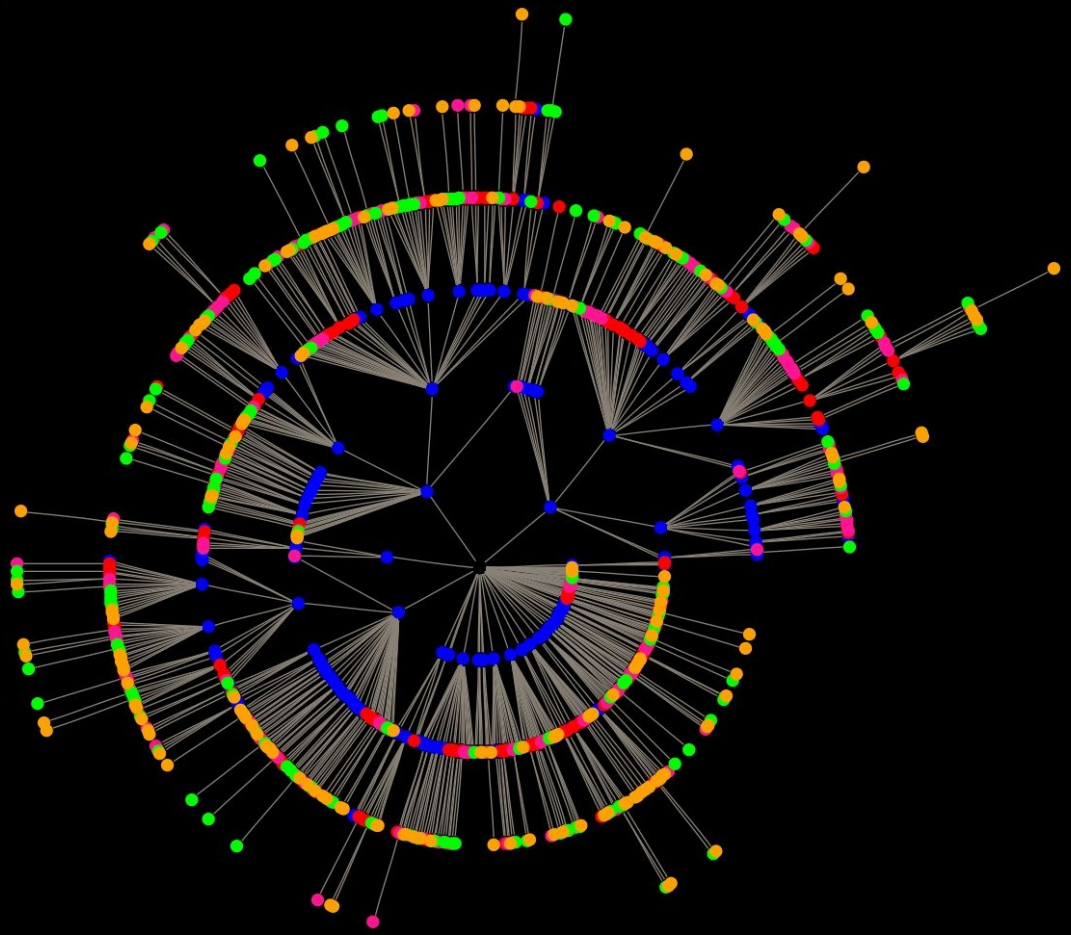
Layers of Networks

(Towards a Science of Networks)



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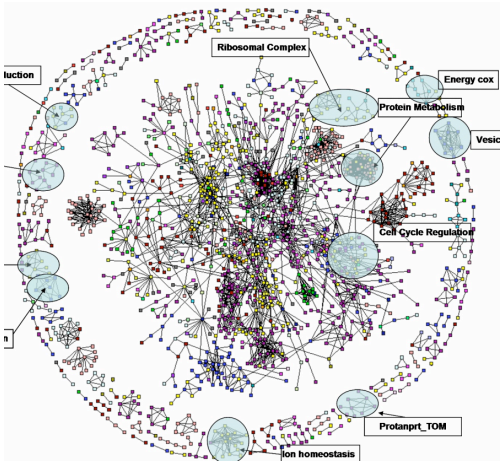
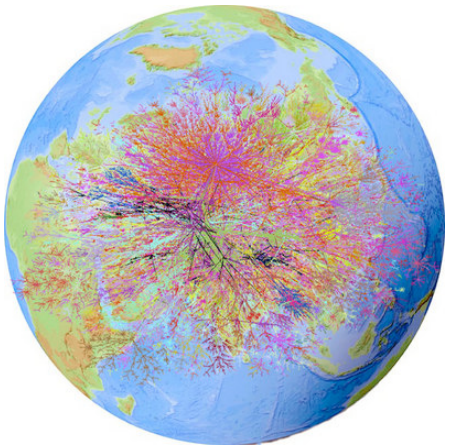
Networks:

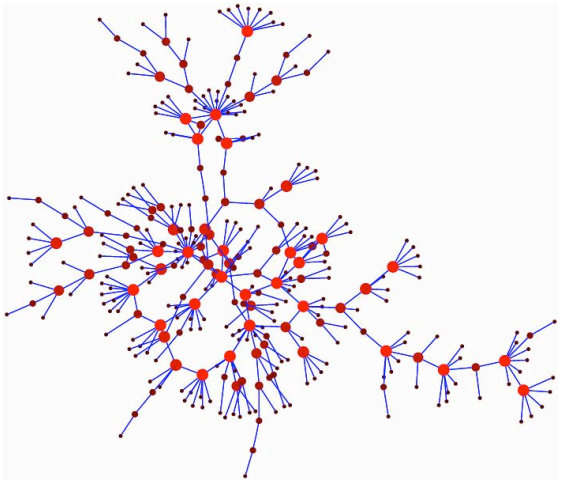
Transportation Networks/ Power grid (distribution/ collection networks)



Biological networks
- protein interaction
- genetic regulation
- drug design

Computer networks





Social networks
- Immunology
- Information
- Commerce

Networks: Physical, Biological, Social

- **Geometric** versus **virtual** (Internet versus WWW).
- **Natural** / spontaneously arising versus **engineered** / built.
- Each network **optimizes** something unique.
- Identifying **similarities** and fundamental **differences** can guide future design/understanding.

  1. How do we build a coherent distributed energy system integrating solar, wind, hydropower, bio-diesel, hydrogen, etc.
  2. Is old infrastructure introducing vulnerabilities in telecom?

- Definition of **node** can depend on level of representation.

Studying each network individually

(Though we know they interact)

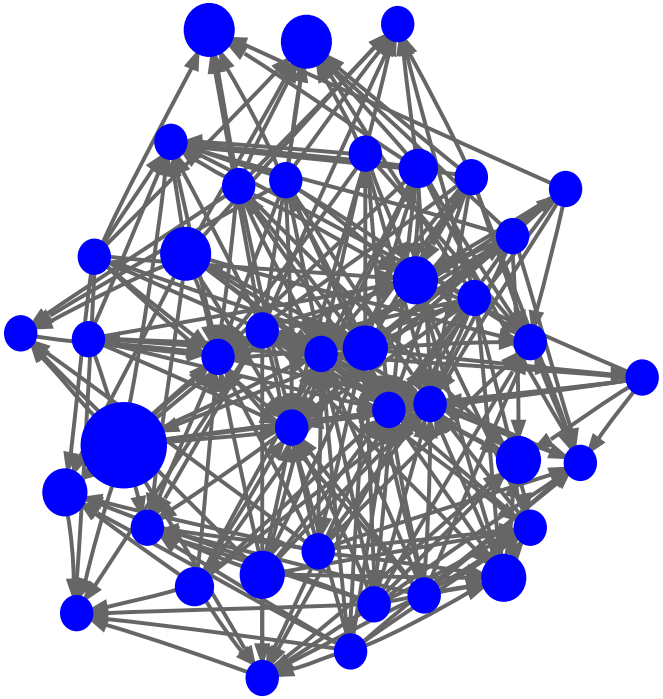
- **Topology** (Statistical properties of node and edges)
  - degree and degree distribution (extremely varied)
  - diameter ("small-world")
  - clustering coefficients
  - assortative mixing
  - betweenness, communities/partitioning, etc.

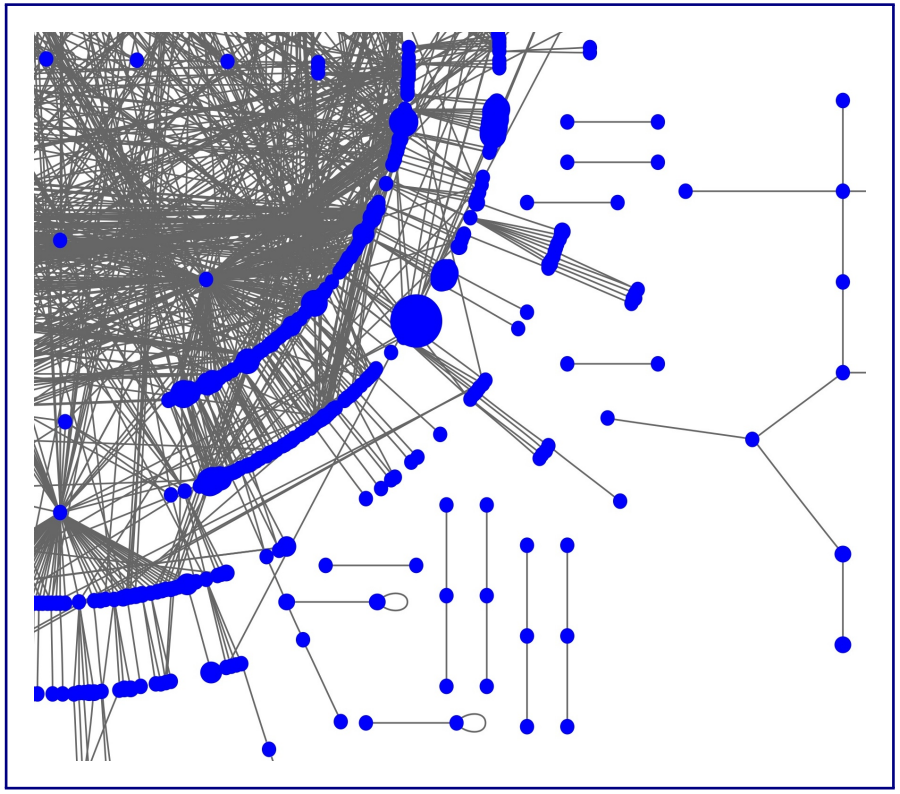
- **Activity** (Information flows)
  - epidemiology (humans and computers)
  - Web search (ranking the web map)
  - consensus formation / tipping points / phase transitions

Interactions between **structure** and **function**.

Software call graphs and OSS Developer networks

- Highly evolveable, modular, robust to mutation, exhibit punctuated eqm
- Open-source software as a “systems” / organization paradigm.



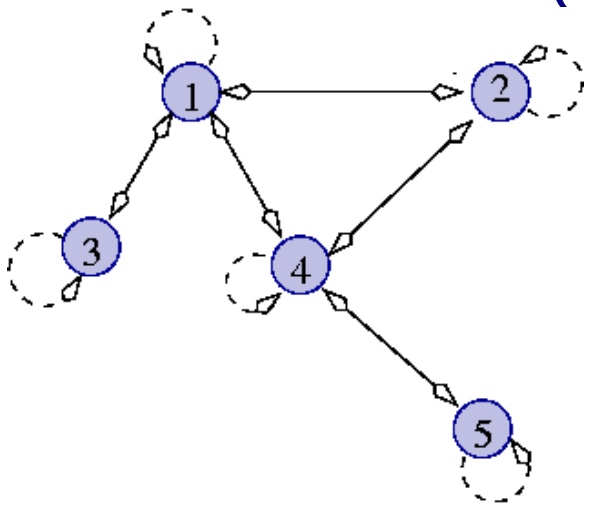


D’Souza, Filkov, Devanbu, Swaminathan, Hsu

NETWORK TOPOLOGY

Connectivity matrix, M:

$$M_{ij} = \begin{cases} 1 \text{ if edge exists between } i \text{ and } j \\ 0 \text{ otherwise.} \end{cases}$$



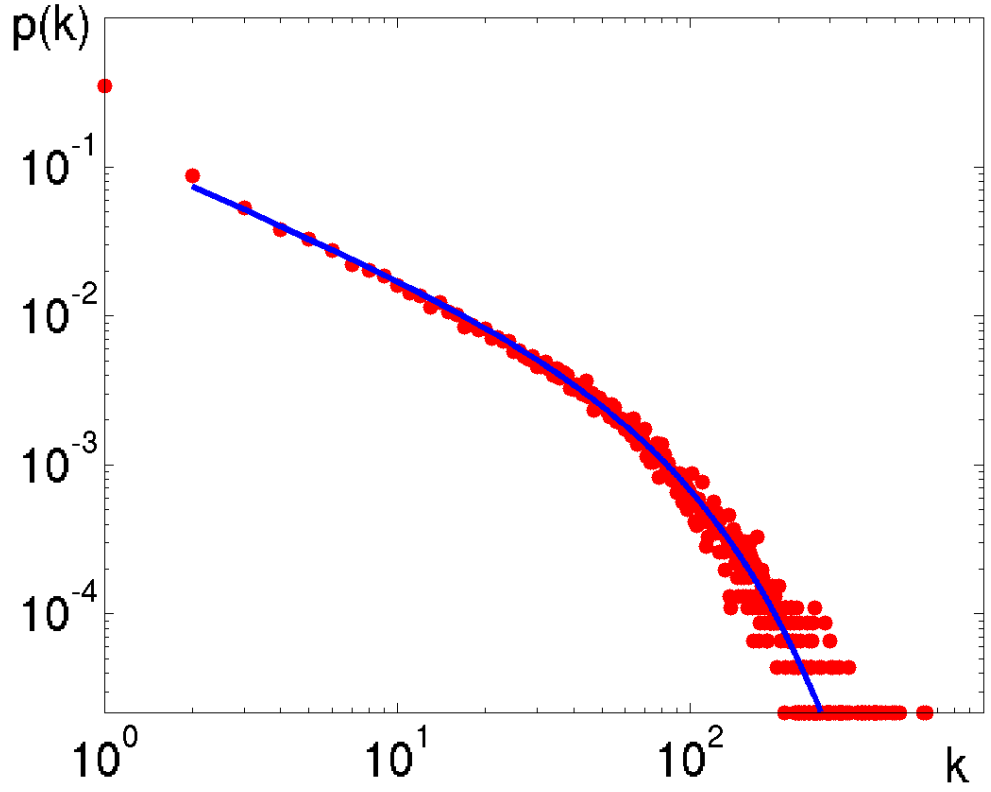
$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix} = M$$

Node **degree** is number of links.

Broad Heterogeneity in node degree

e.g., The “Who-is-Who” network in Budapest

(Balázs Szendröi and Gábor Csányi)

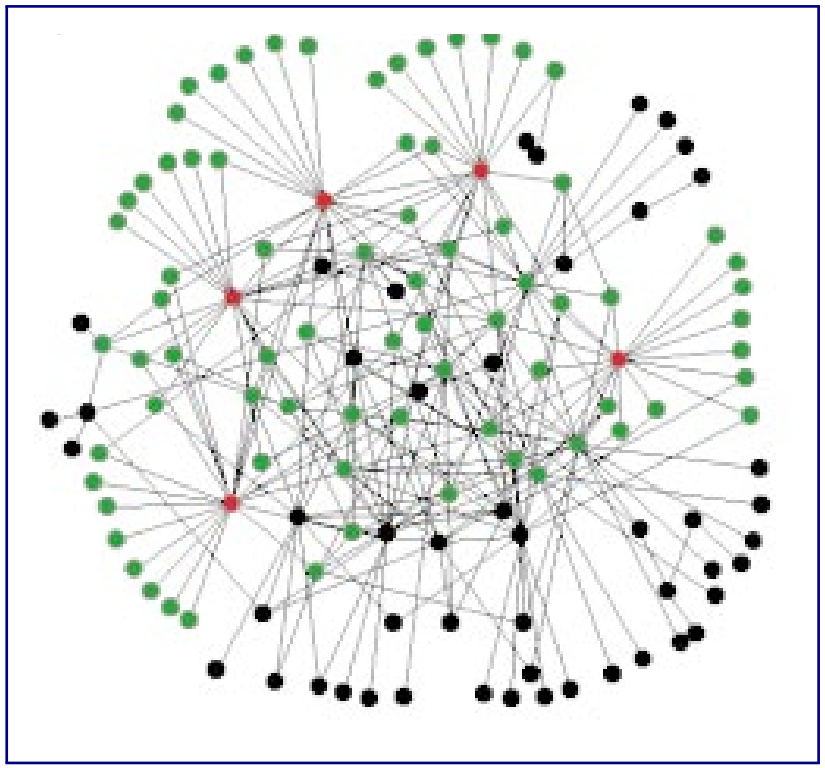


Bayesian curve fitting $\longrightarrow p(k) = ck^{-\gamma}e^{-\alpha k}$

Random Power Law Graphs:

(e.g., "Preferential Attachment", Barabasi and Albert, Science 1999)

Hubs and leaves



Albert, Jeong and Barabasi, Nature, **406** (27) 2000.

N=130, E=215

Red five highest degree nodes;

Green their neighbors.

"Robust" to random failure,

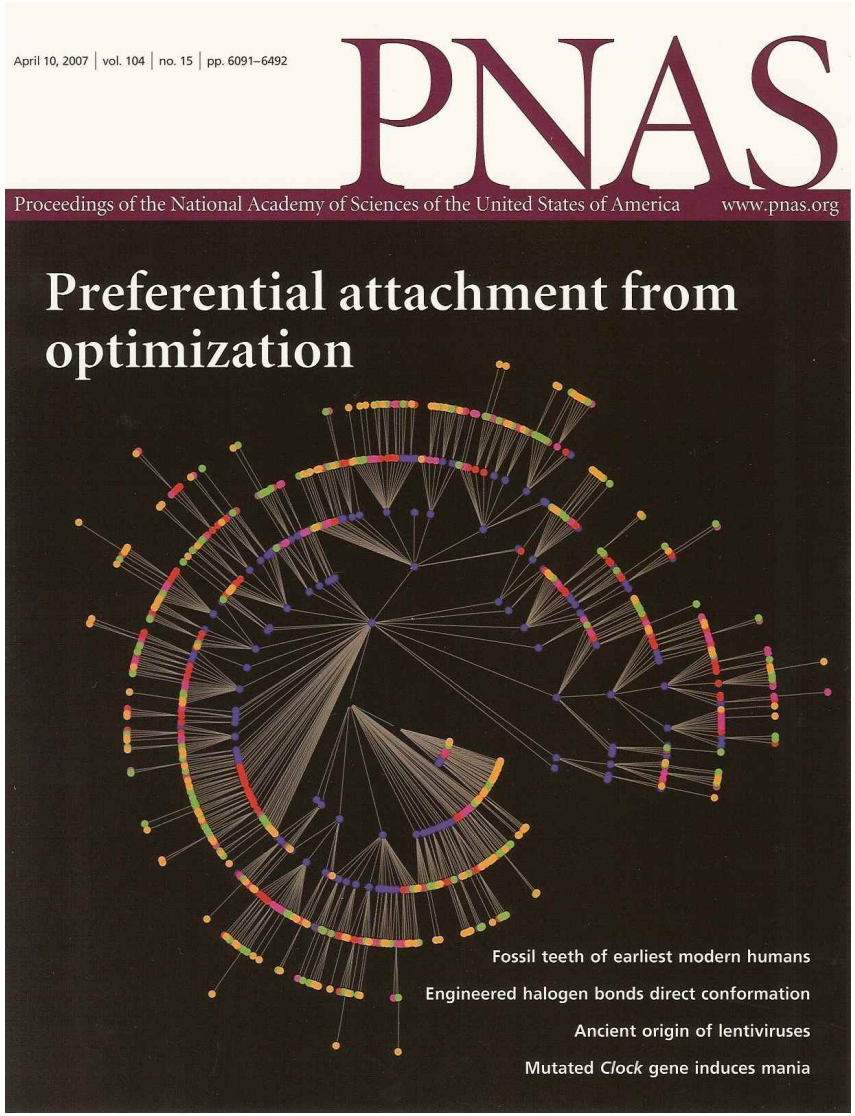
fragile to targeted.

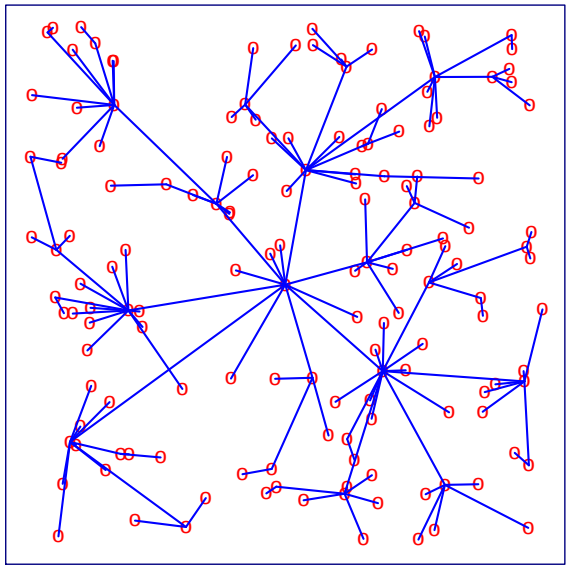
Is connectivity a good thing?

Engineered networks (e.g., the Internet) are not random!

Optimization in network growth

(D'Souza, Borgs, Chayes, Berger, Kleinberg, PNAS 2007)



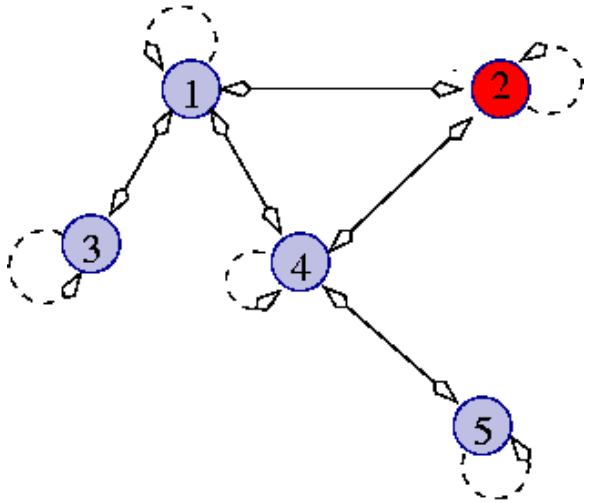


(Competing objectives)

Network Activity: FLOWS on NETWORKS

(Spread of disease, routing data, materials transport/flow, gossip spread/marketing)

Random walk on the network has state transition matrix, P:



$$\begin{pmatrix} 1/4 & 1/3 & 1/2 & 1/4 & 0 \\ 1/4 & 1/3 & 0 & 1/4 & 0 \\ 1/4 & 0 & 1/2 & 0 & 0 \\ 1/4 & 1/3 & 0 & 1/4 & 1/2 \\ 0 & 0 & 0 & 1/4 & 1/2 \end{pmatrix} = P$$

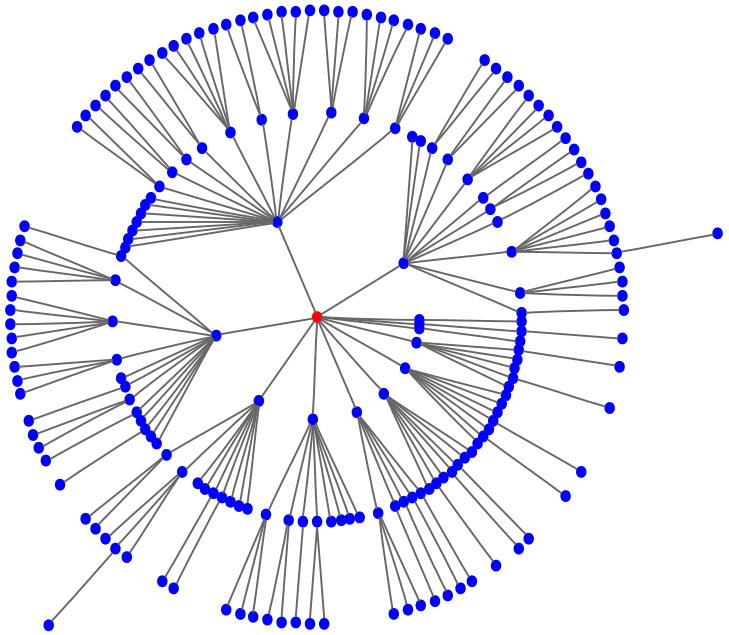
The eigenvalues and eigenvectors convey much information.
Markov Chains, Spectral Gap.

Feedback and network growth of Hierarchical organizations

- **Functional** = efficient information flow **throughout** organization.
- More functional $\longrightarrow$ grow faster
  (but each new attachment less optimal)
- Less functional $\longrightarrow$ grow slower but more balanced
  (each new attachment
  more considered)

(more balanced, efficient structures:

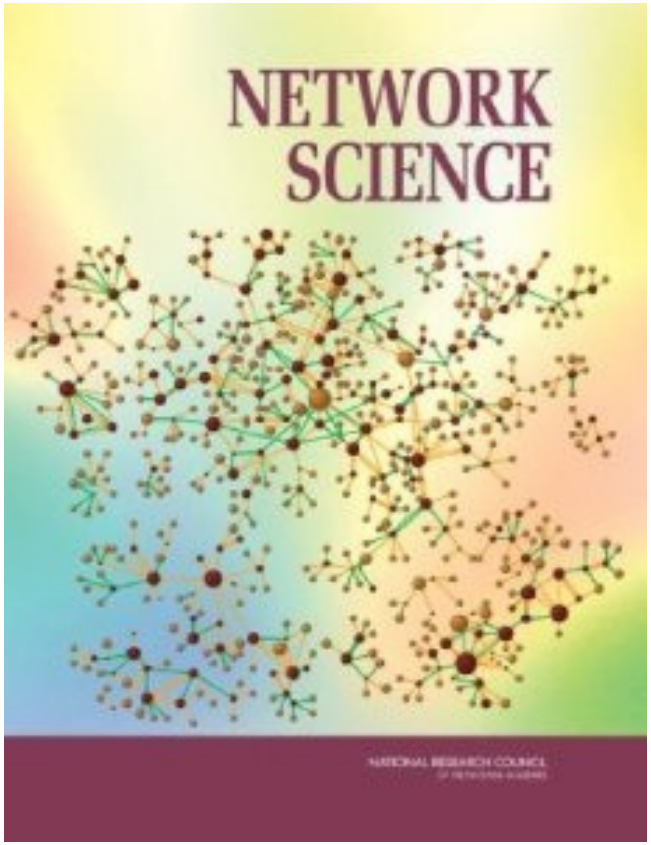
respond to changing circumstances)



Building a "science of networks"

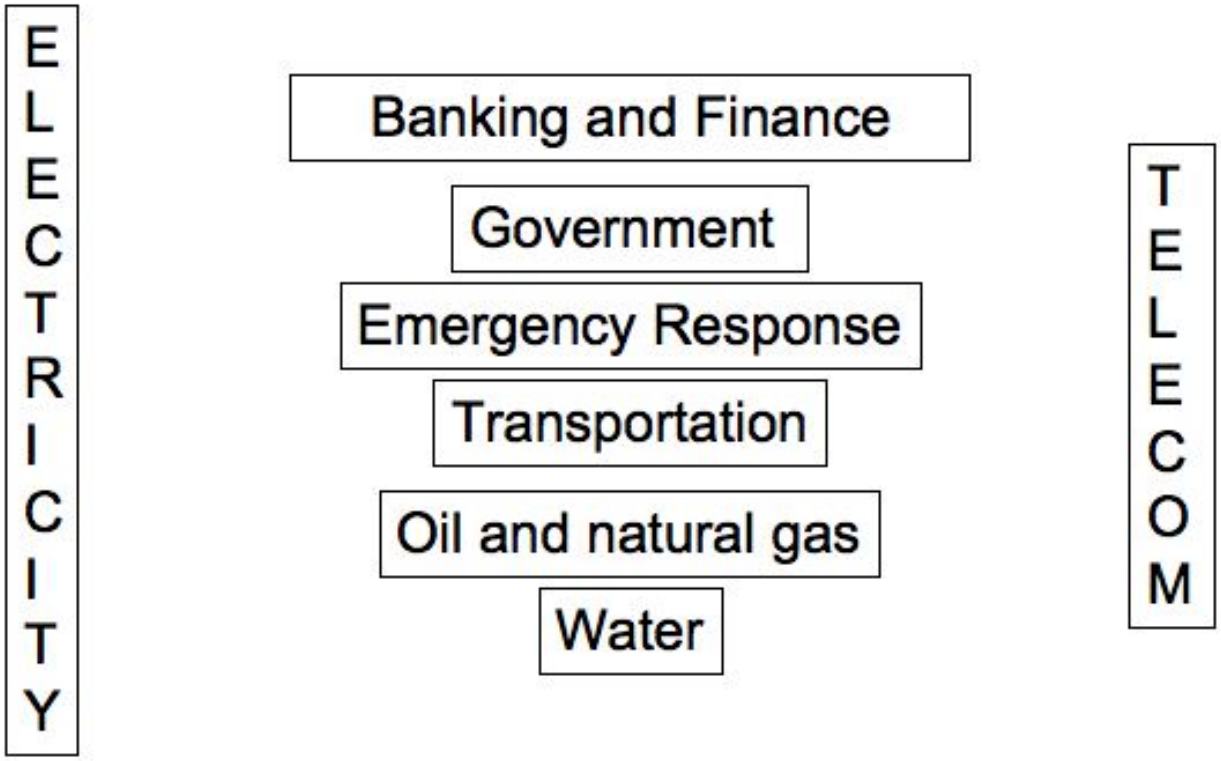
- Last ten years, since 1999.
- Understanding *activity* and *topology* of *individual* networks.
- "Nodes", "Robustness" (e.g., connectivity) context dependent.

"all our modern critical infrastructure relies on networks"



Our modern infrastructure

Layered, interacting networks



- ⋆ MATHEMATICS NEEDED: ⋆
  Multiple info streams; Layered interactions; PDEs (calculus)