Show all of your work. Answers without work will not receive full credit.
Your answers must use the same notation and definitions as in the problem statements.
Grading:

Question 1 is worth 3 points.
Question 2 is worth 6 points.
Question 3 is worth 4 points.
Question 4 is worth 4 points.
Question 5 is worth 5 points.
Question 6 is worth 10 points.

Submit your answers to Gradescope.

Problems:

1. Suppose Chester performs two calculations on the same random variable X: (a) he finds the variance of a random variable; (b) he finds the variance of a (random variable times some quantity). In your own words, explain why these two values will not be the same.

2. Mary investigates two random variables, X and Y, where $Y = X^4$. She finds that $\mathbf{E}(XY) = 0$, $\mathbf{E}(X) = 0$, and $\mathbf{E}(Y) = 4$.
   (a) Using this information, find $\mathbf{COV}(X, Y)$.
   (b) In your own words, describe the association between X and Y.
   (c) Are X and Y independent? Explain your answer using your own words. Include a statement about your covariance calculation in your answer. (You do NOT need to complete any calculations to answer this question.)

3. Answer the following questions about joint random variables.
   (a) The correlation coefficient can tell us a lot about how two variables are related. What type of relationship does the correlation coefficient measure?
   (b) George starts with the joint distribution (X, Y). He finds the marginal distribution for Y. George checks his work, and finds that the sum of the marginal distribution probabilities is equal to 1.25. Is this correct or was an error made (is there enough information to tell?)? If an error was made, what should be the sum of the marginal distribution probabilities?

4. Random Variables:
   (a) What does it mean to "define a random variable"?
   (b) Explain, in your own words, why it is important that we define a random variable before we do any calculations.
   (c) When defining a random variable, we have to state the parameters. Define "parameter" using your own words.

5. (Binomial Distribution). Suppose $X \sim \mathsf{Bin}(n, p)$. Marcy defines a new random variable, X/n, where n is the same constant integer as in the distribution for X.
   (a) Use properties of expected value to help Marcy find $\mathbf{E}(X/n)$. Fully simplify your answer (should not have an expected value in it).
   (b) Use properties of variance to help Marcy find $\mathbf{V}(X/n)$. Fully simplify your answer (should not have a variance in it).

   (Note: We will end up using this information later on in the semester.)

6. For the following problems, write down how you would code the probabilities requested into R. You should use the specialized distribution functions mentioned in the lecture notes from Sections 5.2–5.5. There is one problem for each distribution (Binomial, Poisson, Geometric, and Hypergeometric). You do not need to provide the final answer. It is fine to hand-write your code.
   (a) Suppose $X \sim \mathsf{Bin}(11, 0.67)$. Find $\mathsf{P}(X < 5)$.
   (b) Suppose a truck of books shipped to the Chicago Public Library contains 600 books. For some reason, there are 47 books that have water damage. The librarian randomly chooses 21 books for inspection. What is the probability that 2 or less of the inspected books have water damage?
   (c) Suppose $X \sim \mathsf{Pois}(\lambda t = 11)$. Find $\mathsf{P}(X \geq 15)$ incorporating the lower tail in your calculation.
   (d) Suppose $X \sim \mathsf{Geom}(p = 0.11)$. Define X to be like we did in the Section 5.4 class notes on page 1. We want to find $\mathsf{P}(X = 3)$.