

Question 2.6

The problem

Q2.6 For any real number x , $x + |x - 4| \geq 4$

Logical statement: $\forall x \in \mathbb{R}, x + |x - 4| \geq 4$

Thoughts on the problem: We can prove this problem by cases. We can show that the inequality holds for all real numbers x .

Proof by Cases

Proof. For any real number x , $x + |x - 4| \geq 4$

Case 1: $x + |x - 4| \geq 4$ when $x \geq 4$

$$\begin{aligned} x + |x - 4| &\geq 4 \\ x + x - 4 &\geq 4 \\ 2x - 4 &\geq 4 \\ 2x &\geq 8 \\ x &\geq 4 \end{aligned}$$

This is true because our assumption is that $x \geq 4$

Case 2: $x + |x - 4| \geq 4$ when $x < 4$

$$\begin{aligned} x + |x - 4| &\geq 4 \\ x + 4 - x &\geq 4 \\ 4 &\geq 4 \end{aligned}$$

This is true because our assumption is that $x < 4$

Case 3: $x + |x - 4| \geq 4$ when $x = 4$

$$\begin{aligned} x + |x - 4| &\geq 4 \\ 4 + |4 - 4| &\geq 4 \\ 4 + 0 &\geq 4 \\ 4 &\geq 4 \end{aligned}$$

This is true because our assumption is that $x = 4$

Conclusion: The inequality $x + |x - 4| \geq 4$ holds for all real numbers x .

Therefore, for any real number x , $x + |x - 4| \geq 4$

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