$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$ $\lambda = 1, 4, 6$

$\lambda = 1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = (1) \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$x + 2y + 3z = x$

$4y + 5z = y$

$6z = z$

$z = 0$ #

$y = 0$ #

$x = x$ ให้ $x = 1$ #

$\therefore V_{\lambda=1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ or any non-zero scalar multiple of the same. ✱

$\lambda = 4$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = (4) \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$x + 2y + 3z = 4x$

$4y + 5z = 4y$

$6z = 4z$

$z = 0$ #

$4y = 4y$

$y = y$ #

$x + 2y = 4x$

$\frac{2}{3}y = x$

ให้ $y = 1$; $x = \frac{2}{3}$

$\therefore V_{\lambda=4} = \begin{bmatrix} 2/3 \\ 1 \\ 0 \end{bmatrix}$

or any non-zero scalar multiple of the same. ✱

$\lambda = 6$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 6 \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$x + 2y + 3z = 6x$

$4y + 5z = 6y$

$6z = 6z$

$z = z$ #

$5z = 2y$

let $z = 1$; $y = \frac{5}{2}$ #

$x + 5y + 3z = 6x$

$x + 5 + 3 = 6x$

$x = \frac{8}{5}$ #

$\therefore V_{\lambda=6} = \begin{bmatrix} 8/5 \\ 5/2 \\ 1 \end{bmatrix}$ #

or any non-zero scalar multiple of the same. ✳

$B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{bmatrix}$ $\lambda = 2, \sqrt{3}, -\sqrt{3}$

$\lambda = 2$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 2 \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$z = 2x$

$2y = 2y$

$3x = 2z$

$y = y$ #

$\therefore$ let $y = 1$

$\therefore V_{\lambda=2} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ ✳

$3x = 2(2x)$

$x = 0$ #

$z = 0$ #

$\lambda = \sqrt{3}$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \sqrt{3} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$z = \sqrt{3}x$

$2y = \sqrt{3}y$

$3x = \sqrt{3}z$

$(\sqrt{3} - 2)y = 0$

$y = 0$ #

$3x = \sqrt{3}z$

$\sqrt{3}x = z$

$\sqrt{3}x = \sqrt{3}x$

$x = x$ #

$z = \sqrt{3}x$

let $x = 1$ $\therefore V_{\lambda=\sqrt{3}} = \begin{bmatrix} 1 \\ 0 \\ \sqrt{3} \end{bmatrix}$ ✳

$\lambda = -\sqrt{3}$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\sqrt{3} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$z = -\sqrt{3}x$

$2y = -\sqrt{3}y$

$3x = -\sqrt{3}z$

$(2+\sqrt{3})y = 0$

$y = 0$ #

$3x = -\sqrt{3}(-\sqrt{3}x)$

$3x = 3x$

$x = x$ #

$z = -\sqrt{3}x$

Let $x = 1$; $V_{\lambda=-\sqrt{3}} = \begin{bmatrix} 1 \\ 0 \\ -\sqrt{3} \end{bmatrix}$ or any non-zero scalar multiple of the same. #

$C = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$ $\lambda = 0, 6$

$\lambda = 0$

$$\begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$x + y + z = 0$

ให้ $x = 1, y = 1$ จะได้ $z = -2$ $\therefore V_{\lambda=0} = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$

or any non-zero scalar multiple of the same. #

$\lambda = 6$

$y + z = 2x$

$x + z = 2y$ — ①

$x + y = 2z$ — ②

① − ② ; $z - y = 2y - 2z$

$z = y$ #

$y + z = 2x$

$y + y = 2x$

$y = x$ #

$\rightarrow x = y = z$

ให้ $x = 1$; $V_{\lambda=6} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ #