Question 2.1

The problem

Q2.1 For any integer x, if $x + y > 50$, then $x > 20$ or $y > 30$.

Compound Logic Form: $\forall x \in \mathbb{Z}, \forall y \in \mathbb{Z}, x + y > 50 \to x > 20 \lor y > 30$

Simplify Variables: $x + y = p, x = q, y = r$

Rewrite: $p \to q \lor r$

Thoughts on the problem: We can do a direct proof or a contrapositive proof. I will do both.

Contrapositive: $\neg q \land \neg r \to \neg p$

Proof by Cases

Proof. If $x + y > 50$, then $x > 20$ or $y > 30$.

Case 1: $x \leq 20$ and $y \leq 30$
Let $x = 20$ and $y = 30$ they are both integers.
Then $x + y = 50$ this is True.
Case 2: $x \leq 20$ and $y > 30$
Let $x = 20$ and $y = 31$ they are both integers.
Then $x + y = 51$ this is True.
Case 3: $x > 20$ and $y \leq 30$
Let $x = 21$ and $y = 30$ they are both integers.
Then $x + y = 51$ this is True.
Case 4: $x > 20$ and $y > 30$
Let $x = 21$ and $y = 31$ they are both integers.
Then $x + y = 52$ this is True.

Therefore, since it is true forall cases, the statement is true. □

Proof by Contrapositive

Proof. If $x \leq 20$ and $y \leq 30$, then $x + y \leq 50$.

Let $x = 20$ and $y = 30$ they are both integers.
Then $x + y = 50$ this is True.

Since, the contrapositive is true, the original statement is true. By the law of contrapositive. □