Lecture 9: Nested Quantifiers

Learning Objectives

- interpret nested Quantifiers
- negating nested statements

Nested Quantifiers

Q: What is the difference between

(i) $\forall s \in \mathbb{R} \exists t \in \mathbb{R}$ such that $t > s$

- This is true because we can always pick a number like $t = s + 1$ that is greater than s.

(ii) $\exists t \in \mathbb{R}$ such that $\forall s \in \mathbb{R}$ such that $t > s$

- This is false because we cannot pick a number t that is greater than all real numbers s.

Answer: (i) is true and (ii) is false

Example:

$\forall x \in \mathbb{R} \forall x \in X, \exists y \in Y$ such that $\not\equiv \exists y \in Y$ such that $\forall x \in X, P(x, y)$

$\forall x \in \mathbb{R} \forall x \in X, \forall y \in Y$ such that $\equiv \forall y \in Y$ such that $\forall x \in X, P(x, y)$

Example 2:

Taking the problem and breaking it into shorter single quantified statements.

$\forall x \in X[\exists y in Y$ such that $[\forall z \in Z$ such that $P(x, y, z)]]$

- Break into smaller statements

(i) $\forall x \in X$

(ii) $\exists y \in Y$

(iii) $\forall z \in Z$

Example 3: Epsilon Delta Definition of a Limit

$\forall \epsilon > 0 \exists \delta > 0$ s.t $\forall x \in \mathbb{R}$ s.t $0 < |x - c| < \delta$ we have $|f(x) - L| < \epsilon$

(i) Pick an arbitrary $\epsilon > 0$

(ii) Pick a $\delta > 0$ that depends on ϵ

(iii) Pick an arbitrary $x \in \mathbb{R}$ that satisfies $0 < |x - c| < \delta$

(iv) Check that $|f(x) - L| < \epsilon$ is true or not

Epsilon-Delta Proof

Let $f(x)$ be a function defined on $\mathbb{R}$ and L be the limit of $f(x)$ as x approaches c. We want to prove that:

$$\forall \epsilon > 0, \exists \delta > 0 \text{ such that } \forall x \in \mathbb{R}, 0 < |x - c| < \delta \implies |f(x) - L| < \epsilon$$