Assignment 1

Q1a

convolving f with g: $(I*F)[i,j] = \sum_{k,l} I[i-k, j-l]F[k,l]$

$k,l \in [-1,1]$

$$(I*F)[0,0] = \sum_{k,l} I[0-k, 0-l]F[k,l]$$

$$\begin{aligned}
&= I[1,1]F[-1,-1] + I[1,0]F[-1,0] + I[1,-1]F[-1,1] \\
&+ I[0,1]F[0,-1] + I[0,0]F[0,0] + I[0,-1]F[0,1] \\
&+ I[-1,1]F[1,-1] + I[-1,0]F[1,0] + I[-1,-1]F[1,1] \\
&= (-2)(-1) + (1)(-1) + 0 \\
&+ 0 + (-1)(1) + 0 \\
&+ 0 + 0 + 0 = +2 - 1 - 1 = 0
\end{aligned}$$

$$\begin{aligned}
(I*F)[0,1] &= I[1,2]F[-1,-1] + I[1,1]F[-1,0] + I[1,0]F[-1,1] \\
&+ I[0,2]F[0,-1] + I[0,1]F[0,0] + I[0,0]F[0,1] \\
&+ I[-1,2]F[1,-1] + I[-1,1]F[1,0] + I[-1,0]F[1,1] \\
&= (1)(-1) + (-2)(-1) + (1)(-1) \\
&+ (2)(1) + 0 + (-1)(1) \\
&+ 0 + 0 + 0 = \cancel{-1} + \cancel{2}\,\cancel{-1} + 2 - 1 = 1
\end{aligned}$$

$$\begin{aligned}
(I*F)[0,2] &= I[1,3]F[-1,-1] + I[1,2]F[-1,0] + I[1,1]F[-1,1] \\
&+ I[0,3]F[0,-1] + I[0,2]F[0,0] + I[0,1]F[0,1] \\
&+ I[-1,3]F[1,-1] + I[-1,2]F[1,0] + I[-1,1]F[1,1] \\
&= 0 + (1)(-1) + (-2)(-1) \\
&+ 0 + (2)(1) + 0 \\
&+ 0 + 0 + 0 = -1 + 2 + 2 = 3
\end{aligned}$$

$$F = \begin{array}{c|ccc} k \backslash l & -1 & 0 & 1 \\ \hline -1 & -1 & -1 & -1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{array}$$

$$I = \begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & -1 & 0 & 2 \\ 1 & 1 & -2 & 1 \end{array}$$

$$(I*F)[1,0] = \sum_{k,l} I[0-k, 0-l]F[k,l]$$

$$\begin{aligned}
&= I[2,1]F[-1,-1] + I[2,0]F[-1,0] + I[2,-1]F[-1,1] \\
&+ I[1,1]F[0,-1] + I[1,0]F[0,0] + I[1,-1]F[0,1] \\
&+ I[0,1]F[1,-1] + I[0,0]F[1,0] + I[0,-1]F[1,1] \\
&= 0 + 0 + 0 \\
&+ (-2)(1) + (1)(1) + 0 \\
&+ 0 + (-1)(0) + 0 = -2+1 = -1
\end{aligned}$$

$$(I*F)[1,1] = \sum_{k,l} I[0-k, 0-l]F[k,l]$$

$$\begin{aligned}
&= I[2,2]F[-1,-1] + I[2,1]F[-1,0] + I[2,0]F[-1,1] \\
&+ I[1,2]F[0,-1] + I[1,1]F[0,0] + I[1,0]F[0,1] \\
&+ I[0,2]F[1,-1] + I[0,1]F[1,0] + I[0,0]F[1,1] \Rightarrow 0 \text{ for this row} \\
&= 0 + 0 + 0 \\
&+ (1)(1) + (-2)(1) + (1)(1) = 1-2+1 = 0 \\
&+ 0 + 0 + 0
\end{aligned}$$

$$F = \begin{array}{c|c} k \backslash l & \begin{array}{ccc} -1 & 0 & 1 \end{array} \\ \begin{array}{c} -1 \\ 0 \\ 1 \end{array} & \begin{bmatrix} -1 & -1 & -1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$I = \begin{array}{c|c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} -1 & 0 & 2 \\ 1 & -2 & 1 \end{bmatrix} \end{array}$$

$$(I*F)[1,2] = \sum_{k,l} I[0-k, 0-l]F[k,l]$$

$$\begin{aligned}
&= I[2,3]F[-1,-1] + I[2,2]F[-1,0] + I[2,1]F[-1,1] \\
&+ I[1,3]F[0,-1] + I[1,2]F[0,0] + I[1,1]F[0,1] \\
&+ I[0,3]F[1,-1] + I[0,2]F[1,0] + I[0,1]F[1,1] \\
&= 0 + 0 + 0 \\
&+ 0 + (1)(1) + (-2)(1) \\
&+ 0 + 0 + 0 = 1-2 = -1
\end{aligned}$$

$$(I*F) = \begin{array}{c|c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 0 & 1 & 3 \\ -1 & 0 & -1 \end{bmatrix} \end{array}$$

1b

1D conv: $(I * F_n)[i] = \sum_k I[i-k] F_n[k]$

$$F_1 = \begin{array}{c} -1 \\ 0 \\ 1 \end{array}\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \qquad I = \begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & -1 & 0 & 2 \\ 1 & 1 & -2 & 1 \end{array}$$

$$(I * F_1)[0,0] = \sum_k I[0-k, 0] F_1[k]$$
$$= I[1,0]F_1[-1] + I[0,0]F_1[0] + I[-1,0]F_1[1]$$
$$= (1)(-1) + (-1)(1) + 0 = -2$$

$$(I * F_1)[1,0] = I[2,0]F_1[-1] + I[1,0]F_1[0] + I[0,0]F_1[1]$$
$$= 0 + (1)(1) + (-1)(0) = 1$$

$$(I * F_1)[0,1] = I[1,1]F_1[-1] + I[0,1]F_1[0] + I[-1,1]F_1[1]$$
$$= (-2)(-1) + 0 + 0 = 2$$

$$(I * F_1)[1,1] = I[2,1]F_1[-1] + I[1,1]F_1[0] + I[0,1]F_1[1]$$
$$= 0 + (-2)(1) + 0 = -2$$

$$(I * F_1)[0,2] = I[1,2]F_1[-1] + I[0,2]F_1[0] + I[-1,2]F_1[1]$$
$$= (1)(-1) + (2)(1) + 0 = -1 + 2 = 1$$

$$(I * F_1)[1,2] = I[2,2]F_1[-1] + I[1,2]F_1[0] + I[0,2]F_1[1]$$
$$= 0 + (1)(1) + (2)(0) = 1$$

$$I * F_1 = \begin{array}{c} \\ 0 \\ 1 \end{array}\begin{array}{ccc} 0 & 1 & 2 \\ \hline \end{array}\begin{bmatrix} -2 & 2 & 1 \\ 1 & -2 & 1 \end{bmatrix} = I'$$

$$F_2 = \begin{array}{ccc} -1 & 0 & 1 \\ \left[1\right. & 1 & \left.1\right] \end{array}$$

1D conv: $(I * F_n)[i] = \sum_k I[i-k] F_n[k]$

$$(I' * F_2)[0,0] = \sum_\ell I'[0, j-\ell] F_2[\ell]$$
$$= I'[0,1]F_2[-1] + I'[0,0]F_2[0] + I'[0,-1]F_2[1]$$
$$= (2) + (-2) = 0$$

$$(I' * F_2)[0,1] = I'[0,2] + I'[0,1] + I'[0,0] = 1 + 2 - 2 = 1$$
$$(I' * F_2)[0,2] = I'[0,3] + I'[0,2] + I'[0,1] = 0 + 1 + 2 = 3$$
$$(I' * F_2)[1,0] = I'[1,1] + I'[1,0] + I'[1,-1] = -2 + 1 = -1$$
$$(I' * F_2)[1,1] = I'[1,2] + I'[1,1] + I'[1,0] = 1 - 2 + 1 = 0$$
$$(I' * F_2)[1,2] = I'[1,3] + I'[1,2] + I'[1,1] = 0 + 1 - 2 = -1$$

$$(I * F_1) * F_2 = \begin{bmatrix} 0 & 1 & 3 \\ -1 & 0 & -1 \end{bmatrix}$$

$$(I * F_1)[0,0] = \sum_k I[0-k, 0] F_1[k]$$

$$(I * F_1) * F_2[0,0] = \sum_\ell F_2[\ell] I'[0, j-\ell]$$

$$(I * F)[0,1] = \sum_k I[0-k, 1] F_1[k]$$

1c

Prove for $F = F_1 F_2$, $I * F = (I * F_1) * F_2$

1D conv: $(I * F_n)[i] = \sum_k I[i-k] F_n[k]$

$[I * F_n][i,j] = \sum_k I[i-k, j] F_n[k]$

$\boxed{F = F_1 F_2}$

$F[k,l] = F_1[k] F_2[l]$

↳ def of inner product

$$(I*F)[i,j] = \sum_k \sum_l I[i-k, j-l] F[k,l]$$

$$= \sum_k \sum_l I[i-k, j-l] (F_1[k])(F_2[l])$$

$$= \sum_l F_2[l] \underbrace{\sum_k I[i-k, j-l] F_1[k]}_{(I*F_1)[i,\underbrace{j-l}_{\text{constant}}]}$$

$$\sum_l \left(\left(\sum_k I[i-k, j] F_1[k] \right) [j-l] \right) F_2[l]$$

$$= \sum_l F_2[l] \left[(I*F_1)[i, j-l] \right]$$

$$= (I*F_1) * F_2$$

2a) $g_\sigma(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right)$

$$\frac{d}{dx} g_\sigma(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right)\left(\frac{-2x}{2\sigma^2}\right) = -\frac{x}{\sqrt{2\pi}\,\sigma^3} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

$$\frac{d^2}{dx^2} g_\sigma(x) = -\frac{1}{\sqrt{2\pi}\,\sigma^3} \exp\left(-\frac{x^2}{2\sigma^2}\right) - \frac{x}{\sqrt{2\pi}\,\sigma^3} \cdot \left(\exp\left(-\frac{x^2}{2\sigma^2}\right) \frac{-\cancel{2}x}{\cancel{2}\sigma^2}\right)$$

$$= \exp\left(\frac{-x^2}{2\sigma^2}\right)\left[\frac{x^2}{\sqrt{2\pi}\,\sigma^5} - \frac{1}{\sqrt{2\pi}\,\sigma^3}\right]$$

$$= \exp\left(-\frac{x^2}{2\sigma^2}\right)\left[\frac{x^2 - \sigma^2}{\sqrt{2\pi}\,\sigma^5}\right]$$

5.
Differences:
Without linear blend, the panorama contains artifacts like edges of the other image. We can see where the images are stitched. The location where the key points match aren't exact and we can see the error where the stitching is off.

In linear blend, some of the errors are removed, especially around the fountain area. Where there aren't any object, the stitching is blended so that there's no visible edge.