Recitation Note

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Chapter 1: Set Theory, Probability, and Single Experiment

1. From Set Theory to Probability (of the single experiment)

   (a)

Set Theory	**Probability**
Element	Outcome
Subset	Event
Universal Set	Sample Space

   (b) $\mathbb{P}(Outcome)$ =the possibility that the outcome appears in the sample space. Outcomes are always **Mutually Exclusive**.

   (c) $\mathbb{P}(Event)$ =Event constitutes by different combinations of outcomes, including *null set* and the whole sample space.

   (d) Apparently, $\mathbb{P}(\emptyset) = 0$ since there is no element in *null set*, and $\mathbb{P}(\text{Sample Space}) = 1$

2. Probability Properties

   (a) $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$

   (b) **Mutually Exclusive:** $\mathbb{P}(A \cap B) = 0$ so that $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$

   (c) **Collectively Exhaustive:** $\mathbb{P}(A \cup B) = 1$

   (d) **Partitions:** $\mathbb{P}(\cup_{i=1}^{N} A_i) = \sum_{i=1}^{N} \mathbb{P}(A_i) = 1$

3. Conditional Probability

   (a) $\mathbb{P}(\mathbf{A}|\mathbf{B}) = \frac{\mathbb{P}(\mathbf{AB})}{\mathbb{P}(\mathbf{B})}$

   (b) **If A_i are Mutually Exclusive:** $\mathbb{P}(\mathbf{A}|\mathbf{B}) = \sum_{i=1}^{n} \mathbb{P}(\mathbf{A_i}|\mathbf{B})$

   (c) **If B_i are Partitions of A:** $\mathbb{P}(\mathbf{A}) = \sum_{i=1}^{m} \mathbb{P}(\mathbf{A} \cap \mathbf{B_i}) = \sum_{i=1}^{m} \mathbb{P}(\mathbf{A}|\mathbf{B_i})\mathbb{P}(\mathbf{B_i})-$*Law of Total Number*

4. Baye's Theorem: $\mathbb{P}(\mathbf{A}|\mathbf{B}) = \frac{\mathbb{P}(\mathbf{B}|\mathbf{A})\mathbb{P}(\mathbf{A})}{\mathbb{P}(\mathbf{B})}$

5. **Independent:** $\mathbb{P}(A \cap B) = \mathbb{P}(A)\mathbb{P}(B)$ so that $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A)\mathbb{P}(B)$

Chapter 2: Sequential Experiments

1. Tree Diagrams

2. Counting Methods

   (a) Multiplication: $n \times k_1 \times k_2 \times \ldots$

   (b) Sampling without Replacement

      i. Permutation: $\frac{n!}{(n-k)!}$

      ii. Combination: $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

iii. Combination is Permutation without order. Combination is also called n choose k.

(c) Sampling with Replacement: n^k

(d) Multiple Combination: $\binom{n}{k_1,k_2,\ldots,k_m} = \frac{n!}{k_1!k_2!\ldots k_m!}$ where $n = \sum_{i=1}^{m} k_i$

(e) Multiple Permutation?

3. Independent Trails

   (a) *Theorem 2.8:* The Probability of n_0 failures and n_1 successes in $n = n_0 + n_1$ Independent Trails with success rate p is

$$\mathbb{P}(n_0, n_1) = \binom{n}{n_1}(1-p)^{n0}p^{n_1} = \binom{n}{n_0}(1-p)^{n0}p^{n_1}$$

   (b) *Theorem 2.9:* $n = n_0 + n_1 + \ldots$ and success rates are $p_0, p_1, \ldots$

4. Reliability Analysis

   (a) Components in series

   (b) Components in parallel

Chapter 3: View Graphs

1. Discrete Random Variables: Assign numerical value to outcomes

2. Probability Mass Function: $\sum_{x\in X} \mathbb{P}_X(x) = 1$