Show all of your work. Answers without work will not receive full credit.
Your answers must use the same notation and definitions as in the problem statements.
Grading:

Question 1 is worth 6 points.
Question 2 is worth 5 points.
Question 3 is worth 4 points.
Question 4 is worth 5 points.

Submit your answers to Gradescope.

Problems:

1. Alfred defines the sample space S to consist of all real numbers x such that $2 < x \leq 8$. We take two events in the sample space labeled B_1 and B_2, where $B_1 = \{x \mid 2 < x \leq 4\}$ and $B_2 = \{x \mid 3 < x \leq 5\}$.

   (a) Find the union of the two events inside the sample space.

   (b) Find the intersection of the two events inside the sample space.

   (c) Explain the errors in Alfred's calculation when they found the complement of B_2 inside the sample space. Don't just state where the errors are, explain what the correct statement actually should be. *Hint: There are 3 errors.*

   Alfred's Answer:
   The complement is B_2'. We want to include everything outside of B_2 that is still in the sample space S.

$$\begin{aligned} B_2' &= (\{x \mid 3 < x < 5\})' \\ &= \{x \mid 2 < x \leq 3 \cup 5 < x \leq 8\} \\ &= \{3, 6, 7, 8\} \leftarrow \text{my final answer.} \end{aligned}$$

2. The Rotary Club of Kalamazoo holds an annual rubber duck race. Ducks are divided into one of four categories, C_1, C_2, C_3, C_4. Each duck is a member of only one of these groups, and each duck is placed into a group. Overall, there are 1000 ducks that are in the race. There are 200 ducks in the first category, 60 in the second category, 500 in the fourth category, and the remainder are in the third category.

   Elizabeth sponsors 450 out of the 1000 ducks for the race. We are interested in the probability that Elizabeth sponsors 90 from the first category, 25 from the second category, 225 from the fourth category, and the remainder of the 450 from the third category.

   Questions to Answer:

   (a) Show the setup / calculations required to find the probability required. No final answer needed.

   (b) Write `R` code to perform the calculations in the previous part. Provide your code and your final answer from `R`. You may hand-write your code. Provide the exact output from `R`: do not round your final answer.

3. Suppose Francis can take one of two classes (statistics and economics). Francis knows the probability that they pass each class. Francis is trying to calculate a conditional probability, and is not sure which statement is true. Decide which statement is true, and explain why your choice is correct.

   - Option 1: $1 - \mathsf{P}(\text{Fail} \mid \text{Economics}) = \mathsf{P}(\text{Pass} \mid \text{Economics})$
   - Option 2: $1 - \mathsf{P}(\text{Fail} \mid \text{Economics}) = \mathsf{P}(\text{Fail} \mid \text{Economics})$
   - Option 3: $1 - \mathsf{P}(\text{Fail} \mid \text{Economics}) = \mathsf{P}(\text{Pass} \mid \text{Stats})$
   - Option 4: $1 - \mathsf{P}(\text{Fail} \mid \text{Economics}) = \mathsf{P}(\text{Fail} \mid \text{Stats})$

4. Melanie takes two events, A and B. Assume that $\mathsf{P}(A) \neq 0$ and $\mathsf{P}(B) \neq 0$. Melanie wants to know if it possible for these events to be both mutually exclusive and independent at the same time. Provide a mathematical example illustrating your viewpoint. Your example should include

   - Definitions of the events A and B. Either start with them as mutually exclusive, or as independent of each other. When you are constructing these events, make sure they meet the criteria for one of mutually exclusive or independent.
   - The probability for A, and the probability for B.
   - If you started with A and B as mutually exclusive, provide a calculation to show if they are independent or not.
   - If you started with A and B as independent, provide a calculation to show if they are mutually exclusive or not.