# **Sage Reference Manual**

Release 9.5

**The Sage Development Team** 

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Welcome to the Sage reference manual. Here you find documentation for all of Sage's features, illustrated with lots of examples. A thematic index follows.

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# ONE

# **USER INTERFACES**

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- Jupyter Notebook Interface

# TWO

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## **THREE**

#### **MATHEMATICS**

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- Coercion
- Categories

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- Arithmetic Subgroups of  $SL_2(\mathbf{Z})$
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