1.8 De Morgan's Law for Quantified Statements

De Morgan's Law for Quantified Statements

The negation operation can be applied to a quantified statement, such as $\neg\forall x F(x)$ or $\neg\exists x F(x)$. If the domain for the variable x is the set of all birds and the predicate is $F(x)$ is "x can fly", then $\forall x F(x)$ is "All birds can fly" and $\neg\forall x F(x)$ is "Not all birds can fly". Which is logically equivalent to "There exists a bird that cannot fly". This is the same as $\exists x \neg F(x)$

- $\neg\forall x F(x) \equiv \exists x \neg F(x)$ In other words, "Not all x are $F(x)$" is the same as "There exists an x that is not $F(x)$"
- $\neg\exists x F(x) \equiv \forall x \neg F(x)$ In other words, "There doesn't exist x are $F(x)$" is the same as "All x are not $F(x)$"

1.8.1: De Morgan's Law for universally Quantified Statements

Domain $= a_1, a_2, a_3, a_4$

$\neg\forall x P(x) \equiv \exists x \neg P(x)$

Thus:

$\neg(P(a_1) \wedge P(a_2) \wedge P(a_3) \wedge P(a_4)) \equiv \neg P(a_1) \vee \neg P(a_2) \vee \neg P(a_3) \vee \neg P(a_4)$

1.8.2: De Morgan's Law for Existentially Quantified Statements

Domain $= a_1, a_2, a_3, a_4$

$\neg\exists x P(x) \equiv \forall x \neg P(x)$

Thus:

$\neg(P(a_1) \vee P(a_2) \vee P(a_3) \vee P(a_4)) \equiv \neg P(a_1) \wedge \neg P(a_2) \wedge \neg P(a_3) \wedge \neg P(a_4)$

Example: Using a statement $P(x)$ set of children enrolled in class and x is absent today.

$\neg\exists x P(x)$ is the same as "There is no child absent today"

$\forall x \neg P(x)$ is the same as "All children are not absent today"