Basic Idea : Principles of Induction on Natural numbers (10/6)

English

Infinite

① $P(0)$ ② $P(i) \Rightarrow P(i+1)$

$P(0) \rightsquigarrow P(1) \rightsquigarrow P(2) \ldots$

for all $\boxed{\mathbb{N}}$ are always true

$\therefore \forall m \in \mathbb{N}, P(m)$.

[Slide P.10]

$$P_j \triangleq \left(\sum_{i=0}^{j} i = \frac{j(j+1)}{2} \right)$$

predicate (either true or false)!

To Prove $\boxed{\forall j \in \mathbb{N}, P_j \text{ is true}}$.

1) P_0 : $(j=0)$ $\sum_{i=0}^{0} i = 0 = \boxed{\frac{0(0+1)}{2}}$ $\therefore P_0$ is true!

2) Assume P_K is true. (Note that you do not prove P_K!)

Using this assumption, you have to prove P_{K+1}.

$$P_{K+1} \triangleq \left(\sum_{i=0}^{K+1} i = \frac{(K+1)(K+1+1)}{2} \right)$$

$$\sum_{i=0}^{K+1} i = \sum_{i=0}^{K} i + (K+1)$$

Here, the assumption is used. $= \frac{K(K+1)}{2} + (K+1)$ $\Leftarrow$ P_K is true : $\left(\sum_{i=0}^{K} i = \frac{K(K+1)}{2} \right)$

$$= \frac{(K+1)(K+1+1)}{2}$$

$$P_{K+1} \triangleq \left(\sum_{i=0}^{K+1} i = \frac{(K+1)(K+2)}{2} \right)!$$ by assuming P_K.

is true

By 1) and 2) with principle of Induction,

we conclude that $\boxed{\forall j \in \mathbb{N}, P_j \text{ is true}}$.

"Plain" principle of Induction : $\boxed{P(0)} \wedge (P(0) \Rightarrow P(1))$
$\wedge (P(1) \Rightarrow P(2))$
$\vdots$
$\wedge (P(i) \Rightarrow P(i+1))$
$\wedge \cdots$

☐ "Strong" principle of Induction :

$P(0) \wedge (P(0) \Rightarrow P(1)) \wedge (P(0) \wedge P(1) \Rightarrow P(2))$
$\wedge (P(0) \wedge P(1) \wedge P(2) \Rightarrow P(3))$
$\wedge (P(0) \wedge P(1) \wedge P(2) \wedge P(3) \Rightarrow P(4))$
$\vdots$

1) $P(0)$ is true
2) Assume $\forall j \leq k$, $P(j)$ is true.
You have to show $P(k+1)$ is true.

P : Any positive number $n \geq 2$ is either a prime or a product of prime numbers.

1) $P(2)$: 2 is a prime number! The proof is done.
2) Assume $P(i)$ is true for all $i \leq k$.
To prove $P(k+1)$ is true. ← Goal!

For example, $1024 = 16 \times 64$

Assume $P(16)$ is true and $P(64)$ is true.

$16 = p_1 \cdot \cdots \cdot p_a \qquad 64 = q_1 \cdot \cdots \cdot q_b$

$16 \times 64 = (p_1 \cdot \cdots \cdot p_a \times q_1 \cdot \cdots \cdot q_b)$

$= 1024$ ∴ 1024 is a product of prime numbers!

$k+1$ is a prime number or is not a prime number.

The Proof is done!

$\exists c, d < k+1$. $\boxed{k+1 = c \times d.}$

The assumption is used!

The Proof is done!

Induction (귀납법) - 증명 방법 - 무한히 많은 정수의 수에서 증명! (10/6)
Korean

Principle of Induction on natural numbers : 명제 $P(i)$

- Step1 : Base case) $P(0)$ 가 참!
- Step2 : Inductive case) $\left(P(i) \xrightarrow{} P(i+1) \right)$ 이 참!
  함축

→ $P(i)$ 가 참이다라는 가정 (≠ 증명)
이 가정을 이용해서 $P(i+1)$ 이 참이라고 증명!

$\Longrightarrow \forall i \in \mathbb{N},\ P(i)$!

☆ $P(j) \triangleq \left(\sum_{i=0}^{j} i = \frac{j(j+1)}{2} \right)$
명제 / Predicate

Base case) $P(0) \triangleq \sum_{i=0}^{0} i = 0 = \frac{0(0+1)}{2}$. P(0)가 참이라!

Inductive case) $P(k)$ 가 참이라고 가정 (≠ 증명)
이 가정을 이용해서 $P(k+1)$ 이 참임을 증명

$P(k)$ 가 참: $\sum_{i=0}^{k} i = \frac{k(k+1)}{2}$
참

$P(k+1)$ 이 참? $\sum_{i=0}^{k+1} i = \left(\sum_{i=0}^{k} i\right) + (k+1)$

by 가정

가정 사용! ⇒ $= \frac{k(k+1)}{2} + (k+1)$

$= \frac{(k+1)(k+2)}{2} = \frac{(k+1)((k+1)+1)}{2}$

$\therefore P(k+1) \triangleq \left(\sum_{i=0}^{k+1} i = \frac{(k+1)(k+1+1)}{2} \right)$ 이 참이라!

"Plain" principle of Induction

: $P(0) \underset{\text{and}}{\wedge} (P(0) \Rightarrow P(1)) \wedge (P(1) \Rightarrow P(2)) \wedge (P(2) \Rightarrow P(3)) \wedge \cdots$

"Strong" principle of Induction

: $P(0) \wedge (P(0) \Rightarrow P(1)) \wedge ([P(0) \wedge P(1)] \Rightarrow P(2))$

$\wedge ([P(0) \wedge P(1) \wedge P(2)] \Rightarrow P(3)) \wedge \cdots$

Strong principle of Induction

Step1 - Base case) $P(0)$

Step2 - Inductive case) 가정 : $\forall j \le k$, $P(j)$가 참이라

이 가정을 이용해서 $P(k+1)$이 참임을 증명.

$P(n)$: n이 소수 (prime) 이거나 (OR) 소수들의 곱이다.

(증명 Goal) : $\forall n \ge 2$, $P(n)$. 참이다!

Base case $n=2$) $P(n) = P(2)$: 2는 소수! ∴ $P(2)$는 참!

Inductive case $n=k+1$)

가정 : $\forall j \le k$, $P(j)$가 참이다. (≠ 증명)

목표 : $P(k+1)$이 참?

예) $\underset{k+1}{1024} = \underset{c}{16} \times \underset{d}{64}$ 가정을 이용하면 $P(16)$ 참! 가정을 이용해 $P(64)$ 참!

$16 = p_1 \times \cdots \times p_a$ $64 = q_1 \times \cdots \times q_b$

$= (p_1 \times \cdots \times p_a) \times (q_1 \times \cdots \times q_b)$

$= p_1 \times \cdots \times p_a \times q_1 \times \cdots \times q_b$

∴ 1024는 소수들의 곱! ⇒ $P(1024)$ 참이다!

$k+1$ < 소수(O) → 증명 끝 $P(k+1)$.

소수(X) → $(k+1) = c \times d$ s.t. $\begin{pmatrix} k+1 > c \\ k+1 > d \end{pmatrix}$

$= \underbrace{p_1 \times \cdots \times p_a}_{(1)} \times \underbrace{q_1 \times \cdots \times q_b}_{(2)}$

(1) 가정 $P(c)$와

(2) 가정 $P(d)$ 2개를 사용