HW 11

Group 1 **Grade:**

The expectation of $\mathsf{Geo}(p)$ can be proved by conditional expectation as follows:

Proof. For $X \sim \mathsf{Geo}(p)$,

$$\begin{aligned} \mathbb{E}[X] &= \mathbb{E}[X \mid X = 1]\mathbb{P}(X = 1) + \mathbb{E}[X \mid X > 1]\mathbb{P}(X > 1) \\ &= 1 \cdot p + (\mathbb{E}[X] + 1)(1 - p) \\ \Rightarrow \mathbb{E}[X] &= \frac{1}{p}. \end{aligned}$$

□

Following the same logic, derive the Variance of $\mathsf{Geo}(p)$.

Group 2 **Grade:**

Let Y_1 denote the weight (in tons) of a bulk item stocked by a supplier at the beginning of a week and suppose that Y_1 has a uniform distribution over the interval $0 \leq y_1 \leq 1$. Let Y_2 denote the amount (by weight) of this item sold by the supplier during the week and suppose that Y_2 has a uniform distribution over the interval $0 \leq y_2 \leq y_1$, where y_1 is a specific value of Y_1.

1. Find the joint density function for Y_1 and Y_2.
2. If the supplier stocks a half-ton of the item, what is the probability that she sells more than a quarter-ton?
3. If it is known that the supplier sold a quarter-ton of the item, what is the probability that she had stocked more than a half-ton?

Group 3 **Grade:**

A quality control plan calls for randomly selecting three items from the daily production (assumed large) of a certain machine and observing the number of defectives. However, the proportion p of defectives produced by the machine varies from day to day and is assumed to have a uniform distribution on the interval (0, 1). For a randomly chosen day, find the unconditional probability that exactly two defectives are observed in the sample.

Group 4 **Grade:**

Suppose that the probability that a head appears when a coin is tossed is p and the probability that a tail occurs is $q = 1 - p$. Person A tosses the coin until the first head appears and stops. Person B does likewise. The results obtained by persons A and B are assumed to be independent. What is the probability that A and B stop on exactly the same number toss? Explain how you think. The calculation is not mandatory.

Group 5 **Grade:**

Let Y_1 and Y_2 be independent exponentially distributed random variables, each with mean 1. Find $P(Y_1 > Y_2 \mid Y_1 < 2Y_2)$ and $P(Y_1 < 2Y_2 \mid Y_1 > Y_2)$.

Hint:

1. Find the joint probability of Y_1 and Y_2.
2. Determine the range of Y_1 and Y_2 that satisfies the condition.
3. Use the definition of conditional probability.
4. Surprisingly, $P(Y_1 < 2Y_2 \mid Y_1 > Y_2)$ is much easier to calculate than $P(Y_1 = 2Y_2 \mid Y_1 > Y_2)$.