Chapter 24: Relations and Functions

Homework 2: Function Composition, One-one & Onto, Inverse Functions & Properties

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Key Concepts

1. **Composition** (f g)(x) = f(g(x)) — Domain of g must be compatible with domain of f.
2. **One-one (Injective)**: f(a) = f(b) a = b
3. **Onto (Surjective)**: Every element in codomain is image of at least one element in domain.
4. **Bijective**: Both one-one and onto — has an inverse.
5. **Inverse function** f^{1}: Exists only if f is bijective; satisfies $f(f^{1}(x)) = x$ and $f^{1}(f(x)) = x$.
6. **Properties**: (f g) h = f (g h), $(f g)^{1} = g^{1} f^{1}$ (if both invertible).

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1. If f(x) = 2x + 3 and g(x) = x – 5, find (f g)(x).
2. If f(x) = x^{2} and g(x) = 3x – 1, find (g f)(x).
3. Let h(x) = (f g)(x) where f(x) = x and g(x) = x^{2} + 4. Find h(x).
4. If f(x) = 1/x and g(x) = x + 2, find (f g)(3).
5. Compute (f g)(–2) where f(x) = x^{3} – x and g(x) = —x—.
6. If f(x) = 4 – x and g(x) = x^{2}, find (g f)(x) and (f g)(x). Are they equal?
7. Let f(x) = 2x, g(x) = x/3, h(x) = x + 1. Find (f g h)(x).
8. If (f g)(x) = 5x – 7 and g(x) = 2x + 1, find f(x).
9. If f(x) = x^{2} + 3x and (f g)(x) = x + 6x^{2} + 9, find g(x).
10. Find (f f)(x) if f(x) = 3 – 2x.
11. Let f(x) = x + 1/x (x 0). Find (f f)(x).
12. If f(x) = (x + 5) and g(x) = x^{2} – 9, find the domain of (f g)(x).
13. Find the domain of (g f)(x) where f(x) = 1/(x – 2) and g(x) = x.

14. If f(x) = —x – 3— and g(x) = x^2, find (f g)(–4).
15. Let f(x) = 2x + 1, g(x) = x – 3, h(x) = 1/x. Find (f g h)(2).
16. Is f(x) = 3x + 7 one-one on ? Justify.
17. Is f(x) = x^2 one-one on ? Justify using definition.
18. Is f(x) = x^3 one-one on ? Prove it.
19. Is f(x) = 2x + 5 onto ? Justify.
20. Is f(x) = $e^x one - one and onto from to (0,)$?
21. Show that f(x) = 4 – x^2 is neither one-one nor onto on .
22. Is the function f: → given by f(n) = 2n one-one? Onto?
23. Is f: → given by f(x) = —x— one-one? Onto?
24. Let f(x) = x + 1/x for x ¿ 0. Is it one-one?
25. Is the constant function f(x) = 5 one-one? Onto? (codomain)
26. If f is one-one and g is one-one, is f g one-one? Justify.
27. If f is onto and g is onto, is f g onto? Give reason.
28. Is f(x) = x^3 – 3x one-one on ? (Hint: check derivative or horizontal line test)
29. Let f: [0,) → [0,) be f(x) = x^2. Is it one-one? Onto?
30. Determine whether f(x) = 1/(x^2 + 1) is one-one on .
31. Find the inverse of f(x) = 5x – 12.
32. Find f^1(x) if f(x) = (x + 3)/4.
33. If f(x) = x^3 + 2, find f^1(x).
34. Find the inverse of f(x) = 7 – 2x.
35. Let f(x) = (x + 4) (domain x –4). Find f^1(x) and its domain.
36. If f(x) = $e^{2x-1}, findf(x)$.
37. Find the inverse of f(x) = (2x + 3)/(x – 1).
38. Does f(x) = x^2 – 4 have an inverse on ? Why?
39. If f(x) = —x – 3—, does it have an inverse on ? Restrict domain to make it invertible.
40. Verify that f(x) = 3x + 1 and g(x) = (x – 1)/3 are inverses.
41. If f(x) = x^3 – 1, find f^1(26).
42. Find the inverse of f(x) = 4/(x – 2) + 3.
43. Let f(x) = 2x + 5. Find f^1(17).
44. If g(x) = 1 – x^2 (x 0), find g^1(x).
45. Show that f(x) = (x + 1)/(x – 1) is its own inverse.

46. If f and g are both invertible, prove that $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.
47. If f is invertible, what is $(f \circ f^{-1})(x)$?
48. Let $f(x) = 2x$, $g(x) = x + 3$. Find $(f \circ g)^{-1}(x)$.
49. Show that function composition is associative: $(f \circ g) \circ h = f \circ (g \circ h)$.
50. If $f(x) = x + 1$ and $g(x) = x - 1$, find $(f \circ g)^{-1}(x)$ and $(g \circ f)^{-1}(x)$. Compare.
51. Let $f(x) = x^2$ $(x \neq 0)$ and $g(x) = x$. Are f and g inverses?
52. If f is one-one, prove that $f \circ g$ is one-one only if g is one-one.
53. If f is onto, under what condition is $g \circ f$ onto?
54. Let $f(x) = 3x - 2$, $g(x) = x/3 + 2/3$. Show that $g = f^{-1}$.
55. Find $(f \circ g \circ f)(x)$ where $f(x) = x + 4$, $g(x) = 2x$.
56. If $f(x) = x^3$ and $g(x) = x^{1/3}$, $find (fg)(x) and (gf)(x)$.
57. Let $f(x) = 1/x$ $(x \neq 0)$. Find $f^{-1}(x)$.
58. If f is bijective, then f^{-1} is also bijective. True or false?
59. Give an example of two functions f and g such that $f \circ g = g \circ f$ but $f \neq g$.
60. Let $f(x) = x + \sin x$. Is f one-one on ? Does it have an inverse?