Project: Bertrand's Paradox

Abstract

In the studying of continuous random variable and joint random variable, we are going to learn that the probability of an event is defined as the area under the probability density function (PDF) and the PDF is derived from Cumulated Distribution Function (CDF). Surprisingly, for the same problem, the definition of CDF can also involve randomness. In this project, we will discuss Bertrand's Paradox, which is a problem within the classical interpretation of probability theory. We will also discuss the real-world scenarios where Bertrand's Paradox might take effect.

Bertrand's Paradox is a problem within the classical interpretation of probability theory. It was proposed by Joseph Bertrand in 1889 and reveals a paradox within probability theory.

Consider an equilateral triangle inscribed in a circle. Suppose a chord of the circle is chosen at random. What is the probability that the chord is longer than a side of the triangle?

The paradox is that there are different ways to define what is meant by "chosen at random", and different methods can give three different answers: 1/2, 1/3, or 1/4.

1. Choose a random radius of the circle and then choose a random point on this radius for the midpoint of the chord. In this case, the chord will be longer than the side of the triangle when the midpoint lies in the smaller circle whose radius is half of the original circle's radius. So, the probability is 1/4.

2. Choose two random points on the circle's circumference, and use these as the endpoints of the chord. In this case, the chord will be longer than a side of the triangle if and only if the two points lie in the same semicircle. So, the probability is 1/2.

3. Choose a random point inside the circle and draw the chord with this as one endpoint and the circle center as the other endpoint. The chord will be longer than a side of the triangle when the chosen point lies within the circle inscribed in the triangle. The area of this smaller circle is 1/3 of the area of the original circle, so the probability is 1/3.

Here are two real world scenarios where Bertrand's Paradox might apply:

1. **Uncertainty in Simulation Modeling:** When creating a simulation model for any real-world phenomenon (like climate change, financial markets, *etc.*), defining what is "random" can have significant impacts on the results. Different interpretations of randomness can lead to vastly different simulation outcomes, much like in Bertrand's paradox.

2. **Polling and Statistics:** Consider a company that wants to poll its customers about their satisfaction with a product. How the company defines a "random" sample of customers can dramatically affect the results. For instance, if they randomly select from their online customers, from customers who have bought recently, or from customers within a certain geographic region, they might get different responses in each case.

The paradox is a classic example of the importance of clear definitions in the application of probability, as well as the potential issues with intuition and ambiguity in the interpretation of "random".

Here are three concepts need to be covered in the presentation:

1. Explain the Bertrand's Paradox with figures and calculations.

2. Run a simulation to verify each of the three answers (You may want to work with the team who choose the Monte Carlo).

3. Maybe you can come up with the forth answer or more?