# Managing Market Exposure

*Levering your covariance matrix to control market risk.*

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For most investment managers, the fundamental portfolio management problem is to select portfolios of assets that outperform their benchmarks. The benchmark can be a liability stream (for a pension fund manager); a performance index (for an investment plan counselor); or cash (for a trader). Naturally, solutions to this problem depend on security valuation, but solving the problem also requires a robust risk management framework.

Two developments have amplified the complexity of risk management for many investors in recent years. First, investors have increased their purchases of non-domestic securities. Second, many are using derivatives more frequently. Both these activities have exposed investors to risks with which they have had little previous experience. Investors thus need more and more to understand their global market exposures and manage unfamiliar combinations of risk.

The development of risk management tools has not kept pace with these investment trends. It is becoming harder for investors to answer the question: How much risk does my portfolio have relative to my benchmark? This article introduces a risk management tool we have developed at Goldman Sachs to help manage our own portfolios, as well as to help our customers manage theirs. We call this tool the "market exposure."

In this article, we show that this simple concept can be very useful for understanding and managing global portfolio risk. Indeed, it plays the same role for global and multiasset portfolios that duration and beta

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have historically played for domestic fixed-income and equity portfolios.

Intuitively, a portfolio's market exposure measures its *sensitivity to market moves.*[1] Strictly speaking, the definition of our market exposure is the coefficient in a regression of the return of a portfolio on the return of the market.[2]

The definition of the "market" is not really the issue and can be adjusted as different contexts might warrant. We do not require that the "market" in our definition of market exposure refer to the market portfolio that plays a central role in the capital asset pricing model. While that is the most natural market exposure for the individual investor to be concerned about from the perspective of economic theory (because that is the exposure for which a risk premium should be paid), most investment managers are more narrowly focused on a particular class of assets. For most investors, the market should represent the normal mix of securities the investor invests in.

For example, for a domestic equity investor, the market should refer to the domestic equity market. In this case, the market exposure is exactly the usual definition of the market beta. Alternatively, for a domestic fixed-income investor, the market might refer to one of the standard domestic fixed-income indexes. In this case, the market exposure roughly corresponds to the ratio of the duration of the investor's portfolio to that of the benchmark (adjusted for the relative yield volatilities).

In the domestic portfolio management examples mentioned above, market exposure corresponds to familiar risk measures. Determining market exposure becomes more important, and more difficult, however, in the global context in which investors increasingly find themselves.

For example, the best measure of the market for a global bond manager is a global bond index. In this case, the manager's market exposure is a coefficient that relates the returns on the portfolio to those of the global bond index and provides an answer to the most important question an investor faces: "If the market that I invest in goes up, am I likely to outperform my index?" As we will show, the usual approach to this question, which is to compare portfolio and index durations, can be very misleading in the global context.

Not all investors compare their performance against that of an index. Hedge fund managers or traders, for example, usually try to maximize their returns and minimize risk in an absolute sense. In other words, their benchmark is cash. Such investors may use currencies, commodities, bonds, and equities as asset classes. For investors with such portfolio management objectives, there is no obvious choice of the "market." Nonetheless, the concept of market exposure may still be useful.

For example, a trader may well have a portfolio with long and short positions in many sectors of the world's fixed-income markets. The trader may want to know whether the portfolio will make money if all fixed-income markets rally. The usual approach, to sum the durations, can give a very misleading answer. A better approach is to calculate the market exposure of the positions to an appropriately weighted "global fixed-income market," as we demonstrate.

Similarly, a currency trader may want to measure exposure to a dollar rally. A better measure than simply summing long and short positions in foreign currencies is to measure exposure to a market defined as a set of appropriate weights in foreign currencies. By predefining a set of such "markets," a trader can quickly and easily monitor net long and short exposures to a wide variety of risks.

We first discuss the relationships among risk factors, benchmarks, and market exposure. We examine market exposure in the context of a portfolio manager measured against a performance index, and contrast it with more traditional risk management tools. Our examples show how market exposure improves risk management for traders or hedge fund managers.

## RISK FACTORS, BENCHMARKS, AND MARKET EXPOSURE

Intuitively, most portfolio managers think about asset returns in terms of exposure to risk factors that are either underlying fundamentals or related to underlying fundamentals. For example, an equity manager might consider industry classification an important determinant of equity returns. In general, sources of value are also sources of risk; for this manager, therefore, the exposure to different industries is a factor that will affect the portfolio's risk.

Alternatively, a U.S. fixed-income portfolio manager may regard credit ratings as an important characteristic influencing bond returns. For this manager, exposures to different credit ratings become an important determinant of portfolio performance and risk.

Similarly, a U.S. fixed-income manager may treat bond returns as dependent on three term structure factors — the level, slope, and curvature of the yield curve — that explain virtually all the returns in the U.S. Treasury market. These three risk factors, then, determine the performance of the portfolio relative to the benchmark. In this context, it is natural to think about portfolio strategies that might trade off exposures to one risk factor against another.

For example, the portfolio might have 90% of its risk come from being overexposed to one term structure factor, "level." Or — holding the risk fixed relative to its benchmark — we could restructure the portfolio to focus the risk on the "slope" factor or on the "curvature" factor.

This approach of defining risk factors is especially useful for categorizing the types of positions the manager is taking. In the examples discussed above, the manager finds a portfolio's risk relative to the benchmark by looking at the exposures to the risk factors. We can attribute any volatility of the performance in the portfolio relative to the benchmark to differences in exposures to risk factors plus a residual that, if this approach is to be useful, should be relatively small.

Of course, the risk factors described in the examples so far have the advantage of being easy to identify. Consequently, it is usually relatively straightforward to structure portfolios with the desired factor exposure. Portfolio management and risk measurement become more problematic in portfolios of global assets, however, where there are many more factors affecting return, and the factors are more complex and thus not easy to describe or measure.

Nonetheless, there are certain basic exposures that most managers need to understand. In particular, almost every manager wants to know the answer to the question, "If the market rallies, will I outperform my benchmark?" Put differently, a manager wants to know a portfolio's market exposure.

### Market Exposure of a Global Portfolio

Let's consider a global bond portfolio. Clearly, this portfolio's performance and volatility are affected by the level and direction of changes in interest rates for each country in the portfolio. Suppose for the moment that we are in the simplest of all worlds, and that all interest rates move together. In this case, the portfolio's performance is affected not by the levels of and changes in interest rates in each country, but rather by a common factor called, perhaps, the "global interest rate factor." Consequently, a portfolio's risk relative to another portfolio (or benchmark) could be measured in terms of its relative sensitivity to the global interest rate factor; that is, its relative duration.

In this simple context, a portfolio's market exposure relative to a global benchmark is merely the ratio of the portfolio's duration to that of the benchmark. A portfolio is more exposed to the global interest rate factor if it has a longer duration than the benchmark. In this case, if the global interest rate factor rallies, the portfolio will outperform the benchmark, and vice versa.

The real world is not so simple. Interest rates do not all move together, and the total duration of a global portfolio will probably not be a good proxy for our market exposure measure. Consider a slightly more complicated and realistic view of the global fixed-income markets. Suppose that global interest rate movements can be described in terms of three separate blocs, and that interest rate movements are perfectly correlated within each bloc and completely uncorrelated across blocs. One bloc could be a "Europe" bloc, while a second could be a "dollar" bloc, and a third a "Japan" bloc. In this case, the blocs themselves constitute separate "factors" (e.g., a "Europe" factor). We measure one portfolio's risk relative to another portfolio (or benchmark) in terms of exposures to the blocs.

Suppose that the Europe factor is more volatile than the dollar or Japan factors, and that the portfolio is overweighted in the Europe bloc and underweighted in the dollar bloc, relative to the benchmark weights. Since the Europe factor is more volatile and the dollar factor is less volatile, and since all three factors are uncorrelated, the portfolio's returns are more volatile than those of the benchmark.

Furthermore, we can describe the volatility of the portfolio's performance relative to that of the benchmark in terms of the relative exposures to the Europe, dollar, and Japan factors. Finally, since the factors are uncorrelated, we can describe the portfolio's market exposure as a linear combination of the factor volatilities, with the consequence that higher exposure to the more volatile Europe factor leads to more market exposure. If, on the other hand, the dollar block is more volatile, the market exposure of a portfolio overweighted in the Europe bloc will be lower than that of the benchmark.

In both these examples, we start with an assumption about the structure of the underlying sources of risk. We then use this structure to describe the correlation between asset returns. The final step is to show that the market exposure is a function of the relative exposures to the risk factors.

Some readers familiar with equity valuation theories may recognize this type of distinction between risk factors and market exposure in the context of equity portfolios, where the risk factor approach forms the basis for the arbitrage pricing theory while the market exposure approach is the basis for the capital asset pricing model.

### Market Factors versus Residual Factors

In the real world, unfortunately, trying to identify sources of risk by uncovering a set of "factors" from an actual covariance matrix of asset returns is a very complicated process, particularly in the context of global fixed-income securities. For instance, instability in the correlation matrix of asset returns suggests that identification of factors is time-dependent. Moreover, even if we can isolate factors, finding an economic interpretation for these factors is difficult.

Even though a stable set of "factors" cannot easily be identified, we can obtain a very natural and useful decomposition of risk of a portfolio by measuring 1) the exposure of a portfolio to the risk factors that affect the overall market, versus 2) the "residual" exposure to factors that affect the performance of the portfolio but do not affect the returns of the market.

In this approach, we avoid the difficult and unnecessary step of trying to decompose the covariance structure of returns into underlying risk factors. Rather, we directly quantify the exposure of a portfolio to the market risk factors.

Measuring a portfolio's market risk is fundamentally a forecasting problem. Consequently, several tradeoffs arise in developing market risk measures. We would like measures that are easy to understand but that simultaneously encompass exposure to all risk factors.

Since our measure of market exposure is statistical in nature, it provides a more accurate, though more complex, answer than accounting measures (such as duration) to the forecasting question, "How is my portfolio likely to perform when the market rallies?" While our measure does not mitigate the estimation issues surrounding time-varying correlations (and volatilities), it has the virtue of being more easily interpreted than more complicated multifactor models.

### The Relevant "Market"

In general, no single definition of the "market" portfolio will suffice. For the purpose of understanding risks, most investment managers will want to focus on a capitalization-weighted portfolio that includes securities in which they transact. A fixed-income manager is likely to care about exposure to the fixed-income markets but not about exposure to the equity market. In such a context, the manager's benchmark is likely to be a good proxy for the relevant subcomponent of the market portfolio.

More generally, however, a diversified investor will care about exposures to a broad aggregation of asset classes. Such an investor may indeed care about any equity risks embedded in the fixed-income portfolio. Our feeling is that while the broadest possible definition of the market is the best one for this investor, for many purposes it makes sense to measure exposures to a subcomponent. Moreover, in certain contexts, customizing a portfolio and referring to it as a "market" portfolio — even though it is not all-inclusive — is a natural thing to do. For the sake of brevity, we henceforth refer to measures of exposure to both broad aggregates and subcomponents as market exposures.

Assume a manager whose benchmark serves as a proxy for the manager's "market." If the manager's portfolio has the same aggregate exposure to the unobservable factors that affect the market as does the benchmark portfolio, the two portfolios should move equally in response to broad market moves, so the market exposure equals 1.0. If the portfolio is less exposed in the aggregate to the unobservable factors than the benchmark, then the portfolio's performance (in absolute terms) will be less than that of the benchmark, and the market exposure will be less than 1.0. Finally, if the portfolio is more exposed in the aggregate to the unobservable factors than the benchmark, the portfolio's performance will exceed that of the benchmark (in absolute terms), and the market exposure will exceed 1.0.

Note that a benchmark portfolio is not always a proxy for the relevant market portfolio. The benchmark is sometimes a proxy for liabilities, as opposed to a measure of the universe of investable assets. For example, a pension manager could easily define a benchmark portfolio of long bonds to represent the interest rate risk of the plan's liability stream. In this case, while it may be quite interesting to measure and manage the exposure of the asset portfolio to the factors affecting the valuation of the liabilities — and the approach we describe could be used to do that — we might not want to call this quantity a market exposure, in our terms.

Although it is very difficult to find a stable set of risk factors in global markets, we feel that market exposure is one of a small set of fundamental risk measures that all portfolio managers should understand. Together with portfolio volatility, portfolio tracking error relative to a benchmark, and benchmark volatility, the market exposure of the portfolio is something that all portfolio managers should monitor on a regular basis. Market exposure is the best statistical measure to use to answer the basic question: "How will my portfolio perform if my market rallies?"

We have found the concept of market exposure useful not only in portfolio analysis but also in managing our market-making and proprietary trading positions. Appendix A illustrates how we use market exposure to help manage our positions.

## MARKET EXPOSURE AND THE GLOBAL BOND FUND MANAGER

Here we address market exposure assuming that the performance of an active global bond portfolio manager is measured against an index that provides a relevant proxy for the market component the manager is concerned with. We first discuss how the portfolio management objectives of the fund manager introduce risk. After defining the manager's risk, we demonstrate the insight that is gained into the risk management process by using market exposure in addition to traditional risk measures. While we present the example in terms of a currency hedged index, the same principles apply to a manager who is measured against an unhedged index.

Active global fixed-income portfolio managers are selected on the basis of their ability to provide superior returns relative to an index of global bond market performance. To duplicate the index's performance, the manager needs to match the portfolio's characteristics to those of the index. For instance, the so-called indexed portfolio would match the country weightings and duration of the index within each country. If the index were unhedged (or partially hedged), then the indexed portfolio would also match the currency allocations.

Since the active manager's goal is to outperform the index, however, he or she must choose a portfolio with characteristics that deviate from those of the index. For example, a manager expecting a decline in U.K. interest rates might overweight the long end of the U.K. gilt market. Should the manager's view come true, the portfolio's performance would exceed that of the index. Should a rate increase in the U.K. materialize, the portfolio could be expected to underperform the index.

Consequently, to meet investment objectives, the active manager must take on risk. This risk is measured by the volatility of the return differences of the portfolio relative to the index; it is called the portfolio's *tracking error.*

Higher tracking error implies a wider dispersion in the potential performance for the portfolio relative to the index. For example, an annualized tracking error of 1.00% means that the portfolio's return will be within 100 basis points (bp) of the index's return approximately two-thirds of the time at the end of a year, irrespective of the return on the index. We can compute a portfolio's tracking error by using the deviations of the portfolio from its benchmark's weights, the volatilities of the assets in the portfolio and benchmark, and the correlations between asset returns.

To outperform the index, the manager can either change exposure to those factors that affect the index or change exposure to factors that are not reflected in the index's performance. By changing exposure to either of these factors, the manager changes the tracking error. Thus, we want to distinguish between two sources of tracking error: the *market exposure*, or exposure to factors that influence the index return; and the *residual risk,* or exposure to factors that do not influence the index. We examine each of these in turn.

### Market Exposure and Portfolio Performance

A portfolio's market exposure in our terms is a coefficient that quantifies the expected performance of the portfolio for a given index performance.

An equation shows the relationship between expected portfolio return, index return, and market exposure. Portfolio and index returns are expressed relative to cash — i.e., as returns over (or under) the cash rate.

Expected Portfolio Return =

Market Exposure × Index Return

Notice that if the market exposure equals 1.0, we can anticipate that the portfolio's performance will match the index return, all else equal. If the market exposure is greater than 1.0, however, we can expect the portfolio to outperform the index in rallies and underperform the index in sell-offs.

For example, if global interest rate changes have a greater impact on the portfolio than on the index, the market exposure would exceed 1.0. As global interest rates decline, the portfolio will outperform the index, while in sell-offs the portfolio will underperform the index.

Suppose that the portfolio's market exposure is 1.20, and that the declines in interest rates lead to an index return of 10%. In this case, we expect the portfolio's return to be 12%, meaning that the portfolio will outperform the index by 20%. If the index return is −10%, however, the portfolio's expected return is −12%, indicating that the portfolio will underperform the index by 20%.

In a domestic portfolio management context, the active manager who anticipates a rate decline will implement this view by selecting a portfolio whose duration exceeds that of the index. For example, suppose that the portfolio and the index durations are 5.00 and 4.00, respectively. In this case, a 100-bp decline in all interest rates is translated into the portfolio outperforming the index by 1 percentage point, or a 25% outperformance. Similarly, if all interest rates increase by 100 bp, the portfolio will underperform the index by 1 percentage point, or a 25% underperformance.

Because in a domestic market all interest rate changes are highly correlated, the market exposure is approximately the ratio of the portfolio and index durations. In our example, the market exposure is about 1.25. Of course, in practice, not all domestic interest rates will move by the same amount. Therefore, as a measure of exposure for a domestic fixed-income portfolio, duration may on occasion be highly misleading. Nonetheless, duration is widely used for risk management in this context.

An attempt to extend duration to risk measurement for a global fixed-income portfolio requires two conditions that are not even approximately true. These are 1) that yield volatilities are similar across markets, and 2) that interest rate movements are highly correlated across markets. If both the portfolio and the benchmark are unhedged (i.e., include currency risk), then a third condition is required: that currency movements and interest rate movements are highly correlated.

**EXHIBIT 1**
GS LMI Yield Volatilities

| Country | 1–3 Years | 10+ Years |
|---|---|---|
| France | 15.12 | 14.96 |
| Germany | 13.50 | 13.00 |
| Japan | 30.11 | 12.83 |
| U.K. | 15.95 | 14.74 |
| U.S. | 17.22 | 12.36 |

Let's consider the first condition. Exhibit 1 shows the yield volatilities for two of the sectors of the Goldman Sachs Liquid Market Index™ (GS LMI) in the French, German, Japanese, U.K., and U.S. government bond markets. The two sectors shown in Exhibit 1 are the one- to three-year and the greater-than-ten-year sectors. (We calculate volatilities using daily data covering the period from February 1988 through March 1995, with a 10% monthly decay.)

As Exhibit 1 illustrates, volatilities vary both within markets and across markets. For instance, while the one- to three-year volatility roughly equals the greater-than-ten-year volatility in France and Germany, the short-sector volatility exceeds the longer-sector volatility in Japan and the United States. Similarly, the longer-sector volatility in Japan is less than that in the other markets.

Now let's consider the second condition, that of high correlations both across and within markets. As Exhibit 2 makes clear, this condition is also violated in practice. Exhibit 2 summarizes the correlations for the GS LMI (full index) for the five countries shown in Exhibit 1 (correlations calculated using the same data and procedure as in Exhibit 1). Again, inspection of the data in Exhibit 2 indicates that most of the major markets are not highly correlated.

**EXHIBIT 2**
Correlations Across Markets

| | France | Germany | Japan | U.K. | U.S. |
|---|---|---|---|---|---|
| France | 1.00 | | | | |
| Germany | 0.75 | 1.00 | | | |
| Japan | 0.17 | 0.21 | 1.00 | | |
| U.K. | 0.62 | 0.64 | 0.05 | 1.00 | |
| U.S. | 0.36 | 0.41 | −0.02 | 0.36 | 1.00 |

For example, the correlation between the Japanese market and the other markets ranges from −0.02 to +0.21. The correlation between the German and French markets is higher than all other correlations. By way of comparison, the correlations in the U.S. market range from 0.80 between the one- to three-year and greater-than-ten-year sectors to 0.97 between the seven- to ten-year and greater-than-ten-year sectors.

The final condition required to justify using duration in the context of unhedged global fixed-income management is that currency and interest rate movements are highly correlated. Exhibit 3 summarizes the correlations between currencies and the GS LMI, using the U.S. dollar as the base currency. The exhibit shows the correlation between the return to each currency and the return to the GS LMI (hedged) for all countries.

For example, the correlation between the return to the yen and the return to the GS LMI for the United Kingdom is −0.10; that is, as the yen rallies, the U.K. gilt market sells off. As the exhibit clearly shows, currencies and bonds have been far from highly correlated.

The numbers in Exhibits 1-3 are quite compelling and raise two questions. First, does it make any practical difference if duration is used as a risk measure in a global context? Second, how is our market exposure to be computed if duration is not used?

Looking at the first question, let's suppose that actual yield volatilities and correlations are as shown in Exhibits 1-3. At the risk of complicating the issue, let's consider a real-world example. Suppose that a global manager uses the global duration to calculate the exposure of the portfolio to interest rate changes, and that the manager is measured against an index whose weights and duration within each country (as of January 3, 1994) are shown in Exhibit 4. The index — assumed to be fully hedged — consists of the market capitalization weights in the GS LMI for France, Germany, Japan, the United Kingdom, and the United States. Combining the weights and durations of Exhibit 4 leads to a benchmark duration of 5.2; i.e., for a 100-bp yield decline in all markets, the benchmark's return will be 5.2%.

**EXHIBIT 3**
Currency and Bond Correlations

| One- to Three-Year Sector | France | Germany | Japan | U.K. |
|---|---|---|---|---|
| France | −0.19 | −0.23 | −0.13 | −0.17 |
| Germany | −0.16 | −0.15 | −0.01 | −0.19 |
| Japan | 0.18 | 0.21 | 0.26 | 0.04 |
| U.K. | −0.19 | −0.21 | −0.10 | −0.14 |
| U.S. | −0.05 | −0.04 | −0.08 | −0.03 |

**EXHIBIT 4**
Benchmark Asset Weights

| Country | Country Weight | Country Duration |
|---|---|---|
| France | 7.70 | 5.42 |
| Germany | 12.08 | 4.36 |
| Japan | 22.56 | 5.66 |
| U.K. | 6.92 | 5.89 |
| U.S. | 50.75 | 5.04 |

Now suppose that the manager is bullish on global bond markets and expects yields to decline in all five markets. The manager also believes, however, that there will be relative performance differences between markets. In particular, suppose the manager believes that the German market will outperform cash by 208 bp, that the Japanese market will outperform cash by 159 bp, that the excess return on the U.K. market is 125 bp, that the excess return in the United States is 98 bp, and that the worst-performing market will be the French, which is expected to outperform cash by only 76 bp.

To implement these views, the manager constructs a portfolio whose duration is 10% longer than that of the benchmark; i.e., the portfolio's duration is 5.72 years. Since the manager is concerned about risk control, the portfolio's tracking error is constrained to be 100 bp. The manager believes that all yields will decline by 100 bp, the benchmark will rally by 5.2%, and the portfolio will outperform it by at least 52 bp. Since the manager has views on relative performance between markets (or spread views), the portfolio could outperform the benchmark by more than 52 bp.

On balance, then, the manager believes that two sources of risk have been controlled. First, volatility relative to the index has been controlled, since the tracking error has been constrained at 100 bp. Second, risk from market movements has been controlled, since the portfolio's duration has been constrained to 10% more than the benchmark's duration.

How would these constraints have worked in practice? Suppose that the manager had developed a portfolio using these views and constraints at the beginning of 1994. In this case, the optimal portfolio would have 42.02% of its weight in German bonds, 17.42% of its weight in Japanese bonds, 1.10% in U.K. bonds, and 39.45% in U.S. bonds. Combining these weights with the expected return views means that the portfolio is expected to outperform the benchmark by 81 bp.

Of course, 1994 was a very volatile period for the fixed-income markets; rather than rally, most markets actually sold off. Reflecting the market sell-off, the benchmark underperformed cash by 7.20%. The manager's exposure to market moves, as measured by the relative durations, is 1.1 (5.67/5.15). Thus, the manager would have anticipated underperforming the benchmark by 73 bp, all else equal.

In fact, the portfolio's actual performance in 1994 was −8.52%. In other words, the portfolio underperformed the benchmark by 132 bp, 61 bp more than would have been predicted simply by looking at the relative durations.

What went wrong? In developing the optimal portfolio, the manager used the relative durations as the measure of exposure to market moves. As discussed above, this approach implicitly assumes that all markets are perfectly correlated; in other words, all curves move in the same direction at the same time by the same amount. Exhibits 1-3 clearly demonstrate that this condition is violated in practice.

Now suppose that the manager had decided to recognize the limitations of using duration, and used market exposure instead of duration. That is, rather than develop a portfolio whose duration was 10% longer than that of the benchmark, the manager developed a portfolio whose market exposure was 1.1. To achieve this end, it is necessary to know the market exposures for each of the proposed assets in the portfolio.

Exhibit 5 shows the market exposures for each country in the benchmark, calculated with the volatilities and correlations of Exhibits 1-3 (i.e., explicitly recognizing that interest rate movements are not perfectly correlated across and within markets). We can also calculate market exposures for each of the maturity sectors in each country. (Appendix B provides the market exposures for each of the maturity sectors against the capitalization-weighted benchmark.)

**EXHIBIT 5**
GS LMI Market Exposures

| Country | Market Exposure |
|---|---|
| France | 1.14 |
| Germany | 0.84 |
| Japan | 0.41 |
| U.K. | 1.35 |
| U.S. | 1.23 |

The market exposures in Exhibit 5 show the impact on each country's bond market conditioned on a unit change in the benchmark's return. Let's look, for example, at the Japanese and U.K. bond markets. The durations in each country are approximately the same (5.7-5.9), yet the market exposures are different. A 100-bp rally in the benchmark predicts a 41-bp rally in the Japanese market versus a 135-bp rally in the U.K. market, all else equal. A manager who wants to trade the U.K. for Japan, yet wants to keep portfolio market exposure constant, would need to receive 3.3 "units" of Japan for each "unit" of the U.K. (or 1.35/0.41). By contrast, if duration is used, the manager would be led to trade Japan for the U.K. on a one-for-one basis (since duration is the same in each country).

After calculating the market exposure for each of the assets in the portfolio, the manager would need to develop a portfolio whose market exposure hits some target. Since the views are bullish on bond markets, the manager should be "long" relative to the benchmark.

In the duration case, the portfolio is developed to have a duration 10% longer than the benchmark duration. The analogue in using market exposure as the measure of market risk is to choose a target value of 1.1; that is, when the benchmark's return is 1.0%, the portfolio's return is 1.1%, or 10% more.

With a market exposure target of 1.1, and using the same views, we find the new portfolio weights to be 43.48% in Germany, 24.87% in Japan, and 31.65% in the United States (versus 42% in Germany, 17% in Japan, 39% in the United States, and 1% in the United Kingdom for the duration-constrained portfolio). In addition to changing the aggregate weights within each market (relative to the duration-constrained portfolio), the distribution across maturities in each country is different. (Appendix B contrasts the maturity distribution for each portfolio.)

The 1994 performance characteristics of the new portfolio are strikingly different from those of the previous portfolio. Using our market exposure measure, the portfolio underperforms cash by 7.99%, or by 79 bp more than the benchmark. Notice, however, that the portfolio's performance is 113% of the benchmark's, or roughly what the market exposure would predict. Furthermore, the actual tracking error is within the 100-bp band specified by the tracking error constraint.

These results are instructive for determining what went wrong with the earlier portfolio. Using the market exposures for each of the assets in the initial portfolio, we calculate the overall market exposure as 1.18. Consequently, the actual performance of the portfolio relative to the benchmark is in line with the performance that would be *predicted by using market exposure instead of relative durations.*

In this example, the manager was nearly twice as long relative to his benchmark as he wanted to be. As a result, he suffered nearly twice as much! This example illustrates that correctly measuring market exposure can have a major impact on portfolio risk management.

### Residual Risk and Portfolio Performance

Another source of portfolio risk beyond market exposure is residual risk. Suppose a domestic fixed-income manager is measured against a government bond index and adopts the strategy: Maintain duration equal to the index duration, but purchase corporate bonds (assuming that this is permissible in the investment mandate). Since this is a domestic portfolio, the government bond market exposure is likely to be neutralized by setting the duration equal to the index duration, but the portfolio is nonetheless exposed to risk attributable to movements in the corporate bond spread.

Alternatively, a global fixed-income manager who is measured against a government bond index could pursue the strategy: Maintain a duration equal to the index duration in each country, but adjust holdings to account for differences in curve shape across countries. The strategy might be to position for the convergence of yield curve slopes of countries within a currency block. In this case, the manager's portfolio again is exposed to risk beyond the pure movements in the index. The risk this time is a function of curve shapes.

As a final example, consider again a global fixed-income manager whose benchmark is a fully hedged index. In this case, suppose that the manager's portfolio matches the index weight, duration, and maturity composition in each country. The manager seeks to enhance return by taking on currency exposure (even though the benchmark is a fully hedged index). Again, the portfolio is exposed to risks beyond those inherent in the index. In this case, the risk is attributable to the currency exposure.

A consistent feature of all of these examples is that exposure to pure index movements can be neutralized (i.e., market exposure equals 1.0), yet the portfolio is still exposed to risk. In this case, the risk is what we have called *residual risk.*

Traditional portfolio management focuses only on risk and expected return. The risk measure in that context does not distinguish between market risk and residual risk. We believe the distinction is important. Most investors today do not manage their own funds. Individuals and pensions typically make basic asset allocation decisions, such as the proportions to invest in equities versus bonds and domestic versus foreign securities, but they hire investment managers to actually invest the funds in the particular asset class. In this context, an investment manager who chooses a market exposure substantially different from 1.0 relative to the benchmark can potentially expose the investor to more market risk than was intended by the original asset allocation decision. Since market risks are non-diversifiable, the investor should decide how much of such risk is wanted.

Residual risks are diversifiable, and thus the investor should be much less concerned about the amount of such risks taken by a particular portfolio manager. Since residual risks are diversifiable, there is no risk premium hurdle required to justify taking such risk. Thus, these are the types of risks that portfolio managers who are hired to add value through active management should be encouraged to take.

## MARKET EXPOSURE AND THE GLOBAL BOND TRADER

Traders and portfolio managers very often try to create trade weights that they believe are market neutral. In the fixed-income markets, common examples are yield curve steepening or flattening trades; butterfly positions, long at two points in the curve and short in between, or vice versa; and international spread trades. In the equity market, examples include being long one stock and short related stocks, or being long one industry and short another. In currencies, a typical example would be a position long one currency against a related basket of other currencies. In each of these cases, the motivation is the same: to capture the special relative value of one security versus another, or one sector versus another, without being exposed to the general risk affecting all related securities.

In the absence of a definition of "market neutral," or even of "the market," traders have improvised many schemes for weighting relative value trades. For example, a standard approach to creating relative value trades in fixed-income markets is to weight the legs in such a way that the total position has zero duration. This weighting is motivated by the observation that with such a position, a small parallel shift in yields has no impact on its value. Of course, the problem with such an approach is that market moves are most often not associated with parallel shifts — typically yield curves steepen in rallies and flatten in sell-offs — and traders with zero duration weights find that their positions have a market-directional bias.

We describe here a general approach to creating market-neutral trade weights, and then compare this approach to some alternatives that are commonly used. While we do not argue that the "market-neutral" weights are necessarily better than alternative weights, we do maintain that for *any* set of weights there exists a set of implied views — i.e., expected excess returns for each security in the trade — for which those weights are optimal.

What is special about the views that motivate a "market-neutral" trade is that they are intended to express a view about the relative value of individual securities or sectors of the market; they are not intended to express a view on the market as a whole. Thus, for a market portfolio, the expected excess returns implied by the relative value trade should be zero. We conclude that the market-neutral weighting, as defined here, is appropriate whenever the desire is to express an opinion about relative value.

### Creating "Market-Neutral" Weights for Relative Value Trades

We define a market-neutral trade as a set of positions for which the aggregate returns are uncorrelated with the returns of the market — i.e., with zero market exposure. Thus, the first step in a procedure to create market-neutral trade weights is to identify the market to which the trade is designed to be neutral. This is actually the most difficult issue.

For example, if the desire is to create a trade that will profit when German bonds outperform French bonds, then it is not clear to which market the trade should be neutral: the German market, the French market, some combination of the two, or a larger market such as all of Europe. There is no right or wrong answer to this question. We have found, however, that in practice a simple definition of the market works best.

For example, in this context we might define the market as an equally weighted combination of French and German ten-year bonds. In most contexts, the simplicity of such a definition far outweighs the potential benefits of a more precise measurement.

Given a defined market portfolio, we can state the condition for market neutrality of a spread trade quite simply, in terms of covariances. By a spread trade, we mean a portfolio of two assets — e.g., the German bond and the French bond in the example above. We will call these two assets "x" and "y." We label the market portfolio "z."

Let $\sigma_{xz}$ be the covariance of asset x and the market portfolio, and let $\sigma_{yz}$ be the covariance of asset y with the market portfolio. Normalizing on the amount of asset x, we solve for a weight, w, of asset y such that the portfolio $[x - (w \times y)]$ has zero covariance with the market portfolio, z. In other words, we solve for a weight, w, such that if we short w units of y for each unit of x we obtain a market-neutral portfolio.

It is apparent that the correct value for w is the ratio of $\sigma_{xz}$ to $\sigma_{yz}$.[3] How would this work in practice?

We start with the problem of constructing a market-neutral portfolio long German ten-year bonds and short French ten-year bonds. We suppose that the portfolio to which we want to remain market neutral is an equally weighted sum of German and French ten-year bonds. Using daily data from February 1, 1988, through October 6, 1995, and downweighting older data at a rate of 10% per month, we estimate the annualized covariance of the German bond with this portfolio to be 35.63. The covariance with the French bond is 42.99.

Thus, using the formula above, we should short 35.63/42.99 = 0.8288 units of market value of French bonds for every unit of market value of German bonds. For example, a position long \$100 million of German bonds should be hedged with a short position of \$82.88 million of French bonds.

More generally, when the market-neutral portfolio that we are trying to construct has more than two assets, there is no unique set of weights. In general, market neutrality imposes a condition, a linear constraint, that the relative weights of other assets in the portfolio must satisfy. Again, we normalize the weights relative to one unit of security x.

Consider a portfolio of assets x and y(1), y(2)...y(n) defined by $[x - \Sigma a_j \times y(j)]$. Again we let z represent the market portfolio. Let $\sigma_{y(j)z}$ represent the covariance of asset y(j) with the market and $\sigma_{xz}$ represent the covariance of asset x with the market. In this case, the condition for market neutrality is that $\Sigma a_j \sigma_{y(j)z} = \sigma_{xz}$. Notice that the situation of two-asset weights is a special case where $a_j$ equals zero for all but one asset.

In many cases, traders may have access to volatilities of individual assets but not to the covariances of those assets with each other or a market portfolio. In the context of a spread trade, we can simplify matters further and avoid this problem if we adopt as the market portfolio a positively weighted average of the two assets where the weights are inversely proportional to their volatilities — that is, if we define the market portfolio to be a linear combination of the assets in which each asset contributes equally to volatility. In this special case, which is actually a rather intuitive definition of the market portfolio, the market-neutral portfolio is also a portfolio in which each asset contributes equally to volatility — except that while the market portfolio is long both assets, the market-neutral portfolio is long one asset and short the other.

For example, in the case of German and French bonds, the volatilities of the bonds are 6.03 and 7.14, respectively. The ratio of German volatility to French volatility is 0.8436. Thus, using a market portfolio with 1.0 unit of German bond market value per 0.8436 unit of French bond market value, the market-neutral portfolio must short 0.8436 unit of French bonds for each unit of German bonds. For a position long $100 million of German bonds, we require a short of $84.36 million of French bonds. Notice that the size of the hedge position is relatively insensitive to the composition of the market portfolio.

As another example, let us consider a butterfly trade in U.S. bonds long the five-year benchmark and short the two-year and ten-year benchmarks. This position will look attractive when the yield curve is unusually curved — i.e., the five-year yield is high relative to a weighted average of the two- and ten-year yields, and the yield curve is expected to straighten. To simplify matters, we consider a market portfolio defined as equal weights of each bond.

The covariances of the two-year, the five-year, and the ten-year bond with this market portfolio are 10.55, 24.84, and 37.48, respectively. Thus, using the formula given above, for each unit of the five-year bond long, we must short a weighted average of two- and ten-year bonds using weights w2 and w10 such that $w2 \times 10.55 + w10 \times 37.48 = 24.84$.

There are many such pairs of relative weights that we could use. For example, we could pick weights to match the value in both sides of the trade — i.e., we could also require that $w2 + w10 = 1$. Using this additional constraint, we would require short positions of 46.94 in two-year bonds and 53.06 in ten-year bonds against a long position of $100 million in the five-year bond. Although this weighting is market neutral, as we shall see, it may not express the relative value view that is desired.

Let us revisit this butterfly trade and consider another possible constraint. Remember that the motivation for the trade is to benefit from a straightening of the yield curve. Thus, in addition to being market neutral, we might also want to insulate ourselves from a flattening or a steepening of the yield curve. A natural way to do this is to weight the trade so that it is also neutral relative to a portfolio constructed to have returns sensitive to such moves.

To accomplish this, we follow a procedure analogous to that used in creating a market-neutral set of weights. First, we construct a market-neutral portfolio long two-year bonds and short ten-year bonds. This portfolio, which we call the "steepening" portfolio, has 10.55/37.48 = 0.2814 unit of ten-year per unit of two-year (based on the covariances reported above).

We then measure the covariances between the individual bonds and the steepening portfolio returns. We find that the two-, five-, and ten-year bonds have covariances of 57.38, 18.70, and −105.51, respectively, with the steepening portfolio. Thus, the "market" and "steepening" neutral portfolio must have weights w2 and w10 such that, as above:

$$w2 \times 10.55 + w10 \times 37.48 = 24.84, \text{ and also that}$$

$$w2 \times 57.38 + w10 \times -105.51 = 18.70.$$

Simple algebra reveals that the appropriate short positions are $101.77 million in two-year bonds and $37.63 million in ten-year bonds against the long position of $100 million in the five-year bond. Notice that neutralizing with respect to steepening or flattening of the yield curve has a substantial effect on the weights.

**Alternative Approaches**

How do these market-neutral weights compare with more familiar trade weightings? We return to the German bond versus French bond example. The most common weighting would be to match the durations of each side of the trade. Given durations of 6.72 and 6.61, respectively, for the German and French bonds, the "duration-neutral" weight in French bonds is $101.66 million against a $100 million long position in German bonds. The problem with such weights, as many traders could testify, is that the French market is generally more volatile than the German market. Thus, such a trade will perform well in sell-offs and suffer in rising markets. That is, such weights are not market neutral.

Another common approach to hedging is to use "regression" weights. This approach is motivated by the desire to find the "best hedge" using French bonds against the long position in German bonds. The best hedge — that is, the position that minimizes volatility — is given by the coefficient in the regression of German bond returns on French bond returns. That regression coefficient is the ratio of the covariance of German and French bonds to the variance of the French bonds. That covariance, using the same data as mentioned above, has a value of 34.91, and the variance of French bonds is 51.07. Thus, the regression coefficient is 0.6835, and the best hedge of the German $100 million position is a short French position of $68.35 million.

Notice, first, that such an approach is not symmetric between the markets. The best hedge of a short position in French bonds using German bonds does not lead to the same relative weights in the markets as does the best hedge of a long position in German bonds using French bonds. In other words (given that the German bond variance is 36.35), the best hedge of a short French bond position of $68.35 is not a long German position of $100 million, but rather a long German bond position of only $65.63 million.

Although it may not be obvious, it turns out that relative to any market portfolio consisting of positive weights in the two bonds, the best hedge of the long German position using French bonds will always create a portfolio with a long market-directional bias. And the best hedge of the short French position using German bonds will always create a portfolio with a short market-directional bias. (We derive this result in Appendix C.)

A third common approach to hedging a spread trade is to match volatility-weighted durations. This approach is motivated by the well-known problem of market directionality observed above with respect to trades that match durations directly. The volatility-weighted duration trade scales the duration of one side of the trade by the relative volatilities of the yield changes in the two markets.

To illustrate using the same example, we find that French ten-year bond yields are 20.6% more volatile than German bond yields (measured in basis points, the German annualized volatility is 89.7, and the French is 108.2). Thus, to remove the market-directional bias, it seems natural to use 20.6% less duration in French bonds than in German bonds. For a position long $100 million of German bonds, we short $84.34 million of French bonds rather than $101.66 million.

Perhaps surprisingly, this approach actually works quite well. Notice first, that in this example, the weight in French bonds is almost exactly what we discovered when we used the return volatility ratio — which was motivated by the simplicity obtained from defining the market portfolio to have equal volatility contributions from each bond. This is not an accident. Yield volatility (measured in basis points) times duration is a reasonably good approximation of return volatility. Thus, the volatility-weighted duration approach will generally be a quite good approximation of a market-neutral weighting — at least relative to this particular market weighting!

We have now gone through several examples of market-neutral weightings of trades and compared them with alternative weightings, but we do not wish to argue that market-neutral weights are always better than alternative weights. Rather, we show in Appendix C that there is a mapping between views — defined as expectations of future excess returns on assets — and optimal weights. Thus, just as we can find an optimal portfolio for any set of expected returns, we can back out a set of implied views for which any given portfolio weighting is optimal. In this sense, we cannot argue that a market-neutral portfolio is better than some alternative weighting without considering the implications of the alternative weights for expected returns.

What we can show in Appendix C is an intuitive result: that a portfolio is market neutral if, and only if, it implies a zero expected excess return on the market portfolio. A portfolio with a positive market exposure in our terms will imply a positive expected excess return on the market, while a portfolio with a negative market exposure will imply a negative expected excess return on the market. Thus, we can make precise the sense in which a market-neutral portfolio may be desirable.

Market neutrality is a desirable condition if, and only if, we do not wish to express a view about the expected excess returns of the market. We would characterize this condition of market neutrality as a minimum condition for a "relative value" trade, and we thus conclude that the use of market-neutral weights, as defined here, is an appropriate consideration for *all* relative value trades.

## SUMMARY AND CONCLUSIONS

We have addressed the measurement and management of market risk from the perspective of the global bond fund manager and the global bond trader. We began by describing the fund manager's (or trader's) problem as seeking to outperform some benchmark portfolio while simultaneously controlling risk. We then discussed how, under ideal circumstances, the manager would determine a set of "risk factors" that would be responsible for the returns of the manager's portfolio. While this ideal is not easily achieved, managers (or traders) can nonetheless identify what is generally the most important risk factor, their portfolio's market exposure.

After discussing the advantages of making broader use of correlations across markets in describing market risk, we demonstrated how the use of our measure of market exposure could improve portfolio performance and risk control. We showed that for a global bond fund manager, our market exposure measure protected a portfolio's performance, even given views that were the exact opposite of actual market moves. In contrasting *market risk* with *residual risk*, we argue that active managers can add value by exploiting opportunities that add residual risk while controlling market risk. One advantage of this approach is that residual risk can be easily diversified, while market risk cannot.

Our conclusion is straightforward. By explicitly accounting for the volatilities and correlations of different assets, our measure of market exposure can result in improved portfolio management.

## APPENDIX A

How Goldman Sachs Uses Market Exposure in Monitoring its Bond Trading Risk

Traders in the fixed-income markets have traditionally measured their exposure to interest rates in terms of duration-equivalent measures. One common approach is to quantify the amount of "ten-year equivalents" in a position — that is, to measure the marginal impact on position value of a small change in interest rates such as a 1-bp move, and then to express that impact in terms of the quantity of the current ten-year note that would have the same sensitivity to rate moves. Thus, over time, traders have become comfortable with a "ten-year-equivalent" unit of account for measuring their exposure.

The shortcomings of a duration-based measure of interest rate exposure for a bond trader are the same as those that apply to a bond portfolio manager. Thus, the benefit of using a statistically based market exposure — accounting for different volatilities and correlations — is clearly applicable. To make the market exposure measure meaningful in the environment of bond trading, however, we clearly need to express it differently from the way we do for portfolio managers.

Bond traders do not think about allocations of percentages of portfolio value to different sectors of the fixed-income markets, and they do not think about a coefficient relating their exposure to that of the market. Because they are often focused on hedging, what they want is to know how long or short they are in terms of a common unit such as ten-year equivalents. For this reason, we define a market portfolio that is simply the current most liquid ten-year bond in each country, and then express a trader's — or a desk's — market exposure not as a coefficient but rather as a quantity of "ten-year equivalents."

The ten-year-equivalent measure of market exposure for a position is simply the quantity of ten-year bonds that has the same market exposure (where the market is the ten-year bond) as does the position itself. Defining the market this way ensures that if a trader has zero market exposure, the returns on his positions will be uncorrelated with those of the ten-year bond. For this reason, a trader using ten-year bonds as a hedge will find that he should sell (or buy back for negative values) the quantity of ten-year bonds equal to his current market exposure, measured in ten-year equivalents, in order to minimize risk.

In many cases, the market exposure measure of ten-year equivalents is very different from the duration-based measure of ten-year equivalents. For example, in Exhibit A we show the Goldman Sachs positions in different sectors on a particular day

**EXHIBIT A**
Amount of Ten-Year Equivalents ($ millions)

| U.S. Bond Market Sector | Duration-Based | Market Exposure-Based |
|---|---|---|
| Less than one-year | −6.0 | −68.3 |
| One- to five-year | 686.3 | 732.4 |
| Five- to fifteen-year | 389.2 | 389.4 |
| Greater than fifteen-year | −782.4 | −944.0 |
| Swaps | −51.1 | −38.3 |
| Total | 236.0 | 71.2 |

according to each of these measures. The duration-based and market-exposure-based measures of position size in each sector are clearly related in terms of magnitude. When summed, however, the duration measure shows the desk to be more than three times longer, relative to the market exposure measure.

Which measure is more accurate? The answer is that they measure different entities. The duration measure quantifies the impact of a parallel shift in all yields, an unlikely scenario. The market exposure measure quantifies statistically an estimate of the impact on the portfolio of the most likely shift in bond yields, conditional on a shift in the ten-year yield.

Because it takes into account the different volatilities and correlations of different securities, market exposure is more likely to quantify accurately the impact of a future shift in yields — and therefore, compared with a duration-based measure, market exposure is a more useful guide for hedging.

## APPENDIX B
Optimal Portfolio Constraints and Weights

Exhibit B-1 shows the maturity sector weights for the benchmark and each of the two optimal portfolios. The table also lists the duration and market exposure for each maturity sector. We calculated market exposures for each sector against the benchmark. The portfolio weights developed using the duration constraint are labeled DP Weights, while the portfolio weights developed with the market exposure constraint are labeled MEP Weights. Durations are as of January 3, 1994.

We obtain the overall portfolio duration and market exposure for each portfolio by taking appropriate weighted averages of the duration and market exposure columns. For example, the duration of the benchmark portfolio is 5.15, while the duration of the duration-constrained portfolio is 5.67 and that of the market exposure-constrained portfolio is 6.53. Similarly, the market exposure of the market exposure-constrained portfolio is 1.10, while that of the duration-constrained portfolio is 1.17.

In developing the optimal portfolios shown in Exhibit B-1, we make several assumptions. In both portfolios, we constrain the tracking error (standard deviation of the excess return over the benchmark) to be 100 bp. We also assume that both portfolios are currency-hedged into U.S. dollars, as is the benchmark. To control exposure to market movements, we constrain the first portfolio's duration to be 10% more than that of the index and the second portfolio's market exposure to be 1.1.

Of course, optimization requires expected excess returns. We use a three-step process to develop excess returns across each market. In the first step, we project yield changes for the seven- to eleven-year (or ten-year for the United States) sec-

**EXHIBIT B–1**
Portfolio Weights, Durations, and Market Exposures

| Country | Sector | Benchmark Weight | DP Weight | MEP Weight | Duration | Market Exposure |
|---|---|---|---|---|---|---|
| France | 1-3 | 1.54 | 0.00 | 0.00 | 1.54 | 0.23 |
| | 3-7 | 2.66 | 0.00 | 0.00 | 3.87 | 0.78 |
| | 7-11 | 2.20 | 0.00 | 0.00 | 6.62 | 1.49 |
| | 11+ | 1.30 | 0.00 | 0.00 | 11.14 | 2.36 |
| Germany | 1-3 | 3.04 | 1.19 | 9.95 | 1.74 | 0.25 |
| | 3-7 | 5.29 | 0.00 | 0.00 | 3.87 | 0.72 |
| | 7-11 | 3.75 | 40.83 | 33.53 | 6.26 | 1.30 |
| Japan | 1-3 | 3.92 | 0.00 | 0.00 | 1.81 | 0.14 |
| | 3-7 | 7.97 | 5.15 | 0.00 | 4.44 | 0.38 |
| | 7-11 | 7.80 | 12.27 | 14.51 | 6.90 | 0.53 |
| | 11+ | 2.86 | 0.00 | 10.36 | 10.95 | 0.51 |
| U.K. | 1-3 | 0.64 | 0.00 | 0.00 | 1.74 | 0.37 |
| | 3-7 | 2.32 | 1.10 | 0.00 | 3.76 | 0.87 |
| | 7-11 | 1.59 | 0.07 | 0.00 | 6.26 | 1.56 |
| | 11+ | 2.36 | 0.00 | 0.00 | 8.87 | 1.94 |
| U.S. | 1-3 | 17.60 | 0.00 | 0.00 | 1.81 | 0.46 |
| | 3-7 | 14.82 | 21.86 | 17.13 | 3.76 | 1.05 |
| | 7-10 | 4.54 | 15.79 | 0.00 | 6.15 | 1.69 |
| | 10+ | 13.78 | 1.80 | 14.52 | 10.18 | 2.27 |

**EXHIBIT B-2**
Excess Returns

| Country | Sector 1-3 | 3-7 | 7-11 | 11+ |
|---|---|---|---|---|
| France | −2.97 | −9.17 | −14.53 | −21.08 |
| Germany | −1.90 | −6.99 | −10.32 | — |
| Japan | −1.18 | −4.24 | −6.01 | −6.17 |
| U.K. | −3.34 | −8.18 | −13.17 | −15.37 |
| U.S. | −3.89 | −7.05 | −9.45 | −11.14 |

tor in each market. For France and Germany, we assume that yields in the seven- to eleven-year sector decline by 10 and 20 bp, respectively, while we project that Japanese yields will not change from their January 1994 levels. In the United Kingdom and the United States, we project yields to decline by 5 and 13 bp, respectively.

In the second step, we use the projected yield changes in the seven- to eleven-year (or U.S. ten-year) sector to determine projected excess returns for the sector. Finally, we combine the expected excess returns for the seven- to eleven-year sector with the correlation matrix of excess returns (as of January 1994) to develop excess returns for each remaining sector.[4] We combine these projected excess returns with the market capitalization weights to obtain the projected returns across each market discussed in the body of the article.

Exhibit B-2 shows the actual excess returns for 1994 (i.e., returns over cash) for each maturity sector. We calculate excess returns for each of the portfolios (including the benchmark) shown in Exhibit B-1 by combining the weights in Exhibit B-1 with the excess returns of Exhibit B-2. Using this procedure, we find the excess return on the benchmark over 1994 to be −7.20, while the excess return on the duration-constrained portfolio is −8.52, and the excess return on the market exposure portfolio is −7.99.

## APPENDIX C
The Mapping Between Views and Portfolio Weights

Even in its simplest form, portfolio optimization is generally considered mathematically as a quadratic optimization subject to linear constraints. When we wish to consider the mapping between views and portfolio weights, though, we can solve for a unique mapping only in the case where constraints are not binding. We generally make this assumption in solving for implied views, which allows us to simplify the problem considerably.

Let us assume that a portfolio manager has a set of expected excess returns given by the vector $\mu$ for a given set of assets. Suppose the covariance matrix of those assets is given by $\Sigma$. Then, if we assume that no constraints are binding, the optimal portfolio weights — those that provide the greatest expected excess return for a given degree of risk — are proportional to a vector w, where $w = (\Sigma)^{-1}m$. Of course, these weights are not unique unless we specify a particular level of risk.

Clearly, we can invert this mapping, at least up to a scale factor. That is, given a set of weights, w, and assuming that no constraints are binding, we can solve for a vector $\mu$, where $\mu = \Sigma w$. This vector, and all positive scalar multiples of it, will provide a set of expected excess returns such that the given portfolio weights, w, are optimal relative to those views.

The first point to make is that a portfolio is market neutral if, and only if, the implied view of the return on the market portfolio is zero. This follows directly from the formula for implied views. Let the market portfolio weights be given by a vector, m. A market-neutral portfolio is one for which the covariance of the returns of the portfolio with the returns of the market portfolio is zero. This covariance is given by the expression $[m'\Sigma w]$. Notice that the expected return on the market portfolio is given by $[m'\mu]$. By the expression for $\mu$, it is clear that this expected return on the market portfolio is also given by the expression $[m'\Sigma w]$. Thus, the result follows.

The next point concerns the implied view of a risk-minimizing spread portfolio, relative to a market portfolio that includes positive weights in each of the two assets. As above, for a given portfolio, w, the implied expected excess return on the market is given by $[m'\Sigma w]$. Here we assume that the market weight vector, m, consists of two positive weights, $m_1$ and $m_2$, and that the portfolio weights in the trade are given by $w_1$ and $w_2$, where either $w_1 = 1$ and $w_2 = -\beta$ (in which case we wish to show that the implied view on the market is positive), or $w_1 = -1$ and $w_2 = \beta$ (in which case we wish to show that the implied view on the market is negative). The regression coefficient, $\beta$, is given by the ratio $\sigma_{xy}$ to $\sigma_{yy}$ where again we adopt the notation x and y to represent the first and second assets, respectively.

First consider the case in which the weights are $w_1 = 1$ and $w_2 = -\beta$. The form $[m'\Sigma w]$ can be written as a sum, $m_1 \times \sigma_{xw} + m_2 \times \sigma_{yw}$. Here we use $\sigma_{xw}$ and $\sigma_{yw}$ to represent the covariances of the returns of asset x and asset y, respectively, with the trade returns using the weights, w.

To demonstrate that this sum is positive, we show first that $\sigma_{yw}$ is, by construction, zero. We then show that $\sigma_{xw}$ is non-negative and can be zero only if the two assets have a correlation of 1.0, which, because they are distinct, we assume is not the case. That $\sigma_{yw}$ is zero will not surprise those familiar with regression theory. This is the covariance of a variable with the residuals of a regression on that variable.

In particular, given the weights in the spread trade, the expression for this covariance is given by $\sigma_{yw} = (\sigma_{xy} - \beta \times \sigma_{yy})$. Substitution for $\beta$ leads to the result. Similarly, if we write out the expression for $\sigma_{xw}$, we obtain $\sigma_{xw} = (\sigma_{xx} - \beta \times \sigma_{xy}) = \sigma_{xx} \times (1 - \rho^2)$, where $\rho$ is the correlation coefficient between x and y. Thus, we see that the implied expected excess return on the market portfolio must be positive. The case where the signs are reversed follows similarly, with the conclusion that regression hedging will always create a positive implied view on the market when the variable being hedged is long, and a negative view on the market when the variable being hedged is short.

## ENDNOTES

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[1]While market participants often speak of "market exposure" in a general sense when discussing risk, we use the term here in the precise sense of our statistical measure of risk.

[2]The regression coefficient, in this simple context, is the covariance of these returns divided by the variance of the market return.

[3]Let $p = [x - (w \times y)]$. The covariance between p and the market, z, is given as $\sigma_{pz} = \sigma_{xz} - w\sigma_{yz}$. We want $\sigma_{pz} = 0$, so we want $\sigma_{xz} - w\sigma_{yz} = 0$. Solving for w gives us $w = \sigma_{xz}/\sigma_{yz}$.

[4]The procedure for implementing this calculation is described in Fischer Black and Robert Litterman, "Global Asset Allocation With Equities, Bonds, and Currencies," Goldman, Sachs & Co., October 1991.