Assignment 3 - Solutions

Q1.

integer means + and -

a. $S = \{\pm 15, \pm 30, \pm 45, \ldots\}$

$S = \{x \in \mathbb{Z} : x \text{ is divisible by } 3 \text{ and } 5\}$

b. The set of factors of 12.

$S = \{n \in \mathbb{N} : 12 \text{ is divisible by } n\}$

c. The set of integer powers of 4.

$S = \{\ldots, \frac{1}{16}, \frac{1}{4}, 1, 4, 16, \ldots\}$

integer means 4^+ and 4^-.

Q2.

a. i. Statement.

$\neg$: There does not exist a pair of irrational numbers a, b such that $a \cdot b$ is rational.

or

(preferred version) For every pair of irrational number a, b, $a \cdot b$ is irrational.

ii. Not a statement.

iii. Statement

$\neg$: The sum of interior angles of a triangle is not 180°.

iv. I decided this one was too ambiguously worded so I did not count it.

b. non-quantified: The square root of 16 is 4.

$\neg$: The square root of 16 is not 4.

quantified: For any two points $P \neq Q$ in the xy-plane, there is a unique line passing through P and Q.

$\neg$: $\exists$ two points $P \neq Q$ in the xy-plane such that there is multiple lines pass through P and Q.

non-statements: $x + y = 10$.

Find the critical points of $f(x) = e^{2x}$.

Q3.

a. The proof aims to be a proof by contradiction, meaning it wants to arrive at a logical impossiblity (e.g. p^2 is both even and odd). We know by the statement that $ab > cd$. If the proof was in fact by contradiction, then the logical impossibility should involve $ab > cd$, namely the steps of the proof should have led to something like $ab \leq cd$. Instead, ChatGPT suggests that $ab < b(c+d)$ is the contradictory statement to $ab > cd$, which it in fact is not.

b. The prompted proof is (likely to be) exactly the same as the proof we saw in class.

c. ChatGPT might mention that it is a "classic proof by contradiction". ChatGPT is able reproduce simple well-known proofs, but struggles to prove (or disprove) lesser known or more complicated statements.

d. Answers may vary.

Ex. ChatGPT would be very useful for getting easy access to proofs of "classic theorems".