Math 420, Spring 2021
Team Homework 6

due Thursday, 13 April, 2021

In the following exercises consider the risky assets in groups (A) (B) and (C) of your final project. Consider one-year histories of daily share price data for each asset over the years ending December 31 of 2016-2020. There are 20 quarters within this five year period.

Use adjusted closing prices to compute the return for each trading day over the five years 2016-2020. For each of the five years ending December 31 of 2016-2020 compute $\mathbf{m}$ and $\mathbf{V}$ for the assests in group (A), groups (A) and (B) combined, and groups (A), (B), and (C) combined using one-year histories with uniform weights. Use the U.S. T-Bill rate available at the beginning of each year as the safe investment for the data from that year. Assume that the credit line for each year is three points higher than the U.S. T-Bill rate.

Exercise 1. For each of the five years plot a graph that shows

- the points (m_i, σ_i) associated with each asset,
- the frontier hyperbola associated with each of the three pairs $(\mathbf{m}, \mathbf{V})$,
- the points $(\mu_{\rm st}, \sigma_{\rm st})$ and $(\mu_{\rm ct}, \sigma_{\rm ct})$ on the frontier hyperbola associated with each of the three pairs $(\mathbf{m}, \mathbf{V})$,
- the efficient frontier associated with each of the three pairs $(\mathbf{m}, \mathbf{V})$,
- the long frontier associated with each of the three pairs $(\mathbf{m}, \mathbf{V})$,
- the point $(\mu_{\rm lt}, \sigma_{\rm lt})$ on the long frontier associated with each of the three pairs $(\mathbf{m}, \mathbf{V})$,
- the efficient long frontier associated with each of the three pairs $(\mathbf{m}, \mathbf{V})$.

Exercise 2. For each of the five years and each of the three pairs $(\mathbf{m}, \mathbf{V})$ compute

- the allocation $\mathbf{f}_{\rm st}$ for the tangent portfolio on the frontier hyperbola associated with the safe investment,
- the allocation $\mathbf{f}_{\rm ct}$ for the tangent portfolio on the frontier hyperbola associated with the credit line,
- the allocation $\mathbf{f}_{\rm lt}$ for the tangent portfolio on the long frontier associated with the safe investment.

Compute the signal-to-noise ratio of $\hat{\mu}$ for the portfolios with allocations $\mathbf{f}_{\rm st}$, $\mathbf{f}_{\rm ct}$, and $\mathbf{f}_{\rm lt}$ for each of the five years and each of the three pairs $(\mathbf{m}, \mathbf{V})$. For each portfolio plot this ratio as a function of quarters. There should be nine plots, one for each portfolio. How do these ratios compare with those of the individual assets? For which of these portfolios are you most certain of the expected value of its return? Give your reasoninng.

Exercise 3. Compute the metrics $\omega^{\rm m}$, $\omega^{\rm v}$, $\omega^{\rm KS}$, $\omega^{\rm ar}$, and $\omega^{\rm ac}$ to the portfolios with allocations $\mathbf{f}_{\rm st}$, $\mathbf{f}_{\rm ct}$, and $\mathbf{f}_{\rm lt}$ for each of the five years and each of the three pairs $(\mathbf{m}, \mathbf{V})$. For each portfolio plot these five metrics as a function of quarters. There should be nine plots, one for each portfolio. How do these metrics compare with those of the individual assets? Which of these portfolios is best described by an IID model? Give your reasoning.