
Sage 9.1 Reference Manual: Parallel Computing

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The Sage Development Team

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DECORATE INTERFACE FOR PARALLEL COMPUTATION

```
class sage.parallel.decorate.Fork (timeout=0, verbose=False)
    Bases: object
```

A fork decorator class.

```
class sage.parallel.decorate.Parallel (p_iter='fork', ncpus=None, **kws)
    Bases: object
```

Create a parallel-decorated function. This is the object created by `parallel()`.

```
class sage.parallel.decorate.ParallelFunction (parallel, func)
    Bases: object
```

Class which parallelizes a function or class method. This is typically accessed indirectly through `Parallel.__call__()`.

```
sage.parallel.decorate.fork (f=None, timeout=0, verbose=False)
```

Decorate a function so that when called it runs in a forked subprocess. This means that it won't have any in-memory side effects on the parent Sage process. The pexpect interfaces are all reset.

INPUT:

- `f` – a function
- `timeout` – (default: 0) if positive, kill the subprocess after this many seconds (wall time)
- `verbose` – (default: False) whether to print anything about what the decorator does (e.g., killing the subprocess)

Warning: The forked subprocess will not have access to data created in pexpect interfaces. This behavior with respect to pexpect interfaces is very important to keep in mind when setting up certain computations. It's the one big limitation of this decorator.

EXAMPLES:

We create a function and run it with the `fork` decorator. Note that it does not have a side effect. Despite trying to change the global variable `a` below in `g`, the variable `a` does not get changed:

```
sage: a = 5
sage: @fork
....: def g(n, m):
....:     global a
....:     a = 10
....:     return factorial(n).ndigits() + m
sage: g(5, m=5)
```

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```
8
sage: a
5
```

We use `fork` to make sure that the function terminates after one second, no matter what:

```
sage: @fork(timeout=1, verbose=True)
....: def g(n, m): return factorial(n).ndigits() + m
sage: g(5, m=5)
8
sage: g(10^7, m=5)
Killing subprocess ... with input ((10000000,), {'m': 5}) which took too long
'NO DATA (timed out)'
```

We illustrate that the state of the pexpect interface is not altered by forked functions (they get their own new pexpect interfaces!):

```
sage: gp.eval('a = 5')
'5'
sage: @fork()
....: def g():
....:     gp.eval('a = 10')
....:     return gp.eval('a')
sage: g()
'10'
sage: gp.eval('a')
'5'
```

We illustrate that the forked function has its own pexpect interface:

```
sage: gp.eval('a = 15')
'15'
sage: @fork()
....: def g(): return gp.eval('a')
sage: g()
'a'
```

We illustrate that segfaulting subprocesses are no trouble at all:

```
sage: cython('def f(): print(<char*>0)')
sage: @fork
....: def g(): f()
sage: print("this works"); g()
this works...

-----
Unhandled SIG...
-----
'NO DATA'
```

`sage.parallel.decorate.normalize_input(a)`
Convert a to a pair (args, kwds) using some rules:

- if already of that form, leave that way.
- if a is a tuple make (a, {})
- if a is a dict make (tuple([], a))

- otherwise make `((a,), {})`

INPUT:

- `a` – object

OUTPUT:

- `args` – tuple
- `kwds` – dictionary

EXAMPLES:

```
sage: sage.parallel.decorate.normalize_input( (2, {3:4}) )
((2, {3: 4}), {})
sage: sage.parallel.decorate.normalize_input( (2,3) )
((2, 3), {})
sage: sage.parallel.decorate.normalize_input( {3:4} )
((), {3: 4})
sage: sage.parallel.decorate.normalize_input( 5 )
((5,), {})
```

`sage.parallel.decorate.parallel(p_iter='fork', ncpus=None, **kwds)`

This is a decorator that gives a function a parallel interface, allowing it to be called with a list of inputs, whose values will be computed in parallel.

Warning: The parallel subprocesses will not have access to data created in pexpect interfaces. This behavior with respect to pexpect interfaces is very important to keep in mind when setting up certain computations. It's the one big limitation of this decorator.

INPUT:

- **`p_iter` – parallel iterator function or string:**
 - 'fork' – (default) use a new forked subprocess for each input
 - 'multiprocessing' – use multiprocessing library
 - 'reference' – use a fake serial reference implementation
- `ncpus` – integer, maximal number of subprocesses to use at the same time
- `timeout` – number of seconds until each subprocess is killed (only supported by 'fork'; zero means not at all)

Warning: If you use anything but 'fork' above, then a whole new subprocess is spawned, so none of your local state (variables, certain functions, etc.) is available.

EXAMPLES:

We create a simple decoration for a simple function. The number of cpus (or cores, or hardware threads) is automatically detected:

```
sage: @parallel
....: def f(n): return n*n
sage: f(10)
100
```

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```
sage: sorted(list(f([1,2,3])))
[((1,), {}), 1), ((2,), {}), 4), ((3,), {}), 9)]
```

We use exactly two cpus:

```
sage: @parallel(2)
....: def f(n): return n*n
```

We create a decorator that uses three subprocesses, and times out individual processes after 10 seconds:

```
sage: @parallel(ncpus=3, timeout=10)
....: def fac(n): return factor(2^n-1)
sage: for X, Y in sorted(list(fac([101,119,151,197,209]))): print((X,Y))
(((101,), {}), 7432339208719 * 341117531003194129)
(((119,), {}), 127 * 239 * 20231 * 131071 * 62983048367 * 131105292137)
(((151,), {}), 18121 * 55871 * 165799 * 2332951 * 7289088383388253664437433)
(((197,), {}), 7487 * 26828803997912886929710867041891989490486893845712448833)
(((209,), {}), 23 * 89 * 524287 * 94803416684681 * 1512348937147247 *
↪ 5346950541323960232319657)

sage: @parallel('multiprocessing')
....: def f(N): return N^2
sage: v = list(f([1,2,4])); v.sort(); v
[((1,), {}), 1), ((2,), {}), 4), ((4,), {}), 16)]
sage: @parallel('reference')
....: def f(N): return N^2
sage: v = list(f([1,2,4])); v.sort(); v
[((1,), {}), 1), ((2,), {}), 4), ((4,), {}), 16)]
```

For functions that take multiple arguments, enclose the arguments in tuples when calling the parallel function:

```
sage: @parallel
....: def f(a,b): return a*b
sage: for X, Y in sorted(list(f([(2,3), (3,5), (5,7)]))): print((X, Y))
((2, 3), {}), 6)
((3, 5), {}), 15)
((5, 7), {}), 35)
```

For functions that take a single tuple as an argument, enclose it in an additional tuple at call time, to distinguish it as the first argument, as opposed to a tuple of arguments:

```
sage: @parallel
....: def firstEntry(aTuple): return aTuple[0]
sage: for X, Y in sorted(list(firstEntry([(1,2,3,4), (5,6,7,8)]))): print((X,
↪ Y))
(((1, 2, 3, 4), {}), 1)
(((5, 6, 7, 8), {}), 5)
```

The parallel decorator also works with methods, classmethods, and staticmethods. Be sure to apply the parallel decorator after (“above”) either the classmethod or staticmethod decorators:

```
sage: class Foo(object):
....:     @parallel(2)
....:     def square(self, n):
....:         return n*n
....:     @parallel(2)
```

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```
.....: @classmethod
.....: def square_classmethod(cls, n):
.....:     return n*n
sage: a = Foo()
sage: a.square(3)
9
sage: sorted(a.square([2,3]))
[(((2,), {}), 4), (((3,), {}), 9)]
sage: Foo.square_classmethod(3)
9
sage: sorted(Foo.square_classmethod([2,3]))
[(((2,), {}), 4), (((3,), {}), 9)]
sage: Foo.square_classmethod(3)
9
```

Warning: Currently, parallel methods do not work with the multiprocessing implementation.

REFERENCE PARALLEL PRIMITIVES

These are reference implementations of basic parallel primitives. These are not actually parallel, but work the same way. They are good for testing.

`sage.parallel.reference.parallel_iter(f, inputs)`
Reference parallel iterator implementation.

INPUT:

- `f` – a Python function that can be pickled using the `pickle_function` command.
- `inputs` – a list of pickleable pairs (`args`, `kwds`), where `args` is a tuple and `kwds` is a dictionary.

OUTPUT:

- iterator over 2-tuples `(inputs[i], f(inputs[i]))`, where the order may be completely random

EXAMPLES:

```
sage: def f(N,M=10): return N*M
sage: inputs = [((2,3),{}), (tuple([]), {'M':5, 'N':3}), ((2,),{})]
sage: set_random_seed(0)
sage: for a, val in sage.parallel.reference.parallel_iter(f, inputs):
....:     print((a, val))
((2,), {}), 20
((), {'M': 5, 'N': 3}), 15
((2, 3), {}), 6
sage: for a, val in sage.parallel.reference.parallel_iter(f, inputs):
....:     print((a, val))
((), {'M': 5, 'N': 3}), 15
((2,), {}), 20
((2, 3), {}), 6
```


PARALLEL ITERATOR BUILT USING THE `FORK()` SYSTEM CALL

```
class sage.parallel.use_fork.WorkerData (input, starttime=None, failure="")
    Bases: object
```

Simple class which stores data about a running `p_iter_fork` worker.

This just stores three attributes:

- `input`: the input value used by this worker
- `starttime`: the walltime when this worker started
- `failure`: an optional message indicating the kind of failure

EXAMPLES:

```
sage: from sage.parallel.use_fork import WorkerData
sage: W = WorkerData(42); W
<sage.parallel.use_fork.WorkerData object at ...>
sage: W.starttime # random
1499330252.463206
```

```
class sage.parallel.use_fork.p_iter_fork(ncpus, timeout=0, verbose=False,
                                         reset_interfaces=True)
```

Bases: object

A parallel iterator implemented using `fork()`.

INPUT:

- **`ncpus`** – the maximal number of simultaneous subprocesses to spawn
- **`timeout`** – (float, default: 0) wall time in seconds until a subprocess is automatically killed
- **`verbose`** – (default: False) whether to print anything about what the iterator does (e.g., killing subprocesses)
- **`reset_interfaces`** – (default: True) whether to reset all pexpect interfaces

EXAMPLES:

```
sage: X = sage.parallel.use_fork.p_iter_fork(2,3, False); X
<sage.parallel.use_fork.p_iter_fork object at ...>
sage: X.ncpus
2
sage: X.timeout
3.0
sage: X.verbose
False
```


PARALLEL COMPUTATIONS USING RECURSIVELYENUMERATEDSET AND MAP-REDUCE

There is an efficient way to distribute computations on a set S of objects defined by `RecursivelyEnumeratedSet()` (see `sage.sets.recursively_enumerated_set` for more details) over which one would like to perform the following kind of operations:

- Compute the cardinality of a (very large) set defined recursively (through a call to `RecursivelyEnumeratedSet of forest type`)
- More generally, compute any kind of generating series over this set
- Test a conjecture, e.g. find an element of S satisfying a specific property, or check that none does or that they all do
- Count/list the elements of S that have a specific property
- Apply any map/reduce kind of operation over the elements of S

AUTHORS:

- Florent Hivert – code, documentation (2012–2016)
- Jean Baptiste Priez – prototype, debugging help on MacOSX (2011-June, 2016)
- Nathann Cohen – some documentation (2012)

4.1 Contents

- *How can I use all that stuff?*
- *Advanced use*
- *Profiling*
- *Logging*
- *How does it work ?*
- *Are there examples of classes?*

4.2 How is this different from usual MapReduce ?

This implementation is specific to `RecursivelyEnumeratedSet of forest type`, and uses its properties to do its job. Not only mapping and reducing but also **generating the elements** of S is done on different processors.

4.3 How can I use all that stuff?

First, you need to set the environment variable `SAGE_NUM_THREADS` to the desired number of parallel threads to be used:

```
sage: import os # not tested sage: os.environ["SAGE_NUM_THREADS"] = '8' # not tested
```

Second, you need the information necessary to describe a `RecursivelyEnumeratedSet` of `forest` type representing your set S (see `sage.sets.recursively_enumerated_set`). Then, you need to provide a “map” function as well as a “reduce” function. Here are some examples:

- **Counting the number of elements.** In this situation, the map function can be set to `lambda x: 1`, and the reduce function just adds the values together, i.e. `lambda x, y: x + y`.

We count binary words of length ≤ 16 :

```
sage: seeds = [[]]
sage: succ = lambda l: [l + [0], l + [1]] if len(l) < 16 else []
sage: S = RecursivelyEnumeratedSet(seeds, succ,
....:     structure='forest', enumeration='depth')
sage: map_function = lambda x: 1
sage: reduce_function = lambda x, y: x + y
sage: reduce_init = 0
sage: S.map_reduce(map_function, reduce_function, reduce_init)
131071
```

This matches the number of binary words of length ≤ 16 :

```
sage: factor(131071 + 1)
2^17
```

Note that the map and reduce functions here have the default values of the `sage.combinat.backtrack.SearchForest.map_reduce()` method so that the number of elements can be obtained more simply with:

```
sage: S.map_reduce()
131071
```

Instead of using `RecursivelyEnumeratedSet()`, one can directly use `RESetMapReduce`, which gives finer control over the parallel execution (see *Advanced use* below):

```
sage: from sage.parallel.map_reduce import RSetMapReduce
sage: S = RSetMapReduce(
....:     roots=[[]],
....:     children=lambda l: [l + [0], l + [1]] if len(l) < 16 else [],
....:     map_function=lambda x: 1,
....:     reduce_function=lambda x, y: x + y,
....:     reduce_init=0)
sage: S.run()
131071
```

- **Generating series.** For this, take a Map function that associates a monomial to each element of S , while the Reduce function is still equal to `lambda x, y: x + y`.

We compute the generating series for counting binary words of each length ≤ 16 :

```
sage: S = RecursivelyEnumeratedSet(
....:     [[]], lambda l: [l + [0], l + [1]] if len(l) < 16 else [],
....:     structure='forest', enumeration='depth')
```

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```

sage: x = polygen(ZZ)
sage: sp = S.map_reduce(
.....:     map_function=lambda z: x**len(z),
.....:     reduce_function=lambda x, y: x + y,
.....:     reduce_init=0)
sage: sp
65536*x^16 + 32768*x^15 + 16384*x^14 + 8192*x^13 + 4096*x^12
+ 2048*x^11 + 1024*x^10 + 512*x^9 + 256*x^8 + 128*x^7 + 64*x^6
+ 32*x^5 + 16*x^4 + 8*x^3 + 4*x^2 + 2*x + 1

```

This is of course $\sum_{i=0}^{16} (2x)^i$:

```

sage: sp == sum((2*x)^i for i in range(17))
True

```

Here is another example where we count permutations of size ≤ 8 (here we use the default values):

```

sage: S = RecursivelyEnumeratedSet(
.....:     [[]],
.....:     lambda l: ([l[:i] + [len(l)] + l[i:]
.....:         for i in range(len(l) + 1)] if len(l) < 8 else []),
.....:     structure='forest',
.....:     enumeration='depth')
sage: x = polygen(ZZ)
sage: sp = S.map_reduce(lambda z: x**len(z)); sp
40320*x^8 + 5040*x^7 + 720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1

```

This is of course $\sum_{i=0}^8 i!x^i$:

```

sage: sp == sum(factorial(i)*x^i for i in range(9))
True

```

- **Post Processing.** We now demonstrate the use of `post_process`. We generate the permutation as previously, but we only perform the map/reduce computation on those of even `len`. Of course we get the even part of the previous generating series:

```

sage: S = RecursivelyEnumeratedSet(
.....:     [[]],
.....:     lambda l: ([l[:i] + [len(l) + 1] + l[i:]
.....:         for i in range(len(l) + 1)] if len(l) < 8 else []),
.....:     post_process=lambda l: l if len(l) % 2 == 0 else None,
.....:     structure='forest',
.....:     enumeration='depth')
sage: sp = S.map_reduce(lambda z: x**len(z)); sp
40320*x^8 + 720*x^6 + 24*x^4 + 2*x^2 + 1

```

This is also useful for example to call a constructor on the generated elements:

```

sage: S = RecursivelyEnumeratedSet(
.....:     [[]],
.....:     lambda l: ([l[:i] + [len(l) + 1] + l[i:]
.....:         for i in range(len(l) + 1)] if len(l) < 5 else []),
.....:     post_process=lambda l: Permutation(l) if len(l) == 5 else None,
.....:     structure='forest',
.....:     enumeration='depth')
sage: x = polygen(ZZ)

```

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```
sage: sp = S.map_reduce(lambda z: x**z.number_of_inversions()); sp
x^10 + 4*x^9 + 9*x^8 + 15*x^7 + 20*x^6 + 22*x^5 + 20*x^4 + 15*x^3 + 9*x^2 + 4*x + 1
↪1
```

We get here a polynomial which is the q -factorial (in the variable x) of 5, that is, $\prod_{i=1}^5 \frac{1-x^i}{1-x}$:

```
sage: x = polygen(ZZ)
sage: prod((1-x^i)/(1-x) for i in range(1, 6))
x^10 + 4*x^9 + 9*x^8 + 15*x^7 + 20*x^6 + 22*x^5 + 20*x^4 + 15*x^3 + 9*x^2 + 4*x + 1
↪1
```

Compare:

```
sage: from sage.combinat.q_analogues import q_factorial
sage: q_factorial(5)
q^10 + 4*q^9 + 9*q^8 + 15*q^7 + 20*q^6 + 22*q^5 + 20*q^4 + 15*q^3 + 9*q^2 + 4*q + 1
↪1
```

- **Listing the objects.** One can also compute the list of objects in a `RecursivelyEnumeratedSet` of `forest` type using `RESetMapReduce`. As an example, we compute the set of numbers between 1 and 63, generated by their binary expansion:

```
sage: S = RecursivelyEnumeratedSet(
.....:     [1],
.....:     lambda l: [(l<<1)|0, (l<<1)|1] if l < 1<<5 else [],
.....:     structure='forest',
.....:     enumeration='depth')
```

Here is the list computed without `RESetMapReduce`:

```
sage: serial = list(S)
sage: serial
[1, 2, 4, 8, 16, 32, 33, 17, 34, 35, 9, 18, 36, 37, 19, 38, 39, 5, 10,
20, 40, 41, 21, 42, 43, 11, 22, 44, 45, 23, 46, 47, 3, 6, 12, 24, 48,
49, 25, 50, 51, 13, 26, 52, 53, 27, 54, 55, 7, 14, 28, 56, 57, 29, 58,
59, 15, 30, 60, 61, 31, 62, 63]
```

Here is how to perform the parallel computation. The order of the lists depends on the synchronisation of the various computation processes and therefore should be considered as random:

```
sage: parall = S.map_reduce(lambda x: [x], lambda x, y: x + y, [])
sage: parall # random
[1, 3, 7, 15, 31, 63, 62, 30, 61, 60, 14, 29, 59, 58, 28, 57, 56, 6, 13,
27, 55, 54, 26, 53, 52, 12, 25, 51, 50, 24, 49, 48, 2, 5, 11, 23, 47,
46, 22, 45, 44, 10, 21, 43, 42, 20, 41, 40, 4, 9, 19, 39, 38, 18, 37,
36, 8, 17, 35, 34, 16, 33, 32]
sage: sorted(serial) == sorted(parall)
True
```

4.4 Advanced use

Fine control over the execution of a map/reduce computation is achieved via parameters passed to the `RESetMapReduce.run()` method. The following three parameters can be used:

- `max_proc` – (integer, default: None) if given, the maximum number of worker processors to use. The actual number is also bounded by the value of the environment variable `SAGE_NUM_THREADS` (the number of cores by default).
- `timeout` – a timeout on the computation (default: None)
- `reduce_locally` – whether the workers should reduce locally their work or sends results to the master as soon as possible. See [RESetMapReduceWorker](#) for details.

Here is an example or how to deal with timeout:

```
sage: from sage.parallel.map_reduce import (RESetMPExample, AbortError)
sage: EX = RESetMPExample(maxl=100)
sage: try:
.....:     res = EX.run(timeout=0.01)
.....: except AbortError:
.....:     print("Computation timeout")
.....: else:
.....:     print("Computation normally finished")
.....:     res
Computation timeout
```

The following should not timeout even on a very slow machine:

```
sage: EX = RESetMPExample(maxl=8)
sage: try:
.....:     res = EX.run(timeout=60)
.....: except AbortError:
.....:     print("Computation Timeout")
.....: else:
.....:     print("Computation normally finished")
.....:     res
Computation normally finished
40320*x^8 + 5040*x^7 + 720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

As for `reduce_locally`, one should not see any difference, except for speed during normal usage. Most of the time one should leave it set to `True`, unless one sets up a mechanism to consume the partial results as soon as they arrive. See [RESetParallelIterator](#) and in particular the `__iter__` method for an example of consumer use.

4.5 Profiling

It is possible to profile a map/reduce computation. First we create a [RESetMapReduce](#) object:

```
sage: from sage.parallel.map_reduce import RESetMapReduce
sage: S = RESetMapReduce(
.....:     roots=[[]],
.....:     children=lambda l: [l + [0], l + [1]] if len(l) < 16 else [],
.....:     map_function=lambda x: 1,
.....:     reduce_function=lambda x, y: x + y,
.....:     reduce_init=0)
```

The profiling is activated by the `profile` parameter. The value provided should be a prefix (including a possible directory) for the profile dump:

```
sage: prof = tmp_dir('RESetMR_profile') + 'profcomp'
sage: res = S.run(profile=prof) # random
```

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```
[RESetMapReduceWorker-1:58] (20:00:41.444) Profiling in
/home/user/.sage/temp/.../32414/RESetMR_profilewRCRAx/profcomp1
...
[RESetMapReduceWorker-1:57] (20:00:41.444) Profiling in
/home/user/.sage/temp/.../32414/RESetMR_profilewRCRAx/profcomp0
...
sage: res
131071
```

In this example, the profiles have been dumped in files such as `profcomp0`. One can then load and print them as follows. See `cProfile.Profile` for more details:

```
sage: import cProfile, pstats
sage: st = pstats.Stats(prof+'0')
sage: st.strip_dirs().sort_stats('cumulative').print_stats() # random
...
Ordered by: cumulative time

ncalls  tottime  percall  cumtime  percall filename:lineno(function)
      1    0.023    0.023    0.432    0.432 map_reduce.py:1211(run_myself)
    11968   0.151    0.000    0.223    0.000 map_reduce.py:1292(walk_branch_locally)
...
<pstats.Stats instance at 0x7fedea40c6c8>
```

See also:

[The Python Profilers](#) for more detail on profiling in python.

4.6 Logging

The computation progress is logged through a `logging.Logger` in `sage.parallel.map_reduce.logger` together with `logging.StreamHandler` and a `logging.Formatter`. They are currently configured to print warning messages to the console.

See also:

[Logging facility for Python](#) for more detail on logging and log system configuration.

Note: Calls to logger which involve printing the node are commented out in the code, because the printing (to a string) of the node can be very time consuming depending on the node and it happens before the decision whether the logger should record the string or drop it.

4.7 How does it work ?

The scheduling algorithm we use here is any adaptation of [Wikipedia article Work_stealing](#):

In a work stealing scheduler, each processor in a computer system has a queue of work items (computational tasks, threads) to perform. [...]. Each work items are initially put on the queue of the processor executing the work item. When a processor runs out of work, it looks at the queues of other processors and “steals” their work items. In effect, work stealing distributes the scheduling work over idle processors, and as long as all processors have work to do, no scheduling overhead occurs.

For communication we use Python's basic `multiprocessing` module. We first describe the different actors and communication tools used by the system. The work is done under the coordination of a **master** object (an instance of `RESetMapReduce`) by a bunch of **worker** objects (instances of `RESetMapReduceWorker`).

Each running map reduce instance works on a `RecursivelyEnumeratedSet` of `forest` type called here C and is coordinated by a `RESetMapReduce` object called the **master**. The master is in charge of launching the work, gathering the results and cleaning up at the end of the computation. It doesn't perform any computation associated to the generation of the element C nor the computation of the mapped function. It however occasionally perform a reduce, but most reducing is by default done by the workers. Also thanks to the work-stealing algorithm, the master is only involved in detecting the termination of the computation but all the load balancing is done at the level of the workers.

Workers are instances of `RESetMapReduceWorker`. They are responsible for doing the actual computations: element generation, mapping and reducing. They are also responsible for the load balancing thanks to work-stealing.

Here is a description of the attributes of the **master** relevant to the map-reduce protocol:

- `_results` – a `SimpleQueue` where the master gathers the results sent by the workers.
- `_active_tasks` – a `Semaphore` recording the number of active tasks. The work is complete when it reaches 0.
- `_done` – a `Lock` which ensures that shutdown is done only once.
- `_aborted` – a `Value()` storing a shared `ctypes.c_bool` which is `True` if the computation was aborted before all workers ran out of work.
- `_workers` – a list of `RESetMapReduceWorker` objects. Each worker is identified by its position in this list.

Each **worker** is a process (`RESetMapReduceWorker` inherits from `Process`) which contains:

- `worker._iproc` – the identifier of the worker that is its position in the master's list of workers
- `worker._todo` – a `collections.deque` storing of nodes of the worker. It is used as a stack by the worker. Thiefs steal from the bottom of this queue.
- `worker._request` – a `SimpleQueue` storing steal request submitted to worker.
- `worker._read_task, worker._write_task` – a `Pipe` used to transfert node during steal.
- `worker._thief` – a `Thread` which is in charge of stealing from `worker._todo`.

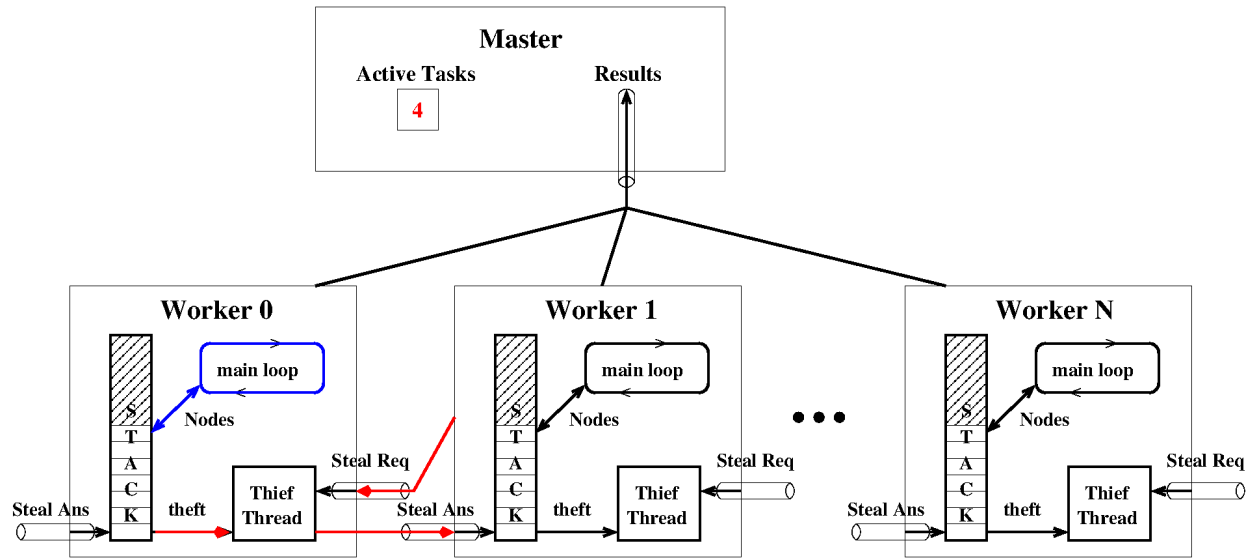
Here is a schematic of the architecture:

4.8 How thefts are performed

During normal time, that is, when all workers are active, a worker W is iterating through a loop inside `RESetMapReduceWorker.walk_branch_locally()`. Work nodes are taken from and new nodes $W._todo$ are appended to $W._todo$. When a worker W runs out of work, that is, when `worker._todo` is empty, it tries to steal some work (i.e., a node) from another worker. This is performed in the `RESetMapReduceWorker.steal()` method.

From the point of view of W , here is what happens:

- W signals to the master that it is idle: `master._signal_task_done`;
- W chooses a victim V at random;
- W sends a request to V : it puts its identifier into `V._request`;
- W tries to read a node from `W._read_task`. Then three things may happen:



- a proper node is read. Then the theft was a success and *W* starts working locally on the received node.
- None is received. This means that *V* was idle. Then *W* tries another victim.
- *AbortError* is received. This means either that the computation was aborted or that it simply succeeded and that no more work is required by *W*. Therefore an *AbortError* exception is raised leading *W* to shutdown.

We now describe the protocol on the victim's side. Each worker process contains a *Thread* which we call *T* for thief which acts like some kind of Trojan horse during theft. It is normally blocked waiting for a steal request.

From the point of view of *V* and *T*, here is what happens:

- during normal time, *T* is blocked waiting on *V*._request;
- upon steal request, *T* wakes up receiving the identification of *W*;
- *T* signals to the master that a new task is starting by *master*._signal_task_start;
- Two things may happen depending if the queue *V*._todo is empty or not. Remark that due to the GIL, there is no parallel execution between the victim *V* and its thief thread *T*.
 - If *V*._todo is empty, then None is answered on *W*._write_task. The task is immediately signaled to end the master through *master*._signal_task_done.
 - Otherwise, a node is removed from the bottom of *V*._todo. The node is sent to *W* on *W*._write_task. The task will be ended by *W*, that is, when finished working on the subtree rooted at the node, *W* will call *master*._signal_task_done.

4.9 The end of the computation

To detect when a computation is finished, a synchronized integer is kept which counts the number of active tasks. This is essentially a semaphore but semaphores are broken on Darwin OSes so we ship two implementations depending on the OS (see *ActiveTaskCounter* and *ActiveTaskCounterDarwin* and the note below).

When a worker finishes working on a task, it calls *master*._signal_task_done. This decreases the task counter *master*._active_tasks. When it reaches 0, it means that there are no more nodes: the work is completed.

The worker executes `master._shutdown` which sends *AbortError* to all `worker._request` and `worker._write_task` queues. Each worker or thief thread receiving such a message raises the corresponding exception, therefore stopping its work. A lock called `master._done` ensures that shutdown is only done once.

Finally, it is also possible to interrupt the computation before its ends, by calling `master.abort()`. This is achieved by setting `master._active_tasks` to 0 and calling `master._shutdown`.

Warning: The macOS Semaphore bug

Darwin OSes do not correctly implement POSIX's semaphore semantic. Indeed, on these systems, `acquire` may fail and return `False` not only when the semaphore is equal to zero but also **because someone else is trying to acquire** at the same time. This makes using Semaphores impossible on macOS so that on these systems we use a synchronized integer instead.

4.10 Are there examples of classes?

Yes! Here they are:

- *RESetMPExample* – a simple basic example
- *RESetParallelIterator* – a more advanced example using non standard communication configuration.

4.11 Tests

Generating series for the sum of strictly decreasing lists of integers smaller than 15:

```
sage: y = polygon(ZZ, 'y')
sage: R = RSetMapReduce(
....:     roots=[([], 0, 0)] + [[(i, i, i) for i in range(1, 15)],
....:     children=lambda list_sum_last:
....:         [(list_sum_last[0] + [i], list_sum_last[1] + i, i)
....:          for i in range(1, list_sum_last[2])],
....:     map_function=lambda li_sum_dummy: y**li_sum_dummy[1])
sage: sg = R.run()
sage: sg == prod((1 + y**i) for i in range(1, 15))
True
```

4.12 Classes and methods

exception `sage.parallel.map_reduce.AbortError`

Bases: `Exception`

Exception for aborting parallel computations.

This is used both as exception or as abort message.

`sage.parallel.map_reduce.ActiveTaskCounter`

alias of `sage.parallel.map_reduce.ActiveTaskCounterPosix`

class `sage.parallel.map_reduce.ActiveTaskCounterDarwin(task_number)`

Bases: `object`

Handling the number of active tasks.

A class for handling the number of active tasks in a distributed computation process. This is essentially a semaphore, but Darwin OSes do not correctly implement POSIX's semaphore semantic. So we use a shared integer with a lock.

abort()

Set the task counter to zero.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import ActiveTaskCounterDarwin as ATC
sage: c = ATC(4); c
ActiveTaskCounter(value=4)
sage: c.abort()
sage: c
ActiveTaskCounter(value=0)
```

task_done()

Decrement the task counter by one.

OUTPUT:

Calling `task_done()` decrements the counter and returns its new value.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import ActiveTaskCounterDarwin as ATC
sage: c = ATC(4); c
ActiveTaskCounter(value=4)
sage: c.task_done()
3
sage: c
ActiveTaskCounter(value=3)

sage: c = ATC(0)
sage: c.task_done()
-1
```

task_start()

Increment the task counter by one.

OUTPUT:

Calling `task_start()` on a zero or negative counter returns 0, otherwise increment the counter and returns its value after the incrementation.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import ActiveTaskCounterDarwin as ATC
sage: c = ATC(4); c
ActiveTaskCounter(value=4)
sage: c.task_start()
5
sage: c
ActiveTaskCounter(value=5)
```

Calling `task_start()` on a zero counter does nothing:

```
sage: c = ATC(0)
sage: c.task_start()
0
```

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```
sage: c
ActiveTaskCounter(value=0)
```

class sage.parallel.map_reduce.ActiveTaskCounterPosix(*task_number*)

Bases: object

Handling the number of active tasks.

A class for handling the number of active tasks in a distributed computation process. This is the standard implementation on POSIX compliant OSes. We essentially wrap a semaphore.

Note: A legitimate question is whether there is a need in keeping the two implementations. I ran the following experiment on my machine:

```
S = RecursivelyEnumeratedSet(
    [[]],
    lambda l: ([l[:i] + [len(l)] + l[i:]
                for i in range(len(l) + 1)]
               if len(l) < NNN else []),
    structure='forest',
    enumeration='depth')
%time sp = S.map_reduce(lambda z: x**len(z)); sp
```

For NNN = 10, averaging a dozen of runs, I got:

- Posix compliant implementation: 17.04 s
- Darwin implementation: 18.26 s

So there is a non negligible overhead. It will probably be worth it if we try to cythonize the code. So I'm keeping both implementations.

abort()

Set the task counter to zero.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import ActiveTaskCounter as ATC
sage: c = ATC(4); c
ActiveTaskCounter(value=4)
sage: c.abort()
sage: c
ActiveTaskCounter(value=0)
```

task_done()

Decrement the task counter by one.

OUTPUT:

Calling `task_done()` decrements the counter and returns its new value.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import ActiveTaskCounter as ATC
sage: c = ATC(4); c
ActiveTaskCounter(value=4)
sage: c.task_done()
3
```

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```
sage: c
ActiveTaskCounter(value=3)

sage: c = ATC(0)
sage: c.task_done()
-1
```

task_start()

Increment the task counter by one.

OUTPUT:

Calling `task_start()` on a zero or negative counter returns 0, otherwise increment the counter and returns its value after the incrementation.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import ActiveTaskCounterDarwin as ATC
sage: c = ATC(4); c
ActiveTaskCounter(value=4)
sage: c.task_start()
5
sage: c
ActiveTaskCounter(value=5)
```

Calling `task_start()` on a zero counter does nothing:

```
sage: c = ATC(0)
sage: c.task_start()
0
sage: c
ActiveTaskCounter(value=0)
```

class `sage.parallel.map_reduce.RESetMPEExample(maxl=9)`

Bases: `sage.parallel.map_reduce.RESetMapReduce`

An example of map reduce class.

INPUT:

- `maxl` – the maximum size of permutations generated (default to 9).

This computes the generating series of permutations counted by their size up to size `maxl`.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPEExample
sage: EX = RESetMPEExample()
sage: EX.run()
362880*x^9 + 40320*x^8 + 5040*x^7 + 720*x^6 + 120*x^5
+ 24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

See also:

This is an example of `RESetMapReduce`

children(l)

Return the children of the permutation `l`.

INPUT:

- `l` – a list containing a permutation

OUTPUT:

The lists with `len(l)` inserted at all possible positions into `l`.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RSetMPExample
sage: RSetMPExample().children([1,0])
[[2, 1, 0], [1, 2, 0], [1, 0, 2]]
```

map_function(*l*)

The monomial associated to the permutation `l`.

INPUT:

- `l` – a list containing a permutation

OUTPUT:

The monomial `x^len(l)`.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RSetMPExample
sage: RSetMPExample().map_function([1,0])
x^2
```

roots()

Return the empty permutation.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RSetMPExample
sage: RSetMPExample().roots()
[[]]
```

```
class sage.parallel.map_reduce.RSetMapReduce (roots=None, children=None,
                                              post_process=None,
                                              map_function=None, re-
                                              duce_function=None, reduce_init=None,
                                              forest=None)
```

Bases: object

Map-Reduce on recursively enumerated sets.

INPUT:

Description of the set:

- either `forest=f` – where `f` is a `RecursivelyEnumeratedSet` of `forest` type
- or a triple `roots, children, post_process` as follows
 - `roots=r` – The root of the enumeration
 - `children=c` – a function iterating through children nodes, given a parent node
 - `post_process=p` – a post-processing function

The option `post_process` allows for customizing the nodes that are actually produced. Furthermore, if `post_process(x)` returns `None`, then `x` won't be output at all.

Description of the map/reduce operation:

- `map_function=f` – (default to `None`)
- `reduce_function=red` – (default to `None`)
- `reduce_init=init` – (default to `None`)

See also:

the `Map/Reduce` module for details and examples.

abort()

Abort the current parallel computation.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetParallelIterator
sage: S = RESetParallelIterator([[]],
....:     lambda l: [l + [0], l + [1]] if len(l) < 17 else [])
sage: it = iter(S)
sage: next(it) # random
[]
sage: S.abort()
sage: hasattr(S, 'work_queue')
False
```

Cleanup:

```
sage: S.finish()
```

finish()

Destroy the workers and all the communication objects.

Communication statistics are gathered before destroying the workers.

See also:

`print_communication_statistics()`

get_results(timeout=None)

Get the results from the queue.

OUTPUT:

The reduction of the results of all the workers, that is, the result of the map/reduce computation.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMapReduce
sage: S = RESetMapReduce()
sage: S.setup_workers(2)
sage: for v in [1, 2, None, 3, None]: S._results.put(v)
sage: S.get_results()
6
```

Cleanup:

```
sage: del S._results, S._active_tasks, S._done, S._workers
```

map_function(o)

Return the function mapped by self.

INPUT:

- `o` – a node

OUTPUT:

By default 1.

Note: This should be overloaded in applications.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMapReduce
sage: S = RESetMapReduce()
sage: S.map_function(7)
1
sage: S = RESetMapReduce(map_function = lambda x: 3*x + 5)
sage: S.map_function(7)
26
```

post_process (*a*)

Return the image of *a* under the post-processing function for self.

INPUT:

- *a* – a node

With the default post-processing function, which is the identity function, this returns *a* itself.

Note: This should be overloaded in applications.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMapReduce
sage: S = RESetMapReduce()
sage: S.post_process(4)
4
sage: S = RESetMapReduce(post_process=lambda x: x*x)
sage: S.post_process(4)
16
```

print_communication_statistics (*blocksize=16*)

Print the communication statistics in a nice way.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPExample
sage: S = RESetMPExample(maxl=6)
sage: S.run()
720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1

sage: S.print_communication_statistics() # random
#proc:      0    1    2    3    4    5    6    7
reqs sent:   5    2    3   11   21   19    1    0
reqs rcvs:  10   10    9    5    1   11    9    2
- thefs:     1    0    0    0    0    0    0    0
+ thefs:     0    0    1    0    0    0    0    0
```

random_worker ()

Return a random worker.

OUTPUT:

A worker for `self` chosen at random.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPExample, _
      ↪ RESetMapReduceWorker
sage: from threading import Thread
sage: EX = RESetMPExample(maxl=6)
sage: EX.setup_workers(2)
sage: EX.random_worker()
<RESetMapReduceWorker(RESetMapReduceWorker-..., initial)>
sage: EX.random_worker() in EX._workers
True
```

Cleanup:

```
sage: del EX._results, EX._active_tasks, EX._done, EX._workers
```

`reduce_function(a, b)`

Return the reducer function for `self`.

INPUT:

- `a, b` – two values to be reduced

OUTPUT:

By default the sum of `a` and `b`.

Note: This should be overloaded in applications.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMapReduce
sage: S = RESetMapReduce()
sage: S.reduce_function(4, 3)
7
sage: S = RESetMapReduce(reduce_function=lambda x,y: x*y)
sage: S.reduce_function(4, 3)
12
```

`reduce_init()`

Return the initial element for a reduction.

Note: This should be overloaded in applications.

`roots()`

Return the roots of `self`.

OUTPUT:

An iterable of nodes.

Note: This should be overloaded in applications.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMapReduce
sage: S = RESetMapReduce(42)
sage: S.roots()
42
```

run (*max_proc=None, reduce_locally=True, timeout=None, profile=None*)

Run the computations.

INPUT:

- *max_proc* – (integer, default: None) if given, the maximum number of worker processors to use. The actual number is also bounded by the value of the environment variable `SAGE_NUM_THREADS` (the number of cores by default).
- *reduce_locally* – See `RESetMapReduceWorker` (default: True)
- *timeout* – a timeout on the computation (default: None)
- *profile* – directory/filename prefix for profiling, or None for no profiling (default: None)

OUTPUT:

The result of the map/reduce computation or an exception `AbortError` if the computation was interrupted or timeout.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPEExample
sage: EX = RESetMPEExample(maxl = 8)
sage: EX.run()
40320*x^8 + 5040*x^7 + 720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

Here is an example or how to deal with timeout:

```
sage: from sage.parallel.map_reduce import AbortError
sage: EX = RESetMPEExample(maxl = 100)
sage: try:
....:     res = EX.run(timeout=0.01)
....: except AbortError:
....:     print("Computation timeout")
....: else:
....:     print("Computation normally finished")
....:     res
Computation timeout
```

The following should not timeout even on a very slow machine:

```
sage: from sage.parallel.map_reduce import AbortError
sage: EX = RESetMPEExample(maxl = 8)
sage: try:
....:     res = EX.run(timeout=60)
....: except AbortError:
....:     print("Computation Timeout")
....: else:
....:     print("Computation normally finished")
....:     res
Computation normally finished
40320*x^8 + 5040*x^7 + 720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

run_serial ()

Run the computation serially (mostly for tests).

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPExample
sage: EX = RESetMPExample(maxl = 4)
sage: EX.run_serial()
24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

setup_workers (*max_proc=None, reduce_locally=True*)

Setup the communication channels.

INPUT:

- *max_proc* – (integer) an upper bound on the number of worker processes.
- *reduce_locally* – whether the workers should reduce locally their work or sends results to the master as soon as possible. See [RESetMapReduceWorker](#) for details.

start_workers ()

Launch the workers.

The workers should have been created using [setup_workers\(\)](#).

class `sage.parallel.map_reduce.RESetMapReduceWorker` (*mapred, iproc, reduce_locally*)

Bases: `multiprocessing.context.ForkProcess`

Worker for generate-map-reduce.

This shouldn't be called directly, but instead created by [RESetMapReduce.setup_workers\(\)](#).

INPUT:

- *mapred* – the instance of [RESetMapReduce](#) for which this process is working.
- *iproc* – the id of this worker.
- *reduce_locally* – when reducing the results. Three possible values are supported:
 - `True` – means the reducing work is done all locally, the result is only sent back at the end of the work. This ensure the lowest level of communication.
 - `False` – results are sent back after each finished branches, when the process is asking for more work.

run ()

The main function executed by the worker.

Calls [run_myself\(\)](#) after possibly setting up parallel profiling.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPExample, _
      ↪ RESetMapReduceWorker
sage: EX = RESetMPExample(maxl=6)
sage: EX.setup_workers(1)

sage: w = EX._workers[0]
sage: w._todo.append(EX.roots()[0])

sage: w.run()
sage: sleep(1)
sage: w._todo.append(None)

sage: EX.get_results()
720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1
```


Cleanups:

```
sage: del EX._results, EX._active_tasks, EX._done, EX._workers
```

run_myself()

The main function executed by the worker.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPEExample, ↵
↵RESetMapReduceWorker
sage: EX = RESetMPEExample(maxl=6)
sage: EX.setup_workers(1)

sage: w = EX._workers[0]
sage: w._todo.append(EX.roots()[0])
sage: w.run_myself()

sage: sleep(1)
sage: w._todo.append(None)

sage: EX.get_results()
720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

Cleanups:

```
sage: del EX._results, EX._active_tasks, EX._done, EX._workers
```

send_partial_result()

Send results to the MapReduce process.

Send the result stored in `self._res` to the master and reinitialize it to `master.reduce_init`.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPEExample, ↵
↵RESetMapReduceWorker
sage: EX = RESetMPEExample(maxl=4)
sage: EX.setup_workers(1)
sage: w = EX._workers[0]
sage: w._res = 4
sage: w.send_partial_result()
sage: w._res
0
sage: EX._results.get()
4
```

steal()

Steal some node from another worker.

OUTPUT:

A node stolen from another worker chosen at random.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPEExample, ↵
↵RESetMapReduceWorker
sage: from threading import Thread
sage: EX = RESetMPEExample(maxl=6)
```

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```

sage: EX.setup_workers(2)

sage: w0, w1 = EX._workers
sage: w0._todo.append(42)
sage: thief0 = Thread(target = w0._thief, name="Thief")
sage: thief0.start() # known bug (Trac #27537)

sage: w1.steal() # known bug (Trac #27537)
42
sage: w0._todo # known bug (Trac #27537)
deque([])

```

walk_branch_locally(node)

Work locally.

Performs the map/reduce computation on the subtrees rooted at node.

INPUT:

- node – the root of the subtree explored.

OUTPUT:

Nothing, the result are stored in `self._res`.

This is where the actual work is performed.

EXAMPLES:

```

sage: from sage.parallel.map_reduce import RESetMPEExample, RESetMapReduceWorker
sage: EX = RESetMPEExample(maxl=4)
sage: w = RESetMapReduceWorker(EX, 0, True)
sage: def sync(): pass
sage: w.synchronize = sync
sage: w._res = 0

sage: w.walk_branch_locally([])
sage: w._res
x^4 + x^3 + x^2 + x + 1

sage: w.walk_branch_locally(w._todo.pop())
sage: w._res
2*x^4 + x^3 + x^2 + x + 1

sage: while True: w.walk_branch_locally(w._todo.pop())
Traceback (most recent call last):
...
IndexError: pop from an empty deque
sage: w._res
24*x^4 + 6*x^3 + 2*x^2 + x + 1

```

```

class sage.parallel.map_reduce.RESetParallelIterator (roots=None, children=None,
                                                       post_process=None,
                                                       map_function=None, re-
                                                       duce_function=None, re-
                                                       duce_init=None, forest=None)

```

Bases: `sage.parallel.map_reduce.RESetMapReduce`

A parallel iterator for recursively enumerated sets.

This demonstrates how to use `RESetMapReduce` to get an iterator on a recursively enumerated set for which the computations are done in parallel.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RSetParallelIterator
sage: S = RSetParallelIterator([[]],
....:     lambda l: [l + [0], l + [1]] if len(l) < 15 else [])
sage: sum(1 for _ in S)
65535
```

map_function(z)

Return a singleton tuple.

INPUT:

- z – a node

OUTPUT:

The singleton (z,).

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RSetParallelIterator
sage: S = RSetParallelIterator([[]],
....:     lambda l: [l + [0], l + [1]] if len(l) < 15 else [])
sage: S.map_function([1, 0])
([1, 0],)
```

reduce_init

alias of `builtins.tuple`

`sage.parallel.map_reduce.proc_number`(max_proc=None)

Return the number of processes to use.

INPUT:

- max_proc – an upper bound on the number of processes or None.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import proc_number
sage: proc_number() # random
8
sage: proc_number(max_proc=1)
1
sage: proc_number(max_proc=2) in (1, 2)
True
```


PARALLEL ITERATOR BUILT USING PYTHON'S MULTIPROCESSING MODULE

`sage.parallel.multiprocessing_sage.parallel_iter` (*processes*, *f*, *inputs*)

Return a parallel iterator.

INPUT:

- *processes* – integer
- *f* – function
- *inputs* – an iterable of pairs (*args*, *kwds*)

OUTPUT:

- iterator over values of *f* at *args*, *kwds* in some random order.

EXAMPLES:

```
sage: def f(x): return x+x
sage: import sage.parallel.multiprocessing_sage
sage: v = list(sage.parallel.multiprocessing_sage.parallel_iter(2, f, [(2,), {}],
→ ((3,), {}]))
sage: v.sort(); v
[((2,), {}), 4, ((3,), {}), 6]
```

`sage.parallel.multiprocessing_sage.pyprocessing` (*processes=0*)

Return a parallel iterator using a given number of processes implemented using pyprocessing.

INPUT:

- *processes* – integer (default: 0); if 0, set to the number of processors on the computer.

OUTPUT:

- a (partially evaluated) function

EXAMPLES:

```
sage: from sage.parallel.multiprocessing_sage import pyprocessing
sage: p_iter = pyprocessing(4)
sage: P = parallel(p_iter=p_iter)
sage: def f(x): return x+x
sage: v = list(P(f)(list(range(10)))); v.sort(); v
[(((0,), {}), 0), (((1,), {}), 2), (((2,), {}), 4), (((3,), {}), 6), (((4,), {}), 8),
→ (((5,), {}), 10), (((6,), {}), 12), (((7,), {}), 14), (((8,), {}), 16),
→ (((9,), {}), 18)]
```


PARALLELIZATION CONTROL

This module defines the singleton class *Parallelism* to govern the parallelization of computations in some specific topics. It allows the user to set the number of processes to be used for parallelization.

Some examples of use are provided in the documentation of `sage.tensor.modules.comp.Components.contract()`.

AUTHORS:

- Marco Mancini, Eric Gourgoulhon, Michal Bejger (2015): initial version

class `sage.parallel.parallelism.Parallelism`

Bases: `sage.misc.fast_methods.Singleton`, `sage.structure.sage_object.SageObject`

Singleton class for managing the number of processes used in parallel computations involved in various fields.

EXAMPLES:

The number of processes is initialized to 1 (no parallelization) for each field:

```
sage: Parallelism()
Number of processes for parallelization:
- linbox computations: 1
- tensor computations: 1
```

Using 4 processes to parallelize tensor computations:

```
sage: Parallelism().set('tensor', nproc=4)
sage: Parallelism()
Number of processes for parallelization:
- linbox computations: 1
- tensor computations: 4
sage: Parallelism().get('tensor')
4
```

Using 6 processes to parallelize all types of computations:

```
sage: Parallelism().set(nproc=6)
sage: Parallelism()
Number of processes for parallelization:
- linbox computations: 6
- tensor computations: 6
```

Using all the cores available on the computer to parallelize tensor computations:

```
sage: Parallelism().set('tensor')
sage: Parallelism() # random (depends on the computer)
Number of processes for parallelization:
- linbox computations: 1
- tensor computations: 8
```

Using all the cores available on the computer to parallelize all types of computations:

```
sage: Parallelism().set()
sage: Parallelism() # random (depends on the computer)
Number of processes for parallelization:
- linbox computations: 8
- tensor computations: 8
```

Switching off all parallelizations:

```
sage: Parallelism().set(nproc=1)
```

get (*field*)

Return the number of processes which will be used in parallel computations regarding some specific field.

INPUT:

- *field* – string specifying the part of Sage involved in parallel computations

OUTPUT:

- number of processes used in parallelization of computations pertaining to *field*

EXAMPLES:

The default is a single process (no parallelization):

```
sage: Parallelism().reset()
sage: Parallelism().get('tensor')
1
```

Asking for parallelization on 4 cores:

```
sage: Parallelism().set('tensor', nproc=4)
sage: Parallelism().get('tensor')
4
```

get_all ()

Return the number of processes which will be used in parallel computations in all fields

OUTPUT:

- dictionary of the number of processes, with the computational fields as keys

EXAMPLES:

```
sage: Parallelism().reset()
sage: Parallelism().get_all()
{'linbox': 1, 'tensor': 1}
```

Asking for parallelization on 4 cores:

```
sage: Parallelism().set(nproc=4)
sage: Parallelism().get_all()
{'linbox': 4, 'tensor': 4}
```


get_default()

Return the default number of processes to be launched in parallel computations.

EXAMPLES:

A priori, the default number of process for parallelization is the total number of cores found on the computer:

```
sage: Parallelism().reset()
sage: Parallelism().get_default() # random (depends on the computer)
8
```

It can be changed via `set_default()`:

```
sage: Parallelism().set_default(nproc=4)
sage: Parallelism().get_default()
4
```

reset()

Put the singleton object `Parallelism()` in the same state as immediately after its creation.

EXAMPLES:

State of `Parallelism()` just after its creation:

```
sage: Parallelism()
Number of processes for parallelization:
- linbox computations: 1
- tensor computations: 1
sage: Parallelism().get_default() # random (depends on the computer)
8
```

Changing some values:

```
sage: Parallelism().set_default(6)
sage: Parallelism().set()
sage: Parallelism()
Number of processes for parallelization:
- linbox computations: 6
- tensor computations: 6
sage: Parallelism().get_default()
6
```

Back to the initial state:

```
sage: Parallelism().reset()
sage: Parallelism()
Number of processes for parallelization:
- linbox computations: 1
- tensor computations: 1
sage: Parallelism().get_default() # random (depends on the computer)
8
```

set (field=None, nproc=None)

Set the number of processes to be launched for parallel computations regarding some specific field.

INPUT:

- `field` – (default: `None`) string specifying the computational field for which the number of parallel processes is to be set; if `None`, all fields are considered

- `nproc` – (default: `None`) number of processes to be used for parallelization; if `None`, the number of processes will be set to the default value, which, unless redefined by `set_default()`, is the total number of cores found on the computer.

EXAMPLES:

The default is a single processor (no parallelization):

```
sage: Parallelism()
Number of processes for parallelization:
- linbox computations: 1
- tensor computations: 1
```

Asking for parallelization on 4 cores in tensor algebra:

```
sage: Parallelism().set('tensor', nproc=4)
sage: Parallelism()
Number of processes for parallelization:
- linbox computations: 1
- tensor computations: 4
```

Using all the cores available on the computer:

```
sage: Parallelism().set('tensor')
sage: Parallelism() # random (depends on the computer)
Number of processes for parallelization:
- linbox computations: 1
- tensor computations: 8
```

Using 6 cores in all parallelizations:

```
sage: Parallelism().set(nproc=6)
sage: Parallelism()
Number of processes for parallelization:
- linbox computations: 6
- tensor computations: 6
```

Using all the cores available on the computer in all parallelizations:

```
sage: Parallelism().set()
sage: Parallelism() # random (depends on the computer)
Number of processes for parallelization:
- linbox computations: 8
- tensor computations: 8
```

Switching off the parallelization:

```
sage: Parallelism().set(nproc=1)
sage: Parallelism()
Number of processes for parallelization:
- linbox computations: 1
- tensor computations: 1
```

`set_default` (*nproc=None*)

Set the default number of processes to be launched in parallel computations.

INPUT:

- `nproc` – (default: `None`) default number of processes; if `None`, the number of processes will be set to the total number of cores found on the computer.

EXAMPLES:

A priori the default number of process for parallelization is the total number of cores found on the computer:

```
sage: Parallelism().get_default() # random (depends on the computer)
8
```

Changing it thanks to `set_default`:

```
sage: Parallelism().set_default(nproc=4)
sage: Parallelism().get_default()
4
```

Setting it back to the total number of cores available on the computer:

```
sage: Parallelism().set_default()
sage: Parallelism().get_default() # random (depends on the computer)
8
```


CPU DETECTION

`sage.parallel.ncpus.ncpus()`
Detects the number of effective CPUs in the system.

EXAMPLES:

```
sage: sage.parallel.ncpus.ncpus()  # random output -- depends on machine.  
2
```

See also:

- `sage.interfaces.psage`

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