Question 1.3

The problem

Q1.3 Every dog runs or eats. Every dog that eats but doesn't run, sleeps. Therefore, every dog that doesn't sleep, runs.

Let $R(x)$ be "x runs", $E(x)$ be "x eats", $S(x)$ be "x sleeps", the domain of x consists of all dogs.

Compound Logic Form: $(\forall x((R(x) \vee E(x))) \wedge \forall x((E(x) \wedge \neg R(x)) \rightarrow S(x))) \rightarrow \forall x(\neg S(x) \rightarrow R(x))$

Argument Form:

$$\begin{array}{l} \forall x((R(x) \vee E(x))) \\ \underline{\forall x((E(x) \wedge \neg R(x)) \rightarrow S(x))} \\ \therefore \forall x(\neg S(x) \rightarrow R(x)) \end{array}$$

Validate Argument:

Proof.

$\forall x((R(x) \vee E(x)))$	(Hypothesis)
$\forall x((E(x) \wedge \neg R(x)) \rightarrow S(x))$	(Hypothesis)
$\forall x(\neg S(x) \rightarrow R(x))$	(Conclusion)
Let x be an arbitrary dog	
Assume $\neg S(x)$	(Assumption)
Assume $\neg R(x)$	(Assumption)
$E(x) \wedge \neg R(x)$	(Conjunction Introduction)
$E(x) \wedge \neg R(x) \rightarrow S(x)$	(Universal Instantiation)
$S(x)$	(Modus Ponens)
$\neg S(x) \wedge S(x)$	(Conjunction)
$\neg\neg R(x)$	(Negation)
$R(x)$	(Double Negation)
$\neg S(x) \rightarrow R(x)$	(Conditional)
$\forall x(\neg S(x) \rightarrow R(x))$	(Universal Instantiation)
$\boxed{\therefore \forall x(\neg S(x) \rightarrow R(x))}$	(Conclusion)

$\square$