Question 1

Prove the following statement using a proof by contrapositive.

1. For every non-zero real number x, if x is irrational, then $\frac{1}{x}$ is irrational.

Statement: $p =$ ”x is irrational”, $q = \frac{1}{x}$”is irrational”

Contrapositive: $\neg p \to \neg q \equiv$ If x is rational, then $\frac{1}{x}$ is rational

Proof.

Theorem. *For every non-zero real number x, if x is irrational, then $\frac{1}{x}$ is irrational.*

Definition. *A rational number is a number that can be expressed as the quotient or fraction $\frac{p}{q}$ of two integers, a numerator p and a non-zero denominator q.*

Definition. *An irrational number is a number that cannot be expressed as the quotient or fraction $\frac{p}{q}$ of two integers, a numerator p and a non-zero denominator q.*

Let: x be a non-zero real number, p and q be integers, and $q \neq 0$.

Assume: x is rational.

$$x = \frac{p}{q}$$
$$\frac{1}{x} = \frac{q}{p}$$

Since p and q are integers that can be expressed as a quotient of two integers $\frac{p}{q}$, then $\frac{1}{x}$ is rational given x is a non-zero number.

Since, x is rational, then $\frac{1}{x}$ is rational.

Therefore, for every non-zero real number x, if x is irrational, then $\frac{1}{x}$ is irrational.

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