Pricing and Application of a Commodity-Linked Note

Models for Financial Derivatives: Project Report

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I. EXECUTIVE SUMMARY

Most investors own stocks and bonds, but very few hold commodity products in their investment portfolio. Commodities may benefit from inflation. Commodity linked notes are very different from owning commodity-related stocks or mutual funds (commodity companies routinely hedge their exposure to commodities). Commodity-Linked Notes offer investors exposure to the performance of commodities or commodity indices and provide for the repayment of principal at maturity. Linked notes are tax-efficient and have access to fully taxable investments. They enhanced returns within an investment (depending on the type **(Detail**/of structured products). Commodity Linked notes reduced volatility (or risk) within an investment. Ability to earn a positive return in low-yield or flat equity market environments. This project focused mainly on the approaches to price commodity-linked notes and to conduct risk-return analysis given physical measures by applying Monte-Carlo Simulation with MATLAB and optimal portfolio allocation. To practically and effectively represent the investment strategy analysis for clients, the project takes also calculation speed, customized deals for clients under subject beliefs and diversification purposes into consideration. It helps to get better understanding in putting structured derivatives theory and quantitative analysis methodology into real world work and Commodity-linked notes are used and are in part motivated by principal protected notes that John Hull's textbook introduces.

II. INTRODUCTION

Structured products create to meet specific needs that cannot be met from the standardized financial instruments available in the markets. Structured products can be used as an alternative to a direct investment, as part of the asset allocation process to reduce risk exposure of a portfolio, or to utilize the current market trend. In this project, we are going to develop and analyze principal protected Commodity Linked Notes. In principal protected note if you hold the notes until maturity, you will receive, at a minimum, your principal amount. In addition, you also may receive a Supplemental Redemption Amount. However, if you sell the notes prior to maturity, you may find that the market value of the notes is less than the principal amount of the notes.

In this paper, we describe the methodology of our pricing models and risk analysis, and discuss the numerical results of our commodity linked model where our commodities are copper, aluminum and zinc.

In risk analysis, we are using Monte-Carlo simulation and results from our

risk-neutral Monte-Carlo pricing model to calculate the expected return and standard deviation that comprise the Sharpe ratio and returns on which we base our recommendations. We are using the control variate technique to improve the convergence speed of Monte-Carlo simulation We also include recommendation for a client who wants to buy the note for diversification purposes.
Lastly, we proposed a business opportunity that combines USD-linked principle protected note and a market opportunity.

III. METHODOLOGY:

As per project description, we Assume that we work for a (fictitious) large investment bank called Stuart & Partners. We assumed that we are a (quantitative) structuring analysis who are willing to design a structure note called commodity-linked notes. Let X(t), Y(t), and Z(t) represents copper, aluminum and zinc prices at time t.
Under the physical measure, these follow:

$$dX(t)=\mu X(t)+\sigma X(t)dWx(t)$$
$$dY(t)=\mu Y(t)+\sigma Y(t)dWy(t)$$
$$dZ(t)=\mu Z(t)+\sigma Z(t)dWz(t)$$

Under the risk-neutral measure, these follow:

$$dX(t)=(r-qx)X(t)+\sigma X(t)dWx(t)$$

$$dY(t)=(r-qy)Y(t)+\sigma Y(t)dWy(t)$$

$$dZ(t)=(r-qz)Z(t)+\sigma Z(t)\, dWz(t)$$

where r is the continuously compounded annualized risk-free interest rate; qx, qy, and qz are the continuously compounded annualized net convenience yield; σx, σy, and σzare the annualized volatility; Wx(t), Wy(t), and Wz(t) are correlated Brownian motions.

{$400×max [13min (X(T)X(0) , 1.25) +13min (Y(T)Y(0),1.25) +13min (Z(T)Z(0),1.25) ,1] }

Given data: Time T = 3. (Here, $400 is the Initial Price, 1/3,1/3 and 1/3 are component weights of three commodities. 1.25,1.25 and 1.25 are the market caps of three commodities. And 1 in second argument of max [,1] is minimum guarantee.

Use of following parameters:

X (0.5000); Y(0) =1600 ; Z(0) = 2000;
r =0.03; qx =qy=qz =0.015; σ x=0.2718 ; σ y=0.2143; σ z=0.2817;

Correlation between dWx(t),dWy(t) and dWz(t) is:

{(1 0.8641 0.7786), (0.8641,1,0.7720),(0.7786,0.7720,1)}

Monte Carlo simulation: The Monte Carlo simulation is based on the Geometric Brownian Motion to generate results from the expectation of a many simulation results. The accuracy and efficiency are measured by standard deviation, the confidence interval and the required iteration N. The Monte Carlo method is applied in this project to simulate commodities' prices and to generate the derivative price given the parameters.

Monte Carlo is about integrating. We take advantage of the fact that certain contracts value depends only on the terminal distribution

Vo=E[-exp (-0Tr(s)ds) V(T)]

Sometimes the terminal distribution is known, whereas in other cases we need to integrate the SDE as one stage. To approach the subject of Monte Carlo simulation for option pricing, let's begin by revisiting geometric Brownian motion

dSt\St=μdt+σdW(t)

Which has the following solution

St=So*exp((r- σ 2/2)t+ σ W(t)

Since wiener process W(t) has independent increments which are normally distributed with zero mean and standard deviation $\sqrt{T}$, we can sample from this distribution and using the equation for the solution, obtain a sample of asset at price t

St=So*exp((r- σ 2/2)t+ σ $\sqrt{T}$ ε where ε =N(0,1)

We used Monte-Carlo simulation to calculate the Returns for our clients and the payoff functions. Black-Scholes-Merton Model-related functions are also used in determining what is best for our client.

IV. NUMERICAL RESULTS AND DISCUSSION

The parameters and the corresponding explanations for the commodity linked notes price simulation are:

p	Initial price for the deal
S1	Initial underlying price of copper
S2	Initial underlying price of aluminum
S3	Initial underlying price of zinc
q1	Continuously compounded annualized net convenience yield of copper
q2	Continuously compounded annualized net convenience yield of al
q3	Continuously compounded annualized net convenience yield of zinc
r	Continuously Compounded Annual Risk-free Interest Rate
T	Time to maturity
sigma1	Annualized volatility of copper
sigma2	Annualized volatility of aluminum
sigma3	Annualized volatility of zinc
rho12	correlation between copper and aluminum
rho13	correlation between copper and zinc
rho23	correlation between aluminum and zinc
N	Number of iterations
seed	Random number seed
w1	Component weight of copper
w2	Component weight of aluminum
w3	Component weight of zinc
cap	Caps of the three commodities
minGuarantee	Minimum Guarantee

The number of iteration required to achieve the given precision is calculated as 51368956.
To use this number as N, the fair value generated from Monte Carlo simulation is **$393.5777**,
which is smaller than the actual price $400. As the result, the note is sold *at premium*.
Also, it takes about 12s and the confidence level is from $393.5676 to $393.5877.

V. RECOMMENDATION FOR REDUCING CALCULATION SPEED

To decrease processing time, it is common in Monte Carlo simulation to utilize a variance reduction technique. Variance reduction is the process of decreasing the variance of a sample Monte Carlo simulation that allows for more precise simulations (i.e. narrower confidence intervals) to be run with a significantly lower amount of repetitions.

Initially, we attempted to decrease computation time by utilizing the **antithetic variable** technique. The antithetic technique requires identically distributed variables to be horizontally independent. Additionally, the payoff function utilized in the simulation must be monotonically increasing or decreasing. While our commodity-linked note qualifies on the last requirement, the compound structure of the note utilizes correlated variables across the different component underlying securities. These qualify as horizontally dependent and thus the antithetic technique of variation reduction is not appropriate.

Next, we attempted to create a **control variate** to reduce the variance of the sample Monte Carlo simulation. A control variate is a random variable with a known mean which is exploited to reduce the standard error of an unknown random variable. For the control variate technique to be appropriate, the control variate must have a strong correlation (e.g. strongly positive or negative) to the underlying payoff structure and the expected mean of the control variate must be known.

A basket of European option structures replicating the payoff structure of our commodity-linked note qualifies on both terms: it is strongly correlated to the payoff structure of the commodity-linked note and the mean of a European option is known as European options are easy to price.

To create the correct basket of European options, it was necessary to create identical payoff terms and notional allocations. First, hypothetical strike prices were created that replicate the payoff structure of our commodity-linked note. For example, each commodity is capped at a return of only 25% over the term of the note so a holder of the note is implicitly short a call option 25% above the initial price assumed in the Monte Carlo simulation. Equivalently, the commodity-linked note is principal-protected. That means the holder of the commodity linked note is long and at-the-money put in each commodity. Synthetically, a long put combined with a short call position is called a collar.

Lastly, it is necessary to use the correct notional amount of each commodity when structuring the option collars. Our principal-protected note is equally impacted by all three commodities and is denominated at $400 thus each commodity receives one third of the $400 exposure but scaled to its initial price. This is necessary to find the theoretical expectation of our control variate.

A basket of European option collars is found to be an excellent control variate. The correlation between the payoff of our commodity-linked note and our control variate is 97.72% and the expected price of our control variate is reasonably close to that of our commodity-linked note.
Using the control variate technique, the required number of simulations decreases from 53 million to just over 2 million decreasing simulation time by 99% from 237 seconds to just 1.4 seconds. The 95% confidence interval for the control variate simulation was $380.39-380.41.

VI. RECOMMENDATION FOR CUSTOMIZED DEALS FOR CLIENTS UNDER SUBJECT BELIEFS

The recommendations are based on the risk analysis with the components Mean return and Sharpe Ratio. Mean Return is the mean, of all the likely returns of investments comprising a portfolio therefore an investor would prefer a higher Mean Return. Sharpe Ratio is a metric indicating return for every one unit of risk, therefore investor would prefer a higher Sharpe ratio. Based on these comparisons, we give our clients the best recommendation with highest Mean Return and a decent Sharpe Ratio [Table 1].

VII. RECOMMENDATION FOR A CLIENT WHO WANTS TO BUY THE NOTE FOR DIVERSIFICATION PURPOSES

Miss Brown is a passive investor of S&P 500 index funds. Currently she has a $10,000,000 investment in S&P 500 Index funds. Miss Brown wants to sell some of her funds and buy our commodity-linked notes. We believe our CLN will significantly diversify Miss Brown's portfolio and provide her a much more attractive risk/reward tradeoff.

In our risk and return analysis, we first calculated returns, volatilities, and correlations for copper, aluminum, and zinc to calculate the performance of the commodity-linked note. Our assumptions were verified in Microsoft Excel with all modeling done in MATLAB.

[Table 2] Commodity prices are displayed in (closing prices as of Wednesday, November 23).

[Table 3] Drift (mu) is the average annual log returns since 1994.
[Table 4] Volatility is the average annual log variance since 1994.
[Table 5] Correlation is the correlation coefficient of daily prices since 1994.
We assume a 3-year time horizon for our client and use the 3-year Treasury as the risk-free rate. The 3-year yield was 1.39% as of November 23, the date of our simulations**.**

VIII. RISK ANALYSIS AND RECOMMENDATION

To make our CLN as attractive as possible to Miss Brown, we thought it would be appropriate to increase the cap in the payoff structure for copper. This increase will allow for a better return and risk trade off because copper has the best return per unit of risk for the three commodities.
Expected return= 0.0354

The expected return of the CLN is sufficiently higher than the risk-free rate of return to consider an investment by Miss Brown.

Sharpe ratio= 0.0944

Even though the Sharpe ratio of our commodity-linked note is not high on its own, it is important to consider as a diversifier to equities to enhance Miss Browns investment returns.
To determine if Miss Brown will be served well by our CLN, we must find the correlation of the CLN with the S&P 500.

Correlation between S&P 500 Index and CLN is 0.0298.

Our CLN has a very low correlation coefficient with a positive expected return. It will be a great diversifier for the Mrs. Brown's portfolio [Table 6].

Next, we must assign weights to our two assets. Standard investment practice states each weight should be in proportion to the unique risk contributed to the portfolio. Please see [Figure 1] for an illustration of how different weights of CLN and the S&P 500 in the portfolio will result in different amounts of risk and return in the portfolio.

In a 2 asset, risky portfolio, the most efficient allocation of capital should be where return is the highest per unit of risk. Accordingly, the optimal weight of the CLN is 46.34% with the remaining 53.66% invested in the S&P 500. Miss Brown will now have a portfolio with a better risk and reward tradeoff than an undiversified portfolio. With the reduced volatility expected in her portfolio, Miss Brown could even consider levering up this portfolio to match the volatility of the S&P 500 with an even higher expected return! Stuart & Partners would be happy to help!

IX. PROPOSAL OF ANOTHER BUSINESS OPPORTUNITY

The world is awash in debt. Since 2012, the McKinsey Global Institute has published a series of reports on debt and the path of deleveraging ahead. In February, the IMF reported the world had not de-levered at all since the crisis as world debt stood at a record $152tn.

The recent issue of the Fiscal Monitor by the IMF offered the prescription of more deficit spending in hopes that fiscal policy can create policy space for debt to be paid down safely while avoiding any asset shocks. Unfortunately, AS per Jaime Caruana of the BIS, it may be too little too late.
Caruana believes calm markets only encourage the build-up of more debt. Cutting edge research by Helene Rey reveals that any respite in advanced markets is matched by capital outflows to emerging markets building up their stocks of debt. “It is the stocks, not the shocks” that matter, as per Caruana, and those stocks are denominated in US dollars. If the USD continues to strengthen, many foreign corporates who borrowed in USD could be in trouble.

Usually an appreciating USD, or depreciating home currency, brings export benefits.

This time, Caruana fears, “the exchange rate takes on a financial amplification role” and any export benefits “may be offset or even reversed by stock valuation effects that operate through the balance sheet.”

That’s why, today, Stuart & Partners is excited to offer a USD-linked Principal Protected Note. The generic version of the USDPPN will use the USD trade-weighted index as its proxy. We will be happy to offer custom versions using any combination of Australian dollars, British pounds, euros, yen, or Mexican pesos. With the recent increase in interest rates, we’re not only able to hedge your exchange rate risk, we can now offer a strong return as well.
Using Monte Carlo Simulation with the appropriate drift and volatility terms derived from the necessary spot and forward yield curves, it will be possible for Stuart & Partners to estimate the expected risk and return of our USDPPN while

managing our risk in options.

X. CONCLUSION

We learn about pricing models and risk analysis using Monte-Carlo simulation and optimal portfolio allocation. For recommendation for customized deals for clients under subject beliefs, we compare the mean returns and the Sharpe ratio to provide the best possible result. We successfully propose a recommendation for a client who wants to buy the note for diversification purposes.
Lastly, we propose another business opportunity for the client.

XI. REFERENCE

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TABLES AND FIGURES

[Table 1] Recommendation for Customized Deals for Clients under Subject Beliefs

Client 1: P Measure	**Recommendation:**	**Values after recommendation:**	**Q measure using new payoff:**
Mean Return 0.0424 Mean Excess Return 0.0124 Sharpe ratio 0.0654	Mrs. Smith will not be happy with the performance of the note. We must offer her better terms. Changing weights and caps is not effective because all three commodities have the same return/risk assumptions. We must change the minimum return to get better results.	Mean Return 0.0775 Mean Excess Return 0.0475 Sharpe ratio 0.1488	fair value 1.0602e+06

Client 2: P measure Mean Return 0.0281 Mean Excess Return -0.0019 Sharpe ratio -0.0106	**Recommendation:** Mr. Johnson will not be happy with the performance of the note. We must offer him better terms. Changing weights and caps is not effective because all three commodities have the same return/risk assumptions. We must change the minimum return to get better results.	**Values after recommendation:** Mean Return 0.0436 Mean Excess Return 0.0136 Sharpe ratio 0.1187	**Q measure using new payoff:** fair value 1.0338e+06
Client 3: P measure Mean Return 0.0154 Mean Excess Return -0.0146 Sharpe ratio -0.1142	**Recommendation:** Ms. Williams will not be happy; we need to remove the minimum language to allow her to get full return stream to be realized. Considering correlation is 0, and not worried about all assets taking off at the same time. We must change the minimum return to get better results.	**Values after recommendation:** Mean Return 0.0568 Mean Excess Return 0.0268 Sharpe ratio 0.0744	**Q measure using new payoff:** fair value 1.0522e+06
Client 4: P measure Mean Return	**Recommendation:** Mr. Jones has a low Sharpe ratio. We need to change copper cap to realize more of expected return of copper.	**Values after recommendation:** Mean Return 0.0469	**Q measure using new payoff:** fair value

0.0319 Mean Excess Return 0.0019 Sharpe ratio 0.0101		Mean Excess Return 0.0169 Sharpe ratio 0.0599	1.0061e+06
Client 5: P measure Mean Return 0.0424 Mean Excess Return 0.0124 Sharpe ratio 0.0654	**Recommendation:** Miss Brown might not be happy as the expectations are too low. We must change cap to realize more upside of zinc.	**Values after recommendation:** Mean Return 0.0523 Mean Excess Return 0.0223 Sharpe ratio 0.0907	**Q measure using new payoff:** fair value 1.0003e+06

[Table 2] **Commodity prices**: (closing prices as of Wednesday, November 23)

Commodity	Current Price)
Cooper (Cu_P)	$5,633
Aluminum(Al_P)	$1,736
Zinc(Zn_P)	$2,615

[Table 3] **Drift** (**mu**) is the average annual log returns since 1994

Commodity	Drift(mu)
muCu	0.05
muAl	0.02
muZn	0.04

[Table 4] **Volatility** is the average annual log variance since 1994

Commodity	Volatility	Annualized Volatility
Copper	0.015889798	0.253241707
Aluminum	0.012583542	0.200548661
Zinc	0.016991473	0.270799522

[Table 5] **Correlation** is the correlation coefficient of daily prices since 1994

	Copper	Aluminum	Zinc
Copper	1		
Aluminum	0.809224534	1	
Zinc	0.817956466	0.8216203	1

[Table 6]

	Annual Return	Annual Std Dev	Return/Std Dev
S&P 500	6.98%	21.21%	0.33
CLN	3.54%	22.77%	0.16
Div. Portfolio	5.39%	15.75%	0.34

[Figure 1] Diversification Effects of Commodity Linked Note