1 Problem Definition

In this problem, we investigate the thermodynamics of compressed air energy storage (on a local scale - home-friendly). The goal is to understand the different processes in compressing air. For our scenario of local scale CAES, the requirements for the storage are:

- reliable (provide constant output of energy);
- safe and does not put people or environment at risk;
- not too expensive;
- the amount of stored energy should be greater than the amount of energy used to store it and greater than the amount of an ordinary battery;
- provide short-term storage; and
- made of materials that can withstand high temperatures and pressures.

For our CAES container, we decided to use a piston, so that the amount of moles inside the container stays constant throughout. The container tank will be cylindrical, like those scuba diving oxygen tanks.

1.1 Adiabatic and Isothermal Processes

Adiabatic and isothermal processes are processes used in storing compressed air energy. Both of the processes alone, cannot happen purely in reality. In ideal gases, adiabatic processes have perfect insulation and heat loss in the system is zero. While for isothermal processes, the temperature is the same throughout, therefore all the work done is lost as heat.

2 Energy Storage

2.1 Assumptions

The initial conditions that we have set are:

- Initial Pressure $P_1 = 1$ atm $= 101325$ Pa or 1.01325x10^5 Pa
- Initial Volume $V_1 = 1$ m^3
- Final Pressure $P_2 = 1.379$x10^5 Pa (This final pressure is for a medium-strong container that can withstand medium-high pressures [3].)
- Initial Temperature $T_1 = 283.15K$
- $\gamma = 7/5$ (air can be seen as diatonic)

- Molar Heat Capacity for air $Cv = 20.76$ (78% the air is N2)

Using the formula below, we can also find out the amount of moles of air within our container (See Appendix A).

$$\text{moles } n = \frac{\text{mass in grams}}{\text{molar mass}} \tag{1}$$

Therefore, we get n at $T = 283.15$K is 43.01 moles for $1m^3$. This number stays the same throughout, no matter what the temperature is, since the container is airtight.

2.2 Adiabatic Process

For the adiabatic process, the final volume V_2 can be calculated with:

$$p_1V_1^\gamma = p_2V_2^\gamma \tag{2}$$

Which gives us $V_2 = 0.02991m^3$. With this and the following equation:

$$T_2 = T_1(\frac{V_1}{V_2})^{\gamma-1} \tag{3}$$

we can get the final temperature $T_2 = 1152.623K$, which we need for calculating the work done on the system:

$$W = \frac{1}{\gamma - 1}(p_1V_1 - p_2V_2) \tag{4}$$

With this we get $W = -778022J$.
Checking this with:

$$U = nC_v\Delta T \tag{5}$$

we get approximately the same number, but negative, as expected, since for an adiabatic process $Q = 0$.

2.3 Isothermal Process

For the isothermal process, the final volume can be calculated with:

$$V_1p_1 = V_2p_2 \tag{6}$$

which gives $V_2 = 0.00734m^3$.
The amount of work we can get with:

$$W = nRT\ln(\frac{p_1}{p_2}) \tag{7}$$

Here T is equal to T_1 (the temperature does not change troughout the process). The amount of work done by the system is $-497538J$.

2.4 Discussion

With an efficiency of 60% this would mean that the isothermal process can store 0.082923 kWh and the adiabatic process 0.12967 kWh. For reference, a mobile phone needs between 0.006 and 0.014 kWh to fully charge. This means that with isothermal, a mobile phone can be fully charged for 9 to 23 times; while for adiabatic, 15 to 36 times. We can see that the adiabatic process stores more energy (more work done to reach the same final pressure).

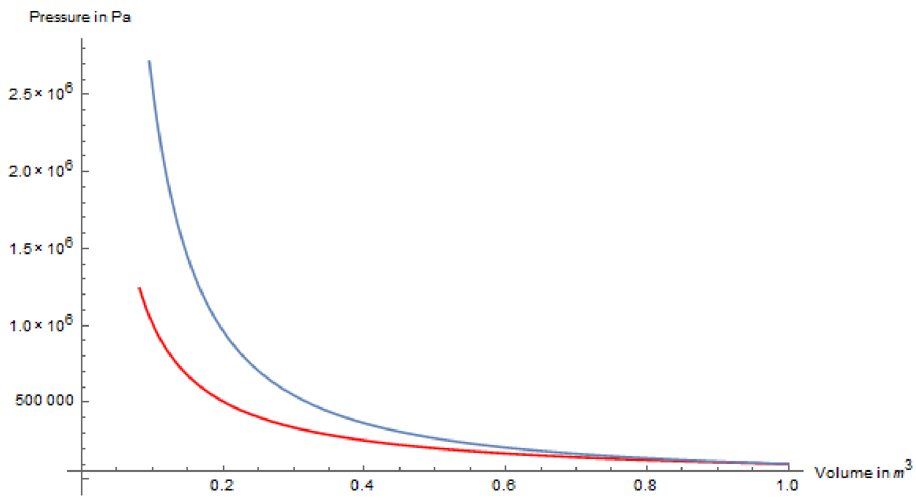

Figure 1: Isothermal (red) and adiabatic (blue) process

In figure 1, the graphs for isothermal process (in red) and adiabatic process (in blue). The area underneath the graph is the amount of work done, which is also the amount of energy the system stores if we assume no energy is lost by friction and whatsoever. As seen for **compression**, adiabatic process can store more energy than isothermal process, which means that our numbers make sense (0.12967 kWh is greater than 0.082923 kWh).

3 Heat Loss

If we take a gas cylinder made of stainless steel, how much heat would the temperature decline over time when compressed? This is important to know, since a decline in temperature means that the pressure also decreases, and so that you 'lose' energy. If we assume that the pressure cylinder does not heat up, and the heat loss is only by conduction on the sides, we get the equation for heat loss:

$$H_{conductivity} = -k 2\pi r h \frac{T(t) - T_{outside}}{b} \tag{8}$$

Here is:

- b = thickness of the gas tank (See Appendix B)

Feedback from teacher

D17:	D19:	D20:	D12:		
T1:	T2:	T3:	M1:	M2:	M3:
R1:	R2:				

- k = thermal conductivity (we took $20Wm^-1K^-1$)
- r = radius of the cylinder (See Appendix B)
- h = height after compression ($0.02991m$)
- $T(t)$ = is the temperature of the gas over time

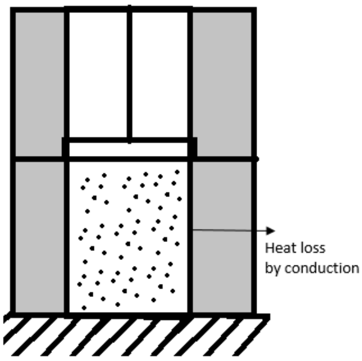

Figure 2: The gas cylinder and its heat loss

And solving the ODE gives us an equation for the temperature of the gas:

$$T(t) = T_{outside} + e^{\alpha t} \cdot C \tag{9}$$

With α being approximately -0.193419. The constant C can be found with the initial condition the end temperature of the adiabatic process. The following graph we see:

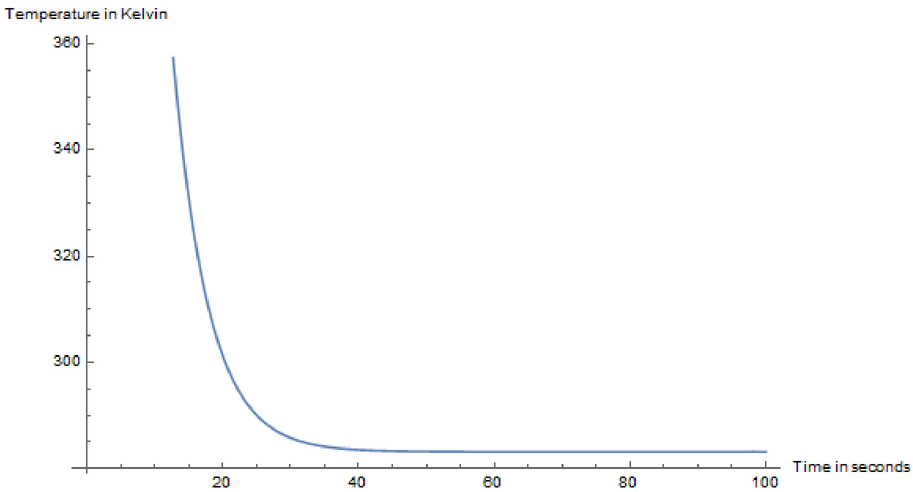

Figure 3: How quick the air in the pressure tank is cooling down

As can be seen it cools down very quickly.

Feedback from teacher

D17:	D19:	D20:	D12:		
T1:	T2:	T3:	M1:	M2:	M3:
R1:	R2:				

4 Discussion

4.1 Requirements

In the beginning we said that it is important to have a constant output of electricity generation. This is a problem with CAES since with a decreasing pressure (when 'extracting' energy from the system), the rate on which work is delivered will also decline. However, you want a constant output. We did not look into this aspect.
We also did not compare CAES with other energy storage systems, like chemical batteries. Comparing the energy per kilogram, the presence of toxic materials and the price for the systems for example could give more insights into the viability of CAES.

4.2 Processes

Because adiabatic and isothermal processes each cannot occur purely on their own in reality, a stepwise process is needed for CAES. Stepwise process combines the different processes as well as determines the efficiency for the system. The more the steps, the more efficient the system. This is because the increased amount of steps, decreases the change in entropy [2]. But more steps also mean a more complicated and more expensive system, because more different types of processes are used.

4.3 Problems Faced

After having calculated the amount of energy an adiabatic and an isothermal process can store, we were faced with the reflection of whether the amount in which adiabatic can store should really be greater than that of isothermal. Most sources online stated that isothermal should be able to store more energy than adiabatic, because adiabatic has a much steeper exponential curve, therefore, adiabatic has less area under the graph and less work was done. We later figured out that this is because those sources started the two processes at the same point (volume and pressure) and expanded it. While what we needed was compression. We needed to start it at the same point and then compress it, which will make the adiabatic curve sit above the isothermal curve (see figure 1). This will then mean that adiabatic can store more energy than isothermal.
Another problem we faced is that we felt that we needed to use the concept of entropy to be able to answer the problem more completely. We could have dived into this matter, but decided not to, since we don't know much about this field yet. However, we think the concept of entropy is important to get a better insight in how CAES systems work.

4.4 Assumptions

We assumed that the outside temperature is constantly 10 degrees Celsius, which is not true. The temperature is constantly changing. The higher the temperature, the more the energy

storage. However, we have seen that increasing the outside temperature with 10 degrees Celsius does not have a big impact on the overall process.
In the heat loss, our assumptions lead to a not so precise result. A decrease of so many degrees in only ten seconds does not sound very plausible. But we still think that the temperature will decrease rapidly if stored in such a tank, because steel has a high thermal conductivity and we set the condition for the thickness of the tank to be not so thick.
For the thickness of the gas cylinders we did some calculations on formulas we found on the manufacturing of these gas containers. However, these vague calculations did not add much, and leaving this out would have saved us (for engineers very valuable) time. Justification for having this in the appendix, can be that we have more certainty on how thick these gas cylinders may be.

4.5 Materials

For the calculations above, we assumed that the container was made out of stainless steel. This is because stainless steel is a quite common metal, therefore it is easy to find the resource. Stainless steel can also withstand high pressures. It also has a high melting point, making it able to withstand high temperatures. We did not look into the price of steel.
Because stainless steel is a good thermal conductor, it would not work well for adiabatic processes, because very good insulation is needed. For adiabatic process, it would be better to use materials like concrete, which can provide better insulation. But 100% insulation is not possible, so the heat loss should be compensated by other approaches, such as burning natural gas. And for isothermal, a very good heat transfer system is necessary to ensure a constant temperature.

4.6 Explosion

Adiabatic process stores around 778.022 kJ of energy and isothermal process stores around 497.538 kJ if energy. A stick of dynamite contains roughly 1 MJ of energy, which is 1000 kJ. This means that if an adiabatic CAES were to explode, the explosion would be $\frac{3}{4}$ of that of a stick of dynamite. While if an isothermal CAES were to explode, the explosion would be half as big as that of a stick of dynamite. A gram of TNT contains 4.184 kJ. This means that adiabatic CAES is equivalent to around 1.86kg of TNT, while isothermal CAES is equivalent to around 1.19kg of TNT. If the CAES were to explode, the explosion would be a very small one. This shows that there is risk in storing large amounts of energy. Although, we don't think that the chances of a small storage exploding is high at all. The problem will just be in convincing people that there are risks, but the probability of it exploding and the amount of energy it can store is quite worth it.

5 Conclusion

Coming to a conclusion, having a pressure cyclinder made of stainless steel, like scuba diving oxygen tank, is not good enough for CAES. It can be seen from both the materials and the insulation. A pressure cylinder does not insulate the heat well enough to store energy and the temperature cools down very quickly, even with a thickness that is much larger than the minimum.
A motor is necessary for this CAES for continual compression, so that the energy output can be constant. Because a motor is needed, efficiency of the motor also has to be taken into account. Because the system loses energy as heat very quickly, the energy can be stored for a very short amount of time.

References

[1] Health Safety Executive. Specification for high-strength seamless steel gas cylinders in a modified chromium molybdenum steel. June 1992.

[2] Paul Sears. Compressed air storage - how viable is it? http://canada.theoildrum.com/node/3473, July 2008. Accessed April 12, 2015.

[3] Engineering Toolbox. Compressed gas and air - storage volumes. http://www.engineeringtoolbox.com/compressed-air-storage-volume-d_843.html. Accessed April 12, 2015.

6 Appendix (Calculations)

6.1 Appendix A: Moles of air

$$\text{moles } n = \frac{\text{mass in grams}}{\text{molar mass}} \tag{10}$$

with:

- ρ_{air} =1.246 kg/m^3 (at 1 atm and $T = 283.15K$)
- molar mass dry air $= 28.97g/mol$

gives $n = 43.01$ moles for $1m^3$.
The values above are for N_2, which makes up 78% of air.

Feedback from teacher					
D17:	D19:	D20:	D12:		
T1:	T2:	T3:	M1:	M2:	M3:
R1:	R2:				

6.2 Appendix B: Heat loss

If we want to calculate the heat loss by conduction of a (stainless) steel gas cylinder, we need to know the thickness. A paper of the Health Safety Executive from 1992 gives formulas to compute this.
The minimum thickness of a steel cylinder can be calculated with

$$t = 2.48\sqrt{\frac{D_i}{T}} \tag{11}$$

With $D_i = \text{internal diameter of cylinder}(mm)$ and $T = 1069N/mm2$ (The "minimum specified tensile strength") [1]
For $D_i = 2r * 10^3$.
Assuming that our simplified pressure cylinder can hold $1m^3$ at max, we get:

$$\pi r^2 h = 1$$

With height $h = 1$m; radius $r = \sqrt{\frac{1}{\pi}}$m.
Putting in all the numbers, we get for the minimum thickness $2.55mm$. To be sure about the ability to withstand the pressure, we take a larger number for the thickness, $0.01m$.