Suffix Trees and Applications

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Exact String Matching

- Input: Two strings over alphabet Σ .
  - A *target* $T[1 \ldots n]$.
  - A *pattern* $P[1 \ldots m]$.
- Output all occurences of P in T.

Exact String Indexing

- Given a string $T[1 \ldots n]$ build a data structure (or index) for T supporting the operation:
- Occur($P[1 \ldots m]$): Report all occurences (if any) of P in T.
- For simplicity assume that $\sigma = O(1)$.

Tries

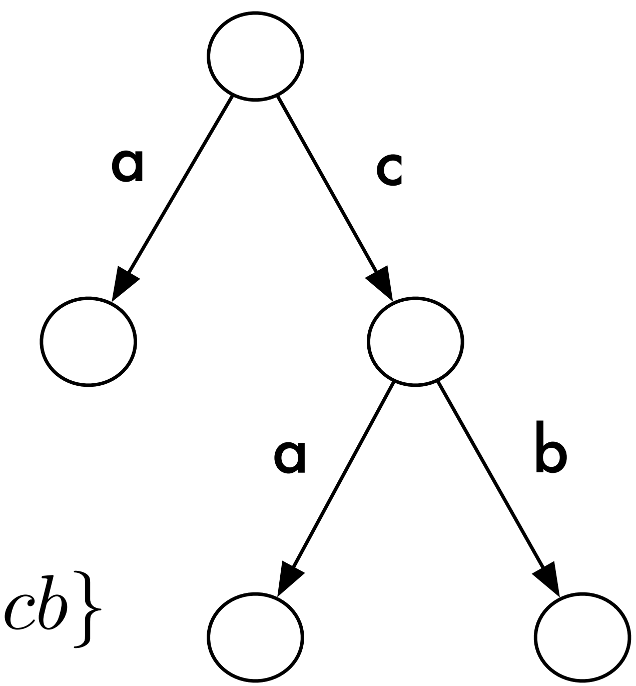
- Membership problem: Build a data structure for a set of strings $S = \{S_1, \ldots, S_k\}$ supporting the following operation:
- Member($P[1 \ldots m]$): Decide if $P \in S$.
- A trie is useful for retrieving this information.

Tries

- A trie for $S = \{S_1, \ldots, S_k\}$ is a rooted tree R such that:
- Each edge is labeled by a character from Σ.
- Child edges of a node have *distinct* labels.
- R has k leaves each corresponding to a string in S.

Example

- Trie for $S = \{a, ca, cb\}$
- Member($P[1 \ldots m]$): Search from top.
- What about $S = \{a, c, ca, cb\}$



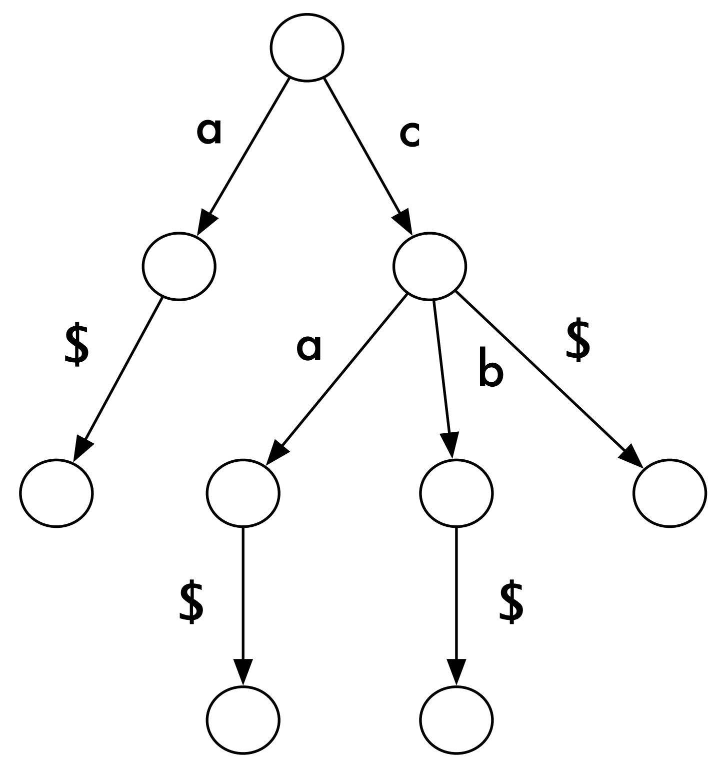
Tries

- Concatenate each string with \$.

$$S = \{a\$, c\$, ca\$, cb\$\}$$

- No string in S is a prefix of another.

- 1-to-1 correspondance between leaves and S.



Constructing tries

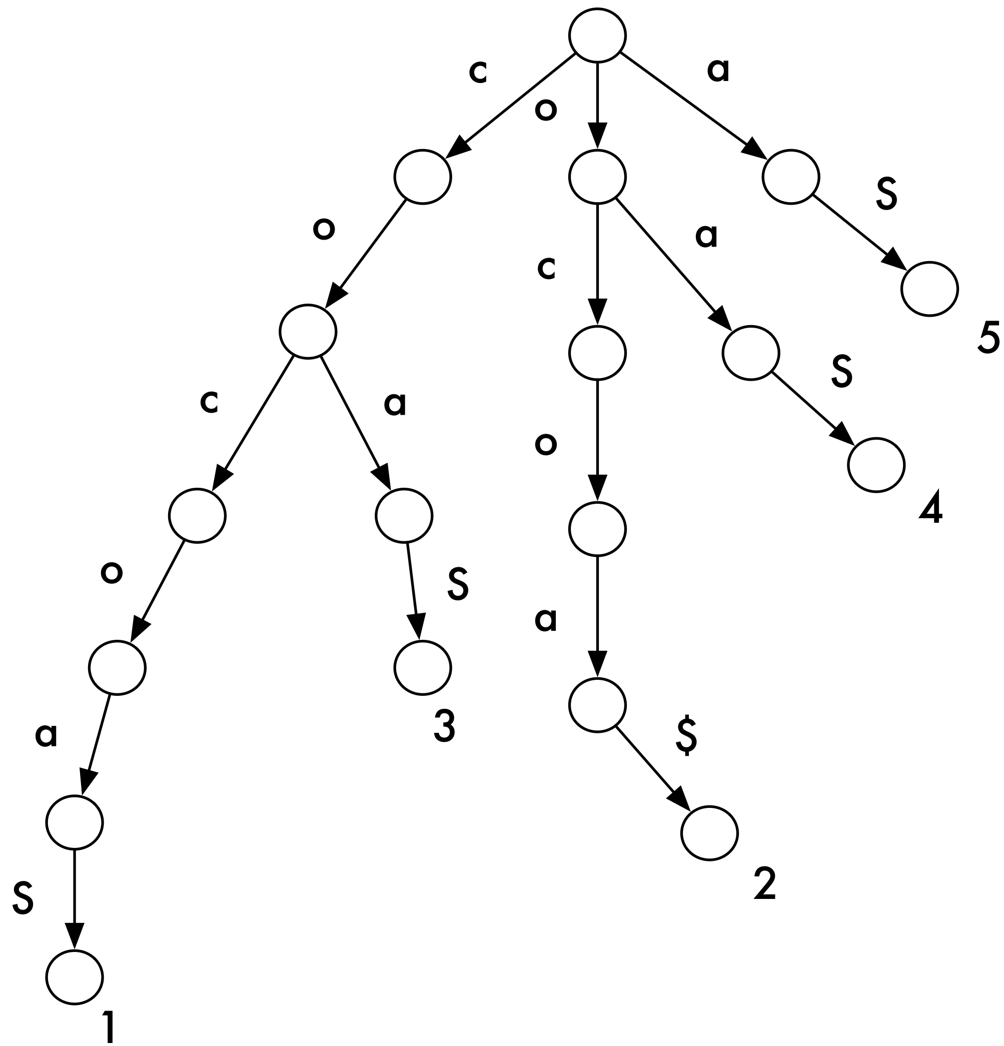
- Let $S = \{S_1, \ldots, S_k\}$.
- How do we build trie for S?
- Add one string at the time.
- To add S_i find the longest prefix of S_i in the current trie consisting of $\{S_1, \ldots, S_{i-1}\}$. Create a new branch representing the remaining part of S_i.

Complexity

- Let $|S| = \sum_{i=1}^{k} S_i$.
- Member($P[1 \ldots m]$): $O(m)$.
- Preprocessing: $O(|S|)$.
- Space: $O(|S|)$.

Tries and string indexing

- Recall: Build index for $T[1 \ldots n]$ supporting Occur($P[1 \ldots m]$).
- Preprocessing: Build trie R for
$$\{T[1..n], T[2..n], \ldots, T[n]\}$$
- Leaf corresponding to $T[i..n]$ is labeled i.
- Occur($P[1 \ldots m]$): Find longest match for P in R. Report labels of all descendent leafs.



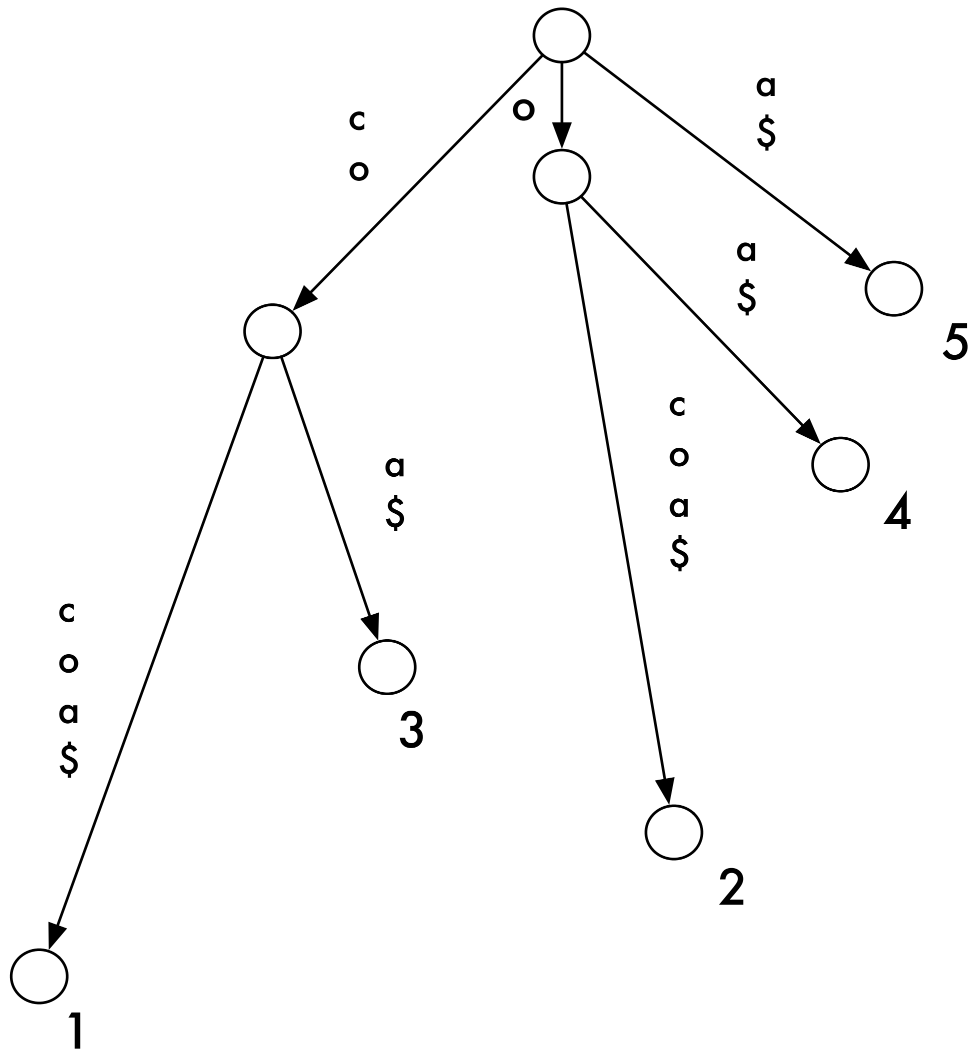
trie of suffixes for “cocoa”

Complexity

- Preprocessing: $O(n^2)$
- Space: $O(n^2)$
- Occur($P[1 \dots m]$): $O(nm)$
- Member is $O(m)$.

Compressed tries

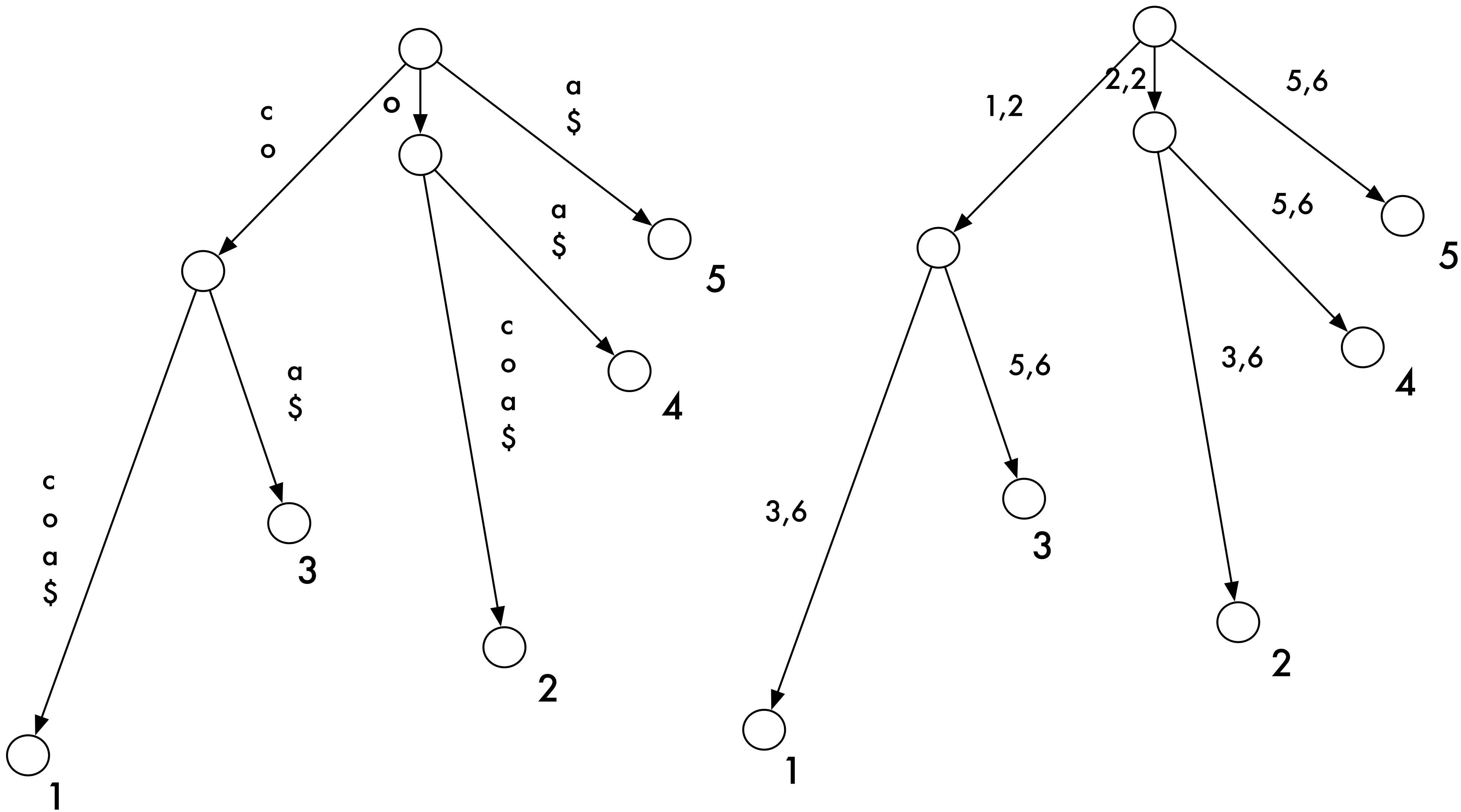
- Idea: Merge paths consiting of nodes with 1 child into a single node.



Compressed trie (or suffix tree) for "cocoa"

Suffix tree

- What is the size of the suffix tree:
- How many nodes and edges are in the suffix tree?
- How much space is needed for the edges?



Edge label compression for “cocoa” suffix tree

Complexity

- Occur($P[1 \ldots m]$): $O(m + z)$, where z is number of occurrences of P.
- Space: $O(n)$
- Preprocessing: $O(n^2)$ using tries.
- Can be done in $O(n)$ time.

- Let S be a string and R be the corresponding suffix tree.
- What the maximum height of R and when it this achieved?
- What is the minimum height of R and when is this achieved?
- What is the degree of the root of R?
- How can we use R find the length of the longest substring occurring more than once in S?

Applications

- String indexing.
- Exact set matching problem.
- Longest common substring.
- Frequent substring

Exact Set Matching

- Given a set of pattern strings $\{P_1, \ldots, P_k\}$ and a target $T[1 \ldots n]$ find all occurrences of pattern strings in T.

Solution

- Build suffix tree for T and lookup all patterns.
- Total time $O(n + \sum_{i=1}^{k} |P_i| + z)$, where the total number of occurrences is z.

Indexing multiple strings

- Suffix trees are useful for indexing a single string.
- How do we index multiple strings, e.g, a set of strings $S = \{S_1, \ldots, S_k\}$?

“Generalized” suffix trees

- A Generalized suffix tree for S is a compressed trie of the all suffixes of each string in S.
- Each leaf is labeled by a pair (i, j), where i indentifies the string and j is the start position in S_i.

Example

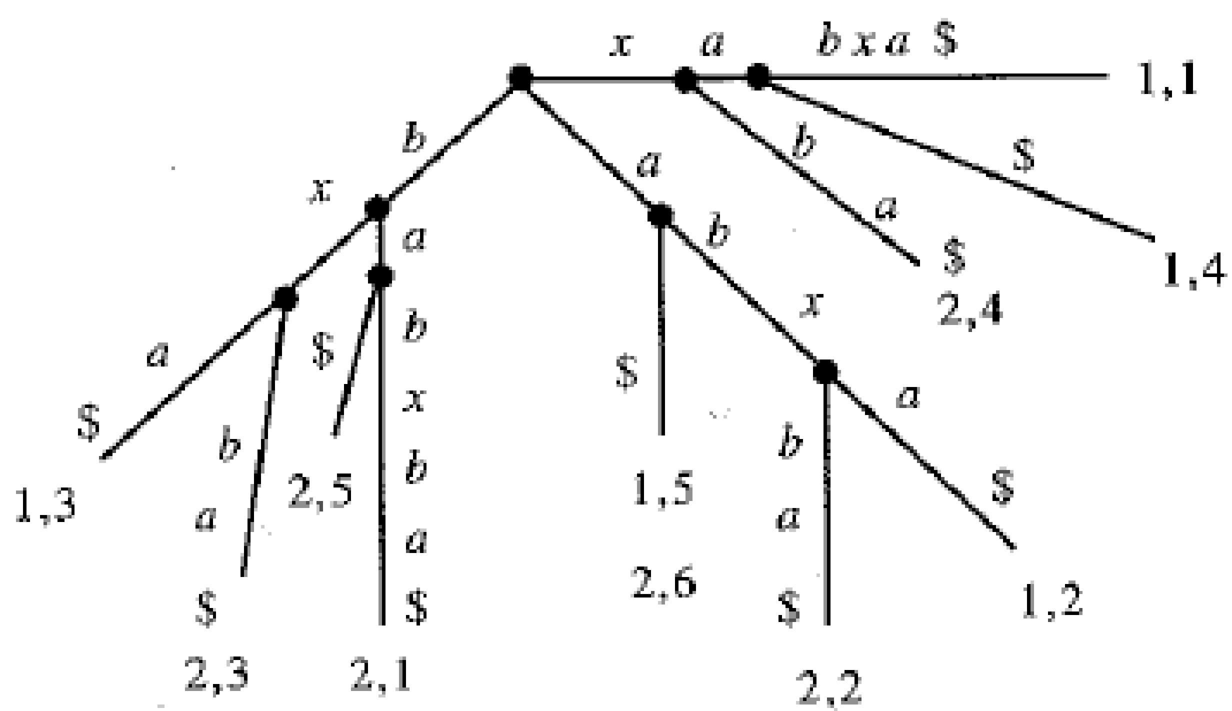


Figure 6.11: Generalized suffix tree for strings $S_1 = xabxa$ and $S_2 = babxba$. The first number at a leaf indicates the string; the second number indicates the starting position of the suffix in that string.

Construction

- Build a suffix tree R for string
$$C = S_1\$_1 \cdots S_k\$_k$$
- Each leaf in R is labeled by a start position of C.
- Remove suffixes that "span" multiple strings.
- Convert leaf labels into (i, j) pairs.
- Total time $O(|C|)$.

Longest common substring

- Given two strings S_1 and S_2 compute a longest common substring of S_1 and S_2.
- Ex: Longest common substring of
  - S_1 = "`superiorcalifornialives`"
  - S_2 = "`sealiver`"

is "`alive`".

Solution

1. Build generalized suffix tree for S_1 and S_2.
2. Mark each internal node v by 1 (2) if the subtree below v contains a leaf for a suffix of S_1 (S_2).
3. Traverse tree to find a node u of maximal string-depth marked by 1 and 2.

u correspond to a longest common substring.

Complexity

- Building the generalized suffix tree for S_1 and S_2 takes linear time.
- Step 2 and 3 can be implemented in linear time using a simple tree traversal. (exercise)

Frequent substrings

- How do we find frequent substrings in a set of string? I.e substrings that are common to many strings in the set.

Frequent substrings

- Let $S = \{S_1, \ldots, S_k\}$ be a set of strings
- Define $l(i)$ for each $i = 2, \ldots, k$ as the length of a longest substring that is common to at least i strings in S.
- The frequent substring problem is to compute $l(i)$ for $i = 2, \ldots, k$.

{sandollar, sandlot, handler, grand, pantry}

i	$l(i)$	substring
2	4	sand
3	3	and
4	3	and
5	2	an

Solution

- Compute generalized suffix tree R for S. Label each leaf with a number $1, \ldots, k$ indentifying the string.
- Compute the number of distinct string identifiers, $C(v)$, that occur below each internal node v in R.

Solution

- With $C(v)$'s compute vector $V(k)$defined as the string depth of the deepest node such that $C(v) = k$.
- $V(k)$ is the length of the longest string that occurs *exactly* k times.
- Finally, compute $l(k)$ values from $V(k)$.

Computing C(v)

- Compute for each internal node v a bit-vector $b[1..k]$ where $b[i] = 1$ iff identifier of string i occurs in a leaf below v.
- Vector for v is computed by ORing the vectors of the child nodes $v_1, \ldots, v_l$.
- Time for v is $O(lk)$.
- In total $O(nk)$ time. How do we get $C(v)$?

More Applications

- Show how to find the longest palindrome in S.
- Preprocess two strings S_1 and S_2 such that longest common extension queries can be solved efficiently. Assume that a nearest common ancestor data structure is availiable.

Conclusion

- Suffix trees can be built in linear time and space.
- Provides fundamental data structure with a huge number of applications.