



Assignment 1

Q1a

convolving f with g : $(I * F)[i, j] = \sum_{k, l} I[i-k, j-l] F[k, l]$

$$k, l \in [-1, 1]$$

$$(I * F)[0, 0] = \sum_{k, l} I[0-k, 0-l] F[k, l]$$

$$= I[1, 1] F[-1, -1] + I[1, 0] F[-1, 0] + I[1, -1] F[-1, 1] \\ + I[0, 1] F[0, -1] + I[0, 0] F[0, 0] + I[0, -1] F[0, 1] \\ + I[-1, 1] F[1, -1] + I[-1, 0] F[1, 0] + I[-1, -1] F[1, 1]$$

$$= (-2)(-1) + (1)(-1) + 0$$

$$+ 0 + (-1)(1) + 0$$

$$+ 0 + 0 + 0 = +2 - 1 - 1 = 0$$

$$(I * F)[0, 1] = I[1, 2] F[-1, -1] + I[1, 1] F[-1, 0] + I[1, 0] F[-1, 1]$$

$$+ I[0, 2] F[0, -1] + I[0, 1] F[0, 0] + I[0, 0] F[0, 1]$$

$$+ I[-1, 2] F[1, -1] + I[-1, 1] F[1, 0] + I[-1, 0] F[1, 1]$$

$$= (1)(-1) + (-2)(-1) + (1)(-1)$$

$$+ (2)(1) + 0 + (-1)(1)$$

$$+ 0 + 0 + 0 = \cancel{-1} + \cancel{2} - 1 + 2 - 1 = 1$$

$$(I * F)[0, 2] = I[1, 3] F[-1, -1] + I[1, 2] F[-1, 0] + I[1, 1] F[-1, 1]$$

$$+ I[0, 3] F[0, -1] + I[0, 2] F[0, 0] + I[0, 1] F[0, 1]$$

$$+ I[-1, 3] F[1, -1] + I[-1, 2] F[1, 0] + I[-1, 1] F[1, 1]$$

$$= 0 + (1)(-1) + (-2)(-1)$$

$$+ 0 + (2)(1) + 0$$

$$+ 0 + 0 + 0 = -1 + 2 + 2 = 3$$

$$I = \begin{matrix} & \begin{matrix} 0 & 1 & 2 \end{matrix} \\ \begin{matrix} 0 \\ 1 \end{matrix} & \begin{bmatrix} -1 & 0 & 2 \\ 1 & -2 & 1 \end{bmatrix} \end{matrix} \quad F = \begin{matrix} & \begin{matrix} k & l \end{matrix} \\ \begin{matrix} -1 \\ 0 \\ 1 \end{matrix} & \begin{bmatrix} -1 & -1 & -1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

$$\begin{aligned}
 (I * F)[1, 0] &= \sum_{k, l} I[0-k, 0-l] F[k, l] \\
 &= I[2, 1] F[1, -1] + I[2, 0] F[-1, 0] + I[2, -1] F[-1, 1] \\
 &\quad + I[1, 1] F[0, -1] + I[1, 0] F[0, 0] + I[1, -1] F[0, 1] \\
 &\quad + I[0, 1] F[1, -1] + I[0, 0] F[1, 0] + I[0, -1] F[1, 1] \\
 &= 0 + 0 + 0 \\
 &\quad + (-2)(1) + (1)(1) + 0 \\
 &\quad + 0 + (-1)(0) + 0 = -2 + 1 = -1
 \end{aligned}$$

$$\begin{aligned}
 (I * F)[1, 1] &= \sum_{k, l} I[0-k, 0-l] F[k, l] \\
 &= I[2, 2] F[1, -1] + I[2, 1] F[-1, 0] + I[2, 0] F[-1, 1] \\
 &\quad + I[1, 2] F[0, -1] + I[1, 1] F[0, 0] + I[1, 0] F[0, 1] \\
 &\quad + I[0, 2] F[1, -1] + I[0, 1] F[1, 0] + I[0, 0] F[1, 1] \Rightarrow 0 \text{ for this row} \\
 &= 0 + 0 + 0 \\
 &\quad + (1)(1) + (-2)(1) + (1)(1) \\
 &\quad + 0 + 0 + 0 = 1 - 2 + 1 = 0
 \end{aligned}$$

$I = \begin{matrix} & \begin{matrix} 0 & 1 & 2 \end{matrix} \\ \begin{matrix} 0 \\ 1 \end{matrix} & \begin{bmatrix} -1 & 0 & 2 \\ 1 & -2 & 1 \end{bmatrix} \end{matrix}$
 $F = \begin{matrix} & \begin{matrix} l & -1 & 0 & 1 \end{matrix} \\ \begin{matrix} k \\ -1 \\ 0 \\ 1 \end{matrix} & \begin{bmatrix} -1 & -1 & -1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{matrix}$

$$\begin{aligned}
 (I * F)[1, 2] &= \sum_{k, l} I[0-k, 0-l] F[k, l] \\
 &= I[2, 3] F[1, -1] + I[2, 2] F[-1, 0] + I[2, 1] F[-1, 1] \\
 &\quad + I[1, 3] F[0, -1] + I[1, 2] F[0, 0] + I[1, 1] F[0, 1] \\
 &\quad + I[0, 3] F[1, -1] + I[0, 2] F[1, 0] + I[0, 1] F[1, 1] \\
 &= 0 + 0 + 0 \\
 &\quad + 0 + (1)(1) + (-2)(1) \\
 &\quad + 0 + 0 + 0 = 1 - 2 = -1
 \end{aligned}$$

$$(I * F) = \begin{matrix} & \begin{matrix} 0 & 1 & 2 \end{matrix} \\ \begin{matrix} 0 \\ 1 \end{matrix} & \begin{bmatrix} 0 & 1 & 3 \\ -1 & 0 & -1 \end{bmatrix} \end{matrix}$$

1b ID conv: $(I * F_1)[i] = \sum_k I[i-k] F_1[k]$ $F_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ $I = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 2 \\ 1 & -2 & 1 \end{bmatrix}$

$$(I * F_1)[0,0] = \sum_k I[0-k,0] F_1[k]$$

$$= I[1,0] F_1[-1] + I[0,0] F_1[0] + I[-1,0] F_1[1]$$

$$= (1)(-1) + (-1)(1) + 0 = -2$$

$$(I * F_1)[1,0] = I[2,0] F_1[-1] + I[1,0] F_1[0] + I[0,0] F_1[1]$$

$$= 0 + (1)(1) + (-1)(0) = 1$$

$$(I * F_1)[0,1] = I[1,1] F_1[-1] + I[0,1] F_1[0] + I[-1,1] F_1[1]$$

$$= (-2)(-1) + 0 + 0 = 2$$

$$(I * F_1)[1,1] = I[2,1] F_1[-1] + I[1,1] F_1[0] + I[0,1] F_1[1]$$

$$= 0 + (-2)(1) + 0 = -2$$

$$(I * F_1)[0,2] = I[1,2] F_1[-1] + I[0,2] F_1[0] + I[-1,2] F_1[1]$$

$$= (1)(-1) + (2)(1) + 0 = -1 + 2 = 1$$

$$(I * F_1)[1,2] = I[2,2] F_1[-1] + I[1,2] F_1[0] + I[0,2] F_1[1]$$

$$= 0 + (1)(1) + (2)(0) = 1$$

$$I * F_1 = \begin{bmatrix} 0 & 1 & 2 \\ -2 & 2 & 1 \\ 1 & -2 & 1 \end{bmatrix} = I'$$

$$F_2 = \begin{bmatrix} -1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

ID conv: $(I * F_1)[i] = \sum_k I[i-k] F_1[k]$

$$(I' * F_2)[0,0] = \sum_l I'[0,j-l] F_2[l]$$

$$= I'[0,1] F_2[-1] + I'[0,0] F_2[0] + I'[0,-1] F_2[1]$$

$$= (2) + (-2) = 0$$

$$(I' * F_2)[0,1] = I'[0,2] + I'[0,1] + I'[0,0] = 1 + 2 - 2 = 1$$

$$(I' * F_2)[0,2] = I'[0,3] + I'[0,2] + I'[0,1] = 0 + 1 + 2 = 3$$

$$(I' * F_2)[1,0] = I'[1,1] + I'[1,0] + I'[1,-1] = -2 + 1 = -1$$

$$(I' * F_2)[1,1] = I'[1,2] + I'[1,1] + I'[1,0] = 1 - 2 + 1 = 0$$

$$(I' * F_2)[1,2] = I'[1,3] + I'[1,2] + I'[1,1] = 0 + 1 - 2 = -1$$

$$(I * F_1) * F_2 = \begin{bmatrix} 0 & 1 & 3 \\ -1 & 0 & -1 \end{bmatrix}$$

$$(I * F_1)[0,0] = \sum_k I[0-k,0] F_1[k]$$

$$(I * F_1)[0,1] = \sum_k I[0-k,1] F_1[k]$$

$$(I * F_1) * F_2[0,0] = \sum_l F_2[l] I'[0,j-l]$$

1c

Prove for $F = F_1 F_2$, $I * F = (I * F_1) * F_2$

$$(I * F)[i, j] = \sum_k \sum_l I[i-k, j-l] F[k, l]$$

$$= \sum_k \sum_l I[i-k, j-l] (F_1[k]) (F_2[l])$$

$$= \sum_l F_2[l] \underbrace{\sum_k I[i-k, j-l] F_1[k]}_{(I * F_1)[i, j-l]}$$

$(I * F_1)[i, j-l]$
constant

$$= \sum_l F_2[l] [(I * F_1)[i, j-l]]$$

$$= (I * F_1) * F_2$$

1D conv: $(I * F_n)[i] = \sum_k I[i-k] F_n[k]$

$$(I * F_n)[i, j]$$

$$F = F_1 F_2$$

$$F[k, l] = F_1[k] F_2[l]$$

↳ def of inner product

$$\rightarrow \sum_l \left(\left(\sum_k I[i-k] F_1[k] \right) [j-l] \right) F_2[l]$$

2a) $g_{\sigma}(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right)$

$$\frac{d}{dx} g_{\sigma}(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right) \left(\frac{-2x}{2\sigma^2}\right) = -\frac{x}{\sqrt{2\pi}\sigma^3} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

$$\frac{d^2}{dx^2} g_{\sigma}(x) = -\frac{1}{\sqrt{2\pi}\sigma^3} \exp\left(-\frac{x^2}{2\sigma^2}\right) - \frac{x}{\sqrt{2\pi}\sigma^3} \cdot \left(\exp\left(-\frac{x^2}{2\sigma^2}\right) \cdot \frac{-2x}{2\sigma^2}\right)$$

$$= \exp\left(-\frac{x^2}{2\sigma^2}\right) \left[\frac{x^2}{\sqrt{2\pi}\sigma^5} - \frac{1}{\sqrt{2\pi}\sigma^3} \right]$$

$$= \exp\left(-\frac{x^2}{2\sigma^2}\right) \left[\frac{x^2 - \sigma^2}{\sqrt{2\pi}\sigma^5} \right]$$