

## HW 14

**Group 1****Grade:**

Suppose that the number of eggs laid by a certain insect has a Poisson distribution with mean  $\lambda$ . The probability that any egg hatches is  $p$ . Assume that the eggs hatch independently of one another. Find the

1. expected value of  $Y$ , the total number of eggs that hatch.
2. variance of  $Y$ .

**Hint:**

Law of Total Expectation and Law of Total Variance.

**Group 2****Grade:**

Let the random variables  $X$  and  $Y$  have a joint PDF which is uniform over the triangle with vertices at  $(0, 0)$ ,  $(0, 1)$ , and  $(1, 0)$ .

1. Find the joint PDF of  $X$  and  $Y$ .
2. Find the marginal PDF of  $Y$ .
3. Find the conditional PDF of  $X$  given  $Y$ .
4. Find  $\mathbb{E}[X \mid Y = y]$ , and use the total expectation theorem to find  $\mathbb{E}[X]$ .

**Group 3****Grade:**

Let  $X$  be a random variable with PDF

$$f_X(x) = \begin{cases} x/4 & 1 \leq x \leq 3, \\ 0 & \text{elsewhere.} \end{cases}$$

let  $A$  be the event  $X \geq 2$ .

1. Find  $\mathbb{E}[X]$ ,  $\mathbb{P}(A)$ ,  $f_{X|A}(x)$ , and  $\mathbb{E}[X \mid A]$ .
2. Let  $Y = X^2$ . Find  $\mathbb{E}[Y]$  and  $\text{Var}[Y]$ .

**Group 4****Grade:**

The random variable  $X$  has the PDF

$$f_X(x) = \begin{cases} cx^{-2} & 1 \leq x \leq 2, \\ 0 & \text{otherwise.} \end{cases}$$

1. Determine the value of  $c$ .
2. Let  $A$  be the event  $X > 1.5$ . Calculate  $\mathbb{P}(A)$  and the conditional PDF of  $X$  given that  $A$  has occurred.

**Group 5****Grade:**

A miner is trapped in a mine containing 3 doors. The first door leads to a tunnel that will take him to safety after 3 hours of travel. The second door leads to a tunnel that will return him to the mine after 5 hours of travel. The third door leads to a tunnel that will return him to the mine after 7 hours. If we assume that the miner is at all times equally likely to choose any one of the doors, what is the expected length of time until he reaches safety?

**Hint:**

Law of Total Expectation.