1.9 Nested Quantifiers

Nested Quantifiers

If a predicate has more that one variable, each variable must be bound by a separate quantifier. For example, the statement A logical expression with more that one quantifier that binds different variables in the same predicate is said to have nested quantifiers.

Examples:

$\forall x \exists y P(x, y)$ is read as "For all x, there exists a y such that $P(x, y)$" x is bound by the universal quantifier and y is bound by the existential quantifier.

$\forall x P(x, y)$ is read as "For all x, $P(x, y)$" x is bound by the universal quantifier and y is free.

$\exists y \exists z T(x, y, z)$ is read as "There exists a y and there exists a z such that $T(x, y, z)$" x is free, y is bound by the first existential quantifier, and z is bound by the second existential quantifier.

1.9.1: Nested Quantifiers of the Same Type

Consider a scenario where the domain is a group of people who are all working on a joint project. Let $M(x, y)$ be the predicate "x has sent an e-mail message to y".

- Domain = {all people working on the project}
- $\forall x \forall y M(x, y)$ reads as "For all x and for all y, x has sent an e-mail message to y"OR "Everyone has sent an e-mail message to everyone"
- The statement $\forall x \forall y M(x, y)$ is true if every pair, x and y, $M(x, y)$ is true. The universal quantifier include the case that $x = y$, so if $\forall x \forall y M(x, y)$ is true, then everyone has sent an e-mail to everyone else and everyone sent an email to themselves.
- The statement $\forall x \exists y M(x, y)$ is false if there is at least one person who has not sent an e-mail message to anyone. If even a single individual has not sent an e-mail message to anyone, then $\forall x \exists y M(x, y)$ is false.
- Now lets consider the statement $\exists x \exists y M(x, y)$
- That can be read as "There is a person who has sent an e-mail to someone" or "Someone has sent an e-mail to someone"
- The statement $\exists x \exists y M(x, y)$ is true if there is a pair x and y in the domain that causes $M(x, y)$ to be true. In particular, $\exists x \exists y M(x, y)$ is true even in the situation that there is a single individual who has sent an e-mail to themselves. The statement is false if no one has sent an e-mail to anyone.

1.9.2: Nested Quantifiers of Different Types

Table 1: 1.9.2: Nested Quantifiers as a two-person game

Player	**Action**	**Goal**
$\exists$ Existential player	selects values for existential bound variables	Tries to make the expression true
$\forall$ Universal player	selects values for universal bound variables	Tries to make the expression false