# HW 6

**Group 1** **Grade:**

The number of eggs laid on a tree leaf by an insect of a certain type is a Poisson random variable with parameter $\alpha$. However, such a random variable can be observed only if it is positive, since if it is 0, then we cannot know that such an insect was on the leaf. If we let $Y$ denote the observed number of eggs, then

$$\mathbb{P}(Y = i) = \mathbb{P}(X > 0 \mid X = i)$$

where $X$ is Poisson with parameter $\alpha$. Find $\mathbb{E}[Y]$.
**Hint:** Regular Poisson $X$ has values starting from 0 while $Y$ starts from 1. We can eliminate $P_X(0)$ from $X$ and re-normalize it to get the PMF of $Y$.

**Group 2** **Grade:**

A jar contains $n$ chips. Suppose that a boy successively draws a chip from the jar, each time replacing the one drawn before drawing another. The process continues until the boy draws a chip that he has previously drawn. Let $X$ denote the number of draws, and compute its probability mass function and Expectation.

**Group 3** **Grade:**

If $X$ is a geometric random variable, show that

$$\mathbb{P}(X = n + k \mid X > n) = \mathbb{P}(X = k).$$

This is called memoryless property. Using the interpretation of a geometric random variable, give a verbal argument as to why the preceding equation is true.

**Group 4** **Grade:**

True or False: For any random variable $X$, $\mathbb{E}[1/X] = 1/\mathbb{E}[X]$.

**Group 5** **Grade:**

Find $\mathbb{P}(K < \mathbb{E}[K])$ when

1. K is $\mathsf{Geom}(1/3)$
2. K is $\mathsf{Binom}(6, 1/2)$
3. K is $\mathsf{Poi}(3)$