SECI1013: DISCRETE STRUCTURES

SESSION 2024/2025 – SEMESTER 1

ASSIGNMENT 2 (CHAPTER 2)

INSTRUCTIONS:

a. This assignment must be conducted in a group. Please clearly write the group members' names & matric numbers on the front page of the submission.
b. Solutions for each question must be readable and neatly written on plain A4 paper. Every step or calculation should be properly shown. Failure to do so will result in the rejection of the submission of the assignment.
c. This assignment has 13 questions (100 marks), contributing 5% of overall course marks.

STRUCTURES:

1. Chapter 2 Part 1: Relation [50 Marks]
2. Chapter 2 Part 2: Function [30 Marks]
3. Chapter 2 Part 3: Recurrence Relation [20 Marks]

Q1. Relation

1. Given A = {2, 3, 4, 5, 6, 7, 8} and R a relation over A. Draw the directed graph of R after realising that xRy iff $x-y = 3n$ for some n ∈ Z. Find all possible equivalence relations for R.

   (5 marks)

2. Let A = {1, 2, 3} and B = {9, 8, 7}.
   Let R: A to B. For all (a, b) ∈ A × B, and given a R b ⇔ a+b is an even number,

   a. Determine R and R^{-1}.
   b. Draw arrow diagrams for both.
   c. Describe R−1 in words.

   (10 marks)

3. Let *A* = {1, 2, 3, 4, 5}, and let *R* be the relation on *A* that has the matrix (given below)

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

   Construct the digraph of *R*, and list in-degrees and out-degrees of all vertices.

   (6 marks)

4. Given A = {0, 1, 2, 3, 4}, and
   R = {(0, 0),(0, 1),(0, 3),(0, 4),(1, 0),(1,1), (1,2), (2,1), (2, 2),(2,3), (3, 0),(3, 2),(3,3),(3,4),(4,0),(4,3),(4,4)}. Draw the relation graph and find is R reflexive, symmetric, or transitive?

   (12 marks)

5. Relation R in the set A = {1, 2, 3…13, 14} defined as R = {(x, y): 3x − y = 0}, Determine whether the relation is
    a. Reflexive
    b. Symmetric
    c. Transitive

   Support your answer with the reason.

(9 marks)

6. Suppose that the given is a relation matrix for R and S,

$$R = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } S = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

   Using Boolean Arithmetric, Find
    a. RS
    b. SR

(8 marks)

Q2. Function

7. What is the different between Relation and Function?

(2 Marks)

8. If A = {2, 3, 4, 5}, then write whether each of the following relations on set A is a function or not. Give reasons also.
   (i) {(2, 3), (3, 4), (4, 5), (5, 2)}
   (ii) {(2, 4), (3, 4), (5, 4), (4, 4)}
   (iii) {(2, 3), (2, 4), (5, 4)}
   (iv) {(2, 3), (3, 5), (4, 5)} (v) {(2, 2), (2, 3), (4, 4), (4, 5)}

(8 marks)

9. Given the relation of $R = \{(x, y) | y = x + 5\ , x\ is\ \mathbb{Z}^{+}\ less\ than\ 6\}$. Depict this relationship using roster form. Write down the domain and the range.

(3 marks)

10. In the following cases, state whether the function is one-one, onto or bijective. Justify your answer.

(v) $f = R \rightarrow R, f(x) = 1 - 2x$

(vi) $f = R \rightarrow R, f(x) = 5x^2 - 1$

(vii) $f = R \quad \rightarrow R, f(x) = x^4$

(viii) $f = R \rightarrow R, f(x) = \left(\frac{x-2}{x-3}\right)$

(8 marks)

11. Given the following functions, find the function $f(g(x))$ and find the value of the function if $x = \{0, 1, 2, 3\}$

(ix) $f(x) = 3x - 1 \, ; g(x) = x^2 - 1$

(x) $f(x) = x^2 \, ; g(x) = 5x - 6$

(xi) $f(x) = x - 1 \, ; g(x) = x^3 + 1$

(9 marks)

Q3. Recurrence Relation

12. Solve the recurrence relation given;

(xii) $a_n = 6a_{n-1} - 9a_{n-2}$; initial conditions $a_0 = 1 \; and \; a_1 = 6$

(xiii) $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$;
initial conditions $a_0 = 2 \, , a_1 = 5 \; and \; a_2 = 15$

(xiv) $a_n = -3a_{n-1} - 3a_{n-2} + a_{n-3}$
initial conditions $a_0 = 1 \, , a_1 = -2 \; and \; a_2 = -1$

(12 marks)

13. A sequence $a_1, a_2 \, , a_3 \, , a_4 \, , \ldots$ is given by

$$a_{n+1} = 5a_n - 3 \; ; \; a_1 = k$$

where k is a non-zero constant.

(i) Find the value of a_4 in terms of k .

(ii) Given that a_4= 7 , determine the value of k .

(8 marks)

Q1. Relation

1. Given A = {2, 3, 4, 5, 6, 7, 8} and R a relation over A. Draw the directed graph of R after realising that xRy iff x−y = 3n for some n ∈ Z. Find all possible equivalence relations for R.

(5 marks)

R: {(2,2), (3,3), (4,4), (5,5), (6,6), (7,7), (8,8), (8,5), (8,2), (7,4), (6,3), (5,2), (5,8), (2,8), (4,7), (3,6), (2,5)}

2. Let A = {1, 2, 3} and B = {9, 8, 7}.
   Let R: A to B. For all (a, b) ∈ A × B, and given a R b ⇔ a+b is an even number,

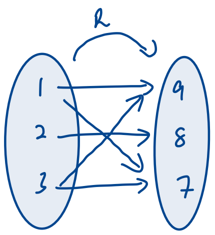
   a. Determine R and R^{-1}.
   b. Draw arrow diagrams for both.
   c. Describe R−1 in words.

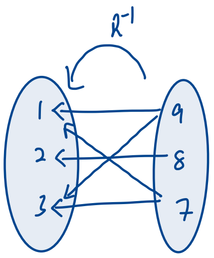
(10 marks)

a) R = {(1,9), (1,7), (2,8), (3,9), (3,7)}

R^{-1}: {(9,1), (7,1), (8,2), (9,3), (7,3)}

b)





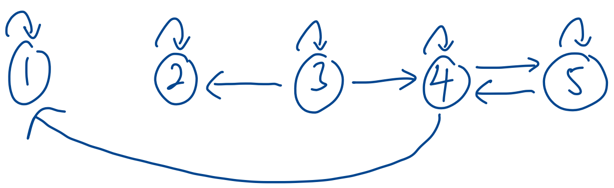
c) For all $(b, a) \in B \times A$, $(b, a) \in R^{-1} \Rightarrow b+a$ is an even.

3. Let *A* = {1, 2, 3, 4, 5}, and let *R* be the relation on *A* that has the matrix (given below)

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

Construct the digraph of *R*, and list in-degrees and out-degrees of all vertices.

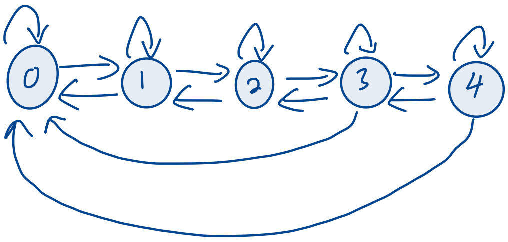
(6 marks)



	1	2	3	4	5
In degree	2	2	1	3	2
Out degree	1	1	3	3	2

4. Given A = {0, 1, 2, 3, 4}, and R = {(0, 0),(0, 1),(0, 3),(0, 4),(1, 0),(1,1), (1,2), (2,1), (2, 2),(2,3), (3, 0),(3, 2),(3,3),(3,4),(4,0),(4,3),(4,4)}. Draw the relation graph and find is R reflexive, symmetric, or transitive?

(12 marks)



Reflexive :

$\forall x \in A, (x,x) \in R$

Symmetric :

$\forall x,y \in A$, if $(x,y) \in R$ then $(y,x) \in R$

Not Transitive :

$(0,1), (1,2) \in R$ but $(0,2) \notin R$.

5. Relation R in the set A = {1, 2, 3…13, 14} defined as R = {(x, y): 3x − y = 0}, Determine whether the relation is
    a. Reflexive
    b. Symmetric
    c. Transitive

Support your answer with the reason.

(9 marks)

$R = \{ (1,3), (2,6), (3,9), (4,12)$

$$R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

a. Reflexive : The Relation is Reflexive because
    i. The diagonal value is 1
    ii. $\forall x \in A, (x,x) \in R$

b. Symmetric : The Relation is Symmetric because
    i. all values except at the diagonal are 0
    ii. $\forall x,y \in A$, if $(x,y) \in R$ then $(y,x) \in R$

c. Transitive : The Relation is Transitive because
    i. all values except at the diagonal are 0.

6. Suppose that the given is a relation matrix for R and S,

$$R = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } S = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Using Boolean Arithmetic, Find
  a. RS
  b. SR

(8 marks)

a)

$$RS = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

b)

$$SR = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Q2. Function

7. What is the different between Relation and Function?

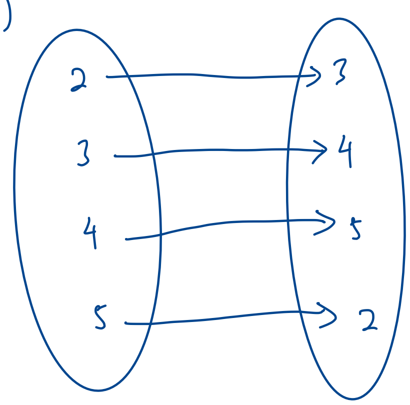
(2 Marks)

Relations are group of ordered pairs from one set of objects to another set of object, while functions are relations that connect one set of inputs to another set of outputs. So, all functions are relations while all relations are not function.

8. If A = {2, 3, 4, 5}, then write whether each of the following relations on set A is a function or not. Give reasons also.
   (i) {(2, 3), (3, 4), (4, 5), (5, 2)}
   (ii) {(2, 4), (3, 4), (5, 4), (4, 4)}
   (iii) {(2, 3), (2, 4), (5, 4)}
   (iv) {(2, 3), (3, 5), (4, 5)} (v) {(2, 2), (2, 3), (4, 4), (4, 5)}

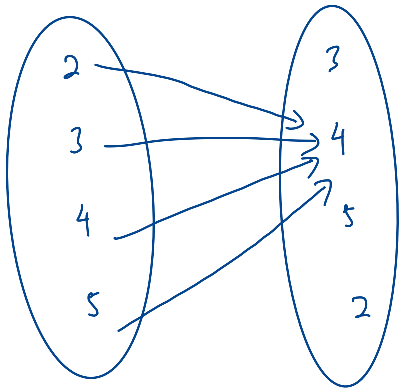
(8 marks)

i)



one to one function

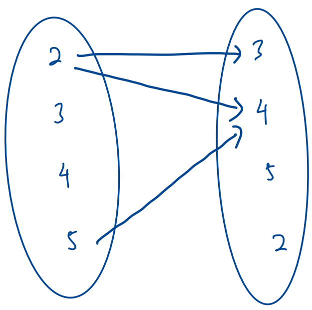
ii)



function because all values in domain has values in codomain

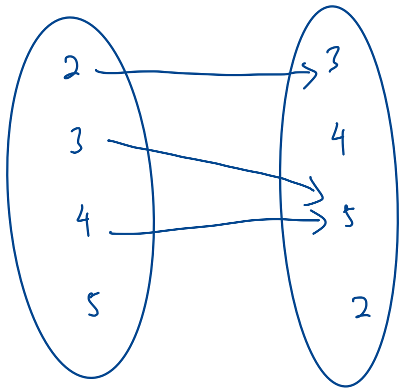
not one to one function

iii)



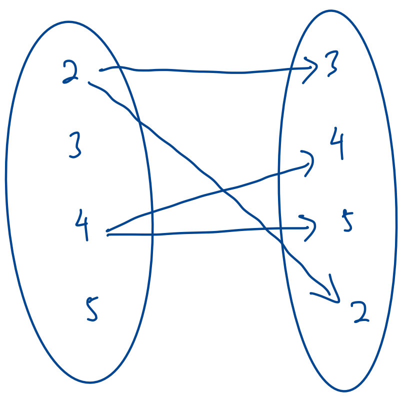
not a function because there are some values in domain has no value in codomain

iv)



not a function because there are some values in domain has no value in codomain

v)



not a function because there are some values in domain has no value in codomain

9. Given the relation of $R = \{(x, y) | y = x + 5, x \text{ is } \mathbb{Z}^+ \text{ less than } 6\}$. Depict this relationship using roster form. Write down the domain and the range.

(3 marks)

$x = \{1, 2, 3, 4, 5\}$

values $y = x + 5$

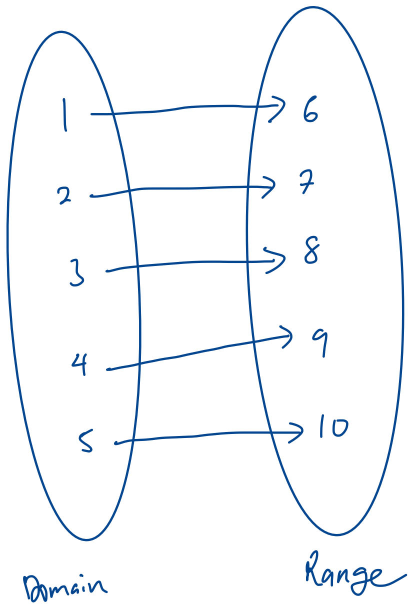
$x = 1, y = 6$

$x = 2, y = 7$

$x = 3, y = 8$

$x = 4, y = 9$

$x = 5, y = 10$



10. In the following cases, state whether the function is one-one, onto or bijective. Justify your answer.

(v) $f = R \to R, f(x) = 1 - 2x$

(vi) $f = R \to R, f(x) = 5x^2 - 1$

(vii) $f = R \to R, f(x) = x^4$

(viii) $f = R \to R, f(x) = \left(\frac{x-2}{x-3}\right)$

(8 marks)

(v) $f: R \to R, f(x) = 1 - 2x$

whether the function is one-to-one

if $f(x_1) = f(x_2)$; $x_1 = x_2$

$f(x_1) = 1 - 2x_1$; $f(x_2) = 1 - 2x_2$

$f(x_1) = f(x_2)$

$1 - 2x_1 = 1 - 2x_2$

$1 - 1 - 2x_1 = -2x_2$

$x_1 = \frac{-2x_2}{-2}$

$x_1 = x_2$

$x_1 - x_2 = 0$

∴ function is one-to-one

whether function is onto

if $y = 1 - 2x$, $x = \frac{1-y}{2}$

$f\left(\frac{1-y}{2}\right) = 1 - 2\left(\frac{1-y}{2}\right)$

$f\left(\frac{1-y}{2}\right) = y$

For each y in codomain R, there exist $\frac{1-y}{2}$, such that $f\left(\frac{1-y}{2}\right) = 1 - 2\left(\frac{1-y}{2}\right) = y$;

∴ the function is onto

Whether the function is Bijective,

∴ The function is bijective because the function is both one to one function and onto function.

(vi) $f: R \to R$, $f(x) = 5x^2 - 1$

Whethere function is one - to one,

if $f(x_1) = f(x_2)$; $x_1 = x_2$

$f(x_1) = 5x_1^2 - 1$; $f(x_2) = 5x_2^2 - 1$

$f(x_1) = f(x_2)$

$5x_1^2 - 1 = 5x_2^2 - 1$

$x_1^2 = x_2^2$

$x_1^2 - x_2^2 = 0$

$x_1^2 - x_2^2 = 0$

$x_1^2 - x_2^2 = (x_1 - x_2)(x_1 + x_2)$

So:

$(x_1 - x_2) = 0$ or $(x_1 + x_2) = 0$

$x_1 = x_2$ or $x_1 = -x_2$

∴ the function is not one to one

Whethere the function is onto function.

if $y = 5x^2 - 1$, $5x^2 = y + 1$

$x^2 = \frac{y+1}{5}$

$x = \sqrt{\frac{y+1}{5}}$

$f\left(\sqrt{\frac{y+1}{5}}\right) = 5\left(\sqrt{\frac{y+1}{5}}\right)^2 - 1 = y$

$f\left(\frac{1-y}{2}\right) = y$

For each y in codomain R, the exist $\sqrt{\frac{y+1}{5}}$, such that $f\left(\sqrt{\frac{y+1}{5}}\right)$ $= 5\left(\sqrt{\frac{y+1}{5}}\right)^2 - 1 = y$

∴ the Function is onto

Whether function is Bijective,

∴ The function is not bijective because the function is not one to one function although it onto function.

(vii) $f: R \to R$, $f(x) = x^4$

Whether the function is not one to one

if $f(x_1) = f(x_2)$; $x_1 = x_2$

$f(x_1) = x_1^4$; $f(x_2) = x_2^4$

$f(x_1) = f(x_2)$

$x_1^4 = x_2^4$

$(x_1^2)^2 = (x_2^2)^2$

$(x_1^2)^2 = (x_2^2)^2$

$(x_1^2)^2 - (x_2^2)^2 = 0$

$(x_1^2 - x_2^2)(x_1^2 + x_2^2) = 0$

$x_1 \neq x_2$

∴ function is not one to one

if $y = x^4$; $x = \sqrt[4]{y} = y^{\frac{1}{4}}$

$f(y^{\frac{1}{4}}) = (y^{\frac{1}{4}})^4 = y$

For each y in codomain R, there exist $y^{\frac{1}{4}}$, such that $f(y^{\frac{1}{4}}) = (y^{\frac{1}{4}})^4 = y$;

∴ the function is onto.

Whether function is Bijective,

∴ the function is Not bijective because the function is Not one to one function although it onto function.

(viii) $f: R \to R,\ f(x) = \left(\frac{x-2}{x-3}\right)$

Whethere function is one to one

if $f(x_1) = f(x_2)$; $x_1 = x_2$

$f(x_1) = \left(\frac{x_1-2}{x_1-3}\right)$; $f(x_2) = \left(\frac{x_2-2}{x_2-3}\right)$

$f(x_1) = f(x_2)$

$\left(\frac{x_1-2}{x_1-3}\right) = \left(\frac{x_2-2}{x_2-3}\right)$

$(x_1-2)(x_2-3) = (x_2-2)(x_1-3)$

$(x_1)(x_2) - 2x_2 + 6 = (x_2)(x_1) - 2x_1 + 6$

$(x_1)(x_2) - 2x_2 + 6 = (x_2)(x_1) - 2x_1 + 6$

$2x_2 = 2x_1$

$x_2 = x_1$

$\therefore$ the function is one - one

Whethere function is onto,

if $y = \left(\frac{x-2}{x-3}\right)$; $x = \left(\frac{3y-2}{y-1}\right)$

$f\left(\frac{3y-2}{y-1}\right) = \left(\frac{\left(\frac{3y-2}{y-1}\right)-2}{\left(\frac{3y-2}{y-1}\right)-3}\right) = y$

$f\left(\frac{3y-2}{y-1}\right) = \frac{\left(\frac{(3y-2)-2(y-1)}{y-1}\right)}{\frac{(3y-2)-2(y-1)}{(3y-2)-3(y-1)}} = y$

$f\left(\frac{3y-2}{y-1}\right) = \frac{(3y-2)-2(y-1)}{(3y-2)-3(y-1)} = y$

$f\left(\frac{3y-2}{y-1}\right) = \frac{y}{1} = y$

For each y in codomain R, there exist $\frac{3y-2}{y-1}$, such that $f\left(\frac{3y-2}{y-1}\right) = y$;

$\therefore$ the function is onto.

Whether function is Bijective,

$\therefore$ The function is bijective because the function is both one to one function and onto function

11. Given the following functions, find the function $f(g(x))$ and find the value of the function if $x = \{0, 1, 2, 3\}$

(ix) $f(x) = 3x - 1$; $g(x) = x^2 - 1$

(x) $f(x) = x^2$; $g(x) = 5x - 6$

(xi) $f(x) = x - 1$; $g(x) = x^3 + 1$

(9 marks)

(ix) $f(x) = 3x - 1$; $g(x) = x^2 - 1$

$f(g(x)) = 3(x^2-1) - 1$

$x = 0,\ f(g(0)) = 3(0^2 - 1) - 1 = -4$

$x = 1,\ f(g(1)) = 3(1^2 - 1) - 1 = -1$

$x = 2,\ f(g(2)) = 3(2^2 - 1) - 1 = 8$

$x = 3,\ f(g(3)) = 3(3^2 - 1) - 1 = 23$

(x) $f(x) = x^2$; $g(x) = 5x - 6$

$f(g(x)) = (5x-6)^2$

$x = 0,\ f(g(0)) = (5(0)-6)^2 = 36$

$x = 1,\ f(g(1)) = (5(1)-6)^2 = 1$

$x = 2,\ f(g(2)) = (5(2)-6)^2 = 16$

$x = 3,\ f(g(3)) = (5(3)-6)^2 = 81$

(xi) $f(x) = x - 1$; $g(x) = x^3 + 1$

$f(g(x)) = (x^3+1) - 1 = x^3$

$x = 0,\ f(g(0)) = 0^3 = 0$

$x = 1,\ f(g(1)) = 1^3 = 1$

$x = 2,\ f(g(2)) = 2^3 = 8$

$x = 3,\ f(g(3)) = 3^3 = 27$

Q3. Recurrence Relation

12. Solve the recurrence relation given;

(xii) $a_n = 6a_{n-1} - 9a_{n-2}$; initial conditions $a_0 = 1$ *and* $a_1 = 6$

(xiii) $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$;
initial conditions $a_0 = 2\,, a_1 = 5$ *and* $a_2 = 15$

(xiv) $a_n = -3a_{n-1} - 3a_{n-2} + a_{n-3}$
initial conditions $a_0 = 1\,, a_1 = -2$ *and* $a_2 = -1$

(12 marks)

(xii) $a_n = 6a_{n-1} - 9a_{n-2}$; initial conditions $a_0 = 1$ and $a_1 = 6$

$a_2 = 6a_{2-1} - 9a_{2-2} = 6a_1 - 9a_0 = 6(6) - 9(1) = 27$

$a_3 = 6a_{3-1} - 9a_{3-2} = 6a_2 - 9a_1 = 6(27) - 9(6) = 108$

$a_4 = 6a_{4-1} - 9a_{4-2} = 6a_3 - 9a_2 = 6(108) - 9(27) = 405$

The sequence :

1, 6, 27, 108, 405, ...

(xiii) $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$;
initial conditions $a_0 = 2$, $a_1 = 5$ and $a_2 = 15$

$a_3 = 6a_{3-1} - 11a_{3-2} + 6a_{3-3} = 6a_2 - 11a_1 + 6a_0 = 6(15) - 11(5) + 6(2) = 47$

$a_4 = 6a_{4-1} - 11a_{4-2} + 6a_{4-3} = 6a_3 - 11a_2 + 6a_1 = 6(47) - 11(15) + 6(5) = 147$

$a_5 = 6a_{5-1} - 11a_{5-2} + 6a_{5-3} = 6a_4 - 11a_3 + 6a_2 = 6(147) - 11(47) + 6(15) = 455$

The sequence : 2, 5, 15, 47, 147, 455, ...

(xiv) $a_n = -3a_{n-1} - 3a_{n-2} + a_{n-3}$
initial conditions $a_0 = 1$, $a_1 = -2$ and $a_2 = -1$

$a_3 = -3a_{3-1} - 3a_{3-2} + a_{3-3} = -3a_2 - 3a_1 + a_0 = -3(-1) - 3(-2) + 1 = 10$

$a_4 = -3a_{4-1} - 3a_{4-2} + a_{4-3} = -3a_3 - 3a_2 + a_1 = -3(10) - 3(-1) + (-2) = -29$

$a_5 = -3a_{5-1} - 3a_{5-2} + a_{5-3} = -3a_4 - 3a_3 + a_2 = -3(-29) - 3(10) + (-1) = 56$

The sequence : 1, -2, -1, 10, -29, 56, ...

13. A sequence $a_1, a_2\,, a_3\,, a_4\,, \ldots$ is given by

$$a_{n+1} = 5a_n - 3\ ;\ a_1 = k$$

where k is a non-zero constant.

(i) Find the value of a_4 in terms of k .
(ii) Given that a_4= 7 , determine the value of k .

(8 marks)

(i) Find the value of a_4 in terms of k.

$n = 1$; $a_{1+1} = a_2 = 5a_1 - 3 = 5(k) - 3 = 5k - 3$

$n = 2$; $a_{2+1} = a_3 = 5a_2 - 3 = 5(5k - 3) - 3 = 25k - 15 - 3 = 25k - 18$

$n = 3$; $a_{3+1} = a_4 = 5a_3 - 3 = 5(25k - 18) - 3 = 125k - 90 - 3 = 125k - 93$

$n = 4$; $a_{4+1} = a_5 = 5a_4 - 3 = 5(125k - 93) - 3 = 625k - 465 - 3 = 625k - 468$

(ii) Given that $a_4 = 7$, determine the value of k

$a_4 = 125k - 93$

$7 = 125k - 93$

$\frac{7 + 93}{125} = k$

$\frac{100}{125} = \frac{4}{5} = k$