Question 1

Answer each of the following.

a.

Write the negation of each of the following quantified statements. (Note: You can use the observations we made in about nested quantifies to determine the negations)

i. **Quantified Statement:** $\forall x \in \mathbb{R}, \exists M \in \mathbb{N}$ s.t $x < M$

Negation with logical operators: $\neg(\forall x \in \mathbb{R}, \exists M \in \mathbb{N}$ s.t $x < M)$

Using De Morgan's Law: $\neg(\forall x \in \mathbb{R}) \vee \neg(\exists M \in \mathbb{N}$ s.t $x < M)$

Negation: $\exists x \in \mathbb{R}, \forall M \in \mathbb{N}$ s.t $x \geq M$

ii. **Quantified Statement:** $\exists q \in \mathbb{Q}, \forall n, m \in \mathbb{Z}, \frac{mq}{n} = 2$

Negation with logical operators: $\neg(\exists q \in \mathbb{Q}, \forall n, m \in \mathbb{Z}, \frac{mq}{n} = 2)$

Using De Morgan's Law: $\neg(\exists q \in \mathbb{Q}) \vee \neg(\forall n, m \in \mathbb{Z}, \frac{mq}{n} = 2)$

Negation: $\forall q \in \mathbb{Q}, \exists n, m \in \mathbb{Z}, \frac{mq}{n} \neq 2$

b.

Explicitly find the negation of the following quantified statement. (Hint: Express the statement in nested layers and follow the same method we used in class/notes to determine the negation.)

i. **Quantified Statement:** $\exists a \in A$ s.t $\forall b \in B$ and $c \notin C, \exists d \notin D$ and $e \in E$ s.t $f \in F, P(a, b, c, d, e, f)$

With logical operators: $((\exists a \in A \rightarrow (\forall b \in B \wedge c \notin C, \exists d \notin D \wedge e \in E)) \rightarrow f \in F, P(a, b, c, d, e, f))$

Negation with logical operators: $\neg((\exists a \in A \rightarrow (\forall b \in B \wedge c \notin C, \exists d \notin D \wedge e \in E)) \rightarrow f \in F, P(a, b, c, d, e, f))$

Using De Morgan's Law: $\neg(\exists a \in A) \vee \neg((\forall b \in B \wedge c \notin C, \exists d \notin D \wedge e \in E)) \vee \neg(f \in F, P(a, b, c, d, e, f))$

Negation: $\forall a \in A, \exists b \in B \vee c \in C, \forall d \in D \vee e \notin E, f \notin F \vee \neg P(a, b, c, d, e, f)$

Question 2

For each statement, write it out in words as a sentence. Is the statement true or false?

a.

$\forall x \in \mathbb{R}, \exists M \in \mathbb{N}$ s.t $x < M$

a. **Statement:** For all x in the set of real numbers, there exists a set of natural numbers M such that x is less than M.

True or False: This statement is True given the case $x = 0$ and $M = 1$.

b.

$\forall \theta \in [0, 2\pi], (\sin(\theta) > cos(\theta)) \vee (cos(\theta) > sin(\theta) \vee (\theta = \frac{\pi}{4}))$

b. **Statement:** For all θ in the interval $[0, 2\pi]$, the sine of θ is greater than the cosine of θ or the cosine of θ is greater than the sine of θ or θ is equal to $\frac{\pi}{4}$.

True or False: This statement is True given the case $\theta = \frac{\pi}{4}$, because $sin\frac{\pi}{4} = cos\frac{\pi}{4}$.

c.

$\exists a \in \mathbb{Z},$ s.t $(\forall b \in \mathbb{N}, ab < b) \wedge (\forall c \in (0,1), ac \notin (0,1))$

c. **Statement:** There exists an integer a such that for all natural numbers b, ab is less than b and for all real numbers c in the exclusive interval of 0 to 1, ac is not in the exclusive interval of 0 to 1.

True or False: This statement is True given the case $a = 0$, because $0(b) < b$ and $0(c) \notin (0,1)$.

Question 3

i. $\neg(A \wedge (B \vee \neg A))$

ii. $\neg B \vee \neg A$

a.

Prove that the statements (i) and (ii) are logically equivalent by comparing their truth tables.

A	B	$\neg A$	$B \vee \neg A$	$A \wedge (B \vee \neg A)$	$\neg(A \wedge (B \vee \neg A))$	$\neg B \vee \neg A$
T	T	F	T	T	F	F
T	F	F	F	F	T	T
F	T	T	T	F	T	T
F	F	T	T	F	T	T

b.

Prove that the statements (i) and (ii) are logically equivalent /textbfwithout using truth tables.

Proof. We can prove that the statements (i) and (ii) are logically equivalent by using the laws of logic.

$$\begin{aligned}
\neg(A \wedge (B \vee \neg A)) &\equiv \neg A \vee \neg(B \vee \neg A) \text{ (De Morgan's Law)} \\
&\equiv \neg A \vee (\neg B \wedge A) \text{ (De Morgan's Law)} \\
&\equiv (\neg A \vee \neg B) \wedge (\neg A \vee A) \text{ (Distributive Law)} \\
&\equiv \neg B \vee \neg A \text{ (Identity Law)}
\end{aligned}$$

Therefore, $\neg(A \wedge (B \vee \neg A)) \equiv \neg B \vee \neg A$. $\square$