Question 2.2

The problem

Q2.2 For any integers x, y, and z with $x \neq y$, if z is divisible by both $x - y$, then z is divisible by $y - x$.

Define: Divisibility: a is divisible by b if there exists an integer c such that $a = bc$.

Compound Proposition: $\forall x, y, z \in \mathbb{Z}, x \neq y(z = (x - y)k \to z = (y - x)l)$

Simple Proposition: $z = (x - y)k = p$ and $z = (y - x)l = q$

Logical Statement: $p \to q$

Thoughts on the problem: We can prove this problem by contradiction. We can assume that z is divisible by $x - y$ and z is not divisible by $y - x$. Then we can show that this assumption leads to a contradiction.

Contradiction: $p \wedge \neg q$ to prove we'll have to show that $p \wedge \neg q \equiv$ False

Proof by Contradiction

Proof. Using the definition of divisibility assume that z is divisible by $x - y$ and z is not divisible by $y - x$. Then there exists an integer k such that $z = (x - y)k$ and there exists an integer l such that $z \neq (y - x)l$.

Defining Set: $x, y, z, k, l \in \mathbb{Z}$ and $x \neq y$

$$
\begin{aligned}
z &= (x - y)k \\
z &= (y - x)l \\
(x - y)k &= (y - x)l \\
k &= \frac{(y - x)l}{(x - y)}
\end{aligned}
$$

Substituting k into the first equation:

$$
\begin{aligned}
z &= (x - y)\frac{(y - x)l}{(x - y)} \\
z &= \cancel{(x - y)}\frac{(y - x)l}{\cancel{(x - y)}} \\
z &= (y - x)l
\end{aligned}
$$

Since $z = (y - x)l$, and $z = (x - y)k$ are divisible, then z is divisible by both $x - y$ and $y - x$.

This is a contradiction. Therefore, z is divisible by $y - x$.

$\square$