ST 501 Project

Given: $Y_i \sim exp(1)$ for $i=1,2,\dots,n$

$$f_{Y_i}(y)=f_Y(y)=e^{-y}\text{, with support } 0<y<\infty$$

$$F_Y(y)=\begin{cases} 1-e^{-y}, & \text{for } 0<y<\infty \\ 0, & \text{otherwise} \end{cases}$$

From Lecture Notes pg 154:

$$F_{Y_{(1)}}(y)=\begin{cases} 1-(1-F_Y(y))^n=1-e^{-ny}, & \text{for } 0<y<\infty \\ 0, & \text{otherwise} \end{cases}$$

The support of $Y_{(1)}$ is the same as Y: $0<y<\infty$, therefore $|Y_{(1)}-0|=Y_{(1)}-0$ and

$P(|Y_{(1)}-0|<\varepsilon)=P(Y_{(1)}-0<\varepsilon)=P(Y_{(1)}<\varepsilon)=F_{Y_{(1)}}(\varepsilon)$

$\varepsilon>0$ by definition, therefore $F_{Y_{(1)}}(\varepsilon)=1-e^{-n\varepsilon}$ for all possible values of ε

$\lim_{n\to\infty} F_{Y_{(1)}}(\varepsilon)=\lim_{n\to\infty}(1-e^{-n\varepsilon})=1$, therefore $\lim_{n\to\infty} P(|Y_{(1)}-0|<\varepsilon)=1$ and

$Y_{(1)}$ converges in probability to 0