Project on OFDM Communications

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Abstract

Orthogonal Frequency Division Multiplexing (OFDM) [1] is a popular modulation scheme in modern wireless communications. It achieves high symbol rates while being robust to frequency-selective fading and intersymbol interference. This project discusses the OFDM technique and evaluates its performance with simulations. The project consists of following parts as required:

(i) A description of the OFDM modulation and demodulation process.

(ii) A discussion of the advantages and disadvantages of OFDM technique.

(iii) The usage of pilot symbols for channel estimation in OFDM system.

(iv) Numerical results to evaluates the OFDM scheme.

1 Modulation and Demodulation

In designing the digital communication system, the complex physical environment (channel) limits the communication capacity and challenges the robustness of the system. A short duration, high symbol rate signal is not robust to multipath environment, which is the common situation, while a long duration , low symbol rate signal cannot satisfy modern communication demand. Thanks to the Uncertainty Principle [2], which states the long duration signal has a narrow bandwidth, if the total bandwidth can be used to transmit multiple narrow bandwidth signals simultaneously, the communication capacity can be improved without sacrificing the robustness. The Orthogonal Frequency Division Multiplexing (OFDM) technique is designed to achieve this goal. In this section we describe the modulation and demodulation process of Orthogonal Frequency Division Multiplexing (OFDM) system.

The Modulation of the OFDM system can be summarized as follows:

(i) A message is encoded into a bit stream with length $N\log_2 M$, where N is the number of narrowband subcarriers and M is the modulation order.

(ii) The bit stream is mapped into N complex information symbols using a modulation scheme such as M-PSK, M-QAM, *etc*. The resulting symbol sequence can be expressed as $[X_0, X_1, \ldots, X_{N-1}]$.

(iii) This sequence passes through a serial-to-parallel converter to form a block as $X = [[X_0], [X_1], \ldots, [X_{N-1}]]$.

(iv) Using the Inverse Fast Fourier Transform (IFFT), the block X is converted into time domain $x = [[x_0], [x_1], \ldots, [x_{N-1}]]$, where

$$x_n = \sum_{i=0}^{N-1} X_i e^{j2\pi \frac{i}{N} n} \qquad n = 0, 1, \ldots, N-1. \tag{1}$$

(v) The cyclic prefix is added by copying the last L samples of x and appending them to the beginning of x, where L is the length of the cyclic prefix. This block passes through a parallel-to-serial converter, and then converts the digital samples, $x[n] = [x_{N-L}, x_{N-L+1}, \ldots, x_{N-1}, x_0, x_1, \ldots, x_{N-1}]$, to an analog baseband signal $x(t)$ by a Digital to Analog Converter (DAC).

The signal $x(t)$ is upconverted to the carrier frequency f_c and transmitted over the channel. Suppose the channel is Linear Time Invariant (LTI), it would have the effect as a filter which can be characterized by Finite Impulse Response (FIR) $h(t)$. The channel also corrupts the signal with Additive White Gaussian Noise (AWGN) $w(t)$. The received signal is downconverted to baseband, passes through a low pass filter to remove the high frequency component (noise), and sampled by the Analog to Digital Converter (ADC), resulting digital samples $y[n] = x[n] \circledast h[n] + w[n]$, which can be expressed as $[y_{N-L}, y_{N-L+1}, \ldots, y_{N-1}, y_0, y_1, \ldots, y_{N-1}]$. Operator $\circledast$ is convolution.

The Demodulation of the OFDM system can be summarized as follows:

(i) The digital samples $y[n]$ contains $N + L$ elements. The cyclic prefix is removed by discarding the first L elements. After a serial-to-parallel converter, the resulting block is $y = [[y_0], [y_1], \ldots, [y_{N-1}]]$.

(ii) Using the FFT, the block y is converted into frequency domain $Y = [[Y_0], [Y_1], \ldots, [Y_{N-1}]]$, where

$$Y_i = \sum_{n=0}^{N-1} y_n e^{-j2\pi i \frac{n}{N}} \qquad i = 0, 1, \ldots, N-1. \tag{2}$$

(iii) Each element in the block is a complex information symbol. After a parallel-to-serial converter, the symbol sequence can be expressed as $[Y_0, Y_1, \ldots, Y_{N-1}]$.

(iv) These symbols are demodulated by the corresponding M-ordered demodulation scheme to recover the original bit stream with length $N \log_2 M$.

(v) The recovered bit stream is decoded to the original message.

2 Advantages and Disadvantages

The OFDM modulation and demodulation look trivial but some non-trivial features are hidden in the process and worth to mention explicitly.

2.1 Orthogonality, Symbol Rate and Guard Interval/Cyclic Prefix

To guarantee each narrowband symbol would not interfere others, the subcarriers in OFDM are designed to be orthogonal to each other. This means that the cross-correlation between any two subcarriers is zero. This property also guarantee the maximum number of subcarriers to be transmitted simultaneously. To achieve this, the subcarriers are spaced at $\triangle f = \frac{1}{T}$ Hz, where T is the OFDM symbol duration. This can be proved as follows.

Proof. Let $f_i = i\triangle f$ and $f_j = j\triangle f$ be the frequencies of two subcarriers, the cross-correlation between them is

$$\begin{aligned}
\langle e^{j2\pi f_i t}, e^{j2\pi f_j t} \rangle &= \frac{1}{T}\int_0^T (e^{j2\pi f_i t})(e^{j2\pi f_j t})^* \, \mathrm{d}t \\
&= \frac{1}{T}\int_0^T e^{j2\pi (f_i - f_j)t} \, \mathrm{d}t \\
&= \frac{1}{T}\int_0^T e^{j2\pi (i-j)\triangle f t} \, \mathrm{d}t \\
&= \frac{\sin(\pi T(i-j)\triangle f)}{\pi T(i-j)\triangle f} e^{j\pi T(i-j)\triangle f}. \\
\Re[\langle e^{j2\pi f_i t}, e^{j2\pi f_j t} \rangle] &= \frac{\sin(\pi T(i-j)\triangle f)}{\pi T(i-j)\triangle f} \cos(j\pi T(i-j)\triangle f) \\
&= \frac{\sin(2\pi T(i-j)\triangle f)}{2\pi T(i-j)\triangle f} \\
&= \mathrm{sinc}(2T(i-j)\triangle f).
\end{aligned}$$

To make the cross-correlation zero, we need to have $2T(i-j)\triangle f = k$ for some integer k when $i \neq j$. This implies that $T \cdot \triangle f = 1$. □

The fig. 1 shows the cross-correlation between two subcarriers with different spacing. The cross-correlation is zero when the subcarriers are spaced at $\triangle f = \frac{1}{T}$ Hz.

With the orthogonality of the subcarriers, the OFDM is able to transmit N information symbols (*i.e.* QAM, PSK, *etc.*) in parallel in each duration T, resulting in high OFDM symbol rate, $R_s = \frac{N}{T}$ symbols per second. And hence the duration T can be longer than other modulation schemes when the same amount of information is transmitted. The longer duration T gives OFDM rooms to add the guard interval in between each OFDM symbol. By setting the length of the guard interval to be longer than the channel delay spread, the OFDM system can combat the intersymbol interference caused by the multipath delay. The guard interval is usually implemented as the cyclic prefix as described above.

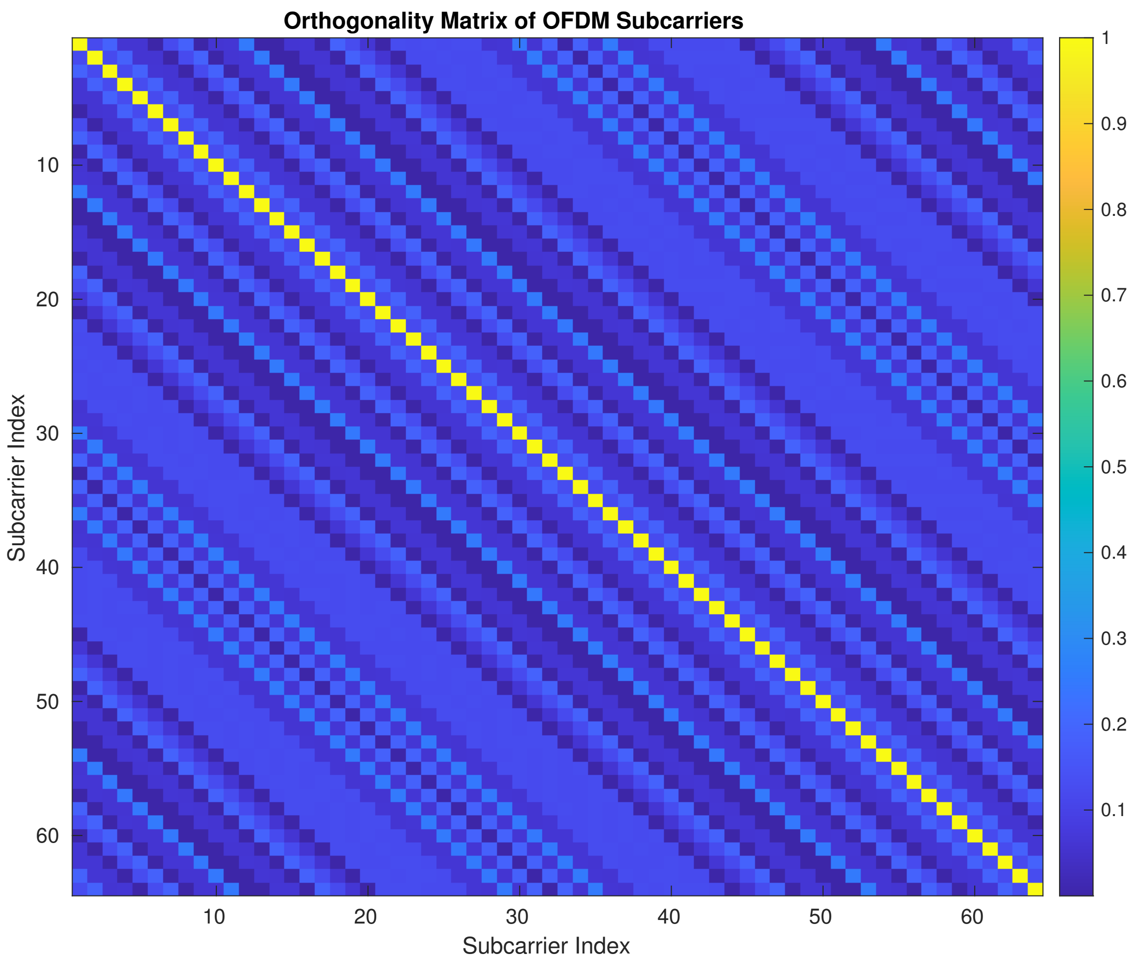


Figure 1: Subcarriers = 64, CP = 16, SNR = 20 dB.

2.2 Circular Convolution, Circulant Matrices and Intersymbol Interference (ISI)

The cyclic prefix leads to circular convolution between the channel impulse response and the transmitted signal. We show this mathematical fact and its properties as follows. The content is based on [3] and is modified with the content in [4].

We use the same notation as defined above. We first show that the cyclic prefix of $y[n]$ can be safely discard. Suppose the channel delay spread is the same as the length of the cyclic prefix L. Then the received signal $y[n]$ can be written as

$$\begin{bmatrix} y_N \\ y_{N-1} \\ \vdots \\ y_2 \\ y_1 \end{bmatrix} = \begin{bmatrix} h_0 & h_1 & \cdots & h_{L-1} & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & h_0 & \cdots & h_{L-2} & h_{L-1} & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots & h_0 & h_1 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & h_0 & \cdots & h_{L-1} \end{bmatrix} \begin{bmatrix} x_N \\ x_{N-1} \\ \vdots \\ x_2 \\ x_1 \\ x_N \\ \vdots \\ x_{N-(L-1)} \end{bmatrix} + \begin{bmatrix} w_N \\ w_{N-1} \\ \vdots \\ w_2 \\ w_1 \end{bmatrix} \tag{3}$$

where h_l is the channel impulse response of the l-th channel. The above matrices show that discarding the cyclic prefix at the receiver does not affect the received signal $y[n]$. It is also clear that the above representation will remain valid as long as the channel delay spread is less than the length of the cyclic prefix, since setting the h_l to be zero will not affect the above representation. But longer delay spread will violate that. This mathematical fact also tells that the ISI of OFDM symbols is eliminated by discarding the cyclic prefix in the OFDM system.

With some algebra, the above representation can be written as

$$\begin{bmatrix} y_N \\ y_{N-1} \\ \vdots \\ y_2 \\ y_1 \end{bmatrix} = \begin{bmatrix} h_0 & h_1 & \cdots & h_{L-3} & h_{L-2} & h_{L-1} & 0 & \cdots & 0 & 0 \\ 0 & h_0 & \cdots & h_{L-4} & h_{L-3} & h_{L-2} & h_{L-1} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ h_2 & h_3 & \cdots & h_{L-1} & 0 & 0 & 0 & \cdots & h_0 & h_1 \\ h_1 & h_2 & \cdots & h_{L-2} & h_{L-1} & 0 & 0 & \cdots & 0 & h_0 \end{bmatrix} \begin{bmatrix} x_N \\ x_{N-1} \\ \vdots \\ x_2 \\ x_1 \end{bmatrix} + \begin{bmatrix} w_N \\ w_{N-1} \\ \vdots \\ w_2 \\ w_1 \end{bmatrix} \tag{4}$$

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{w}. \tag{5}$$

The eq. (3) is the actual signal at the receiver while the eq. (4) is an analytical equivalent, which is circular convolution. The matrix $\boldsymbol{H}$ is an $N \times N$ matrix induced from an $N \times (N+L)$ matrix and is a special type of matrix called circulant matrix. Hence, the $\boldsymbol{H}$ can be expressed as

$$\boldsymbol{H} = h_0\boldsymbol{P}^0 + h_1\boldsymbol{P}^1 + h_2\boldsymbol{P}^2 + \cdots + h_{L-1}\boldsymbol{P}^{L-1}, \tag{6}$$

where $\boldsymbol{P}$ is an $N \times N$ permutation matrix

$$\boldsymbol{P} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ 1 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}. \tag{7}$$

The eq. (6) tells us the eigenvectors of $\boldsymbol{P}$ is the eigenvectors of $\boldsymbol{H}$ since $\boldsymbol{H}$ is a polynomial of $\boldsymbol{P}$. It is also safe to hypothesis that the eigenvectors of $\boldsymbol{H}$ are related to Fourier basis since $\boldsymbol{H}$ is sort of periodic. Substituting eq. (6) in to $\det(\boldsymbol{P} - \lambda\boldsymbol{I}) = 0$ and evaluating different N, we can verify our hypothesis and find the orthonormal eigenvector matrix of $\boldsymbol{H}$

$$\boldsymbol{F} = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \omega & \omega^2 & \cdots & \omega^{N-1} \\ 1 & \omega^2 & \omega^4 & \cdots & \omega^{2(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \omega^{N-1} & \omega^{2(N-1)} & \cdots & \omega^{(N-1)(N-1)} \end{bmatrix} \tag{8}$$

$$\omega = e^{-j2\pi\frac{1}{N}}. \tag{9}$$

The matrix $\boldsymbol{F}$ is the same as the discrete Fourier transform matrix. It has the following properties: (i) $\boldsymbol{F}^{-1} = \boldsymbol{F}^{\mathsf{H}}$; (ii) $\boldsymbol{F}^{\mathsf{H}}\boldsymbol{F} = \boldsymbol{F}\boldsymbol{F}^{\mathsf{H}} = \boldsymbol{I}$; (iii) $\boldsymbol{H} = \boldsymbol{F}\Lambda\boldsymbol{F}^{\mathsf{H}} = \boldsymbol{F}^{\mathsf{H}}\Lambda\boldsymbol{F}$ where Λ is a diagonal matrix with the eigenvalues of $\boldsymbol{H}$, *i.e.* the frequency domain samples of the channel transfer function. And leads to the following result of the demodulated OFDM symbol

$$
\begin{aligned}
\boldsymbol{Y} &= \boldsymbol{F}\boldsymbol{y} \\
&= \boldsymbol{F}(\boldsymbol{H}\boldsymbol{x} + \boldsymbol{w}) \\
&= \boldsymbol{F}(\boldsymbol{F}^{\mathsf{H}}\Lambda\boldsymbol{F}\boldsymbol{F}^{-1}\boldsymbol{X} + \boldsymbol{w}) \\
&= \Lambda\boldsymbol{X} + \boldsymbol{F}\boldsymbol{w}.
\end{aligned} \tag{10}
$$

The eq. (10) states that the orthogonality of the subcarriers is preserved in the demodulated OFDM symbols.

2.3 Equalization, Multipath Propagation and Frequency-Selective Fading

The eq. (10) reveals a simple equalization scheme for the OFDM system. The equalization can be done by multiplying the demodulated OFDM symbols by the inverse of the channel transfer function.

$$
\boldsymbol{X} = \Lambda^{-1}\boldsymbol{Y}. \tag{11}
$$

This is zero-forcing equalization. It is simple but doesn't work well when the channel transfer function has zeros or offsets at the frequency domain samples. In this case, the equalization will amplify the noise at the zeros. Besides, even when the offsets are not existed, the zero-forcing equalization doesn't benefit Signal to Noise Ratio (SNR). The minimum mean square error (MMSE) equalization, adaptive equalization, *etc.* are better approaches.

The existence of offsets in the channel transfer function is referred to frequency selective fading and caused by multipath propagation. This can be illustrated as follows. The channel impulse response can be expressed as

$$
h(t) = \sum_{l=0}^{L-1} h_l \delta(t - \tau_l), \tag{12}
$$

When $L = 0$, the channel transfer function (the Fourier transform of h_0) is flat across whole bandwidth and the channel is said to be flat fading.

$$
\boldsymbol{F} h_0 \delta(t - \tau_0) = h_0 \cdot \boldsymbol{F}\delta(t - \tau_0) = h_0 e^{j2\pi f \tau_0}. \tag{13}
$$

When $L > 0$, *a.k.a.* multipath propagation, the channel transfer function becomes

$$
\begin{aligned}
\boldsymbol{F} h(t) &= \boldsymbol{F} \sum_{l=0}^{L-1} h_l \delta(t - \tau_l) \\
&= \sum_{l=0}^{L-1} h_l \boldsymbol{F}\delta(t - \tau_l) \\
&= \sum_{l=0}^{L-1} h_l e^{j2\pi f \tau_l}.
\end{aligned} \tag{14}
$$

If $|f\tau_{l_i} - f\tau_{l_j}| = \frac{4k-1}{2}$ for some integer k, then $\operatorname{sgn}(e^{j2\pi f \tau_{l_i}}) = -\operatorname{sgn}(e^{j2\pi f \tau_{l_j}})$ and cause offsets in the channel transfer function fig. 2. Frankly, the wideband signals suffer more from the frequency selective fading than narrowband signals because the wideband signals will have more frequency offsets across the bandwidth, while the narrowband signals can be considered flat because the bandwidth may locate in between the frequency offsets (*a.k.a.* coherent bandwidth). The OFDM system is wideband but it is robust to frequency-selective fading because the subcarriers are spaced at $\triangle f = \frac{1}{T}$ Hz. This spacing is usually much smaller than the coherence bandwidth of the channel. Therefore, the channel can be considered flat over each subcarrier. This allows the OFDM system to use a simple equalizer (*i.e.* zero-forcing) to combat the frequency-selective fading.

2.4 Peak to Average Power Ratio, Doppler Shift and Intercarrier Interference (ICI)

We have explored many advantages of the OFDM system above and need to cover its weakness. Historically, the computation overhead of Discrete Fourier Transform (DFT) could be one but has been perfectly solved by Fast Fourier Transform (FFT). The high Peak to Average Power Ratio (PAPR) and sensitivity to Doppler Shift are two remaining issues.

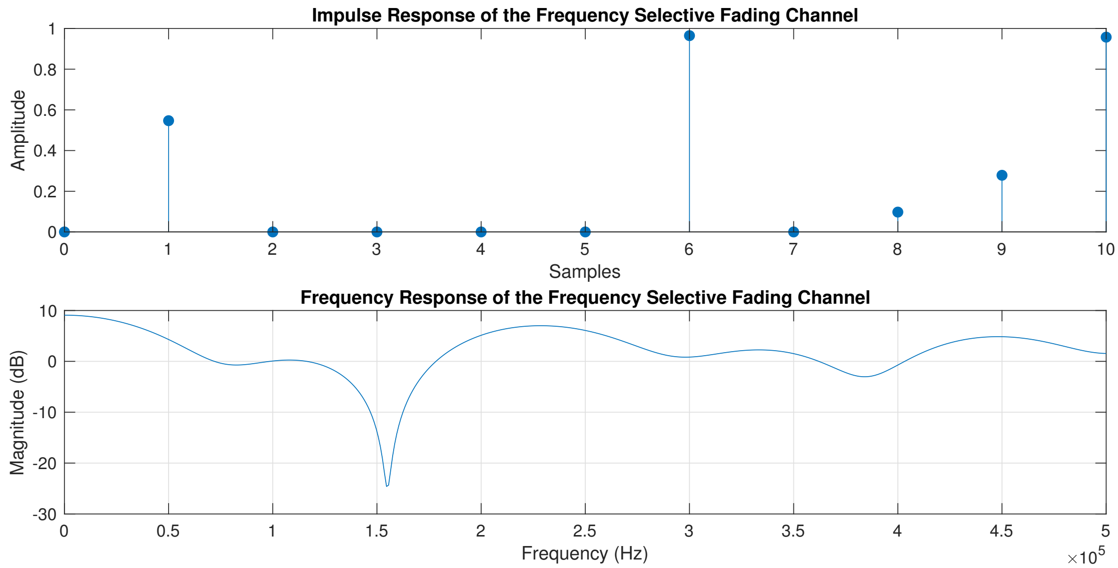


Figure 2: Ten paths with different amplitudes.

The high PAPR is a common issue in the OFDM system. The OFDM system employs a large number of subcarriers in each OFDM symbol duration and has independent information symbols on each subcarrier. The independence means that the phases and amplitudes of the subcarriers can align at certain time instances, resulting high peak power. The high peak power requires the power amplifier to have a large dynamic range, which is expensive and inefficient. The high PAPR also causes the signal to be distorted by the power amplifier, which degrades the system performance. The quantitative analysis of PAPR issue is shown below. The PAPR is defined as

$$\mathrm{PAPR} = \frac{\max|x_n|^2}{\mathbb{E}[|x_n|^2]}, \quad n = 0, 1, \ldots, N-1. \tag{15}$$

Since the subcarriers are orthogonal (independent), the central limit theorem can be applied on eq. (1) and x_n can be approximated as complex Gaussian random variables with zero mean and variance σ^2 when N is sufficiently large (which is preferred in practice). Then the max PAPR can be approximated as

$$\max|x_n|^2 = \sum_{i=0}^{N-1} \sigma^2 = N\sigma^2 \tag{16}$$

$$\mathbb{E}\left[|x_n|^2\right] = \sum_{i=0}^{N-1} \left[\frac{\sigma}{\sqrt{N}}\right]^2 = \sigma^2 \tag{17}$$

$$\max \mathrm{PAPR} = N. \tag{18}$$

The probability of PAPR exceeding certain threshold P_0 is

$$\mathbb{P}(\mathrm{PAPR} > P_0) = 1 - (1 - e^{-P_0})^N. \tag{19}$$

The eq. (19) states the PAPR will exceed certain threshold with high probability when N is large. And the eq. (18) states the PAPR can be as large as N. The high PAPR can be compensated by clipping the OFDM signal above some threshold, using spread spectrum to reduce the peak power, *etc*. The fig. 3 shows the PAPR of the OFDM system with different number of subcarriers.

Another issue is that OFDM is sensitive to Doppler Shift. The Doppler Shift in the channel will cause frequency shift in the received signal. This will reduce the SNR because the matched filter is designed to match the carrier frequency. The OFDM suffers more from the Doppler Shift because the OFDM system has a large bandwidth and the Doppler Shift can cause the subcarriers to shift out of the subcarrier spacing. This will cause the orthogonality of the subcarriers to be

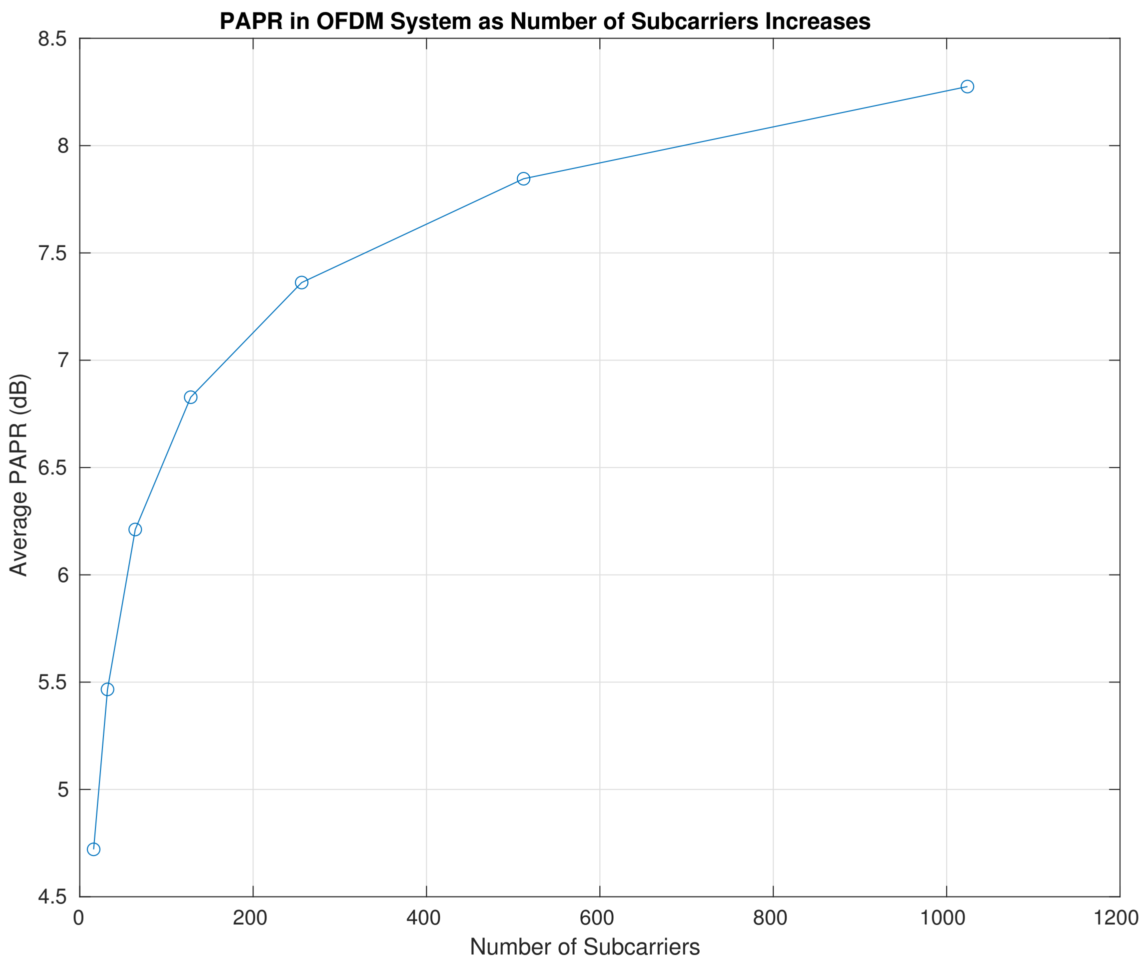


Figure 3: BPSK, Average of 5 simulations for each number of subcarriers.

violated and the intercarrier interference (ICI) to be introduced. From the matrix analysis perspective, the Λ in eq. (10) will not be diagonal under the Doppler Shift. The Doppler Shift can be compensated in many ways.

(i) Increase the subcarrier spacing to reduce the sensitivity to the Doppler Shift. But this will reduce the symbol rate and the system capacity.

(ii) Use equalization to improve the SNR. However, equalization does not work well when the Doppler Shift is large. Besides, the equalization needs to know the channel transfer function, which is not always viable in the system or cannot be accurately estimated in the fast time-varying channel.

(iii) Error Correction Coding (ECC) can be used to improve the system robustness. However, the ECC will introduce additional overhead and reduce the system capacity.

(iv) New modulation schemes such as the Orthogonal Time Frequency Space (OTFS), which designs the waveform to be invariant to the Doppler Shift by using delay-Doppler domain signal representation. However, OTFS is not easy to be equalized as OFDM and the system complexity is higher than OFDM. The delay-Doppler domain channel estimation is also challenging.

In conclusion, although OFDM system is able to overcome ISI and frequency-selective fading while maintain high symbol rate under complex channel condition, the maximum capacity is still bounded by the high PAPR and Doppler Shift. Overcoming these issues is the key to design the next generation wireless communication system.

3 Synchronization and Pilot Symbols Aided Channel Estimation

In the above content, the mathematical properties of OFDM is elegant but the real life application could be complex. For example, how to align the received signal with the transmitted signal? How to detect the number of multipath in the channel? How to detect the frequency offset and align subcarriers? The above questions are referring synchronization and channel estimation. In this section, we will discuss the related techniques.

3.1 Synchronization

In designing the optimal receiver, a matched filter containing possible transmitted waveforms is used to classify the received signal. However, the OFDM baseband signal $x(t)$ is a superposition of multiple subcarriers' data symbols by the Inverse Fourier transform. And the matched filter is actually applied on the Fourier transform of the received signal $\boldsymbol{Y}$. To guarantee the optimal performance, the receiver should be able to align the FFT operator with the observed sample stream. This process is time synchronization.

The time synchronization of the OFDM system can be achieved by a correlation-based approach. Due to the usage of cyclic prefix, signal contains a part of repeated pattern of itself. The receiver can calculate the sliding autocorrelation of the observed symbol stream using a lag commensurate with the length of the cyclic prefix. The magnitude of the result will reach a peak (close to 1) at the instant the corresponds to the end of the cyclic prefix, which is also the start of the desired signal. The limitations of the correlation based synchronization is that it does not optimize SNR. There are other methods to achieve time synchronization, such as the MLE/MAP/MMSE, *etc.* in corporate with the cyclic prefix for balanced performance.

The OFDM system also needs frequency synchronization because the subcarriers are frequency relevant. A common approach is using Phase-locked loop (PLL) [5]. The intuition behind the PLL is that any consecutive two samples are supposed to have the same phase shift in an LTI system. So we can generate a compensation phase shift to align the incoming signals. The compensation phase shift will be used to generate a reference signal to be compared with the next input signal and then update the compensation phase shift by averaging all the previous phase shifts. Once the compensation phase shift stop changing, the optimal result of the PLL is achieved.

The OFDM system is less sensitive to the time synchronization so the mentioned technique is usually adequate. But the frequency synchronization is more critical and challenge. To achieve high resolution frequency synchronization result, the knowledge of the channel is preferred. This process is channel estimation, and it can be done at the receiver with the help of pilot symbols.

3.2 Pilot Symbols

The Pilot Symbols are data symbols, which are known by transmitter and receiver, inserted into the transmitted signal. The receiver can use the known data symbols to estimate the channel and then compensate the channel effect on the received signal or perform synchronization.

There are many ways to insert the pilot symbol. It can be interpolated purely in between time slots, in between frequency spacing, or in between both. (i) If the pilot symbol is added on the k-th subcarrier, while the received signal detect the pilot symbol on the k'-th subcarrier, the receiver can estimate the frequency offset by the difference of k and k'; (ii) If the pilot symbol is added on the l-th time slot, while the received signal detect the pilot symbol on the l'-th time slot, the receiver can estimate the time offset by the difference of l and l'; (iii) If the pilot symbol has magnitude A, while the received signal detect the pilot symbol with magnitude A', the receiver can estimate the channel gain by the ratio of A and A'.

The pilot symbol method is an effective and flexible technique. The occupancy of bandwidth does reduce the spectral efficiency, but the performance gain is significant. The pilot symbol method is widely used in the OFDM system. The channel estimation not only helps the synchronization but also improves the performance of other signal processing techniques, such as equalization, beamforming, and precoding. The fig. 4 shows the channel estimation aided by pilot symbols.

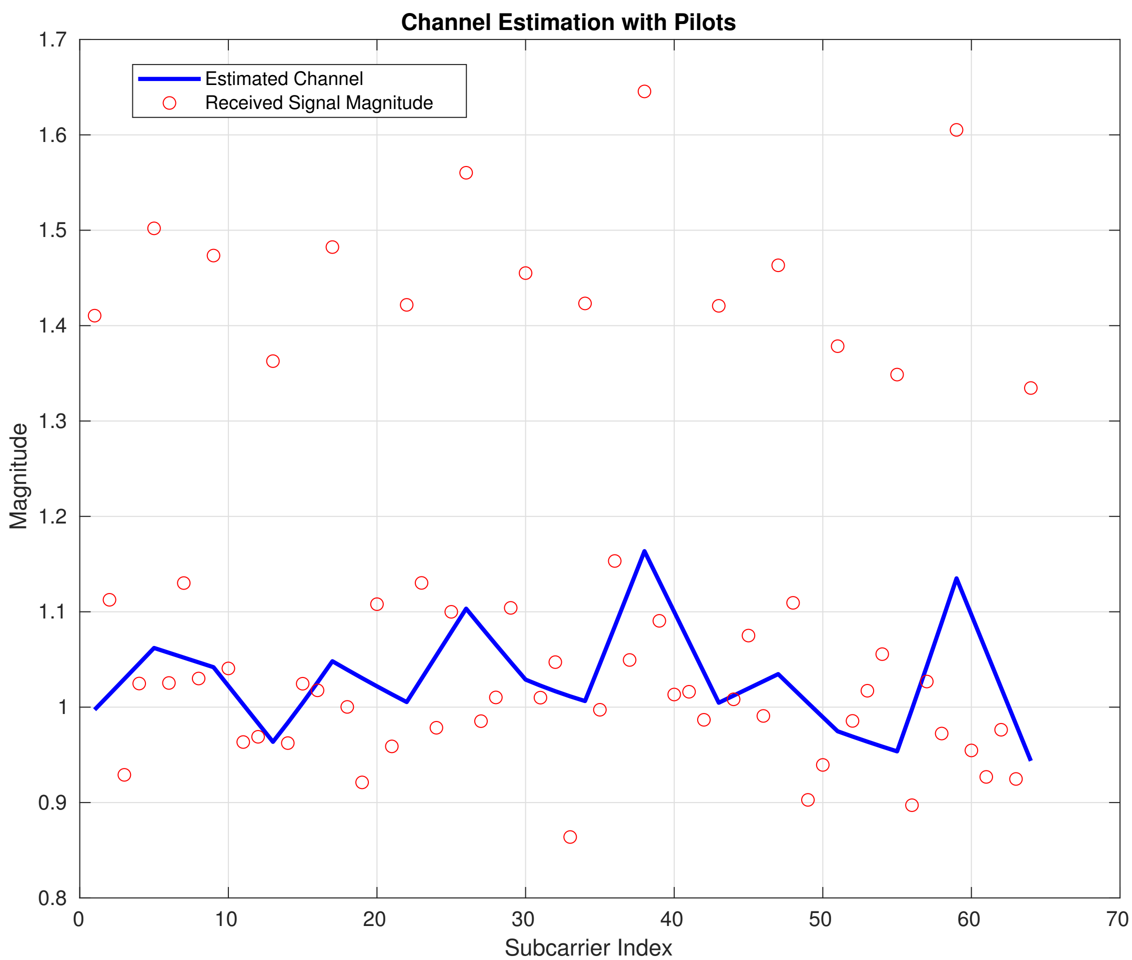


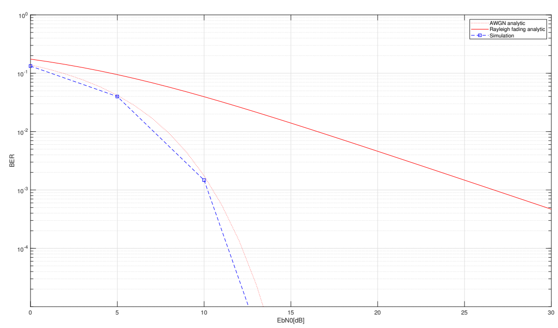
Figure 4: BPSK, 64 subcarriers, 16 CP, 1000 OFDM symbols, pilot value $1 + 1j$

4 Simulation Results

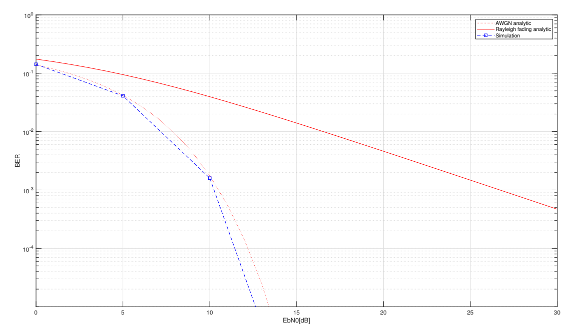
We conduct experiments to illustrate the properties of the OFDM system. The experiment setup is as follows.

- The experiment is done in MATLAB.
- The modulation scheme is 16-QAM.
- The OFDM system has 64 subcarriers.
- There are 10^5 iterations. Each iteration has 3 OFDM symbols.
- The cyclic prefix is added in between OFDM symbols and its length is either 3 or 16.
- The Channel condition is either AWGN or Rayleigh fading. For the Rayleigh fading channel, the channel is set to have 5 taps with the delay spread being $[0, 3, 5, 6, 8]$ and the channel power being $[0, -8, -17, -21, -25]$.
- We evaluate the average Bit Error Rate (BER) vs. the EbN0 (dB) over iteration with EbN0 being $[0 : 5 : 30]$.
- The simulation result is shown in fig. 5.

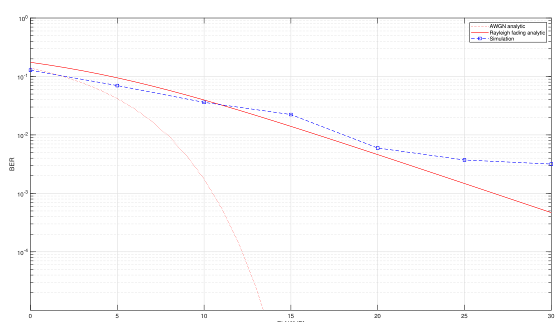
(a) AWGN Channel with 3-tap CP



(b) AWGN Channel with 16-tap CP



(c) Rayleigh Channel with 3-tap CP



(d) Rayleigh Channel with 16-tap CP

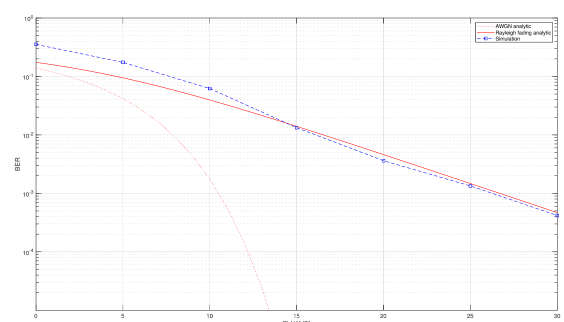


Figure 5: BER performance for OFDM system with 16-QAM. The figs. 5a and 5b are for AWGN channel. The different CP length does not affect the simulation results since the flat fading has constant complex gain on the transmitted signal. The figs. 5c and 5d are for Rayleigh fading channel with 5 taps. The simulation results show that the BER exceeds the analytical result if the CP length is shorter than the delay taps, while the BER will be closed to the analytical result if the CP length is longer than the delay taps. The result implies that the OFDM system is only subject to flat fading as long as the CP length is longer than the channel delay spread.

5 Conclusion

In this report, we introduce the OFDM modulation and demodulation process, discuss its mathematical properties, and evaluate its performance with simulations. We show that the OFDM system is robust to frequency-selective fading and intersymbol interference. The simulation results demonstrate that the OFDM system is only subject to flat fading as long as the cyclic prefix length is longer than the channel delay spread. The OFDM system is widely used in modern wireless communications due to its high symbol rates and robustness to channel impairments.

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