



Linear algebra II

Linear transformation

$$Ax = b$$


1. place x on original grid
2. transform x onto A grid
3. read transformed x
on original grid ($=b$)

example 1

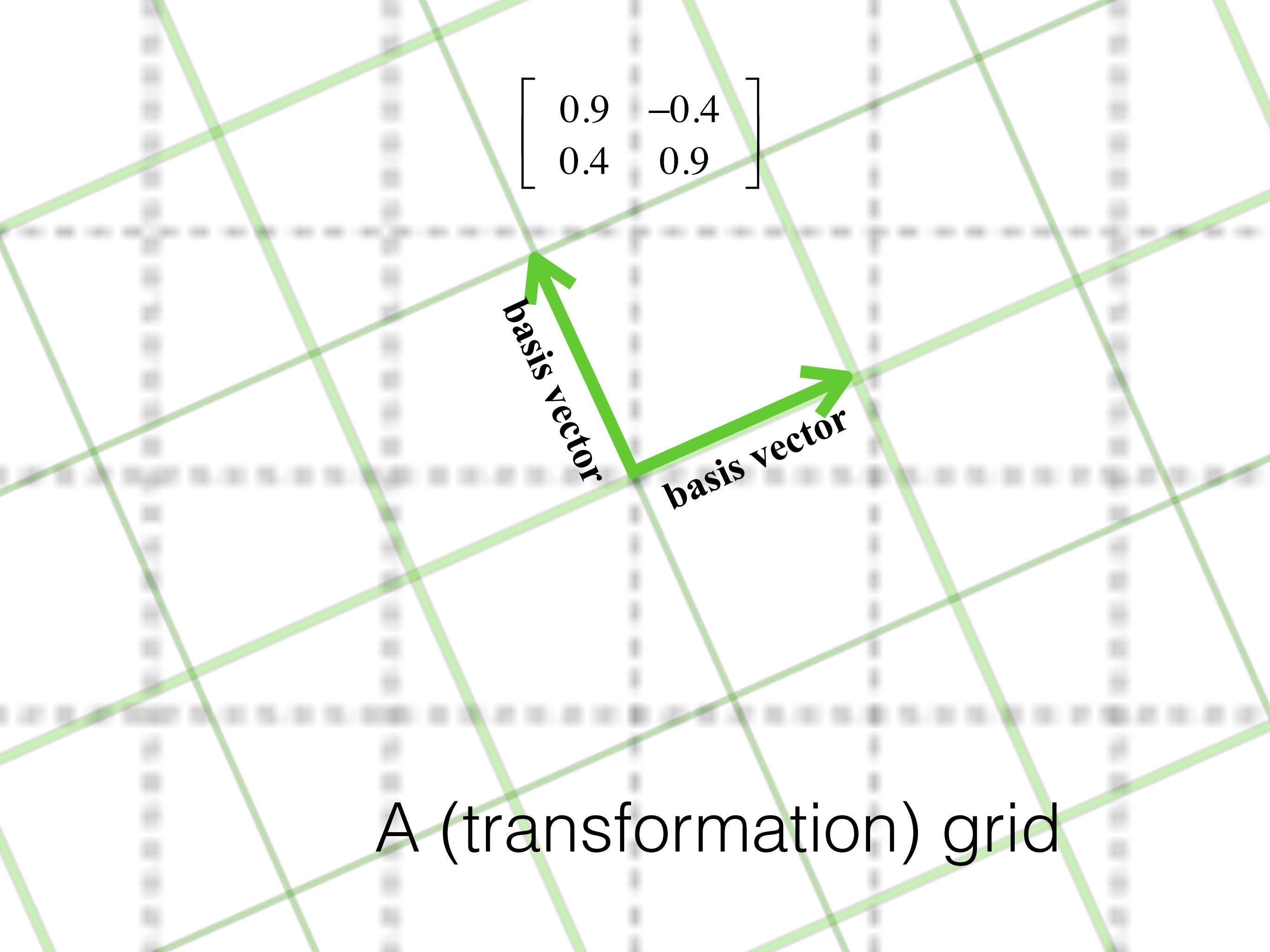
$$\begin{bmatrix} 0.9 & -0.4 \\ 0.4 & 0.9 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 1.3 \end{bmatrix}$$


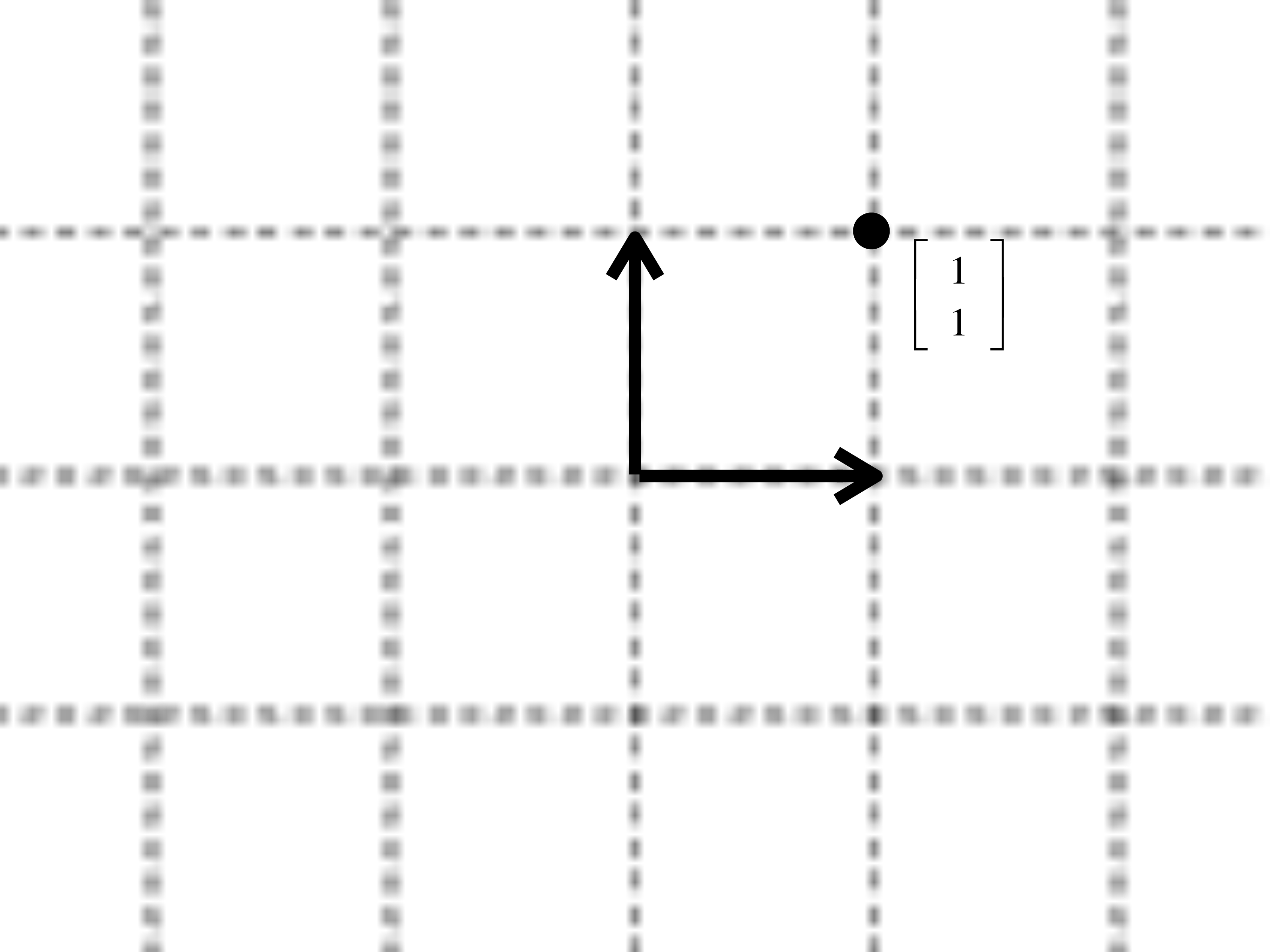
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

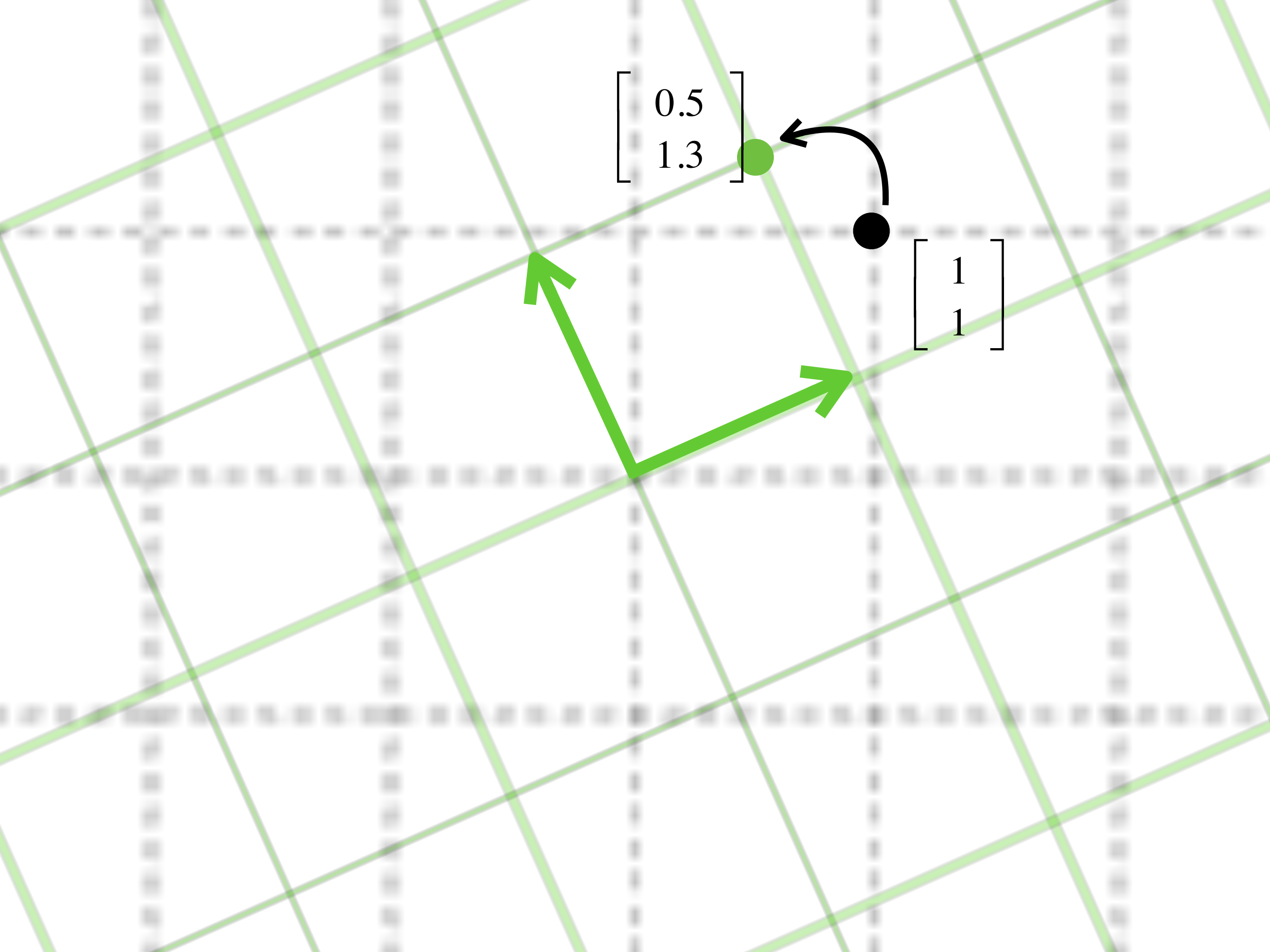
basis vector

basis vector

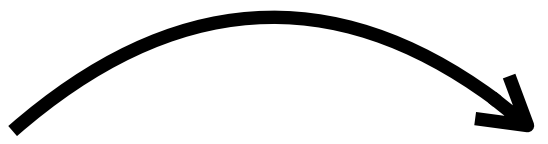
original grid







example 2

$$\begin{bmatrix} 1.25 & 0.25 \\ 0.25 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.5 \\ 1.25 \end{bmatrix}$$


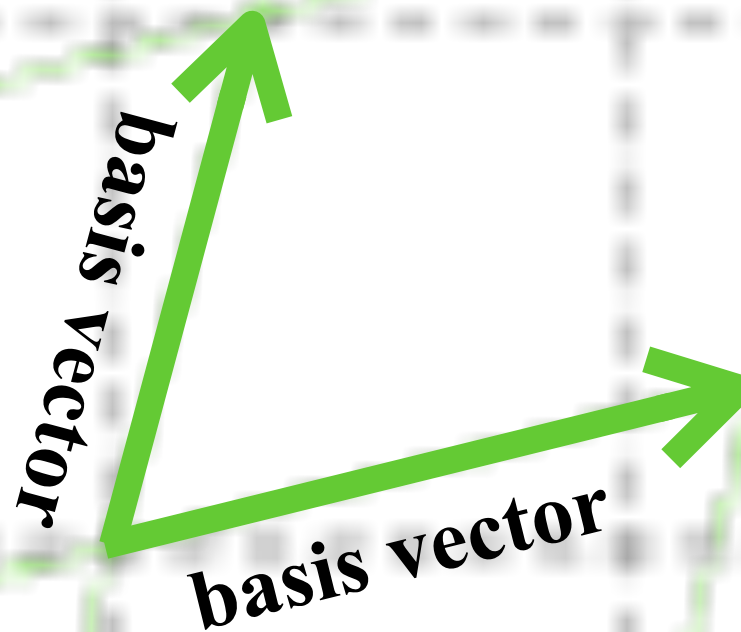
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

basis vector

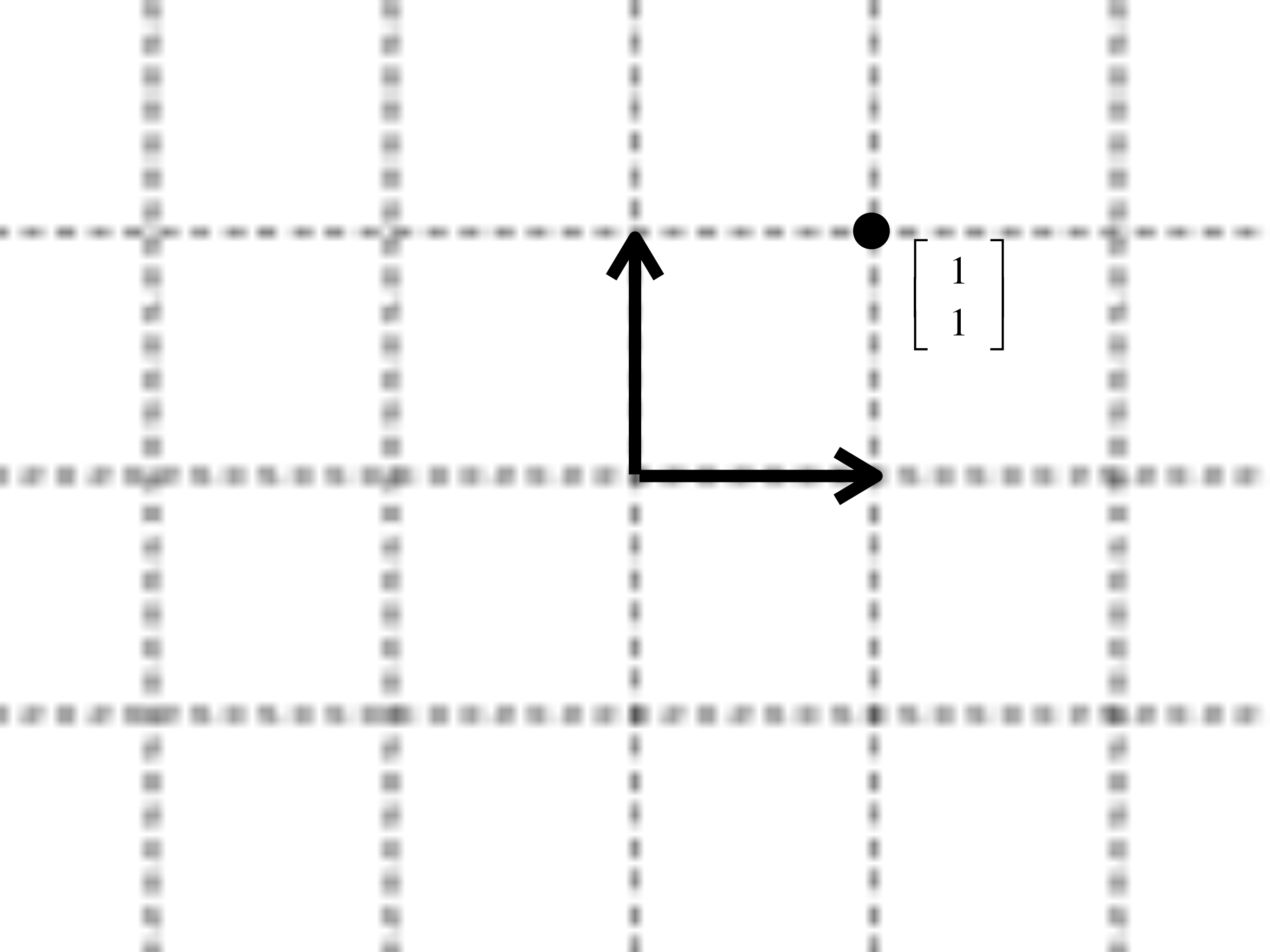
basis vector

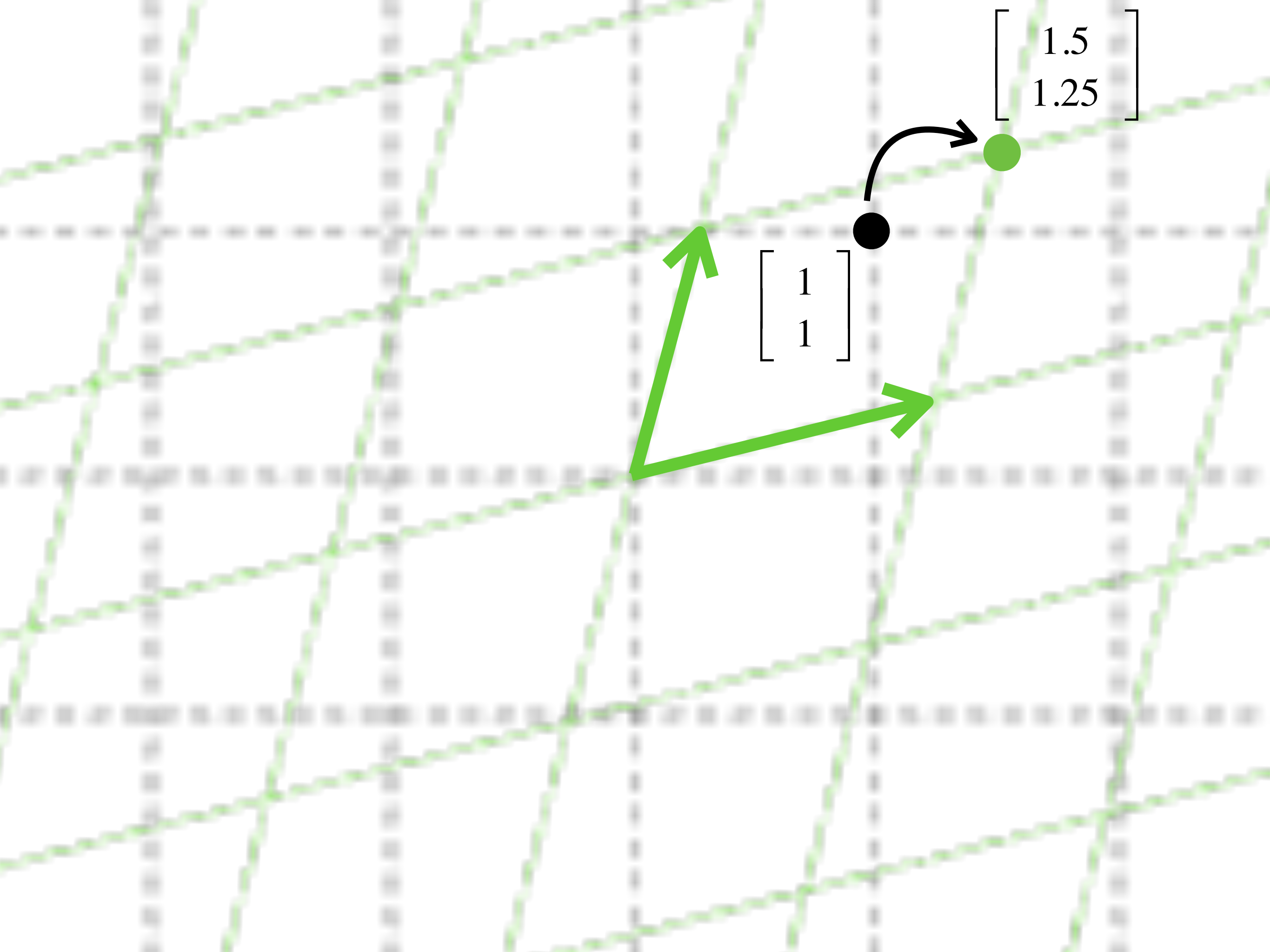
original grid

$$\begin{bmatrix} 1.25 & 0.25 \\ 0.25 & 1 \end{bmatrix}$$




A (transformation) grid





example 3

$$\begin{bmatrix} 1 & 0.5 \\ 0.25 & 0.125 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.5 \\ 0.375 \end{bmatrix}$$


$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

basis vector

basis vector

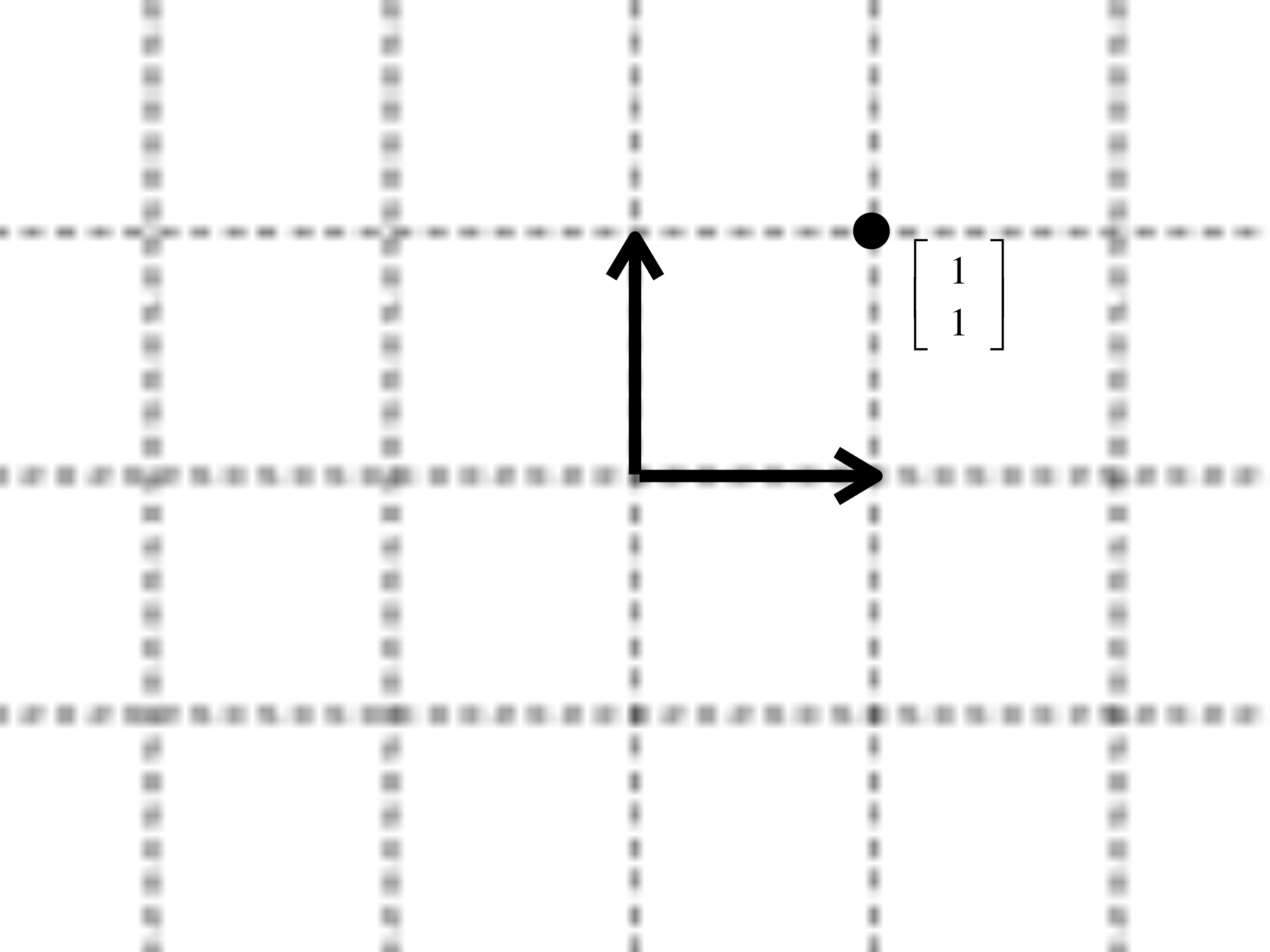
original grid

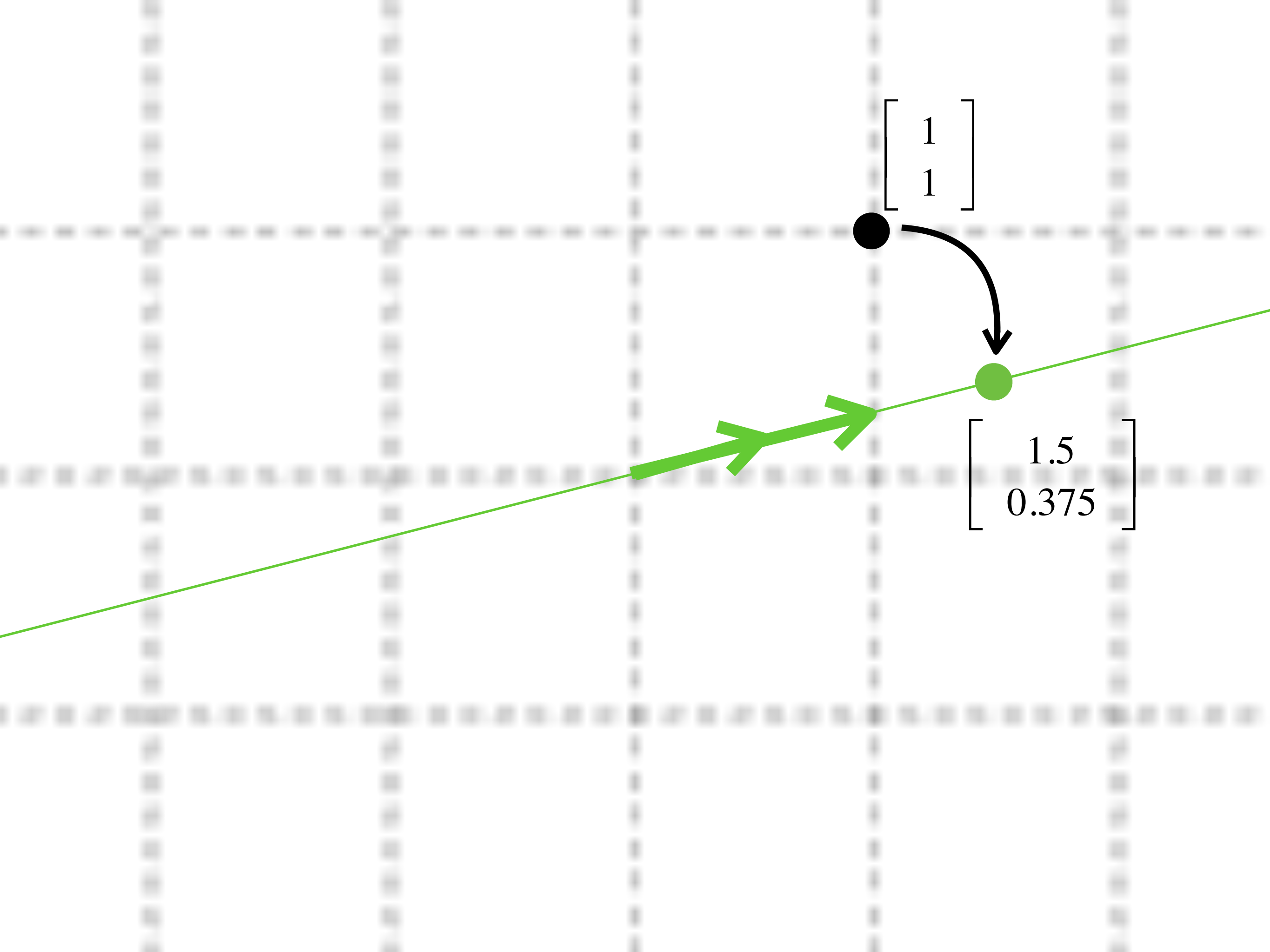
$$\begin{bmatrix} 1 & 0.5 \\ 0.25 & 0.125 \end{bmatrix}$$



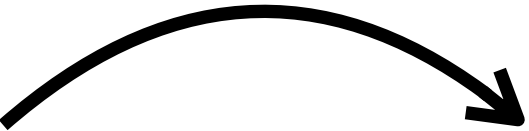
basis vectors

A (transformation) grid




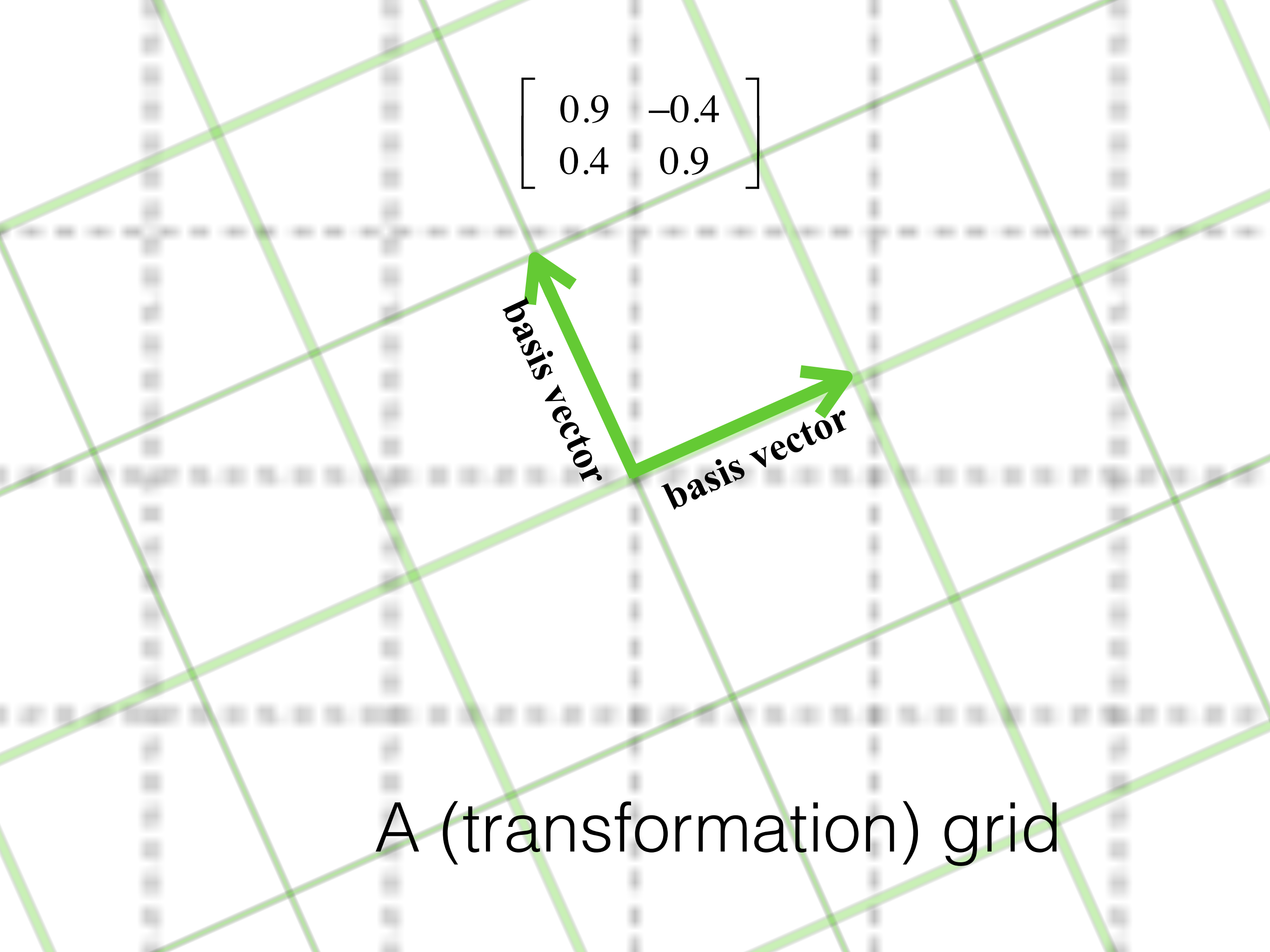


Detransformation:
Inverse matrix

$$A^{-1}b = x$$


example 1

$$\begin{bmatrix} 0.9 & -0.4 \\ 0.4 & 0.9 \end{bmatrix}^{-1} \begin{bmatrix} 0.5 \\ 1.3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$


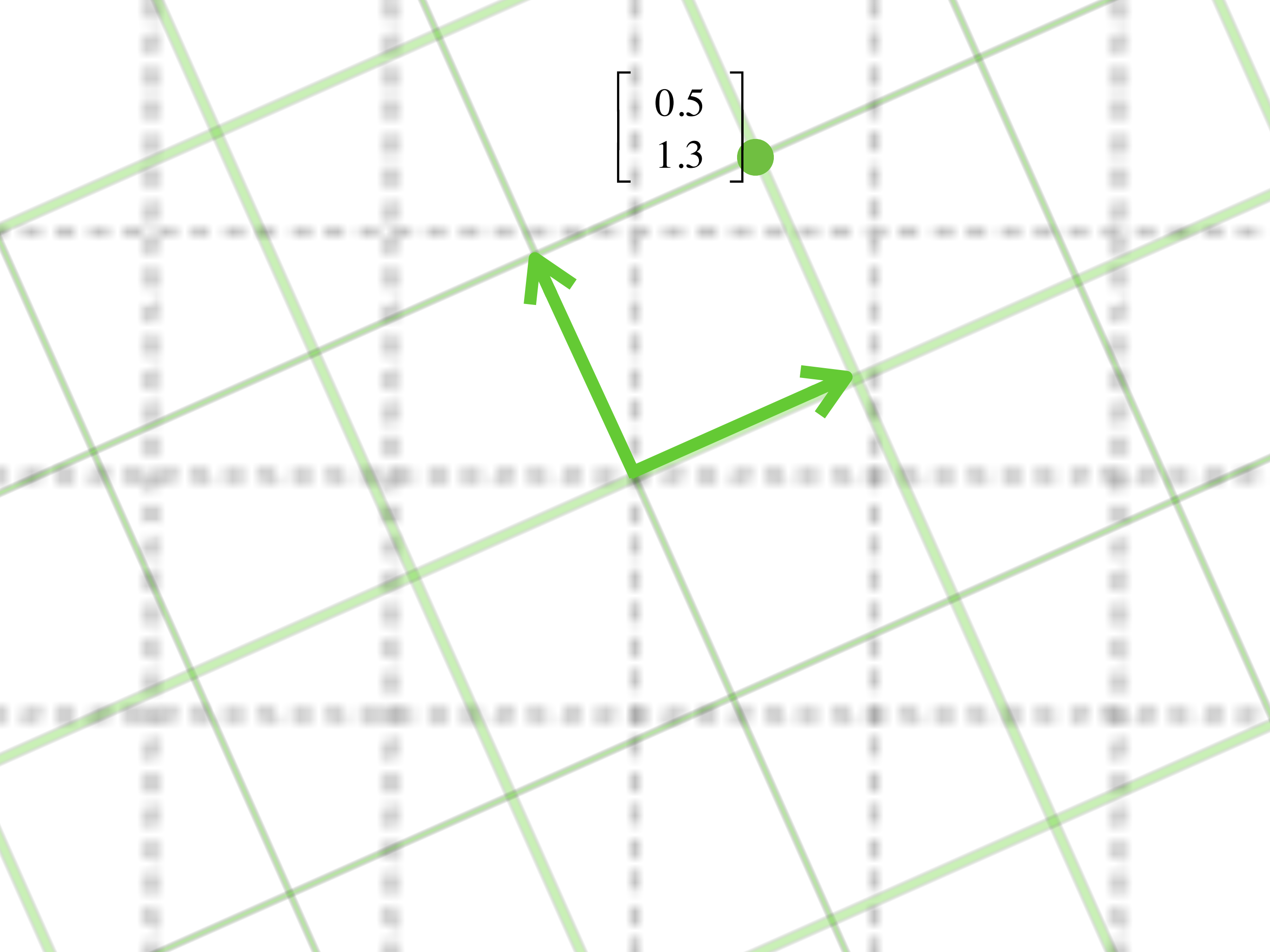


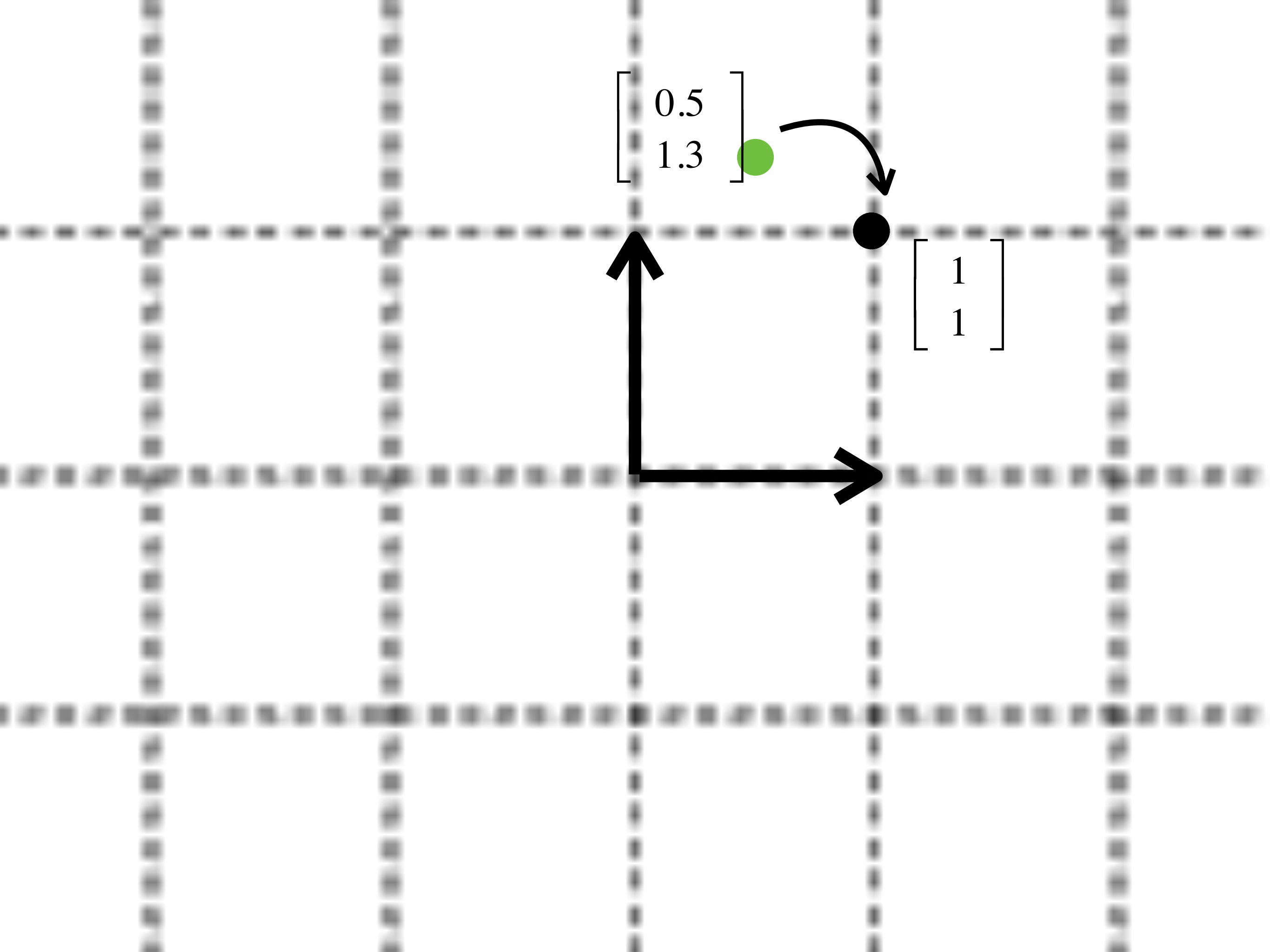
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

basis vector


basis vector

original grid

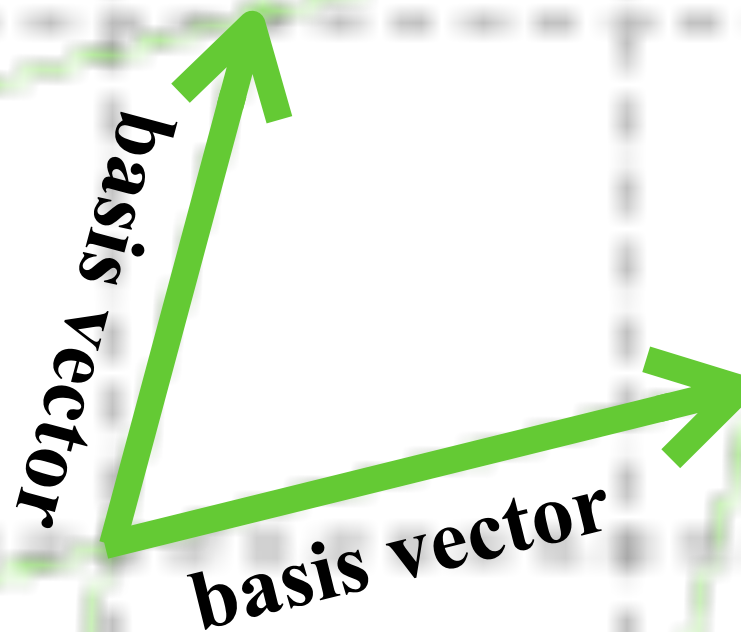




example 2

$$\begin{bmatrix} 1.25 & 0.25 \\ 0.25 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1.5 \\ 1.25 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$


$$\begin{bmatrix} 1.25 & 0.25 \\ 0.25 & 1 \end{bmatrix}$$



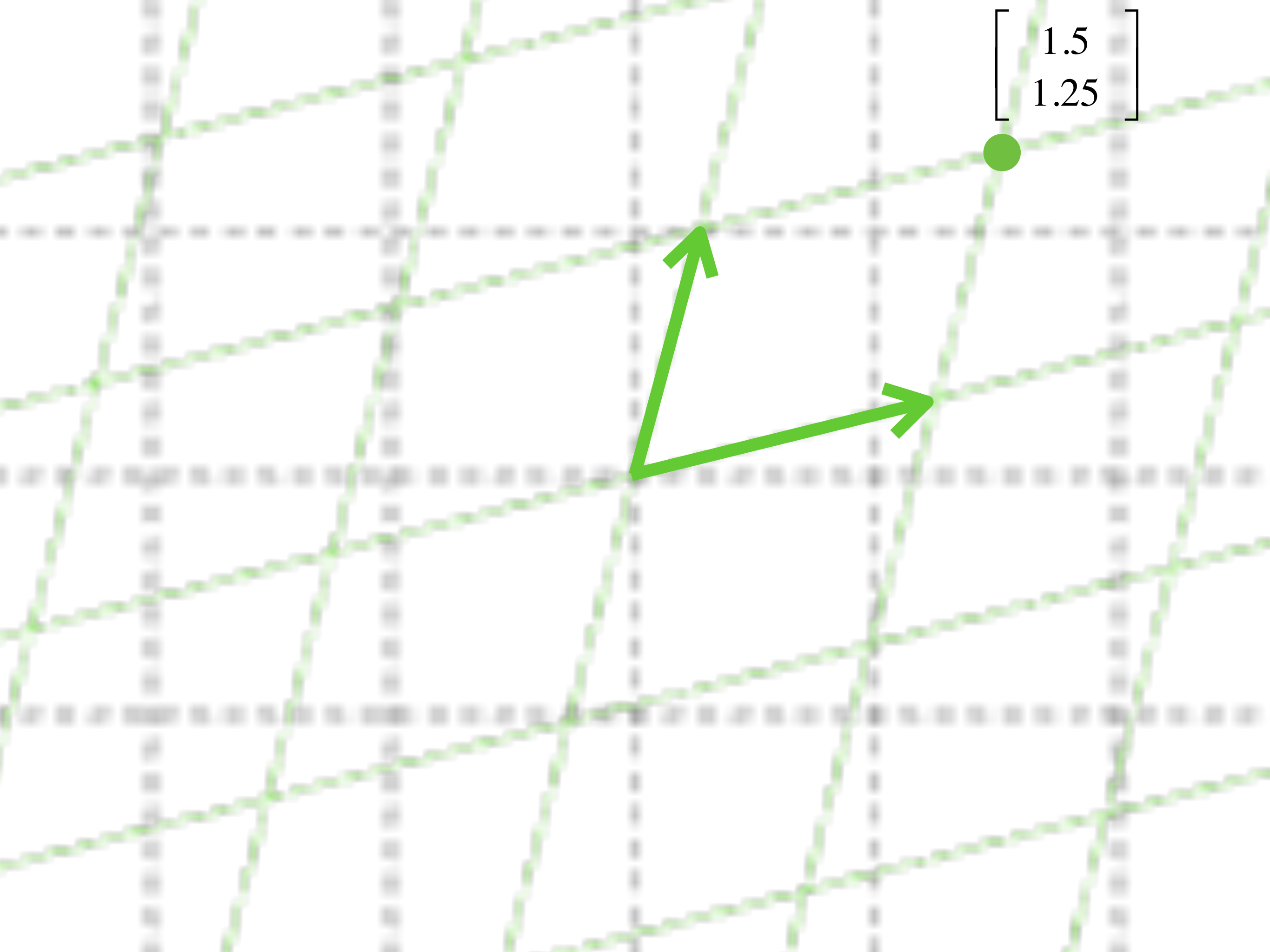
A (transformation) grid

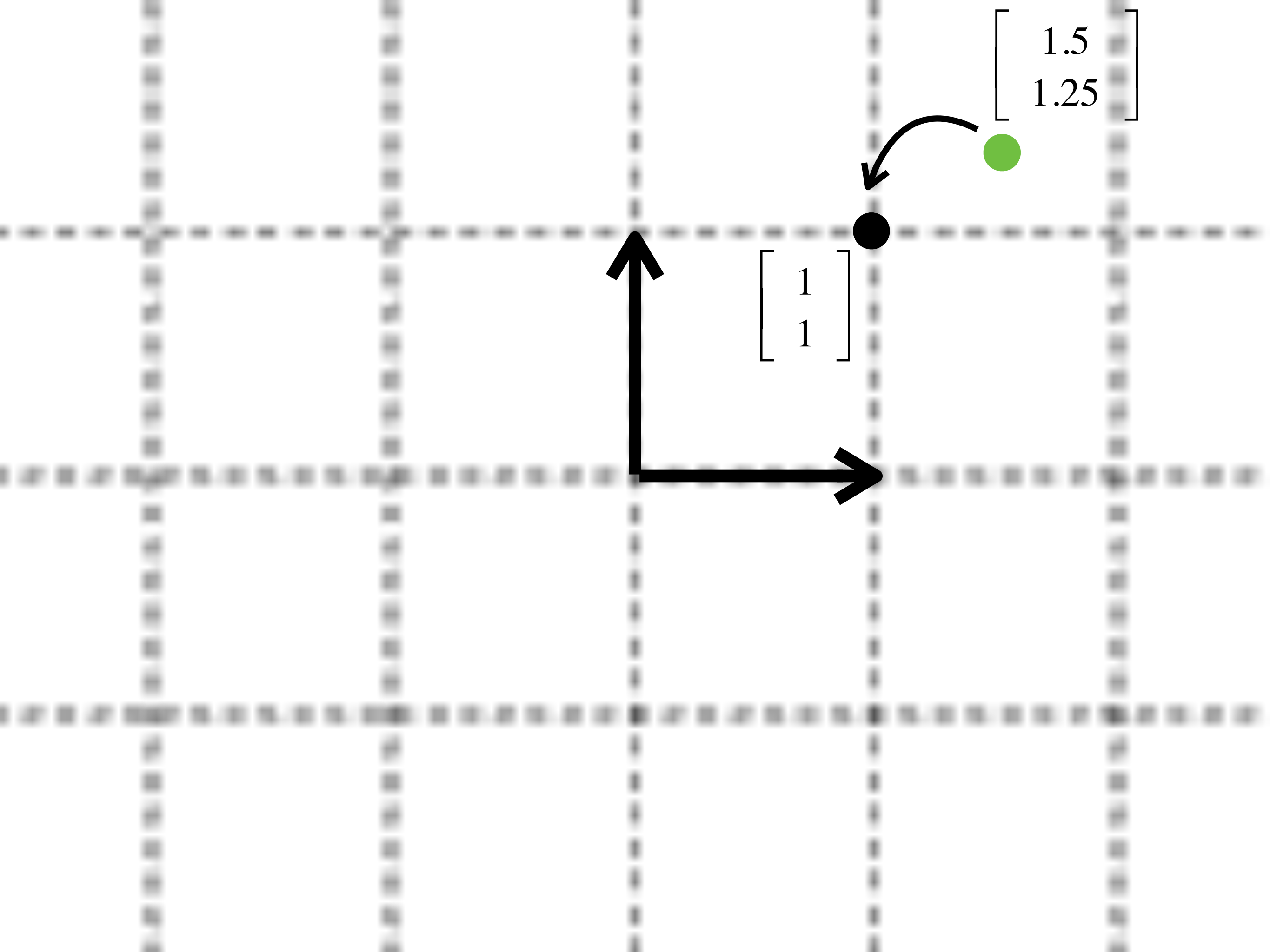
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

basis vector


basis vector

original grid





example 3

$$\begin{bmatrix} 1 & 0.5 \\ 0.25 & 0.125 \end{bmatrix}^{-1} \begin{bmatrix} 1.5 \\ 0.375 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$


$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

basis vector

basis vector

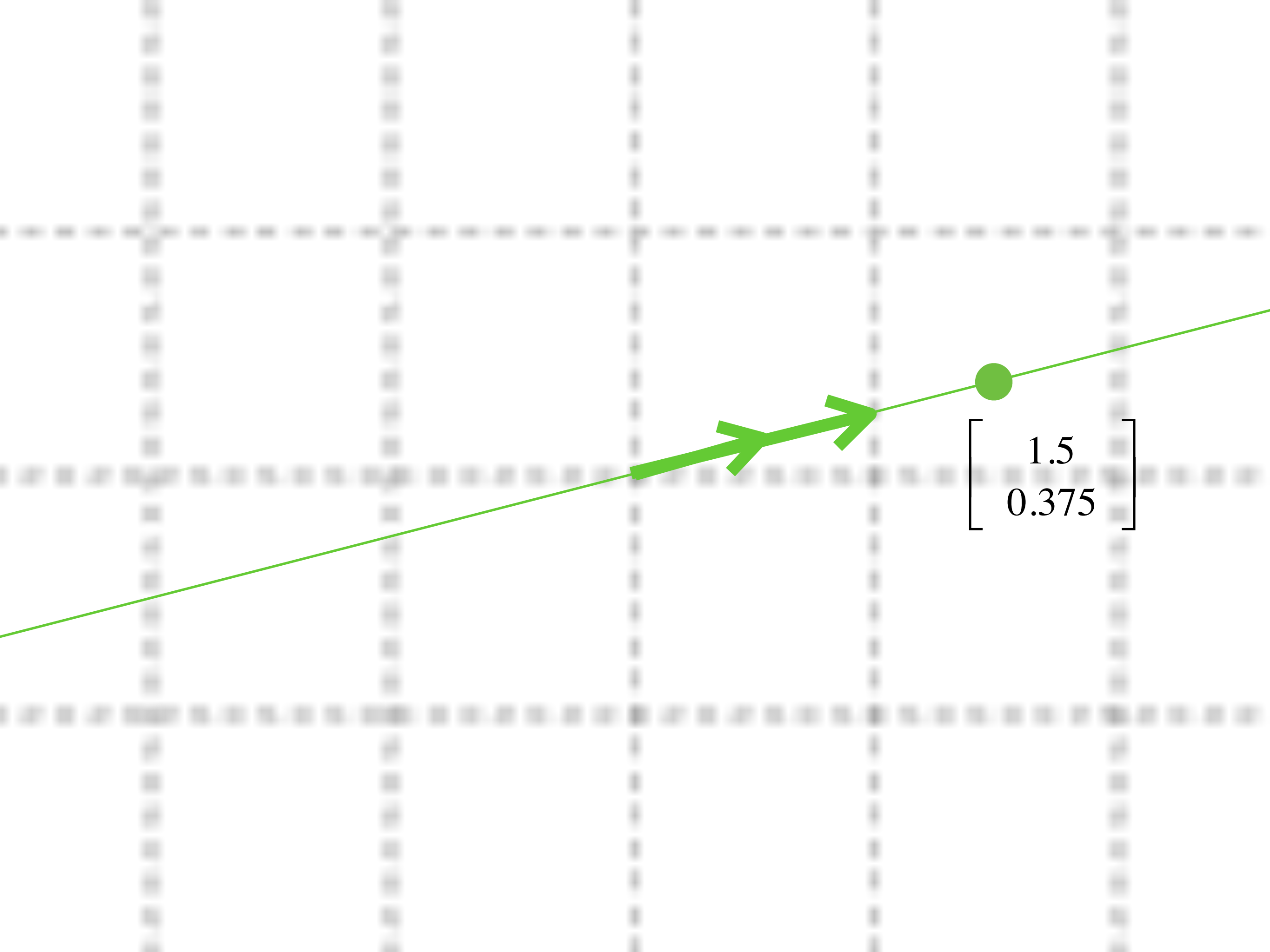
original grid

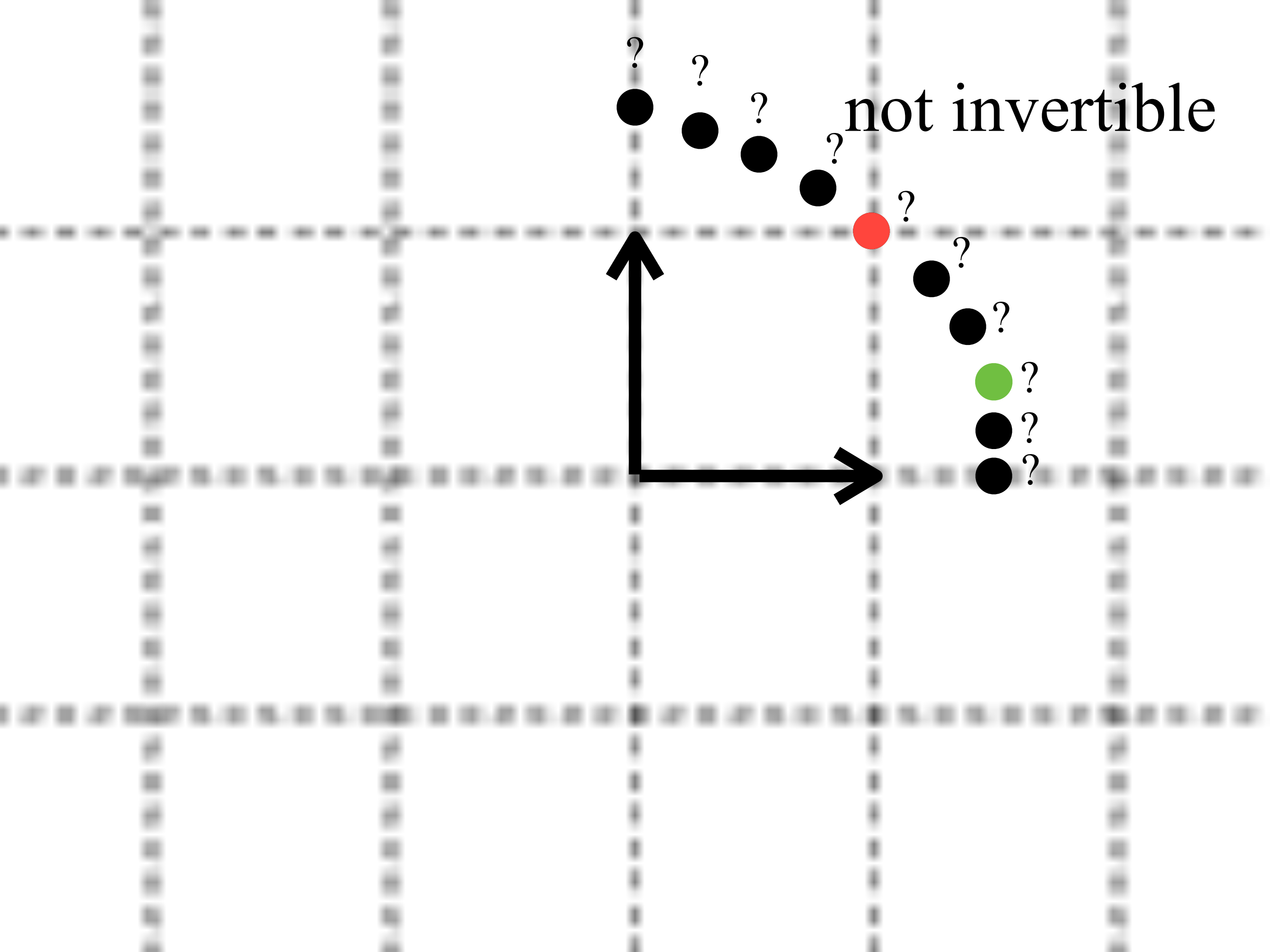
$$\begin{bmatrix} 1 & 0.5 \\ 0.25 & 0.125 \end{bmatrix}$$



basis vectors

A (transformation) grid



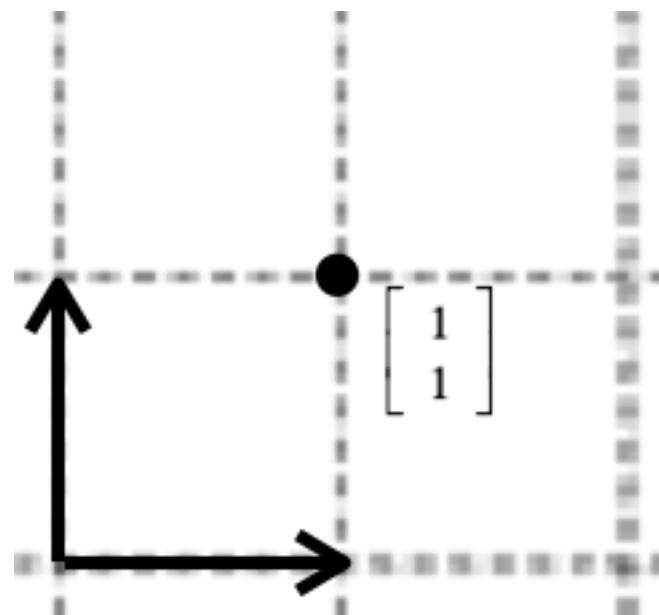


eigenvector

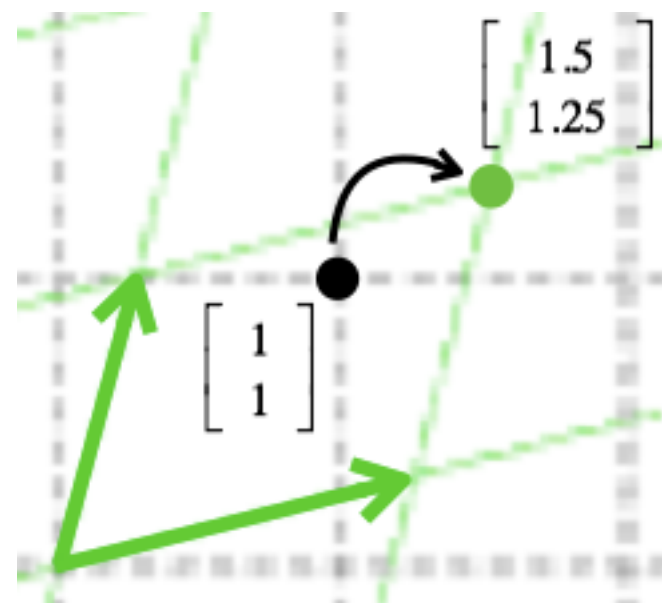
$$Av=b$$

A transforms v to b

$$\begin{bmatrix} 1.25 & 0.25 \\ 0.25 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.5 \\ 1.25 \end{bmatrix}$$



x space



b space

Among all v ,
some v is parallel to Av .
that v is eigenvector

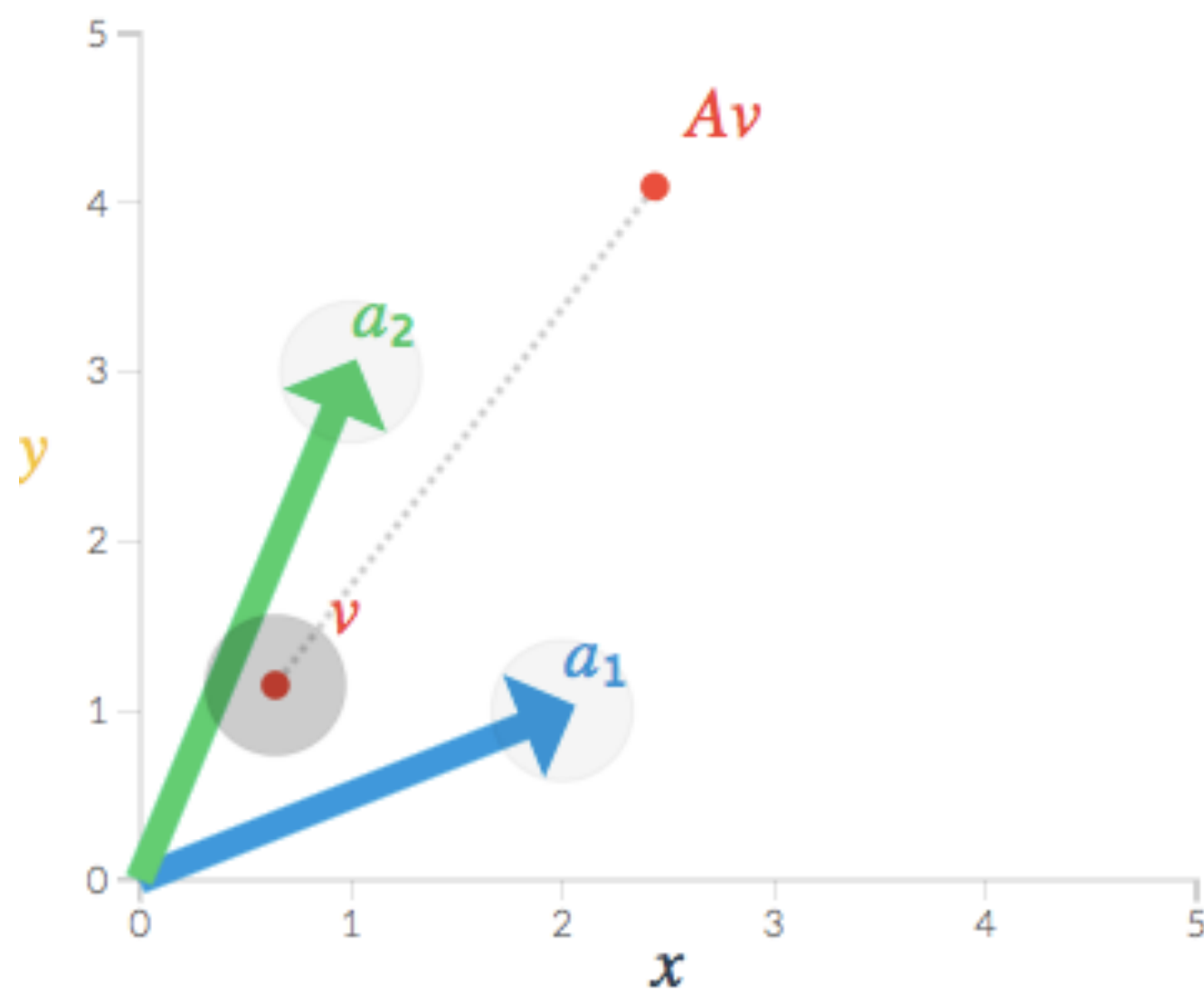
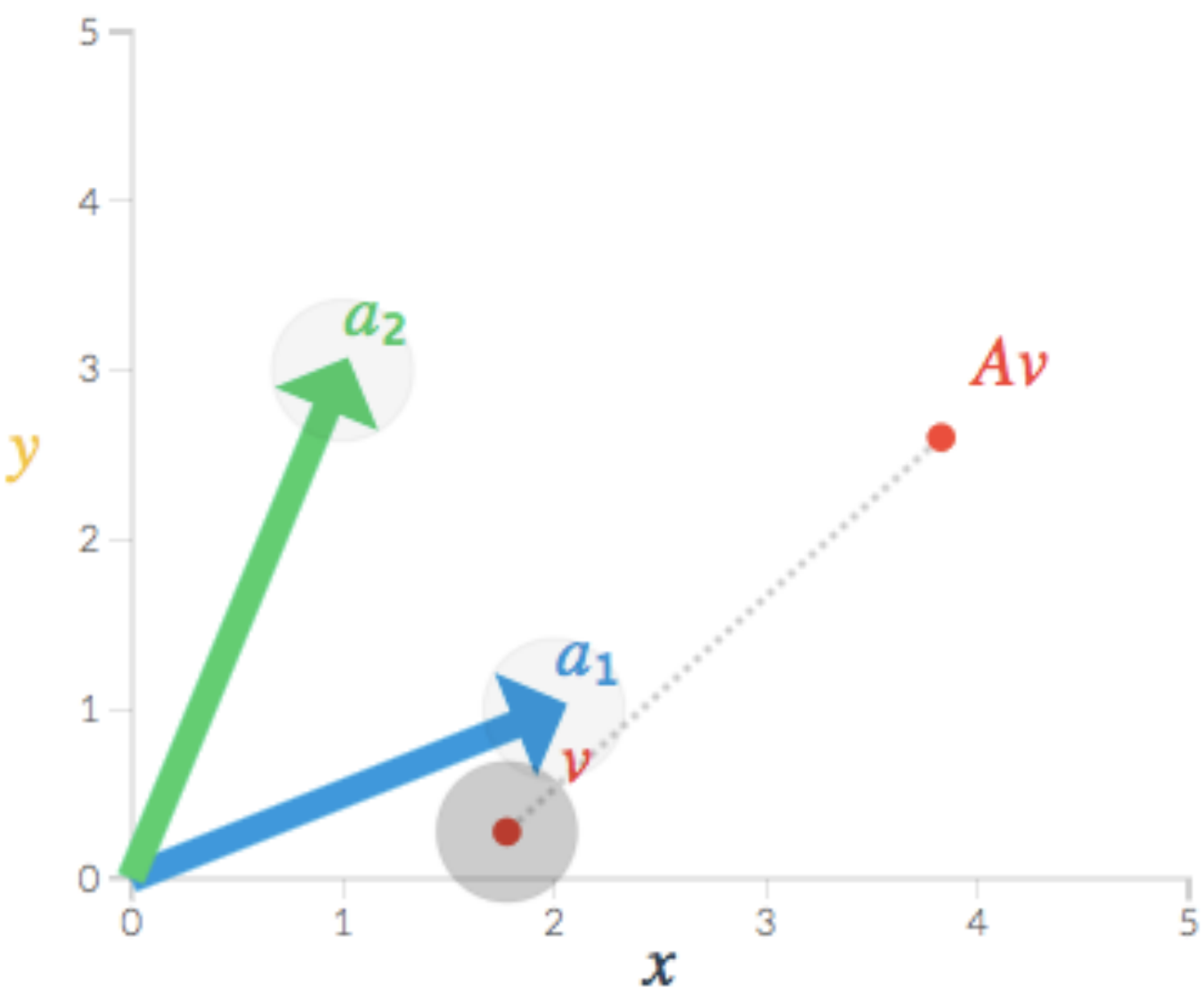
$$Av = \lambda v$$

v : *eigenvectors*

λ : *eigenvalues*

Visualizing eigenvectors

<http://setosa.io/ev/eigenvectors-and-eigenvalues/>



eigenspace

