

# Chapter 24: Relations and Functions

Homework 1: Relations, Functions, Domain, Range, Evaluation & Types

Nalanda Academy, Wardha

## Key Concepts

1. **Relation:** Any subset of  $A \times B$  (Cartesian product).
2. **Function:** A relation where every element in the domain has exactly one output (no two different outputs for the same input).
3. **Domain:** Set of all possible input values.
4. **Range:** Set of all actual output values.
5. **Vertical Line Test:** A graph represents a function if no vertical line intersects it more than once.
6. **Types:** Constant, Identity, Linear, Quadratic, Cubic, Absolute value, Square root, Reciprocal, One-one (injective), Onto (surjective), Bijective, Many-one.

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1. Which of the following sets of ordered pairs is NOT a function? (A)  $\{(1,2), (2,3), (3,4)\}$  (B)  $\{(4,5), (4,6), (5,7)\}$  (C)  $\{(0,0), (1,1), (-1,1)\}$  (D)  $\{(7,7)\}$
  2. Does the relation  $\{(x, y) \mid y = x^2, x \in \mathbb{R}\}$  represent a function?
  3. Does the relation  $\{(x, y) \mid x = y^2, x \in \mathbb{R}\}$  represent a function?
  4. Explain why the graph of a circle is not a function.
  5. Using the vertical line test, determine whether the graph of  $y = -x - 1$  is a function.
  6. Is the mapping diagram where two arrows come out from the same input element a function?
  7. Is “Person  $\rightarrow$  Date of birth” a function? Justify.
  8. Is “Person  $\rightarrow$  Favourite colour” a function? Justify.
  9. Is the relation “is the father of” on the set of people a function?
  10. The relation “student  $\rightarrow$  roll number” in a class: function or not?
  11. Which of the following is a function from  $A$  to  $B$ ? (A)  $f(x) = x + 1$  (B)  $f(x) = x/2$  (C)  $f(x) = 1/x$  (D)  $f(x) = x^2$

12. Give one real-life example of a relation that is a function.
13. Give one real-life example of a relation that is not a function.
14. Does  $y = (x - 3)$  represent a function? What is its natural domain?
15. The graph passes the vertical line test but has a hole: still a function?
16. Find the domain and range of the relation  $\{(-3,4), (-2,4), (-1,5), (0,4)\}$ .
17. Find domain and range of  $f = \{(x, x^2 - 4) \mid x = -2, -1, 0, 1, 2\}$ .
18. Determine the domain of  $y = 1/(x^2 - 4)$ .
19. Find the domain of  $y = (5 - x)$ .
20. What is the domain of  $f(x) = \log(x + 7)$ ?
21. Find the range of the constant function  $f(x) = -6$ .
22. Find the range of  $f(x) = x^2 + 3x + 5$  ( $x \in \mathbb{R}$ ).
23. Find domain and range of  $y = -x - 3$ .
24. What values make  $y = 3/(x - 5)(x + 2)$  undefined?
25. Find domain of  $y = (x^2 - 16)$ .
26. For  $y = x^3 - 8x$ , domain =  $\mathbb{R}$ . What is the range?
27. The function gives the number of handshakes possible among  $n$  people. Find domain and range.
28. Area of a circle with radius  $r$ : domain and range?
29. Distance travelled at constant speed 60 km/h in time  $t$  hours: domain and range?
30. Temperature in Kelvin (always  $> 0$ ): domain and range when converted to  ${}^\circ\text{C}$ ?
31. Find range of  $f(x) = 2x + 5$  where domain is  $-4, -2, 0, 2, 4$ .
32. Find domain and range of the relation “ $x$  is 5 more than  $y$ ” on set  $\{1, 2, 3, 4, 5\}$ .
33.  $y = 1/x$ ,  $x \neq 0$ . Domain and range?
34.  $y = (9 - x^2)$ . Domain and range?
35.  $f(x) = x + 1/x$  for  $x \neq 0$ . What is the range?
36. If  $f(x) = 5x - 8$ , find  $f(3)$  and  $f(-2)$ .
37. Evaluate  $g(x) = x^2 - 7x + 10$  at  $x = 4$ .
38.  $h(x) = -2x - 3$ . Find  $h(-1)$  and  $h(5)$ .
39.  $p(x) = (x + 10)$ . Find  $p(6)$  and  $p(15)$ .
40.  $q(x) = 1/(x - 4)$ . Find  $q(6)$  and  $q(0)$ .
41. If  $f(x) = 3x + 7$ , solve  $f(x) = 22$  for  $x$ .
42. If  $g(x) = x^2 - 5x + 6$ , find  $x$  such that  $g(x) = 0$ .
43.  $r(x) = x^3 - 2x^2 + x$ . Find  $r(2)$ .

44.  $s(x) = 4 - x^2$ . Find  $s(-3)$ .
45.  $t(x) = 2x + 1/x$  for  $x \neq 0$ . Find  $t(2)$ .
46. If  $f(x) = x^2 + 4x - 5$ , find  $f(-5)$  and  $f(1)$ .
47. Solve for  $x$ :  $2x + 5 = 17$ .
48. If  $h(x) = (x - 1)$ , find  $x$  when  $h(x) = 4$ .
49. Evaluate  $f(x) = -x^2 - 7x + 3$  at  $x = 10$  and  $x = 4$ .
50. Find  $x$  if  $g(x) = x^2 - 9 = 16$ .
51. Classify  $f(x) = 12$  as constant / linear / quadratic / other.
52. Classify  $f(x) = 3x + 1$ .
53. Classify  $f(x) = x^2 - 4x + 3$ .
54. Classify  $f(x) = -x + 2$ .
55. Classify  $f(x) = (x - 5)$ .
56. Classify  $f(x) = 1/(x + 3)$ .
57. Classify  $f(x) = x^3 - x$ .
58. Is  $f(x) = x^2$  one-one on ? Justify.
59. Is  $f(x) = 2x + 5$  one-one and onto on ?
60. Is  $f(x) = x^3$  one-one on ?
61. Give an example of a many-one function.
62. Give an example of a function that is one-one but not onto (on ).
63. A constant function is one-one? Onto? (codomain )
64. If  $f$  is bijective, it is both one-one and onto. True or false?
65. Is  $y = \sin x$  one-one on ? Why?
66. Find range of  $f(x) = x^2 - 6x + 10$ .
67. Give an example of a piecewise function.
68. For  $f(x) = x + 2/x$  ( $x \neq 0$ ), find minimum value using calculus or AM-GM (optional).
69. If domain =  $-2, -1, 0, 1, 2$ , find range of  $f(x) = x^2 + x$ .
70. If codomain =  $1, 2, 3, 4, 5$  and function is onto, minimum number of elements in domain?