

---

# **Reference Manual**

***Release 9.7***

**The Sage Development Team**

**Sep 20, 2022**



# CONTENTS

<b>1</b>	<b>User Interfaces</b>	<b>3</b>
<b>2</b>	<b>Graphics</b>	<b>5</b>
<b>3</b>	<b>Mathematics</b>	<b>7</b>
3.1	Parents and Categories . . . . .	7
3.2	Basic Rings and Fields . . . . .	7
3.3	Linear Algebra . . . . .	7
3.4	Calculus and Analysis . . . . .	8
3.5	Probability and Statistics . . . . .	8
3.6	Mathematical Structures . . . . .	8
3.7	Discrete Mathematics . . . . .	8
3.8	Geometry and Topology . . . . .	9
3.9	Homological Algebra . . . . .	9
3.10	Number Fields, Function Fields, and Valuations . . . . .	9
3.11	Number Theory . . . . .	9
3.12	Algebraic and Arithmetic Geometry . . . . .	10
3.13	Miscellaneous . . . . .	10
<b>4</b>	<b>Programming</b>	<b>11</b>
4.1	Facilities . . . . .	11
4.2	Interfaces . . . . .	11
4.3	Documentation System . . . . .	11
<b>5</b>	<b>General Information</b>	<b>13</b>
<b>6</b>	<b>Indices and Tables</b>	<b>15</b>



Here you find documentation for all of [Sage's](#) features, illustrated with lots of examples. A thematic index follows.  
This documentation is licensed under the [Creative Commons Attribution-Share Alike 3.0 License](#).



## USER INTERFACES

- Command Line Interface
- Jupyter Notebook Interface





## GRAPHICS

- 2D Graphics
- 3D Graphics



### 3.1 Parents and Categories

- Parents and Elements
- Coercion
- Categories

### 3.2 Basic Rings and Fields

- Integers and Rational Numbers
- Real and Complex Numbers
- Commutative Polynomials
- Power Series and Laurent Series
- Finite Rings and Fields
- $p$ -adic Numbers
- Noncommutative Polynomials
- Quaternion Algebras

### 3.3 Linear Algebra

- Matrices and Spaces of Matrices
- Vectors and Modules
- Tensors on Free Modules of Finite Rank

## 3.4 Calculus and Analysis

- Symbolic Calculus
- Mathematical Constants
- Elementary and Special Functions
- Asymptotic Expansions
- Numerical Optimization

## 3.5 Probability and Statistics

- Probability
- Statistics
- Quantitative Finance

## 3.6 Mathematical Structures

- Sets
- Monoids
- Groups
- Semirings
- Rings
- Algebras

## 3.7 Discrete Mathematics

- Combinatorics
- Graph Theory
- Quivers
- Matroid Theory
- Discrete Dynamics
- Coding Theory
- Cryptography
- Game Theory
- Symbolic Logic
- SAT solvers

## 3.8 Geometry and Topology

- Euclidean Spaces and Vector Calculus
- Combinatorial and Discrete Geometry
- Cell Complexes, Simplicial Complexes, and Simplicial Sets
- Manifolds and Differential Geometry
- Hyperbolic Geometry
- Parametrized Surfaces
- Knot Theory

## 3.9 Homological Algebra

- Chain Complexes and their Homology
- Resolutions

## 3.10 Number Fields, Function Fields, and Valuations

- Number Fields
- Function Fields
- Discrete Valuations

## 3.11 Number Theory

- Diophantine approximation
- Quadratic Forms
- $L$ -Functions
- Arithmetic Subgroups of  $SL_2(\mathbb{Z})$
- General Hecke Algebras and Hecke Modules
- Modular Symbols
- Modular Forms
- Quasimodular Forms
- Modular Forms for Hecke Triangle Groups
- Modular Abelian Varieties
- Miscellaneous Modular-Form-Related Modules

## 3.12 Algebraic and Arithmetic Geometry

- Schemes
- Plane and Space Curves
- Elliptic and Hyperelliptic Curves

## 3.13 Miscellaneous

- Databases
- Games

## PROGRAMMING

### 4.1 Facilities

- Data Structures
- Utilities
- Test Framework
- Parallel Computing

### 4.2 Interfaces

- Interpreter Interfaces
- C/C++ Library Interfaces
- Python Technicalities

### 4.3 Documentation System

- Documentation System





## GENERAL INFORMATION

- External Packages
- Bibliographic References
- History and License



## INDICES AND TABLES

- `genindex`
- `modindex`
- `search`