HW 10

Group 1 **Grade:**

Suppose your grade in a probability course depends on your exam scores X_1 and X_2. The professor, a fan of probability, releases exam scores in a normalized fashion such that X_1 and X_2 are independent Gaussian ($\mu = 0, \sigma = \sqrt{2}$) random variables. Your semester average is $X = 0.5(X_1 + X_2)$.

1. You earn an A grade if $X > 1$. What is $\mathbb{P}(A)$?
2. To improve his SIRS (Student Instructional Rating Service) score, the professor decides he should award more A's. Now you get an A if $\max(X_1, X_2) > 1$. What is $\mathbb{P}(A)$ now?
3. The professor found out he is unpopular at ratemyprofessor.com and decides to award an A if either $X > 1$ or $\max(X_1, X_2) > 1$. Now what is $\mathbb{P}(A)$?
4. Under criticism of grade inflation from the department chair, the professor adopts a new policy. An A is awarded if $\max(X_1, X_2) > 1$ and $\min(X_1, X_2) > 0$. Now what is $\mathbb{P}(A)$?

Hint:

1. Sum of independent Gaussian and Φ function.
2. $\mathbb{P}(\max(X_1, X_2) > 1) = 1 - \mathbb{P}(\max(X_1, X_2) \leq 1)$.
3. Draw the Venn Diagram to show the relationship between $X > 1$ and $\max(X_1, X_2) > 1$.
4. Draw the Venn Diagram to show the relationship between $\max(X_1, X_2) > 1$ and $\min(X_1, X_2) > 0$.

Group 2 **Grade:**

Let X and Y be continues random variables with joint PDF

$$f(x, y) = \begin{cases} 2 & 0 \leq y \leq x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

1. What are $\mathbb{E}[X]$ and $\mathrm{Var}(X)$?
2. What are $\mathbb{E}[Y]$ and $\mathrm{Var}(Y)$?
3. What are $\mathrm{Cov}(X, Y)$ and $\mathrm{Corr}(X, Y)$?
4. What are $\mathbb{E}[X + Y]$ and $\mathrm{Var}(X + Y)$?

Group 3 **Grade:**

A company receives shipments from two factories. Depending on the size of the order, a shipment can be in

1. 1 box for a small order,
2. 2 boxes for a medium order,
3. 3 boxes for a large order.

The company has two different suppliers. Factory Q is 60 miles from the company. Factory R is 180 miles from the company. An experiment consists of monitoring a shipment and observing B, the number of boxes, and M, the number of miles the shipment travels. The following probability model describes the experiment:

	Factory Q	Factory R
small order	0.3	0.2
medium order	0.1	0.2
large order	0.1	0.1

1. Find $P_{B,M}(b, m)$, the joint PMF of the number of b oxes and the distance.
2. What is $\mathbb{E}[B]$, the expected number of boxes?
3. Are B and M independent?

Group 4 **Grade:**

Random Variables N and K have the joint PMF

$$P_{N,K}(n,k) = \begin{cases} \frac{(1-p)^{n-1}p}{n} & k = 1, 2, \ldots, n; n = 1, 2, \ldots \\ 0 & \text{otherwise.} \end{cases}$$

Find $P_N(n)$, $\mathbb{E}[N]$, $\mathrm{Var}(N)$, $\mathbb{E}\left[N^2\right]$, $\mathbb{E}[K]$, $\mathrm{Var}(K)$, $\mathbb{E}[N+K]$, $r_{N,K}$, $\mathrm{Cov}(N,K)$.

Hint:

Please solve the first four and leave the rest for me.

Group 5 **Grade:**

X and Y are random variables such that X has expected value $\mu_X = 0$ and standard deviation $\sigma_X = 3$ while Y has expected value $\mu_Y = 1$ and standard deviation $\sigma_Y = 4$. In addition, X and Y have covariance $\mathrm{Cov}[X, Y] = -3$. Find the expected value and variance of $W = 2X + 2Y$.