Yates and Goodman 3e Solution Set: 7.3.1

Problem 7.3.1 Solution

X and Y each have the discrete uniform PMF

$$P_X(x) = P_Y(x) = \begin{cases} 0.1 & x = 1, 2, \ldots, 10, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

The joint PMF of X and Y is

$$\begin{aligned} P_{X,Y}(x, y) &= P_X(x)\, P_Y(y) \\ &= \begin{cases} 0.01 & x = 1, 2, \ldots, 10; y = 1, 2, \ldots, 10, \\ 0 & \text{otherwise.} \end{cases} \end{aligned} \tag{2}$$

The event A occurs iff $X > 5$ and $Y > 5$ and has probability

$$\mathrm{P}[A] = \mathrm{P}[X > 5, Y > 5] = \sum_{x=6}^{10} \sum_{y=6}^{10} 0.01 = 0.25. \tag{3}$$

Alternatively, we could have used independence of X and Y to write $\mathrm{P}[A] = \mathrm{P}[X > 5]\,\mathrm{P}[Y > 5] = 1/4$. From Theorem 7.6,

$$\begin{aligned} P_{X,Y|A}(x, y) &= \begin{cases} \frac{P_{X,Y}(x,y)}{\mathrm{P}[A]} & (x, y) \in A, \\ 0 & \text{otherwise,} \end{cases} \\ &= \begin{cases} 0.04 & x = 6, \ldots, 10; y = 6, \ldots, 20, \\ 0 & \text{otherwise.} \end{cases} \end{aligned} \tag{4}$$