
Sage Reference Manual: Standard Semirings

Release 8.5

The Sage Development Team

Dec 29, 2018

CONTENTS

1	Non Negative Integer Semiring	1
2	Tropical Semirings	3
3	Indices and Tables	7
	Python Module Index	9
	Index	11

NON NEGATIVE INTEGER SEMIRING

```
class sage.rings.semirings.non_negative_integer_semiring.NonNegativeIntegerSemiring
    Bases: sage.sets.non_negative_integers.NonNegativeIntegers
```

A class for the semiring of the non negative integers

This parent inherits from the infinite enumerated set of non negative integers and endows it with its natural semiring structure.

EXAMPLES:

```
sage: NonNegativeIntegerSemiring()
Non negative integer semiring
```

For convenience, NN is a shortcut for `NonNegativeIntegerSemiring()`:

```
sage: NN == NonNegativeIntegerSemiring()
True

sage: NN.category()
Category of facade infinite enumerated commutative semirings
```

Here is a piece of the Cayley graph for the multiplicative structure:

```
sage: G = NN.cayley_graph(elements=range(9), generators=[0,1,2,3,5,7])
sage: G
Looped multi-digraph on 9 vertices
sage: G.plot()
Graphics object consisting of 48 graphics primitives
```

This is the Hasse diagram of the divisibility order on NN.

```
sage: Poset(NN.cayley_graph(elements=[1..12], generators=[2,3,5,7,11])).show()
```

Note: as for `NonNegativeIntegers`, NN is currently just a “facade” parent; namely its elements are plain Sage Integers with `Integer Ring` as parent:

```
sage: x = NN(15); type(x)
<type 'sage.rings.integer.Integer'>
sage: x.parent()
Integer Ring
sage: x+3
18
```

```
additive_semigroup_generators()
```

Returns the additive semigroup generators of self.

EXAMPLES:

```
sage: NN.additive_semigroup_generators()  
Family (0, 1)
```

TROPICAL SEMIRINGS

AUTHORS:

- Travis Scrimshaw (2013-04-28) - Initial version

```
class sage.rings.semirings.tropical_semiring.TropicalSemiring(base,  
                                                             use_min=True)  
Bases: sage.structure.parent.Parent, sage.structure.unique_representation.  
UniqueRepresentation
```

The tropical semiring.

Given an ordered additive semigroup R , we define the tropical semiring $T = R \cup \{+\infty\}$ by defining tropical addition and multiplication as follows:

$$a \oplus b = \min(a, b), \quad a \odot b = a + b.$$

In particular, note that there are no (tropical) additive inverses (except for ∞), and every element in R has a (tropical) multiplicative inverse.

There is an alternative definition where we define $T = R \cup \{-\infty\}$ and alter tropical addition to be defined by

$$a \oplus b = \max(a, b).$$

To use the max definition, set the argument `use_min = False`.

Warning: `zero()` and `one()` refer to the tropical additive and multiplicative identities respectively. These are **not** the same as calling `T(0)` and `T(1)` respectively as these are **not** the tropical additive and multiplicative identities respectively.

Specifically do not use `sum(...)` as this converts 0 to 0 as a tropical element, which is not the same as `zero()`. Instead use the `sum` method of the tropical semiring:

```
sage: T = TropicalSemiring(QQ)  
  
sage: sum([T(1), T(2)]) # This is wrong  
0  
sage: T.sum([T(1), T(2)]) # This is correct  
1
```

Be careful about using code that has not been checked for tropical safety.

INPUT:

- `base` – the base ordered additive semigroup R

- `use_min` – (default: `True`) if `True`, then the semiring uses $a \oplus b = \min(a, b)$; otherwise uses $a \oplus b = \max(a, b)$

EXAMPLES:

```
sage: T = TropicalSemiring(QQ)
sage: elt = T(2); elt
2
```

Recall that tropical addition is the minimum of two elements:

```
sage: T(3) + T(5)
3
```

Tropical multiplication is the addition of two elements:

```
sage: T(2) * T(3)
5
sage: T(0) * T(-2)
-2
```

We can also do tropical division and arbitrary tropical exponentiation:

```
sage: T(2) / T(1)
1
sage: T(2) ^ (-3/7)
-6/7
```

Note that “zero” and “one” are the additive and multiplicative identities of the tropical semiring. In general, they are **not** the elements 0 and 1 of R , respectively, even if such elements exist (e.g., for $R = \mathbf{Z}$), but instead the (tropical) additive and multiplicative identities $+\infty$ and 0 respectively:

```
sage: T.zero() + T(3) == T(3)
True
sage: T.one() * T(3) == T(3)
True
sage: T.zero() == T(0)
False
sage: T.one() == T(1)
False
```

Element

alias of *TropicalSemiringElement*

additive_identity()

Return the (tropical) additive identity element $+\infty$.

EXAMPLES:

```
sage: T = TropicalSemiring(QQ)
sage: T.zero()
+infinity
```

gens()

Return the generators of `self`.

EXAMPLES:


```
sage: T = TropicalSemiring(QQ)
sage: T.gens()
(1, +infinity)
```

infinity()

Return the (tropical) additive identity element $+\infty$.

EXAMPLES:

```
sage: T = TropicalSemiring(QQ)
sage: T.zero()
+infinity
```

multiplicative_identity()

Return the (tropical) multiplicative identity element 0.

EXAMPLES:

```
sage: T = TropicalSemiring(QQ)
sage: T.one()
0
```

one()

Return the (tropical) multiplicative identity element 0.

EXAMPLES:

```
sage: T = TropicalSemiring(QQ)
sage: T.one()
0
```

zero()

Return the (tropical) additive identity element $+\infty$.

EXAMPLES:

```
sage: T = TropicalSemiring(QQ)
sage: T.zero()
+infinity
```

class sage.rings.semiringsemirings.tropical_semiring.TropicalSemiringElement

Bases: sage.structure.element.Element

An element in the tropical semiring over an ordered additive semigroup R . Either in R or ∞ . The operators $+$, \cdot are defined as the tropical operators \oplus , \odot respectively.

lift()

Return the value of `self` lifted to the base.

EXAMPLES:

```
sage: T = TropicalSemiring(QQ)
sage: elt = T(2)
sage: elt.lift()
2
sage: elt.lift().parent() is QQ
True
sage: T.additive_identity().lift().parent()
The Infinity Ring
```

multiplicative_order()

Return the multiplicative order of `self`.

EXAMPLES:

```
sage: T = TropicalSemiring(QQ)
sage: T.multiplicative_identity().multiplicative_order()
1
sage: T.additive_identity().multiplicative_order()
+Infinity
```

class `sage.rings.semirings.tropical_semiring.TropicalToTropical`

Bases: `sage.categories.map.Map`

Map from the tropical semiring to itself (possibly with different bases). Used in coercion.

INDICES AND TABLES

- [Index](#)
- [Module Index](#)
- [Search Page](#)

PYTHON MODULE INDEX

r

`sage.rings.semirings.non_negative_integer_semiring`, 1
`sage.rings.semirings.tropical_semiring`, 3

INDEX

A

`additive_identity()` (`sage.rings.semiring.tropical_semiring.TropicalSemiring` method), 4

`additive_semigroup_generators()` (`sage.rings.semiring.non_negative_integer_semiring.NonNegativeIntegerSemiring` method), 1

E

`Element` (`sage.rings.semiring.tropical_semiring.TropicalSemiring` attribute), 4

G

`gens()` (`sage.rings.semiring.tropical_semiring.TropicalSemiring` method), 4

I

`infinity()` (`sage.rings.semiring.tropical_semiring.TropicalSemiring` method), 5

L

`lift()` (`sage.rings.semiring.tropical_semiring.TropicalSemiringElement` method), 5

M

`multiplicative_identity()` (`sage.rings.semiring.tropical_semiring.TropicalSemiring` method), 5

`multiplicative_order()` (`sage.rings.semiring.tropical_semiring.TropicalSemiringElement` method), 5

N

`NonNegativeIntegerSemiring` (class in `sage.rings.semiring.non_negative_integer_semiring`), 1

O

`one()` (`sage.rings.semiring.tropical_semiring.TropicalSemiring` method), 5

S

`sage.rings.semiring.non_negative_integer_semiring` (module), 1

`sage.rings.semiring.tropical_semiring` (module), 3

T

`TropicalSemiring` (class in `sage.rings.semiring.tropical_semiring`), 3

`TropicalSemiringElement` (class in `sage.rings.semiring.tropical_semiring`), 5

`TropicalToTropical` (class in `sage.rings.semiring.tropical_semiring`), 6

Z

`zero()` (`sage.rings.semiring.tropical_semiring.TropicalSemiring` method), 5