Question 1

Prove the following statement using a direct proof.

1. If $m+n$ and $n+p$ are odd integers, where m, n, and p are integers, then $m+p$ is even.

Proof.

Theorem. *If $m+n$ and $n+p$ are odd integers, where m, n, p, j, k, l are integers, then $m+p$ is even.*

Definition. *An integer n is odd if there exists an integer k such that $n = 2k+1$. An integer n is even if there exists an integer k such that $n = 2k$.*

$$\begin{aligned}
m+n &= 2k+1 \\
n+p &= 2j+1 \\
m+p &= (m+n)+(n+p)-n-n \\
&= (2k+1)+(2j+1)-n-n \\
&= 2k+2j+2n \\
&= 2(k+j+n) \\
l &= (k+j+n) \\
m+p &= 2l
\end{aligned}$$

Since $m+p=2l$, where l is an integer, $m+p$ is even. ■