Purely Functional State

Goals

- Purely functional random number generation
- Working with stateful APIs
- The State data type

To write purely functional programs that manipulate state, using the simple domain of **random number generation**.

Learn the basic pattern for how to make **any** *stateful API* purely functional.

Random Number Generation w/ Side Effects

```
scala> val rng = new scala.util.Random

scala> rng.nextDouble
res1: Double = 0.9867076608154569

scala> rng.nextDouble
res2: Double = 0.8455696498024141

scala> rng.nextInt
res3: Int = -623297295

scala> rng.nextInt(10)
 res4: Int = 4
```

Creates a new random number generator seeded with the current system time

Gets a random integer between 0 and 9

APIs are stateful …
so they are not referentally transparent

- Not testable
- Not composable
- Not modular,
- Not easily parallelized

Tests need to be reproducible

```
def rollDie: Int =
  val rng = new scala.util.Random
  rng.nextInt(6) // off by one error
```

- Cure?

```
def rollDie(rng: scala.util.Random): Int = rng.nextInt(6)
```

The "same" generator has to be both created with the same seed, and also be in the same state, which is difficult to guarantee.

Purely functional Generation

- The key to recovering referential transparency is to make the state updates explicit.
- Don't update the state as a side effect.

```
trait RNG:
  def nextInt: (Int, RNG)
```

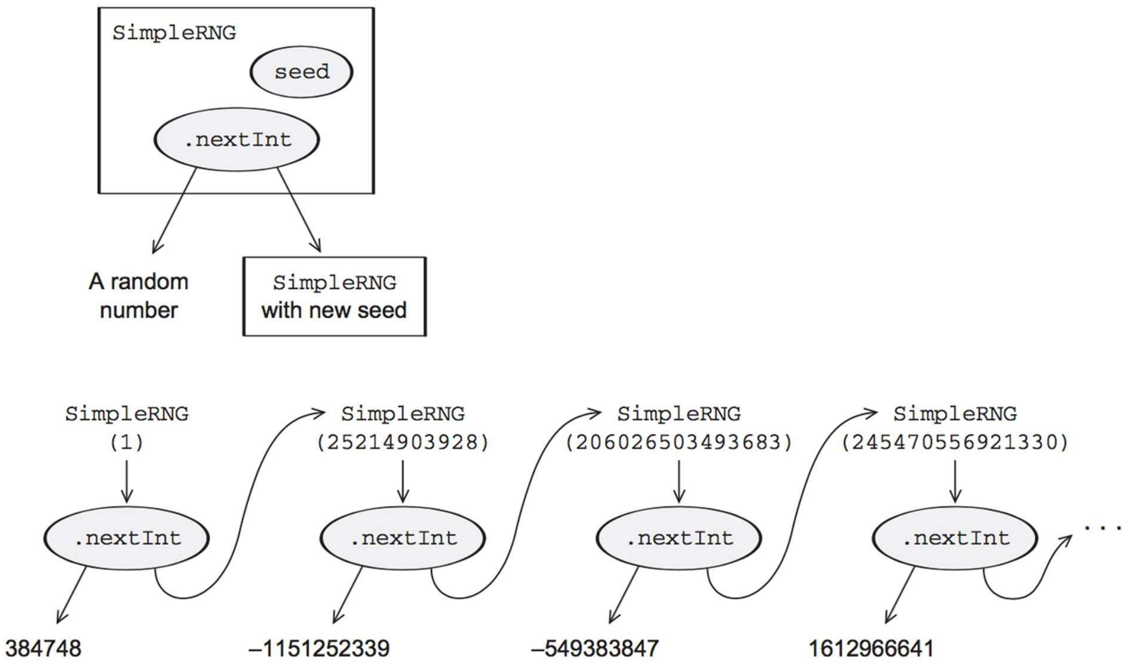
- Separate the concern of computing what the next state is from the concern of communicating the new state to the rest of the program.

A purely functional random number generator

```
/*
 * linear congruential generator
 */
case class SimpleRNG(seed: Long) extends RNG:
  def nextInt: (Int, RNG) =
    val newSeed = (seed * 0x5DEECE66DL + 0xBL) & 0xFFFFFFFFFFFFL
    val nextRNG = SimpleRNG(newSeed)
    val n = (newSeed >>> 16).toInt
    (n, nextRNG)
```

The next state, which is an RNG instance created from the new seed.

The return value is a tuple containing both a pseudo-random integer and the next RNG state.



Each call to SimpleRNG.nextInt returns the next random number in the sequence and the SimpleRNG object needed to continue the sequence.

Using Pure API

```
scala> val rng = SimpleRNG(42)
rng: SimpleRNG = SimpleRNG(42)

scala> val (n1, rng2) = rng.nextInt
n1: Int = 16159453
rng2: RNG = SimpleRNG(1059025964525)

scala> val (n2, rng3) = rng2.nextInt
n2: Int = -1281479697
rng3: RNG = SimpleRNG(197491923327988)
```

Let’s choose an arbitrary seed value, 42.

Syntax for declaring two values by deconstructing the pair returned by `rng.nextInt`.

Common Pattern

- To make pure API, make explicit the transition from one state to the next.

```
class Foo:
  private var s: FooState = ...
  def bar: Bar // somehow modify s
  def baz: Int // somehow modify s, too
```

```
trait Foo:
  def bar: (Bar, Foo)
  def baz: (Int, Foo)
```

- Now, the caller is responsible for passing the computed next state through the rest of the program.

```
def randomPair(rng: RNG): ((Int, Int), RNG) =
  val (i1, _) = rng.nextInt
  val (i2, rng2) = rng.nextInt
  ((i1, i2), rng2)
```

What's the difference?

```
def randomPair(rng: RNG): ((Int, Int), RNG) =
  val (i1, rng2) = rng.nextInt
  val (i2, rng3) = rng2.nextInt
  ((i1, i2), rng3)
```


Try to find common patterns in usages

- [Exercise 6.1]

```
def nonNegativeInt(rng: RNG): (Int, RNG) = ???
```

- [Exercise 6.2]

```
def double(rng: RNG): (Double, RNG) = ???
```

- [Exercise 6.3]

```
def intDouble(rng: RNG): ((Int, Double), RNG) = ???
def doubleInt(rng: RNG): ((Double, Int), RNG) = ???
d4f double3(rng: RNG): ((Double, Double, Double), RNG) = ???
```

- [Exercise 6.4]

```
def ints(count: Int)(rng: RNG): (List[Int], RNG) = ???
```

Better API for State Action

- Each of our functions has a type of the form **RNG => (A, RNG)** for some type **A**.
- Functions of this type are called **state actions** or **state transitions** because they transform RNG states from one to the next.
- These state actions can be combined using **combinators** through which to pass the state from one action to the next *automatically*.

Type Alias for Convenience

```
type Rand[+A] = RNG => (A, RNG)
```

- Informally, a value of type **Rand[A]** as “a randomly generated A.”
- It’s really a state action—a *program* that depends on some RNG, uses it to generate an **A**, and also transitions the RNG to a new state that can be used by another action later.

```
def int(rng: RNG): (Int, RNG) = rng.nextInt
```

```
val int: Rand[Int] = rng => rng.nextInt
```

We want to write functions that let us combine Rand actions while avoiding explicitly passing along the RNG state.

We'll end up with a kind of domain specific language (DSL) that does all of the passing for us.

unit

A simple RNG state transition which passes the RNG state through without using it, always returning a constant value rather than a random value:

```
def unit[A](a: A): Rand[A] =
  rng => (a, rng)
```

map

Transforms the output of a state action without modifying the state itself.

```
def map[A, B](s: Rand[A])(f: A => B): Rand[B] =
  rng => {
    val (a, rng2) = s(rng)
    (f(a), rng2)
  }
```

Example:

```
def nonNegativeEven: Rand[Int] =
  map(nonNegativeInt)(i => i - i % 2)
```


Combining State Actions: map2

- **map2** takes two actions, **ra** and **rb**, and a function **f** for combining their results, and returns a new action that combines them:

```
def map2[A,B,C](
  ra:Rand[A],
  rb:Rand[B]
)(f: (A, B) => C): Rand[C] = ???
```

Application of map2

```
def both[A,B](ra: Rand[A], rb: Rand[B]): Rand[(A,B)] =
  map2(ra, rb)((_, _))

val randIntDouble: Rand[(Int, Double)] =
  both(int, double)

val randDoubleInt: Rand[(Double, Int)] =
  both(double, int)
```


Exercise: *Hard*

EXERCISE 6.7

Hard: If you can combine two RNG transitions, you should be able to combine a whole list of them. Implement `sequence` for combining a `List` of transitions into a single transition. Use it to reimplement the `ints` function you wrote before. For the latter, you can use the standard library function `List.fill(n)(x)` to make a list with `x` repeated `n` times.

```
def sequence[A](fs: List[Rand[A]]): Rand[List[A]]
```

Nesting State Actions

```
def nonNegativeLessThan(n: Int): Rand[Int] =
  map(nonNegativeInt) { _ % n }
```

- The generated numbers may be skewed because **Int.MaxValue** may not be exactly divisible by **n**. Then, how to rectify this?

```
def nonNegativeLessThan(n: Int): Rand[Int] =
  map(nonNegativeInt) { i =>
    val mod = i % n
    if (i + (n-1) - mod >= 0) mod else nonNegativeLessThan(n)(???)
  }
```

Retry recursively if the `Int` we got is higher than the largest multiple of `n` that fits in a 32-bit `Int`.

Manual Passing instead of using "map"

```
def nonNegativeLessThan(n: Int): Rand[Int] =
  rng =>
    val (i, rng2) = nonNegativeInt(rng)
    val mod = i % n
    if i + (n-1) - mod >= 0 then
      (mod, rng2)
    else
      nonNegativeLessThan(n)(rng2)
```

Combinator: flatMap

- But it would be better to have a combinator that does this passing along for us. Neither **map** nor **map2** will cut it. We need a more powerful combinator, **flatMap**.

```
def flatMap[A, B](f: Rand[A])(g: A => Rand[B]): Rand[B]
  = ???
```

- Implement **flatMap**, and then use it to implement **nonNegativeLessThan**.
- Reimplement **map** and **map2** in terms of **flatMap**!

we say that flatMap is more *powerful* than map and map2.

A General State Action Data Type

unit, **map**, **map2**, **flatMap**, and **sequence** are general-purpose functions for working with **state actions**.

Don't care about the type of the state such as **Rand**.

Generalize **Rand** to a new type to handle any type:

```
type State[S,+A] = S => (A,S)   or
case class State[S,+A](run: S => (A,S))
```

We can now just make **Rand** a type alias for **State**:

```
type Rand[A] = State[S,A]
```

State Type defined as Case Class

```scala
case class State[S,+A](run: S => (A,S)) {
  def map[B](f: A => B): State[S, B]
  def map2[B,C](r: State[S, B])(f: (A, B) => C): State[S, C]
  def flatMap[B](f: A => State[S, B]): State[S, B]
}

object State {
  def unit[S, A](a: A): State[S, A]
  def sequence[S,A](sas: List[State[S, A]]): State[S, List[A]]
  …
}
```


Scala3 Only: Opaque Type

```scala
opaque type State[S, +A] = S => (A, S)

object State:
  extension [S, A](underlying: State[S, A])
    def run(s: S): (A, S) = underlying(s)
    def apply[S, A](f: S => (A, S)): State[S, A] = f
```

At the cost of some boilerplate conversions, we get both ***encapsulation*** *and* ***performance****.*

- An opaque type behaves like a type alias inside the defining scope.
  - “Defining scope" refers to the object containing the definition, or if the definition is top level, the package containing it.
- Outside of the defining scope though, the opaque type is unrelated to the representation type.

Purely functional imperative programming

```
val ns: Rand[List[Int]] =
  int.flatMap(x =>
    int.flatMap(y =>
      ints(x).map(xs =>
        xs.map(_ % y))))
```

≡

```
val ns: Rand[List[Int]] =
  for
    x <- int
    y <- int
    xs <- ints(x)
  yield xs.map(_ % y)
```


Two Constructors for Imperative Style

```
// Passes the incoming state along and returns it as the value
def get[S]: State[S, S] = s => (s, s)

// Replaces the state and returns unit as the value
def set[S](s: S): State[S, Unit] = _ => ((), s)

// Modifies the incoming state by the function f
def modify[S](f: S => S): State[S, Unit] = for
  s <- get
  _ <- set(s)
yield ()
```