# Discrete Mathematics Notes

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## 1 Introduction

- Today we will be learning about predicate logic.

## 2 Logic

- Logic is the study of reasoning.
- It is used in mathematics, philosophy, and computer science.
- Logic is used to determine whether an argument is valid or not.
- An argument is a sequence of statements that end with a conclusion.
- A statement is a sentence that is either true or false.
  - Examples of statements:
    - The sky is blue.
    - The sky is red.
    - The sky is green.
  - Examples of non-statements:
    - What time is it?
    - Go to the store.
    - $x + 2 = 5$
  - A valid argument is one where the conclusion is true if the premises are true.
  - An invalid argument is one where the conclusion is false if the premises are true.
  - A tautology is a statement that is always true.
  - A contradiction is a statement that is always false.
  - A contingency is a statement that is neither a tautology nor a contradiction.
  - A proposition is a statement that is either true or false.
  - logical equivalence
    - Two statements are logically equivalent if they have the same truth value.
    - Example: $p \wedge q$ is logically equivalent to $q \wedge p$.
    - Example: $p \vee q$ is logically equivalent to $q \vee p$.
    - Example: $p \to q$ is logically equivalent to $\neg p \vee q$.
    - Example: $p \leftrightarrow q$ is logically equivalent to $(p \to q) \wedge (q \to p)$.
    - Example: $\neg(p \wedge q)$ is logically equivalent to $\neg p \vee \neg q$.
    - Example: $\neg(p \vee q)$ is logically equivalent to $\neg p \wedge \neg q$.
    - Example: $\neg(p \to q)$ is logically equivalent to $p \wedge \neg q$.
  - De Morgan's Laws
    - $\neg(p \wedge q)$ is logically equivalent to $\neg p \vee \neg q$.
    - $\neg(p \vee q)$ is logically equivalent to $\neg p \wedge \neg q$.