# Sage Reference Manual

*Release 9.3*

**The Sage Development Team**

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Welcome to the Sage reference manual. Here you find documentation for all of Sage's features, illustrated with lots of examples. A thematic index follows.

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