HW 5

Group 1 **Grade:**

You are a contestant on a TV game show; there are four identical-looking suitcases containing \$200, \$400, \$800, and \$1600. You start the game by randomly choosing a suitcase. Among the three unchosen suitcases, the game show host opens the suitcase that holds the least money. The host then asks you if you want to keep your suitcase or switch one of the other remaining suitcases. For the following analysis, use the following notation for events:

- C_i is the event that you choose a suitcase with i dollars.
- O_i denotes the event that the host opens a suitcase with i dollars.
- R is the reward in dollars that you keep.

Sketch a tree diagram for this experiment, and answer the following questions.

1. You refuse the host's offer and open the suitcase you first chose. Find the PMF of R and the expected value $\mathbb{E}[R]$.
2. You always switch and randomly choose one of the two remaining suitcases. You receive the R dollars in this chosen suitcase. Find the PMF and expected value of R.
3. Can you do better than either always switching or always staying with your original choice? Explain.

Group 2 **Grade:**

Show that the variance of $\mathsf{Binom}(n, p)$ is

$$\mathrm{Var}(X) = np(1-p).$$

Group 3 **Grade:**

In the New Jersey state lottery, each \$ 1 ticket has six randomly marked numbers out of $1, 2, \ldots, 46$. A ticket is a winner if the six marked numbers match six numbers drawn at random at the end of a week. For each ticket sold, 50 cents are added to the pot for the winners. If there are k winning tickets, the pot is divided equally among the k winners. Suppose you bought a winning ticket in a week in which $2n$ tickets are sold, and the pot is n dollars. Given that your ticket is a winner, what is the probability that there are "$\mathbb{E}[k]$" winners?
Note: For this problem, first explain that the number of winners is a binomial random variable, determine the corresponding parameters and then find the solution. Assume that $2nq$ is an integer, where $q = \frac{1}{\binom{46}{6}}$.
Note: In this statement, there are a lot of redundant information. Try to extract the essential information and express the problem in your own words.

Group 4 **Grade:**

An internet service provider uses 50 modems to serve the needs of 1000 customers. It is estimated that at a given time, each customer will need a connection with probability 0.01, independent of the other customers.

(a) What is the PMF of the number of customers that need a connection at a given time?

(b) Repeat item (a) by approximating the PMF of the number of customers that need a connection with a Poisson PMF

(c) What is the probability that there are more customers needing a connection than there are modems? Provide an exact (item (a)), as well as an approximate formula based on the Poisson approximation of item (b).

Group 5 **Grade:**

Let $X \sim \mathsf{Binom}(n, p)$ and $Y \sim \mathsf{Binom}(n, 1-p)$ represent binomial random variables with parameters n and p, show that

$$\mathbb{P}(X \leq i) + \mathbb{P}(Y \leq n - i - 1) = 1.$$