Question 3

The problem

Q3 Prove that for any integer n, n is odd if and only if $5n+3$ is even. Indicate which proof methods you used, as well as the assumptions (what you suppose) and the conclusions (what you need to show) of the proof.

Logical statement: $\forall n \in \mathbb{Z}, n$ is odd $\Leftrightarrow 5n+3$ is even

Simplify Variables: Let $n = N(x))$ and $5n+3 = E(y)$ $x, y \in \mathbb{Z} x = 2k, y = 2k+1$ for some $k \in \mathbb{Z}$

Rewrite the statement: $\forall x, y \in \mathbb{Z} N(y) \Leftrightarrow E(x)$

Define: Even: $x = 2n$ for some $n \in \mathbb{Z}$

Define: Odd: $x = 2n+1$ for some $n \in \mathbb{Z}$

Thoughts on the problem: We can prove this problem by contradiction. We can assume that n is odd and $5n+3$ is odd

Contradiction: $\neg p \wedge \neg q$ which is

$$\begin{aligned}
&\neg(\forall x, y \in \mathbb{Z} N(y) \Leftrightarrow E(x)) \\
&\neg(\forall x, y \in \mathbb{Z}(N(y) \Rightarrow E(x)) \wedge (E(x) \Rightarrow N(y))) \\
&\neg(\forall x, y \in \mathbb{Z}(\neg N(y) \vee E(x)) \wedge (\neg E(x) \vee N(y))) \\
&\exists x, y \in \mathbb{Z} \neg(\neg N(y) \vee E(x)) \vee \neg(\neg E(x) \vee N(y)) \\
&\exists x, y \in \mathbb{Z}(N(y) \wedge \neg E(x)) \vee (E(x) \wedge \neg N(y))
\end{aligned}$$

More thoughts on the problem: I'm going to need to do two cases for the proof by contradiction one for n being odd and $5n+3$ being even and the other for n being even and $5n+3$ being odd. I will have to show that both cases lead to a contradiction because they are tied together by an "OR" statement.

Proof by Contradiction

Proof. **Hypothesis:** For any integer n, n is odd if and only if $5n+3$ is even.

Contradiction: There exists an integer n such that n is odd and $5n+3$ is odd or there exists an integer n such that n is even and $5n+3$ is even.

Define Set: $n, k, l \in \mathbb{Z}$

Case 1: n is odd and $5n+3$ is odd

$$\begin{aligned}
n &= 2k+1 \\
5n+3 &= 2l+1 \\
5(2k+1)+3 &= 2l+1 \\
10k+5+3 &= 2l+1 \\
10k+8 &= 2l+1 \\
10k+7 &= 2l
\end{aligned}$$

This is a contradiction because $10k+7$ is odd and $2l$ is even.

Case 2: n is even and $5n+3$ is even

$$\begin{aligned}
n &= 2k \\
5n+3 &= 2l \\
5(2k)+3 &= 2l \\
10k+3 &= 2l
\end{aligned}$$

This is a contradiction because $10k + 3$ is odd and $2l$ is even.

Conclusion: The hypothesis is true because both cases lead to a contradiction.

Therefore, for any integer n, n is odd if and only if $5n + 3$ is even.

$\square$