Extra Credit Assignment

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Extra Credit Problem

Prove the following logical equivalence:

$$\neg p \to (q \to r) \equiv q \to (p \vee r)$$

Start with the Left Side:

1. $\neg p \to (q \to r)$
2. $\neg\neg p \vee (q \to r)$ Conditional Identity
3. $p \vee (q \to r)$ Double Negation
4. $p \vee (\neg q \vee r)$ Conditional Identity
5. $(p \vee \neg q) \vee r$ Associative Law

Now the Right Side:

1. $q \to (p \vee r)$
2. $\neg q \vee (p \vee r)$ Conditional Identity
3. $(\neg q \vee p) \vee r$ Associative Law
4. $(p \vee \neg q) \vee r$ Commutative Law

Since the Left Side and the Right Side are logically equivalent. The following statement must be true.

$$(p \vee \neg q) \vee r \equiv (p \vee \neg q) \vee r$$

Since that statement is true, the following statement must also be true.

$$\neg p \to (q \to r) \equiv q \to (p \vee r)$$

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$\neg p \to (q \to r) \equiv q \to (p \vee r)$

$\neg\neg p \vee (q \to r) \quad \neg q \vee (p \vee r)$

$p \vee (\neg q \vee r)$

$p \vee (r \wedge \neg q)$

$(p \vee r) \wedge (p \wedge \neg q)$

$\neg p \to (q \to r) \equiv \neg\neg p \vee (\neg q \vee r)$

$\equiv p \vee (\neg q \vee r)$

$\equiv (\neg q \vee p) \vee r$

$q \to (p \vee r) \equiv \neg q \vee (p \vee r)$

$\equiv (\neg q \vee p) \vee r$

$$(\neg q \vee p) \vee r \equiv (\neg q \vee p) \vee r$$