 [cs, stat]. URL: <http://arxiv.org/abs/1902.06720> (visited on 08/13/2023).
 - [42] Jiaming Xu and Hanjing Zhu. “One-Pass Stochastic Gradient Descent in Overparametrized Two-layer Neural Networks”. In: ().
 - [43] Lenaïc Chizat, Edouard Oyallon, and Francis Bach. *On Lazy Training in Differentiable Programming*. Jan. 7, 2020. arXiv: 1812.07956 [cs, math]. URL: <http://arxiv.org/abs/1812.07956> (visited on 08/17/2023). preprint.

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- [44] Mario Geiger et al. “Disentangling Feature and Lazy Training in Deep Neural Networks”. In: *Journal of Statistical Mechanics: Theory and Experiment* 2020.11 (Nov. 1, 2020), p. 113301. ISSN: 1742-5468. DOI: 10.1088/1742-5468/abc4de. URL: <https://iopscience.iop.org/article/10.1088/1742-5468/abc4de> (visited on 08/17/2023).
- [45] Behrooz Ghorbani et al. “Limitations of Lazy Training of Two-layers Neural Networks”. In: ().