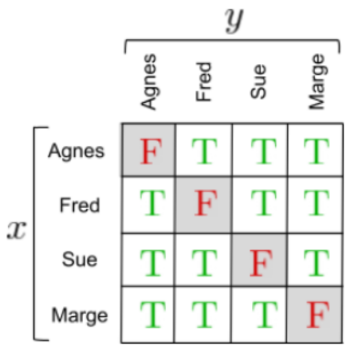
1.10 Nested Quantifiers

More Nested Quantified Statements

Using logic to express "everyone else"

- Consider a scenario where the domain is a group of people who are all working on a joint project. Let $M(x, y)$ be the predicate "x has sent an e-mail message to y". The statement $\forall x \forall y M(x, y)$ asserts that everyone has sent an e-mail message to themselves How can we use logic to express that everyone sent an e-mail to everyone without sending it to themselves? The idea is to use a conditional operation:

  $(x \neq y) \rightarrow M(x, y)$



- The statement $\forall x \forall y M(x, y)$ is false because M(Fred, Fred) and M(Marge, Marge) are both false
- The statement $\forall x \forall y((x \neq y) \rightarrow M(x, y))$ is true because the only false case is when $x = y$ and $M(x, y)$ is false

Expressing uniqueness in quantified statements

An existentially quantified statement evaluates to true even if there is more than one element in the domain that causes the predicate tot evaluate to true. If the domain is a set of people who attended a meeting and the predicate $L(x)$ indicates whether or not x came late to the meeting, the statement $\exists x L(x)$ is true if there are on or more people who came late.