Assignment 4

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Question 1

As has already been mentioned (and seen), finding the negation of a statement will be useful when we start to do actual proofs. The statements and propositions that we will be proving will not always be a basic statement, having one or no quantifiers (or implications, which we will explore later), so we want to continue to build on and improve our knowledge of negation by finding the negation of more complex statements,

Answer each of the following.
[10 marks]

a. Write the negation of each of the following quantified statements. (Note: You can use the observations we made in about nested quantifiers to determine the negations.)

   i. $\forall\ x \in \mathbb{R}, \exists\ M \in \mathbb{N}$ s.t. $x < M$.

   iii. $\exists\ q \in \mathbb{Q}$ s.t. $\forall\ n, m \in \mathbb{Z}, \frac{mq}{n} = 2$.

b. Explicitly find the negation of the following quantified statement. (Hint: Express the statement in nested layers and follow the same method we used in class/notes to determine the negation.)

   $\exists\, a \in A$ s.t. $\forall\, b \in B$ and $c \notin C$, $\exists\, d \notin D$ and $e \in E$ s.t $\forall\, f \in F, P(a, b, c, d, e, f)$.

Question 2

There is no standard practice for when to use symbols and when to use words in written work. Regardless, when one is reading or speaking a mathematical statement (or proposition later on), that can only be done in words. In practice, this means that one needs to be able to readily translate a statement from symbols into words.

For each statement, write it out in words as a sentence. Is the statement true or false? Explain your reasoning.
[8 marks]

a. $\forall\ x \in \mathbb{R}, \exists\ M \in \mathbb{N}$ s.t. $x < M$.

b. $\forall\ \theta \in [0, 2\pi], (\sin\theta > \cos\theta) \vee (\cos\theta > \sin\theta) \vee (\theta = \frac{\pi}{4})$.

c. $\exists\ a \in \mathbb{Z}$ s.t. $\big(\forall\ b \in \mathbb{N}, ab < b\big) \wedge \big(\forall\ c \in (0,1), ac \notin (0,1)\big)$.

Question 3

Outside of studying logic, truth tables and logical algebra are not explicitly used very often. This is often because either the statements are simple or because one is so practiced in manipulating statements, that written algebra is not necessary (the same way we might do a quick calculation in our heads rather than going through the process of writing it out). While we are learning the definitions and rules of logic, however, these are power tools for understanding relationships between various operations (and implications).

Consider the statements

(i) $\neg\big(A \wedge (B \vee \neg A)\big)$

(ii) $\neg B \vee \neg A$

Answer each of the following.
[10 marks]

- Prove that the statements (i) and (ii) are logical equivalent by comparing their truth tables. (Note: Find the truth tables for how they are presented above, i.e. do not simplify.)
- Prove that the statements (i) and (ii) are logically equivalent **without** using their truth tables. Explain your steps. (Hint: What properties about logical operations can we use to simplify?)