Syntax of the arithmetic PL. (BNF: Backus Naur form) English

t ⇒ terms (programs, expressions)

$$t ::= \text{true} \mid \text{false} \mid \text{if } t \text{ then } t \text{ else } t \mid 0 \mid \text{succ } t \mid \text{pred } t \mid \text{iszero } t$$

(Example terms)

if false then 0 else succ 0

⇒ if false then 0 else succ 0

succ 0 ≜ 1 3 ≜ succ (succ (succ 0))

pred (succ 0) = 0

Def: pred 0 ≜ 0

Slide 6

A set of terms S : This exactly contains all terms by the BNF def.

$$S = S_0 \cup S_1 \cup S_2 \cup S_3 \cup \cdots = \bigcup_{i=0}^{\infty} S_i$$

$S_0 = \emptyset$

$$S_{i+1} \triangleq \{\text{true}, \text{false}, 0\} \cup \{\text{succ } t, \text{pred } t, \text{iszero } t \mid t \in S_i\} \cup \{\text{if } t_1 \text{ then } t_2 \text{ else } t_3 \mid t_1, t_2, t_3 \in S_i\}$$

using S_i

Q. $S_1 = \{\text{true}, \text{false}, 0\}$

$\cup \{\text{succ } t, \text{pred } t, \text{iszero } t \mid t \in S_0\} \to \emptyset$

$\cup \{\text{if } t_1 \text{ then } t_2 \text{ else } t_3 \mid t_1, t_2, t_3 \in S_0\} \to \emptyset$

$= \{\text{true}, \text{false}, 0\} \cup \emptyset \cup \emptyset$

$S_2 = \{\text{true}, \text{false}, 0\}$

$\cup \{$ succ true, pred true, iszero true, succ false, pred false, iszero false, succ 0, pred 0, iszero 0 $\}$

$\cup \{ \cdots \}$

(10/13) – R
Einstein

(Study) Consts(t), Size(t), depth(t)

Topic: Induction (proof techniques) on terms (of on natural numbers)

(10/13)-1 Korean

Ch3 Arithmetic 표현식과 연산 { 구문을 정의 (Syntax) / 의미를 정의 (Semantics: Evaluation rules) / Induction proof 기법 }

구문 Syntax t ⇒ terms(t) (programs, expressions)

BNF (Backus-Naur form)

$t_1 ::= true \mid false \mid if\ t_1\ then\ t_2\ else\ t_3 \mid 0 \mid succ\ t \mid pred\ t \mid Iszero\ t$

(Example terms)

succ 0 = succ 0 (이상: 1)

succ(succ(succ 0)) ≜ 3

succ(succ 0) ≜ 2

pred (succ 0) ≡ 0 (≜ 1)

pred (succ (succ 0)) ⇒ succ 0 (2)

pred 0 ≜ 0 (*)

If false then 0 else succ 0 (1)

Slide6 A set of terms S: 위 BNF 구문 정의로 만들 수 있는 모든 항들의 집합

$S = S_0 \cup S_1 \cup S_2 \cup S_3 \cup \cdots = \bigcup_{i=0}^{\infty} S_i$

$S_0 = \emptyset$

$S_{i+1} = \{true, false, 0\}$ (S_i를 사용하여 정의)

$\cup \{ succ\ t,\ pred\ t,\ Iszero\ t \mid t \in S_i \}$

$\cup \{ If\ t_1\ then\ t_2\ else\ t_3 \mid t_1, t_2, t_3 \in S_i \}$

$S_1 = \{true, false, 0\}$

$S_2 = \{true, false, 0\}$

$\cup \{ succ\ true,\ pred\ true,\ Iszero\ true,\ succ\ false,\ pred\ false,\ Iszero\ false,\ succ\ 0,\ pred\ 0,\ Iszero\ 0 \}$

$\cup \{ \cdots \}$

Q. S_3 = ??

(Study 7) Consts(t), size(t), depth(t)

Topic: Induction (proof techniques) on terms
(if on natural numbers)