Sage 9.2 Reference Manual: Standard Semirings

Release 9.2

The Sage Development Team

Oct 25, 2020

CONTENTS

1 Non Negative Integer Semiring 1

2 Tropical Semirings 3

3 Indices and Tables 7

Python Module Index 9

Index 11

CHAPTER

ONE

NON NEGATIVE INTEGER SEMIRING

sage.rings.semirings.non_negative_integer_semiring.**NN = Non negative integer semiring**

class sage.rings.semirings.non_negative_integer_semiring.**NonNegativeIntegerSemiring**

Bases: sage.sets.non_negative_integers.NonNegativeIntegers

A class for the semiring of the non negative integers

This parent inherits from the infinite enumerated set of non negative integers and endows it with its natural semiring structure.

EXAMPLES:

```
sage: NonNegativeIntegerSemiring()
Non negative integer semiring
```

For convenience, NN is a shortcut for NonNegativeIntegerSemiring():

```
sage: NN == NonNegativeIntegerSemiring()
True

sage: NN.category()
Category of facade infinite enumerated commutative semirings
```

Here is a piece of the Cayley graph for the multiplicative structure:

```
sage: G = NN.cayley_graph(elements=range(9), generators=[0,1,2,3,5,7])
sage: G
Looped multi-digraph on 9 vertices
sage: G.plot()
Graphics object consisting of 48 graphics primitives
```

This is the Hasse diagram of the divisibility order on NN.

sage: Poset(NN.cayley_graph(elements=[1..12], generators=[2,3,5,7,11])).show()

Note: as for NonNegativeIntegers, NN is currently just a “facade” parent; namely its elements are plain Sage Integers with Integer Ring as parent:

```
sage: x = NN(15); type(x)
<type 'sage.rings.integer.Integer'>
sage: x.parent()
Integer Ring
sage: x+3
18
```

additive_semigroup_generators()

Returns the additive semigroup generators of `self`.

EXAMPLES:

```
sage: NN.additive_semigroup_generators()
Family (0, 1)
```

CHAPTER

TWO

TROPICAL SEMIRINGS

AUTHORS:

- Travis Scrimshaw (2013-04-28) - Initial version

class sage.rings.semirings.tropical_semiring.**TropicalSemiring**(*base*, *use_min=True*)

Bases: sage.structure.parent.Parent, sage.structure.unique_representation.UniqueRepresentation

The tropical semiring.

Given an ordered additive semigroup R, we define the tropical semiring $T = R \cup \{+\infty\}$ by defining tropical addition and multiplication as follows:

$$a \oplus b = \min(a, b), \qquad a \odot b = a + b.$$

In particular, note that there are no (tropical) additive inverses (except for ∞), and every element in R has a (tropical) multiplicative inverse.

There is an alternative definition where we define $T = R \cup \{-\infty\}$ and alter tropical addition to be defined by

$$a \oplus b = \max(a, b).$$

To use the $\max$ definition, set the argument use_min = False.

Warning: *zero()* and *one()* refer to the tropical additive and multiplicative identities respectively. These are **not** the same as calling T(0) and T(1) respectively as these are **not** the tropical additive and multiplicative identities respectively.

Specifically do not use sum(...) as this converts 0 to 0 as a tropical element, which is not the same as *zero()*. Instead use the sum method of the tropical semiring:

```
sage: T = TropicalSemiring(QQ)

sage: sum([T(1), T(2)]) # This is wrong
0
sage: T.sum([T(1), T(2)]) # This is correct
1
```

Be careful about using code that has not been checked for tropical safety.

INPUT:

- base – the base ordered additive semigroup R

- `use_min` – (default: `True`) if `True`, then the semiring uses $a \oplus b = \min(a, b)$; otherwise uses $a \oplus b = \max(a, b)$

EXAMPLES:

```
sage: T = TropicalSemiring(QQ)
sage: elt = T(2); elt
2
```

Recall that tropical addition is the minimum of two elements:

```
sage: T(3) + T(5)
3
```

Tropical multiplication is the addition of two elements:

```
sage: T(2) * T(3)
5
sage: T(0) * T(-2)
-2
```

We can also do tropical division and arbitrary tropical exponentiation:

```
sage: T(2) / T(1)
1
sage: T(2)^(-3/7)
-6/7
```

Note that "zero" and "one" are the additive and multiplicative identities of the tropical semiring. In general, they are **not** the elements 0 and 1 of R, respectively, even if such elements exist (e.g., for $R = \mathbf{Z}$), but instead the (tropical) additive and multiplicative identities $+\infty$ and 0 respectively:

```
sage: T.zero() + T(3) == T(3)
True
sage: T.one() * T(3) == T(3)
True
sage: T.zero() == T(0)
False
sage: T.one() == T(1)
False
```

Element
alias of *TropicalSemiringElement*

additive_identity()
Return the (tropical) additive identity element $+\infty$.

EXAMPLES:

```
sage: T = TropicalSemiring(QQ)
sage: T.zero()
+infinity
```

gens()
Return the generators of `self`.

EXAMPLES:

```
sage: T = TropicalSemiring(QQ)
sage: T.gens()
(1, +infinity)
```

infinity()

Return the (tropical) additive identity element $+\infty$.

EXAMPLES:

```
sage: T = TropicalSemiring(QQ)
sage: T.zero()
+infinity
```

multiplicative_identity()

Return the (tropical) multiplicative identity element 0.

EXAMPLES:

```
sage: T = TropicalSemiring(QQ)
sage: T.one()
0
```

one()

Return the (tropical) multiplicative identity element 0.

EXAMPLES:

```
sage: T = TropicalSemiring(QQ)
sage: T.one()
0
```

zero()

Return the (tropical) additive identity element $+\infty$.

EXAMPLES:

```
sage: T = TropicalSemiring(QQ)
sage: T.zero()
+infinity
```

class sage.rings.semirings.tropical_semiring.**TropicalSemiringElement**

Bases: sage.structure.element.Element

An element in the tropical semiring over an ordered additive semigroup R. Either in R or ∞. The operators $+, \cdot$ are defined as the tropical operators $\oplus, \odot$ respectively.

lift()

Return the value of `self` lifted to the base.

EXAMPLES:

```
sage: T = TropicalSemiring(QQ)
sage: elt = T(2)
sage: elt.lift()
2
sage: elt.lift().parent() is QQ
True
sage: T.additive_identity().lift().parent()
The Infinity Ring
```

multiplicative_order()

Return the multiplicative order of `self`.

EXAMPLES:

```
sage: T = TropicalSemiring(QQ)
sage: T.multiplicative_identity().multiplicative_order()
1
sage: T.additive_identity().multiplicative_order()
+Infinity
```

class sage.rings.semirings.tropical_semiring.**TropicalToTropical**

Bases: `sage.categories.map.Map`

Map from the tropical semiring to itself (possibly with different bases). Used in coercion.

CHAPTER

THREE

INDICES AND TABLES

- Index
- Module Index
- Search Page

PYTHON MODULE INDEX

r

sage.rings.semirings.non_negative_integer_semiring, 1
sage.rings.semirings.tropical_semiring, 3

INDEX

A

additive_identity() (*sage.rings.semirings.tropical_semiring.TropicalSemiring method*), 4
additive_semigroup_generators() (*sage.rings.semirings.non_negative_integer_semiring.NonNegativeIntegerSemiring method*), 1

E

Element (*sage.rings.semirings.tropical_semiring.TropicalSemiring attribute*), 4

G

gens() (*sage.rings.semirings.tropical_semiring.TropicalSemiring method*), 4

I

infinity() (*sage.rings.semirings.tropical_semiring.TropicalSemiring method*), 5

L

lift() (*sage.rings.semirings.tropical_semiring.TropicalSemiringElement method*), 5

M

module
    sage.rings.semirings.non_negative_integer_semiring, 1
    sage.rings.semirings.tropical_semiring, 3
multiplicative_identity() (*sage.rings.semirings.tropical_semiring.TropicalSemiring method*), 5
multiplicative_order() (*sage.rings.semirings.tropical_semiring.TropicalSemiringElement method*), 5

N

NN (*in module sage.rings.semirings.non_negative_integer_semiring*), 1
NonNegativeIntegerSemiring (*class in sage.rings.semirings.non_negative_integer_semiring*), 1

O

one() (*sage.rings.semirings.tropical_semiring.TropicalSemiring method*), 5

S

sage.rings.semirings.non_negative_integer_semiring
    module, 1
sage.rings.semirings.tropical_semiring
    module, 3

T

TropicalSemiring (*class in sage.rings.semirings.tropical_semiring*), 3
TropicalSemiringElement (*class in sage.rings.semirings.tropical_semiring*), 5
TropicalToTropical (*class in sage.rings.semirings.tropical_semiring*), 6

Z

zero() (*sage.rings.semirings.tropical_semiring.TropicalSemiring method*), 5