1 Predicate Logic

Notes on 1.6

1.1 predicate logic

predicate - a statement that contains a variable

predicate - $P(x)$: This is a predicate with the variable x

variable - x: This is a variable in the predicate $P(x)$

Quantified Statement -is a logical expression that contains universal or existential quantifiers

universal quantifier - $\forall$: This is a universal quantifier what this means is that the statement is true for all values of x

existential quantifier - $\exists$: This is an existential quantifier what this means is that the statement is true for at least one value of x

quantified statement - $\forall x P(x)$: This is a quantified statement what this means is that the statement is true for all values of x

quantified statement - $\exists x P(x)$: This is a quantified statement what this means is that the statement is true for at least one value of x

1.2 Universal Quantifier

What are universal quantifier?

A universal quantifier is a statement that is true for all values of x

$\forall x P(x)$: This is a universal quantifier

example - $\forall x(x > 0)$: This is a universal quantifier because it is true for all values of x that are greater than 0.

The domain of $P(x)$ is all real numbers

Therefore:

$$\forall x P(x) \equiv P(1) \land P(2) \land P(3) \land \ldots$$

Counterexample - A counterexample is a value of x that makes the statement false and because the we are our domain consists of all real numbers, we can use any real number as a counterexample

1.3 Existential Quantifier

What are existential quantifiers?

An existential quantifier is a statement that is true for at least one value of x

$\exists x P(x)$: This is an existential quantifier

example:

$\exists x(x + 1 < 0)$: This is an existential quantifier because it is true for at least one value of x

Therefore:

$$\exists x(x + 1 < 0) \equiv (0 + 1 < 0) \vee (1 + 1 < 0) \vee (2 + 1 < 0) \vee \dots$$

Counterexample - A counterexample is a value of x that makes the statement false.

example:

$$\exists x(x + 1 \geq 0) \forall x \in R$$

As you can see, this statement is false because this is true for all values of x.

2 Quantified Statements

Notes on 1.7

2.1 Quantifiers and Negations

Quantified Statements - A quantified statement is a statement that contains a quantifier such as $\forall$ or $\exists$

example: P(x): x is a prime number and O(x): x is an odd number

$$\exists x(P(x) \wedge \neg O(x))$$

This statement is true because there exists a prime number that is not odd, which is 2.

$$\forall x(P(x) \rightarrow O(x))$$

This statement is false because there exists a prime number that is not odd, which is 2.

Free Variables - A free variable is a variable that is not bound by a quantifier

$$\forall x(P(x) \rightarrow O(x))$$

In this statement, x is a free variable because it is not bound by a quantifier. It is called a free variable because it is free to take on any value.

Bound Variables - A bound variable is a variable that is bound by a quantifier. It is called a bounded variable because it is bound to the quantifier and can only take on values that are in the domain of the quantifier.

What makes something a proposition?

A proposition is a statement that is either true or false

example:

$x + 1 = 2$ is a proposition because it is either true or false

$x + 1 = 2$ is false because x can be any value

$x + 1 = 2$ is true because x can only be 1

What makes something a predicate?

A predicate is a statement that contains a variable

example:

$\forall x(x + 1 = 2)$ is a predicate because it contains a variable

Negation of a Quantified Statement

Define: Negation: $\neg$

example:

$\forall x(x + 1 = 2)$ is a predicate because it contains a variable

$\neg\forall x(x + 1 = 2)$ is the negation of the predicate $\forall x(x + 1 = 2)$

$\neg\forall x(x + 1 = 2)$ is equivalent to $\exists x(x + 1 \neq 2)$

$\exists x(x + 1 \neq 2)$ is the negation of the predicate $\forall x(x + 1 = 2)$

example:

$$\neg\exists x(P(x) \wedge \neg O(x)) \equiv \forall x \neg P(x) \vee O(x) \equiv \forall x(P(x) \rightarrow O(x) \tag{1}$$