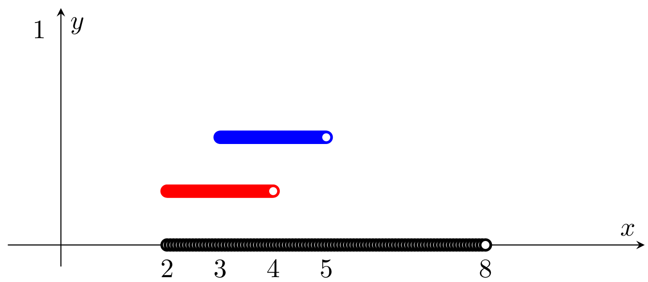
Problem 1

Problem 1: Alfred defines the sample space S to consist of all real numbers x such that $2 < x \leq 8$. We take two events in the sample space B_1 and B_2, where $B_1 = \{x|2 < x \leq 4\}$ and $B_2 = \{x|3 < x \leq 5\}$.



a) Find the union of the two events B_1 and B_2.

Answer: $B_1 \cup B_2 = \{x|2 < x \leq 5\}$

b) Find the intersection of the two events inside the sample space.

Answer: $B_1 \cap B_2 = \{x|3 < x \leq 4\}$

c) Explain the errors in Alfred's calculations when they found the complement of B_2 inside the sample space. Don't just state where the errors are, explain what the correct statement should be. Hint: There are 3 errors.

Answer:

Error 1: The first error is the set of numbers were given as $\mathbb{R}$ (real numbers). What Alfred did was give an answer as whole numbers, So he forgot to include the decimal numbers.

Error 2: The second error is the answer should have been something like this:

$$B_2' = \{x|2 < x < 3 \cup 5 \leq x \leq 8\}$$

This includes all real numbers.

Error 3: The third error is when they wrote out the equation they included the 3.

Problem 2

Problem 2: The Rotary Club in Kalamazoo holds an annual rubber duck race. Ducks are divided into four categories, C_1, C_2, C_3, and C_4. Each duck is a member of only one of these group, and each duck is placed into a group. Overall, there are 1000 ducks that are in the race. There are 200 ducks in the first category, 60 in the second category, 500 in the fourth category, and the remaining ducks are in the third category.

Elizabeth sponsors 450 out of the 1000 ducks for the race. We are interested in the probablility that Elizabeth sponsors 90 from the first category, 25 from the second category, 225 from the fourth category, and the remainder of the 450 ducks form the third category.

Questions to Answer:

(a) Show the Set up of the problem/calculations required to find the probablility required. No final answer is required.

Answer:

$$P(C_1) = \frac{90}{200} = \frac{2}{20}$$
$$P(C_2) = \frac{25}{60} = \frac{5}{12}$$
$$P(C_3) = \frac{[450 - (90 + 25 + 225)]}{[1000 - (200 + 60 + 500)]} = \frac{110}{240} = \frac{11}{24}$$
$$P(C_4) = \frac{225}{500} = \frac{9}{20}$$

(b) Write R code to perform the calculation required in the previous part. Provide your code and your final answer from R.

Answer:

STAT 381 > WtittenHomework > WrittenHwk_1 > wrt1.r > p3

```r
# ducks in each category
n1 <- 200  # Category 1
n2 <- 60   # Category 2
n3 <- 1000 - n1 - n2 - 500  # Category 3 (remaining ducks)
n4 <- 500  # Category 4

# ducks sponsored by Elizabeth in each category
k1 <- 90   # Category 1
k2 <- 25   # Category 2
k3 <- 450 - k1 - k2 - 225  # Category 3 (remaining ducks)
k4 <- 225  # Category 4

# Calculate the probability of each individual category
p1 <- k1 / n1
p2 <- k2 / n2
p3 <- k3 / n3
p4 <- k4 / n4

# print the probability of each individual category
print(p1)
print(p2)
print(p3)
print(p4)
```

PROBLEMS 15 OUTPUT DEBUG CONSOLE TERMINAL PORTS

```
tenHwk_1\\wrt1.r", en$
[1] 0.0001633842
> source("c:\\NoteSchema\\STAT 381\\WtittenHomework\\WrittenHwk_1\\wrt1.r", en$
[1] 0.45
[1] 0.4166667
[1] 0.4583333
[1] 0.45
```

Problem 3

Problem 3: Suppose Francis can take one of two classes (statistics and econ). Francis knows the probablility that they pass each class. Francis is trying to calculate a conditional probablility, and is not sure which statement is true. Decide which statement is true and explain why.

Option 1: $1 - P(\text{Fail}|\text{Econ}) = P(\text{Pass}|\text{Econ})$

Option 2: $1 - P(\text{Fail}|\text{Econ}) = P(\text{Fail}|\text{Econ})$

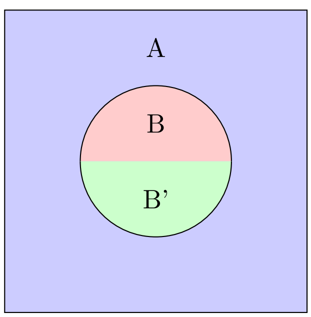
Option 3: $1 - P(\text{Fail}|\text{Econ}) = P(\text{Pass}|\text{Stat})$

Option 3: $1 - P(\text{Fail}|\text{Econ}) = P(\text{Fail}|\text{Stat})$

Answer: Option 1 is true!

Explanation: Relevant equations:

$$1 - P(B|A) = P(B'|A)$$



1 - P(B—A) = P(B'—A)

1 Problem 4:

Problem 4: Melanie takes two events, A and B. Assume that $P(A) \neq 0$ and $P(B) \neq 0$. Melanie wants to know if it is possible for these events to be both mutually exclusive and independent at the same time. Provide a mathematical example illustrating your viewpoint Your example should include:

* Definitions of the events A and B. Either start with them mutually exclusive or as independent of each other. When you are constructing these events, make sure they meet the criteria for one mutually exclusive or independent.
* The probablility for A, and the probablility fo B.
* If you started with A and B as mutually exclusive, provide a a calculation to show if they are independent or not.
* If you started with A and B as independent, provide a calculation to show if they are mutually exclusive or not.

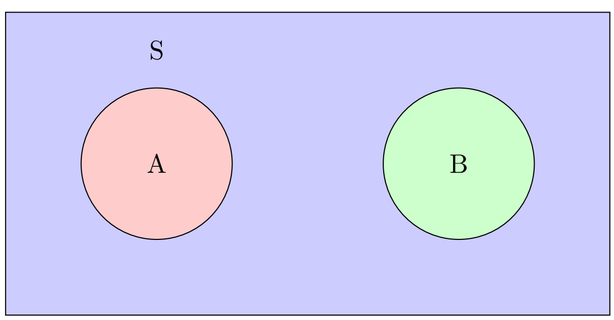
Answer:

Start with A and B as mutually exclusive.

Definitions:

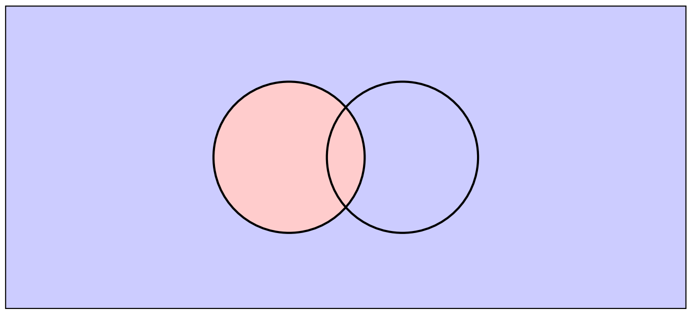
Mutually Exclusive: No shared outcomes.

$$P(A \cap B) = \emptyset$$



Independent: The outcome of one event does not affect the outcome of the other event.

$$P(A \cap B) = P(A)P(B)$$



Calculation:

$$P(A \cap B) = \emptyset \neq P(A)P(B)$$

$$\begin{aligned} P(A \cap B) &= P(A)P(B) \\ \emptyset &= P(A)P(B) \\ \emptyset &\neq P(A)P(B) \end{aligned}$$

Conclusion: Two events with $P(A) \neq 0$ and $P(B) \neq 0$ cannot be both independent and mutually exclusive at the same time.

$P(A \cap B) = \emptyset \neq P(A)P(B)$ So, $P(A \cap B) \neq \emptyset$ If $P(A) \neq 0$ and $P(B) \neq 0$ Which means that A and B would have value. Therefore A and B are not mutually exclusive.

$P(A \cap B) = \emptyset \neq P(A)P(B)$ So, $P(A)P(B) > 0$ Which means that A and B are not mutually exclusive.

If we were to assign we would have a non-zero probability of $P(A \cap B)$, which would mean that A and B are not mutually exclusive and independent. It would have to be either mutually exclusive or independent. By the way in which they are defined.

$$\neg \exists n \epsilon \mathbb{R} [(P(A) \vee P(B)) | P(A) \neq 0, P(B) \neq 0]$$