HW 3

Group 1 **Grade:**

When utilizing theorems of probability theory (and other theorems), it's crucial to carefully distinguish between the generalized form and the special case. For instance, the generalized version of the combination formula, $\binom{n}{k_1, k_2, \ldots k_r}$, mirrors the pattern of $\binom{n}{k}$. However, the generalized form of De Morgan's Law is not as straightforward as its special case. In this problem, we will delve into the concept of the **independence of three events**.

Definition 0.1. A_1, A_2, and A_3 are mutually independent if and only if

1. A_1 and A_2 are independent.
2. A_1 and A_3 are independent.
3. A_2 and A_3 are independent.
4. $\mathbb{P}(A_1 \cap A_2 \cap A_3) = \mathbb{P}(A_1)\mathbb{P}(A_2)\mathbb{P}(A_3)$.

Question: Rolling two dices independently. Let X and Y be the scores on two fair dice taking values in the set $\{1, 2, \ldots, 6\}$. Let $A_1 = \{X + Y = 9\}$, $A_2 = \{X \in \{1, 2, 3\}\}$, and $A_3 = \{X \in \{3, 4, 5\}\}$. Are A_1, A_2, and A_3 are mutually independent.
Note: In upcoming lectures, we will learn that the assumption of independence is both strong and commonly employed in many probability modeling theorems. Nevertheless, in real-world problems, it can be quite challenging to meet this assumption. There is a Ph.D. level course, High Dimensional Probability, that tackles this topic in real-life scenarios. For the moment, let's proceed under the assumption that it holds true.

Group 2 **Grade:**

A diagnostic test for a disease is such that it (correctly) detects the disease in 90% of the individuals who actually have the disease. Also, if a person does not have the disease, the test will report that he or she does not have it with probability .9. Only 1% of the population has the disease in question. If a person is chosen at random from the population and the diagnostic test indicates that she has the disease, what is the conditional probability that she does, in fact, have the disease? Are you surprised by the answer? Would you call this diagnostic test reliable?
Note: Always keep *Law of Total Number* in mind.

Group 3 **Grade:**

The *symmetric difference* between two events A and B is the set of all sample points that are in exactly one of the sets and is often denoted $A \triangle B$. Note that $A \triangle B = (A \cap B^{\mathsf{c}}) \cup (A^{\mathsf{c}} \cap B)$. Prove that $\mathbb{P}(A \triangle B) = \mathbb{P}(A) + \mathbb{P}(B) - 2\mathbb{P}(A \cap B)$.

Group 4 **Grade:**

Of the travelers arriving at a small airport, 60% fly on major airlines, 30% fly on privately owned planes, and the remainder fly on commercially owned planes not belonging to a major airline. Of those traveling on major airlines, 50% are traveling for business reasons, whereas 60% of those arriving at private planes and 90% of those arriving at other commercially owned planes are traveling for business reasons. Suppose that we randomly select one person arriving at this airport. What is the probability that the person

1. is traveling on business?
2. is traveling for business on a privately owned plane?
3. arrived on a privately owned plane, given that the person is traveling for business reasons?
4. is traveling on business, given that the person is flying on a commercially owned plane?

Group 5 **Grade:**

If A_1, A_2, and A_3 are three events and $\mathbb{P}(A_1 \cap A_2) = \mathbb{P}(A_1 \cap A_3) \neq 0$ but $\mathbb{P}(A_2 \cap A_3) = 0$, show that

$$\mathbb{P}(\text{at least one } A_i) = \mathbb{P}(A_1) + \mathbb{P}(A_2) + \mathbb{P}(A_3) - 2\mathbb{P}(A_1 \cap A_2).$$

Note: Venn Diagram is helpful, but we correctly expect something beyond that.