HW 12 Solution

Group 1 **Grade:**

A coin that has probability of heads equal to p is tossed successively and independently until a head comes twice in a row or a tail comes twice in a row. Find the expected value of the number of tosses in terms of p and $q = 1 - p$.

Hint:

The Law of Total Expectation is useful.

Group 2 **Grade:**

X and Y are independent identical discrete uniform $(1, 10)$ random variables. Find the conditional PMF $P_{X,Y|A}(x, y)$. Let A denote the event that

1. $\min(X, Y) > 5$.
2. $\max(X, Y) \leq 5$.

Hint:

For discrete random variables, you can always list all the possible values and their corresponding probabilities.

Group 3 **Grade:**

A supermarket has two customers waiting to pay for their purchases at counter I and one customer waiting to pay at counter II. Let Y_1 and Y_2 denote the numbers of customers who spend more than \$50 on groceries at the respective counters. Suppose that Y_1 and Y_2 are independent binomial random variables, with the probability that a customer at counter I will spend more than \$50 equal to .2 and the probability that a customer at counter II will spend more than \$50 equal to .3. Find the

1. joint probability distribution for Y_1 and Y_2.
2. probability that not more than one of the three customers will spend more than \$50.

Group 4 **Grade:**

The length of life Y for fuses of a certain type is modeled by the exponential distribution, with rate $\lambda = 1/3$ (The measurements are in hundreds of hours.).

(a) If two such fuses have independent lengths of life Y_1 and Y_2, find the joint probability density function for Y_1 and Y_2.

(b) One fuse in item (a) is in a primary system, and the other is in a backup system that comes into use only if the primary system fails. The total effective length of life of the two fuses is then $Y_1 + Y_2$. Find $\mathbb{P}(Y_1 + Y2 \leq 1)$.

Group 5 **Grade:**

A bus arrives at a bus stop at a uniformly distributed time over the interval 0 to 1 hour. A passenger also arrives at the bus stop at a uniformly distributed time over the interval 0 to 1 hour. Assume that the arrival times of the bus and passenger are independent of one another and that the passenger will wait for up to 1/4 hour for the bus to arrive. What is the probability that the passenger will catch the bus?

Hint:

1. Let Y_1 denote the bus arrival time and Y_2 the passenger arrival time.
2. Determine the joint density of Y_1 and Y_2.
3. Find $\mathbb{P}(Y_2 \leq Y_1 \leq Y_2 + 1/4)$.