# HW 1

Name: ____________________ Signature: ____________________

NetID: ____________________ RUID: ____________________

**Quiz Instructions:**

- Indicate your answer in the box provided. Answers indicated elsewhere will not be graded and receive a **zero** grade, irrespective of the correctness.
- **CREDIT** is only given for **CORRECT** answers and **NO PARTIAL CREDIT**.
- Simplify your answer as much as possible.

## Problem 2.3.1 Grade:

Consider a binary code with 5 bits (0 or 1) in each code word. An example of a code word is 01010. In each code word, a bit is a zero with probability 0.8, independent of any other bit.

1. What is the probability of the code word 00111?
2. What is the probability that a code word contains exactly three ones?

## Problem 1.6.9 Grade:

In an experiment with *equiprobable outcomes*, the sample space is $S = 1, 2, 3, 4$ and $P[s] = \frac{1}{4}$ for all $s \in S$. Find three events in $S$ that are pairwise independent but are not independent.

## Problem 2.3.3 Grade:

Suppose each day that you drive to work a traffic light that you encounter is either green with probability 7/16, red with probability 7/16, or yellow with probability 1/8, independent of the status of the light on any other day. If over the course of five days, G, Y, and R denote the number of times the light is found to be greed, yellow, or red, respectively. What is the probability that $P[G = 2, Y = 1, R = 2]$? Also, what is the probability $P[G = R]$?

## Problem 2.2.3 Grade:

Your Starburst candy has 12 pieces, three pieces of each of four flavors: berry, lemon, orange, and cherry, arranged in a random order in the pack. You draw the first three pieces from the pack.

1. What is the probability they are all the same flavor?
2. What is the probability they are all different flavors?

**Problem 3.5.19** **Grade:**

Let $X^{[n]}$ be the $\mathsf{Bin}(n, p)$ discrete random variable with $n$ trails and success rate $p$ of each trail. That says $X^{[n-1]}$ would be $\mathsf{Bin}(n-1, p)$. Prove that $\mathbb{E}\left[X^{[n]}\right] = np$.
Hint:

1. Show that $x\binom{n}{x} = n\binom{n-1}{x-1}$
2. Write down the PMF of $\mathsf{Bin}(n, p)$
3. Use the Definition of $\mathbb{E}\left[X^{[n]}\right]$
4. Use the property of PMF $\sum_{x=0}^{n} P_{X^{[n]}}(x) = \sum_{x-1=0}^{n-1} P_{X^{[n-1]}}(x-1) = 1$

**Problem 3.7.1** **Grade:**

Starting on day $n = 1$, you buy one lottery ticket each day. Each ticket costs 1 dollar and is independently a winner that can be cashed for 5 dollars with probability 0.1; otherwise the ticket is worthless. Let $X_n$ equal your **net profit** after $\boldsymbol{n}$ days. What is $\mathbb{E}[X_n]$?
Hint:

1. What is the PMF ($\mathbb{P}(X)$) and the Expectation ($\mathbb{E}[X]$) of **one day's** net profit?
2. What is the relationship between $\mathbb{E}[X]$ and $\mathbb{E}[X_n]$?

**Problem 3.8.5** **Grade:**

Let $X$ have the PMF

$$P_X(x) = \binom{4}{x}\left(\frac{1}{2}\right)^4.$$

Find $\mathbb{P}(\mu_X - \sigma_X \le X \le \mu_X + \sigma_X)$.
Hint (Hint is not question. Answering hint does not yield credit):

1. Which probability distribution does $X$ follow?

Answers: