FINM 326: Computing for Finance in C++

Lecture 1

Chanaka Liyanarachchi

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Course Info

- Instructor:
  Chanaka Liyanarachchi (chanaka@uchicago.edu)

- Teaching Assistants:
  1. Johnny Roches (jaroches@gmail.com)
  2. Alex Kay (akay@uchicago.edu)
  3. Viren Desai (vdd@uchicago.edu)

What We Cover

- This course is about using C++ to solve computing problems in Finance.
- We learn:
  1. C++ and related programming concepts.
  2. Writing *good quality* code using:
     - appropriate features and techniques
     - good programming practices
  3. Use them to write applications in finance.
- Target audience:
  1. Group 1: No prior C++/programming experience.
  2. Group 2: Have some C++ experience, but you want to learn more and improve.
- Tentative course schedule posted. Take a look to make sure this is a suitable course for you.
  - We won't cover everything about C++ in this course.
  - This course will not make you a "professional C++ developer".
  - We will build a solid foundation on the fundamentals of programming, using C++.

Why C++?

That was a deliberate policy to have the development of C++ problem-driven rather than imitative. – Bjarne Stroustrup

- C++ is very popular and widely used in many application fields, including Finance.
- Huge demand for C++ skills.
- Required skill for all, not just programmers.

Course Structure

- We will learn C++ using a practical and problem-driven approach.
- Hands-on and practical course.
- We learn things step-by-step.

- We use many tools. Each component is important and connected to one another:
  - Lectures and notes
  - Live coding sessions and demos
  - Discussions: This is a discussion oriented class
  - Assignments/homework to practice/experiment what we learn.

Office-hours and TA Sessions and Other Resources

I highly encourage everyone to ask questions.

- Instructor office-hours:
  - Fridays before class (approx. 5 PM CT)
  - Sundays, 6 PM CT (virtual)
- TA Sessions (virtual):
  - Saturdays, 6 PM CT
  - Weekday - announced later
- Pl. make use of office-hours, TA sessions as much as possible so we can use the class time efficiently.
- Other resources:
  - Email
  - Canvas discussion forum

Assignments and Exams

- 5 Required/graded assignments (each one has equal weight). Graded assignments are to be completed individually.
- Exams (more details to follow, after week 3):
  - Mid-term: Feb 3, 6-9 PM CT (Week 5)
  - Final: March 10, 6-9 PM CT (Week 10)
- Should use the concepts discussed in class to get full credit.
- Evaluated based on completeness, correctness, finding the best solution (within what we have discussed), use of correct language constructs, good program design and quality of code.
- Academic honesty: Read: MSFM Academic Honesty Attestation.pdf

Final Grade

Final grade weights:

1. Graded assignments 30%
2. Mid-term 25% ; Final 45%
3. Bonus points (up to 3 points max) awarded to encourage active student participation and engagement that improve the learning experiences of all students:
   - Awarded at the instructor's discretion ONLY
   - Participate class discussions regularly, in a meaningful way.
   - Provide (meaningful) feedback regularly.
   - Share relavent interview questions and experiences.

Final Grade cutoffs: per program guidelines

Reference

- The Internet
  - `www.cplusplus.com`
  - `www.cppreference.com`
  - Microsoft Developer Network `www.msdn.com`
  - `www.stackoverflow.com`
  - and, more ...

- Text books NOT required.
- Following are recommended as sources of further study:
  1. C++:
     - Bjarne Stroustrup (2014). *Programming Principles and Practice Using C++* (2^{nd} Edition). Addison-Wesley.
     - Bjarne Stroustrup (2013). *The C++ Programming Language* (4^{th} Edition). Addison-Wesley.
     - Stanley Lippman, Josee Lajoie and Barbara Moo (2012). *C++ Primer* (5^{th} Edition). Addison-Wesley.
     - David Vandervoorde and Nicolai Josuttis. *C++ Templates - The complete guide.* Addison-Wesley.
  2. C++ Standard Library:
     - Nicolai M Josuttis (2012). *The C++ Standard Library* (2^{nd} Edition). Addison-Wesley.
  3. Good practices:
     - Scott Meyers (2005). *Effective C++*. Addison-Wesley.
     - Herb Sutter (2000). *Exceptional C++*. Addison-Wesley.
  4. Finance:[1]
     - John Hull. *Options, Futures and Other Derivatives*. Addison-Wesley.

[1] This book covers finance theory of applications we use in this course.

Getting Started

Programming Environment

- We use Visual Studio on Windows as our primary programming environment:
  1. Highly user friendly and feature rich programming environment suitable for the beginner and experienced programmer, alike.
  2. Free.
  3. Works on Windows (Most popular O/S among our students).
- We use CLion (works on Windows/Linux/MacOS) as a secondary environment.
- Other development environments (e.g. Xcode) are not discussed. If you decide to use any other development environment, you have to learn it on your own.

Obtaining Software

- Visual Studio 2022 (Community version): `https://visualstudio.microsoft.com/downloads/`
- CLion: `https://www.jetbrains.com/clion/download/`
- See *Annoucements* for details.
- Need help? Contact the TAs.

Getting Started with Visual Studio/CLion

Getting Started

- Creating/writing an application using CLion/Visual Studio involve:
  1. Create a project and write code
  2. Organize code (a project/program may use more than one source file)
  3. Build the application
  4. Run the application
  5. Test to make sure it is working correctly
  6. Debug the application
- We will learn how to do these steps next...

Getting Started: Hello World Program

- We will create a very simple project/program to illustrate how to use Visual-Studio/CLion.
- The program below writes a message, "Hello, World!", to console:

```
#include <iostream>

int main()
{
    std::cout << "Hello, World!" << std::endl;

    return 0;
}
```

Our First C++ Project

- First, we create, build and run this program in Visual Studio/CLion.
- After that, we will explain this program step-by-step and learn C++.
- Steps for Visual Studio are shown in Appendix A; CLion in Appendix B.

Exercise

1. Use the steps shown above to create an application/project in Visual Studio/CLion.
2. Code *Hello World* example. **NOTE: C++ is case sensitive**
3. Build the application.
4. Run it.
5. Change the program to print a different message to Console.

Hello, World Program: Line by Line

```
int main()
```

- `main()` is a special *function*.
- It is the program *entry point*.
- Every application/program should have:
  - a `main()`.
  - we can have only one `main()`.
- We do not pass any arguments to this function.
- This function returns an integer (int) value.
- Note: We can have two different function signatures for `main` function:
  1. `int main()`
  2. `int main(int argc, char* argv[])`. We will discuss the use of the second function signature later.

- The line below shows the body of our main function.

```
{
   std::cout << "Hello, World!" << std::endl;

   return 0;
}
```

- A function body starts with `{` and ends with `}`.
- Body has 2 statements; each statement ends with a semicolon (";").
- First line *writes* a message, "Hello, World!", to console.
- Here we use some *features* from the C++ Standard Library.
- After that, the `main()` returns 0 (an integer value)[2].
- We use the `return` to return from a function. More on this later.

[2] We don't have to return a value in main function. Return 0 (zero) is implied.

The C++ Standard Library: A First Look

The C++ Standard Library

- The C++ Standard Library is a collection of very useful features.
- Today, we briefly look at the features we use in our first *Hello World* example.
- The Standard Library is divided into several *sections* (more precisely headers), based on functionality.
- Each header has a name.
- We need to *include* the appropriate header in order to have access to its components.
- The functionality to handle output to console is defined in the `iostream` header.
- To include it, we type:

  ```
  #include <iostream>
  ```

- We use *angle brackets* to include a header not in our project, e.g. a header from the Standard Library. More on this later.

The Standard Input and Output

- Terminology:
  - Input: a sequence of byte flow coming into the computer memory from an input device (e.g. keyboard, file).
  - Output: a sequence of byte flow from the memory to an output device (e.g. console, file).
  - Stream: a sequence of characters (bytes) read from an input device or written to an output device.
- The `iostream` section of defines objects to read and write a *stream* to *Standard I/O* channels:
  - `cin`
  - `cout`
  - and more..

Standard Output

`cout` (pronounced as *see-out*):

- Known as the **standard output**.
- Attached to the standard output device, which usually is the console
- Uses the stream insertion operator, <<:

```
cout << "Hello, World";
```

Standard Input

`cin` (we pronounce it as *see-in*):

- Known as the **standard input**
- Attached to the standard input device, which usually is the keyboard
- Uses the stream extraction operator (>>), to read a stream from the standard input:

```
int x;
cin >> x;
```

Namespace

- Namespaces allow us to group functions and classes using a name.
- Names (functions, classes) in the C++ Standard Library are defined in the `std` namespace.
- We have to use the fully qualified name to use them.
- Example: We saw the use of `std::cout`.
  - `std` is the namespace.
  - `::` is the scope resolution operator in C++.
  - `cout` is the object in this case.

Using Namespaces

- Having to write the namespace every time we use a function is a bit cumbersome.
- There are different ways to overcome this problem.
- Technique 1: qualify individually, e.g. cout to mean std::cout

```
#include <iostream>
using std::cout;
using std::endl;

int main()
{
   cout << "Hello, World!" << endl;
}
```

- Technique 2: allows us to access all names from std namespace

```
#include <iostream>
using namespace std;

int main()
{
   cout << "Hello, World!" << endl;
}
```

- If we're not careful, the second technique can lead to name clashes in large programs (where the same name is used in more than one namespace, example later).
- I encourage everyone to use the first technique.
- Note:
  - I may not show every namespace in subsequent slides after they are introduced.
  - E.g. From now on, I will just write `cout` (without `using std::cout`).

Data Types

Data Types

- We write programs to handle/manipulate data.
- We use variables to store data.
- A program need to store different *kinds/types* of data.
- In C++:
  - Every variable has to have a type.
  - The type of the variable has to be declared before it is used.
  - Once a type is assigned to a variable, it cannot be changed.
- Data types in C++ belong to two main categories:
  1. Fundamental data types
  2. User defined data types
- Fundamental data types are defined in the C++ language.
- C++ also allows the user to define types (classes).

Fundamental Data Types

Numerical Types

- Numerical values can be whole or real numbers: e.g. 1 or 1.2
- For whole numbers we have several types:
  - short: uses 2 bytes
  - int: uses 4 bytes
  - long: at least 4 bytes
- Signed and unsigned:
  - unsigned types can store positive numbers only
  - signed types can store both positive and negative numbers

Type	Size (Bytes)	Value Range
unsigned short	2	0 to 65,535
short	2	-32,768 to 32,767
unsigned int	4	0 to 4,294,967,295
int	4	-2,147,483,648 to 2,147,483,647

- For real numbers we have two types (in C++):
  - float : uses 4 bytes
  - double : uses 8 bytes
- As a result, max and min values each can store are different.

Type	Size (Bytes)	Value Range
float	4	$\pm$ 3.4 $E^{\pm 38}$
double	8	$\pm$ 1.7 $E^{\pm 308}$

- We can use them to store both negative and positive real numbers.
- Remember: only a very few real values can be stored exactly.

More Types

- Type to hold a true/false (flag) value
  - `bool`
- Type to hold a single character
  - `char`
- Reference: `https://docs.microsoft.com/en-us/cpp/cpp/data-type-ranges?view=msvc-170`

- Type to store a string of characters
  - `std::string`
  - `std::string` is NOT a fundamental data type
  - defined in the C++ Standard Library
  - defined in the `string` header (`<string>`)

C++ Operators

Operators for Fundamental Data Types

Listed below are some operators defined for fundamental types:

- Assignment operator $(=)$:

```
a = 5;      //a is an int
b = 1.5f;   //b is a float
c = 1.4;    //c is a double
d = 'a';    //d s a char
e = true;   //e is a bool
```

- Arithmetic operators $(+, -, *, /)$:

```
a  =  a + 1;
b  =  b - 1.2f;
c  =  c * 2.1 ;
c  =  c / 2.2;
```

- Comparison operators $(<, >, <=, >=)$:

```
a > 3;
a < 3;
a >= 3;
a <= 3;
```

Comparison operators return a bool (true/false) value.

- Equality (==):

  ```
  a == 3;
  ```

- Non Equality (! =) :

  ```
  a != 3;
  ```

- Logical AND (&&) : Returns true if both operands are true; false otherwise[3]

  ```
  a == 3 && d == 'a';
  ```

- Logical OR (||) : Return true if either or both operands is true; false otherwise

  ```
  a == 3 || d == 'a';
  ```

- Modulo % (to find the remainder from integer division):

  ```
  int x = 7 % 3;
  ```

[3] https://en.wikipedia.org/wiki/Truth_table#Logical_conjunction_(AND)

Operators: Short-cuts

We have some short-cuts; e.g. For an `int` value `i`:

- Increment:

```
i++;      // same as, i = i+1;
i += 17; // same as, i = i+17;
```

- Decrement:

```
i--;      // same as, i = i-1;
i -= 21; // same as, i = i-21;
```

- Multiplication:

```
i *= 13; // same as, i = i*13;
```

- Division:

```
i /= 42; // same as, i = i/42;
```

Prefix and Postfix

C++ supports prefix and postfix increment and decrement operators.

- Postfix:
  - increment x and return old x:

    ```
    int x = 3;
    int y = x++;
    ```

  - now, `y == 3` and `x == 4`
- Prefix:
  - increment x and return new value

    ```
    int x = 3;
    int y = ++x;
    ```

  - now, `y == 4` and `x == 4`

Functions

C++ Functions

- We saw the `main()` function – every application has to have a `main()`.
- We do NOT write the entire program in the main function.
- It is easier to solve a problem by breaking it up into smaller tasks/functions:
  - We write a function to do a certain task.
  - Combine functions to solve the problem.
- Using functions has several advantages.

- First, let's learn how to write functions in C++.
- Example: we can write a function to display a greeting message on the screen:

```
void DisplayGreeting()
{
    cout << "Hello World!" << endl;
}
```

- A C++ function:
  - has a name
  - takes zero or more arguments
  - does something in the function body
  - returns some value of a certain type
  - if a function does not return anything, its return type is `void`

Functions: Example 1

- Our first function writes the greeting, "Hello World!" to the screen:

```
void DisplayGreeting()
{
   cout << "Hello World!" << endl;
}
```

- We can call this function from the `main()` or any other function:

```
int main()
{
   DisplayGreeting();
}
```

Functions: Example 2

- How would you write a function to add two integers?

```
int add(int a, int b)
{
  return a+b;
}

int main()
{
  int result = add(2, 3);

  cout << " Result :" << result << endl;

}
```

Functions: Advantages

1. Better/simpler solution: divide the problem into smaller tasks.
2. Better code structure: improve readability/clarity.
3. Reuse: once a function is written, we can use it any number of times.
4. Maintainability: we write a function once, if we want to make a chance, fix a bug, we can do it just in one place.
5. Type safety (avoiding type related errors):
   - Types of arguments must be compatible with the types declared in the function.
   - Program will not compile if there's a mismatch.

Parameter Overloading

- Overloading allows 2 or more functions to have the same name:
  - they must have different input argument **types**
  - they cannot differ only by the return type
- Allows us to write more than one method to do some task in a different way, using different inputs.
- For example, we can write another `add()` function as:

```
double add(double a, double b)
{
  return a+b;
}
```

Implicit Type Conversions

- Suppose we have the following function (i.e. it is the only add function we have):

```
double add(double a, double b)
{
  return a+b;
}
```

- What happens if we use it to add two integer values:

```
int main()
{
   int x = 2;
   int y = 3;
   cout << add(x, y) << endl;
}
```

- Question: does this program compile (build)?

- Yes, this program compiles:
  - Compiler converts the int to double (implicitly) in this case.
  - Size of int: 4 bytes; double: 8 bytes.
  - We can safely store any int value in a double.
- Conversion from a smaller (4 byte) to a larger (8 byte) type is called a *widening, or, promotion*
- Promotions are safe implicit conversions.

- Now, suppose we have the following function (i.e. it is the only add function we have):

```
int add(int a, int b)
{
  return a+b;
}
```

- What happens if we use it to add two double values:

```
int main()
{
   double x = 2.2;
   double y = 3.3;
   cout << add(x, y) << endl;
}
```

- Will this program compile (build)?

- Yes, this program also compiles:
  - Compiler converts the double to int values implicitly in this case
  - Size of double: 8 bytes; int: 4 bytes
  - We cannot safely convert every double value to an int
- Conversion from a larger type to a smaller type is known as *narrowing*.
- Narrowing can be dangerous, and results in a build *Warning*.
- Why is it not an error?

- Why it is not an error:
  - C++ expects users to know what they are doing.
  - You can easily shoot yourself in the foot if we're not careful.
- Lesson: Pay Attention to Build Warnings
- We can use project properties to treat warnings as errors (more on this later).
  - Visual Studio: Properties ⇒ C/C++ ⇒ General ⇒ Treat Warnings as Errors⇒ Yes
  - CLion:
    - CLion compiler-flags: `https://www.jetbrains.com/help/clion/cmake-profile.html#compiler-flags`
    - gcc options: `https://gcc.gnu.org/onlinedocs/gcc/Option-Summary.html`
    - E.g. for gcc compiler, add the following line in your CMakeLists.txt script:
      `set(CMAKE_CXX_FLAGS "${CMAKE_CXX_FLAGS} -Werror -Wconversion")`

Casting

- Explicit type conversions are known as *casting*.
- C++ supports several types of casts (known as *named casts*).
- We'll look at `static_cast` now:
- What's the result from integer division below?

```
int a = 3, b = 2;
cout << a / b << endl;
```

- We must explicitly convert at least one operand to a real value to get the correct answer.
- `static_cast`:
  - Used to force conversions, e.g. `int->double`

    ```
    int a = 3; int b = 2;
    cout << static_cast<double>(a) / b << endl;
    ```
  - `static_cast` returns a double value in this example.

The const keyword

- The `const` keyword is used to define a constant value (i.e. value does not change):
  - clearly communicates the intent to the reader
  - prohibits unintentional changes
- If you try to change the value of a `const` member, the compiler catches it.

```
const double pi = 3.14;

pi = 4.5;  <= This is an error, compiler will catch this error
```

- This is another example of the benefits of type safety in C++.
- Following notation is also equally correct:

```
double const pi = 3.14;
```

- We will discuss more uses of the `const` keyword later in the course.

Type Alias

- We use type aliasing to define more meaningful names for existing types (i.e. type aliasing) to make code:
  1. clear and readable
  2. less error prone
- Consider the function declaration below which calculates the Black Scholes price of an Option.

```
double CallPrice(double s,
                 double k,
                 double r,
                 double v,
                 double t);
```

- Error prone: it is not clear what the arguments are
- Instead, we can use a type alias:
  1. typedef
  2. using
- typedef:

```
typedef double StockPrice;
typedef double Strike;
typedef double Expiration;
typedef double Rate;
typedef double Volatility;
```

- using:

```
using StockPrice = double;
using Strike = double;
using Expiration = double;
using Rate = double;
using Volatility = double;
```

- Now we can write:

```
double CallPrice(StockPrice s,
          Strike k,
          Expiration t,
          Rate r,
          Volatility v);
```

- This is an example of writing "clear and readable" code.

Comments

- We can write comments using C style or C++ style.
- C Style:

```
/* This is a C-style comment.
    Useful when we comment ...
    more than one line.
*/
```

- C++ Style:

```
// This is a C++ style comment
// Each line has to be commented ..
// when we use this style.
```

- Use good comments to improve readability and clarity.

.cpp Source and *.h Header Files

- In practice we never write all code in one file:
  - Difficult to read a very large file (with thousands of lines of code)
  - Doesn't promote reuse
  - Build process takes longer
- Solution is to use separate files:
  - .cpp source files
  - .h header files

- Let's use the add functions as an example:
  - We will implement the `add()` functions in a separate file, let's name it Add.cpp.
  - When we move the add functions to a new file our program won't build.
  - The `main()` function doesn't know about the `add()` functions.
  - We need to declare a function before we can use it in another source file.
  - We could add the following two declarations before we use them in main:

    ```
    int add(int, int);
    double add(double, double);
    ```

  - It works, but this solution is not ideal: it requires us to have these declarations in every file we use them.

- Proper C++ way:
  - Write the function declarations in a header file (e.g. Add.h):

    ```
    int add(int, int);
    double add(double, double);
    ```

  - *Include* it in source files, using `#include` preprocessor directive:

    ```
    #include "Add.h"
    ```

  - We use " " to include headers in the same project.

C++ Build Process

C++ build process uses several steps:

1. Preprocessing:
   - Uses the preprocessor
   - The preprocessor handles the preprocessor directives (e.g. `#include`)
   - The output of this step is a C++ file without preprocessor directives
2. Compiling:
   - Uses the compiler.
   - Checks syntax.
   - Checks type safety.
   - Does other things (e.g. implicit type conversions). More on this later.
   - Can do computations (we won't discuss this topic in this course, but we will talk about it in my HPC course).
   - Each .cpp file is compiled to create an object file (.obj).
3. Linking:
   - Uses the linker
   - Creates the executable (.exe) file using the object files

- We can have build errors in each stage.
- Having an idea about each stage will help us fix build problems (more on this later).

Lecture 1: Key Points

- Creating a project, using .h and .cpp files to structure code
- Fundamental types and operations.
- Functions.
- Writing clear/readable code.
- Build process - first look.
- Understand (some) things compiler can do.
- Should pay attention to warnings.
- How to submit an assignment.

Assignment 0 (Submission Optional)

Goals:

1. To make sure everyone has a C++ development environment.
2. Learn the proper structure of a C++ program.
3. Use fundamental types, operators and functions.
4. To make sure everyone knows how to submit an assignment.

Problem:

1. Write a function to add two integers.
2. Read any two integer values from the keyboard.
3. Add them using the function you wrote.
4. Write the result to console.

Submission steps:

- Clean the solution using *Clean Solution* under *Build* tab (in Visual Studio/CLion).
- Compress (e.g. zip on Windows) the folder containing the project.
- Use *Assignments* -> *Assignment 0* link in canvas to upload the compressed/zipped file.

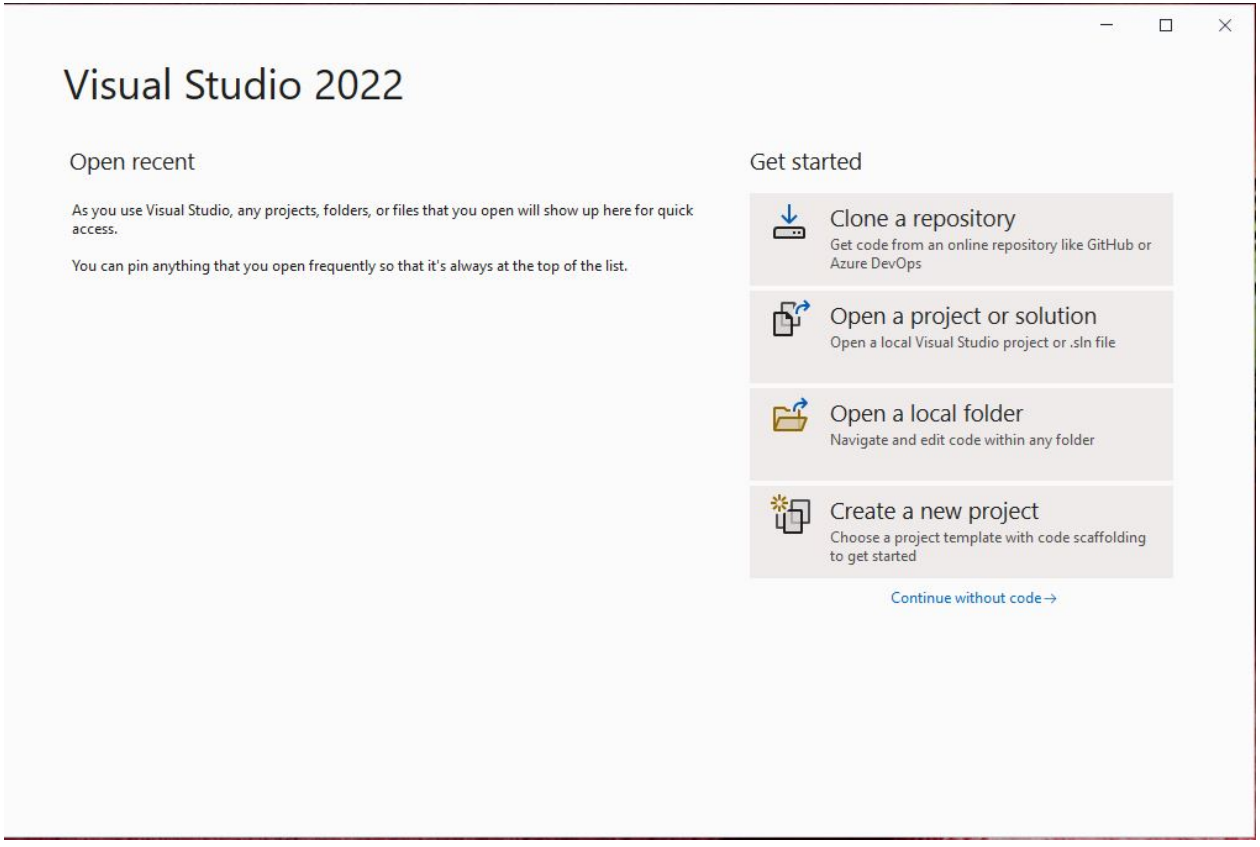
- PI. use following guidance on how to name your solutions:
  - Visual Studio users: Name the solution using the following format. "Lastname_Firstname_Assignment#".
  - CLion Users: Name the project using the following format. "Lastname_Firstname_Assignment#".
- This is not a requirement, but it helps TAs sort all of the source code that gets submitted.

- Individual Assignment.
- Due: January 13 by 6 PM (Central Time).
- Points: 0 (Zero)

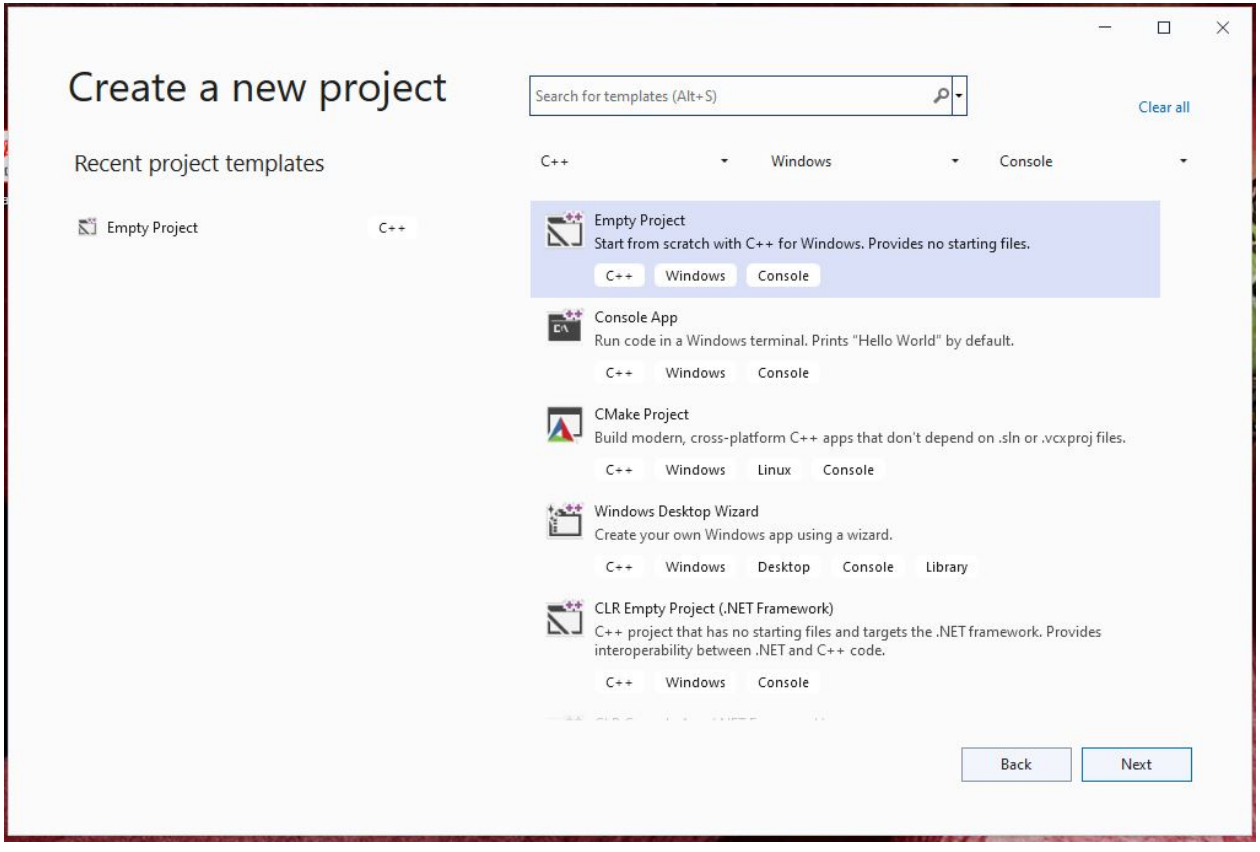
Appendix A: Getting Started with Visual Studio

Steps

- Step-by-step instructions to create an application are shown below.
- Start Visual Studio. Select *Create a new project.*

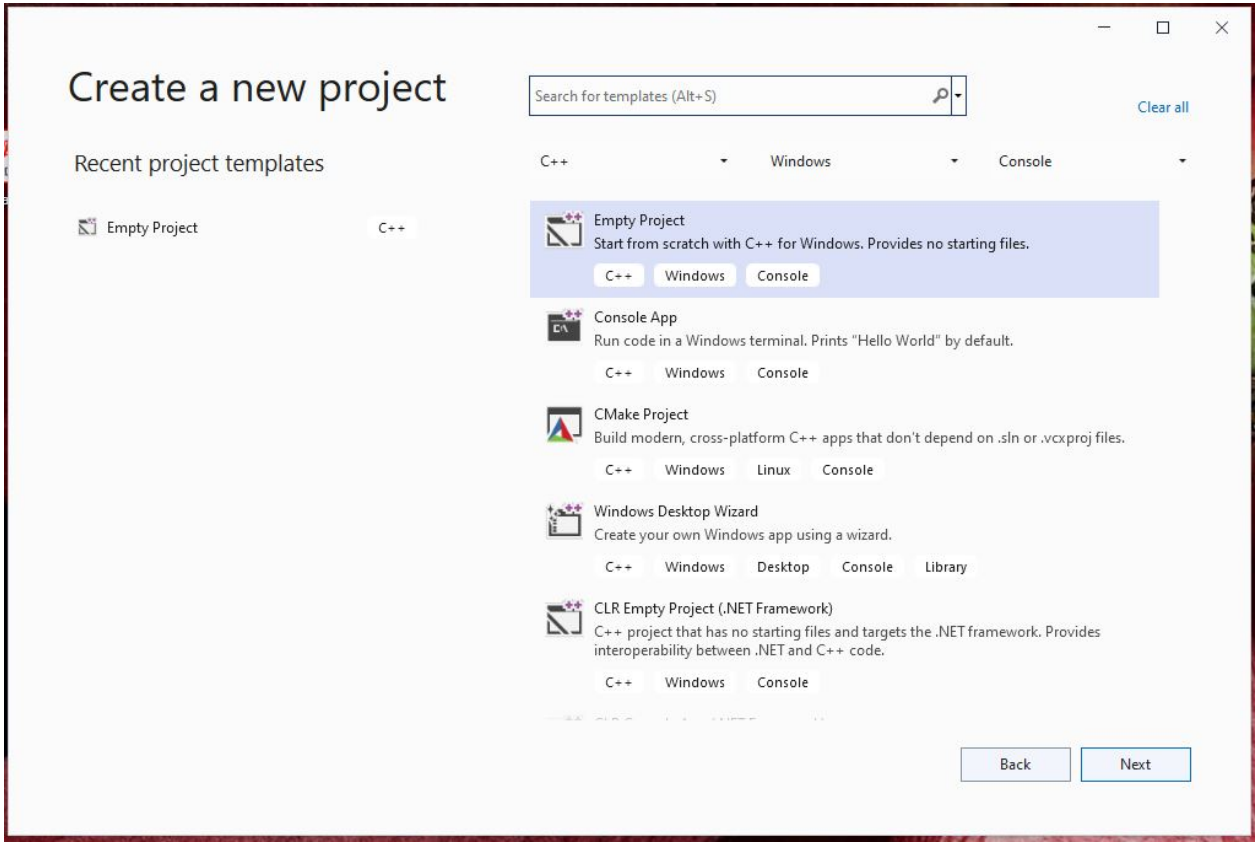


- Select "C++" -> first and then "Empty Project"

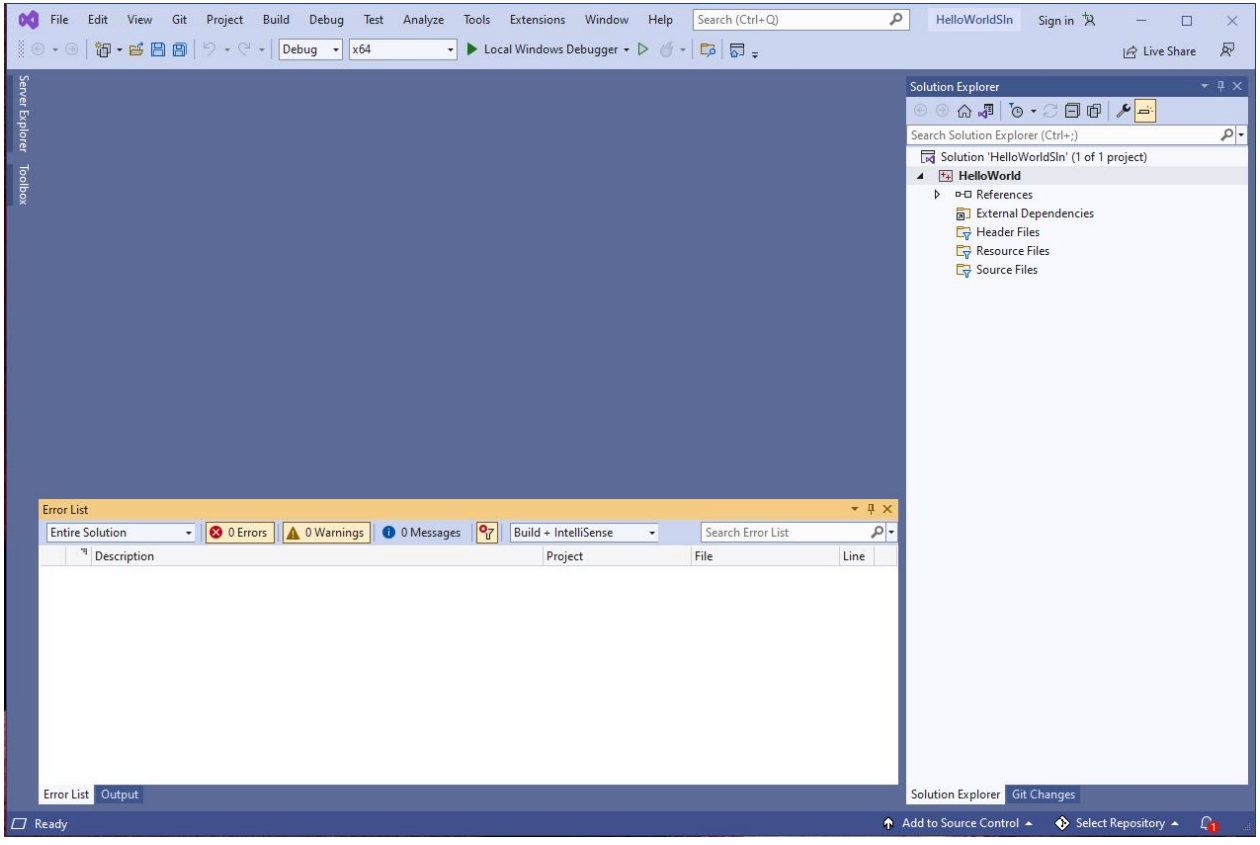


Enter:

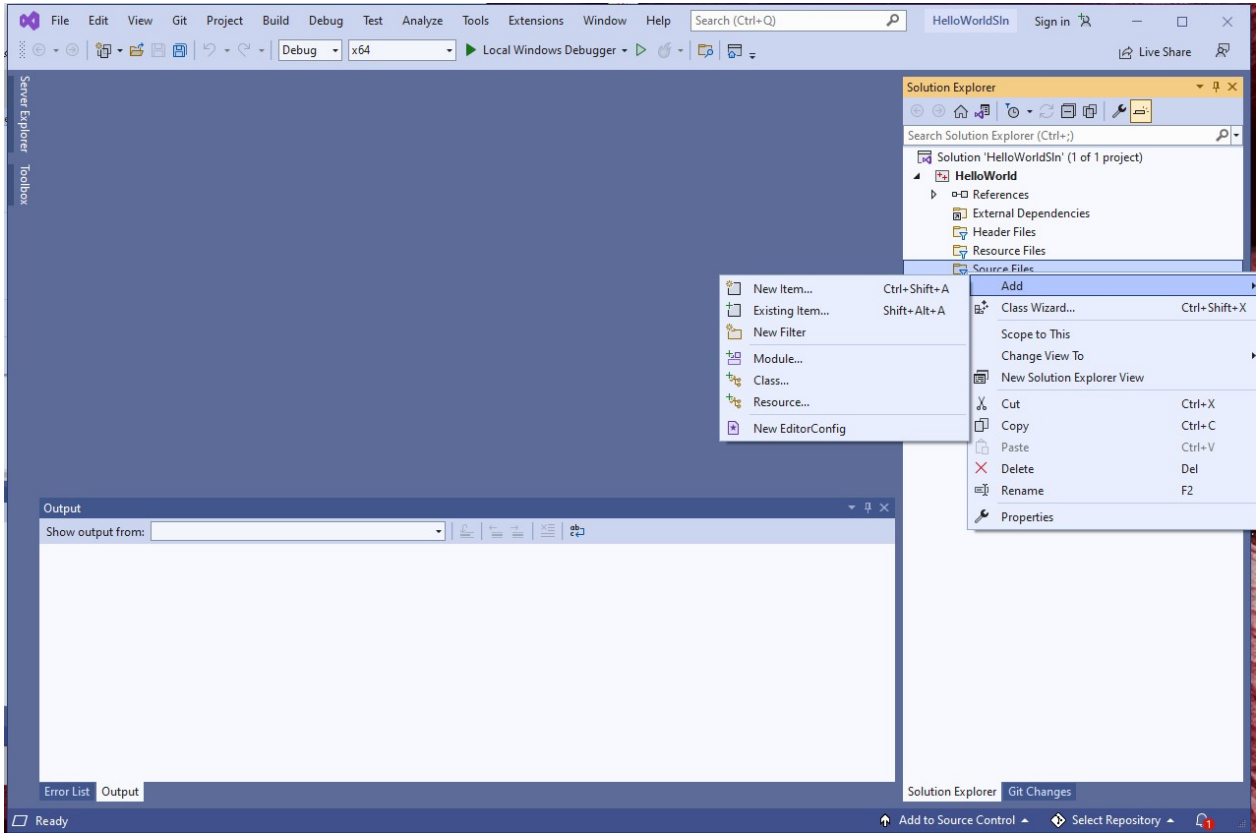
1. (project/application) name
2. directory location for the project
3. solution (place holder for one or more projects) name for the project.



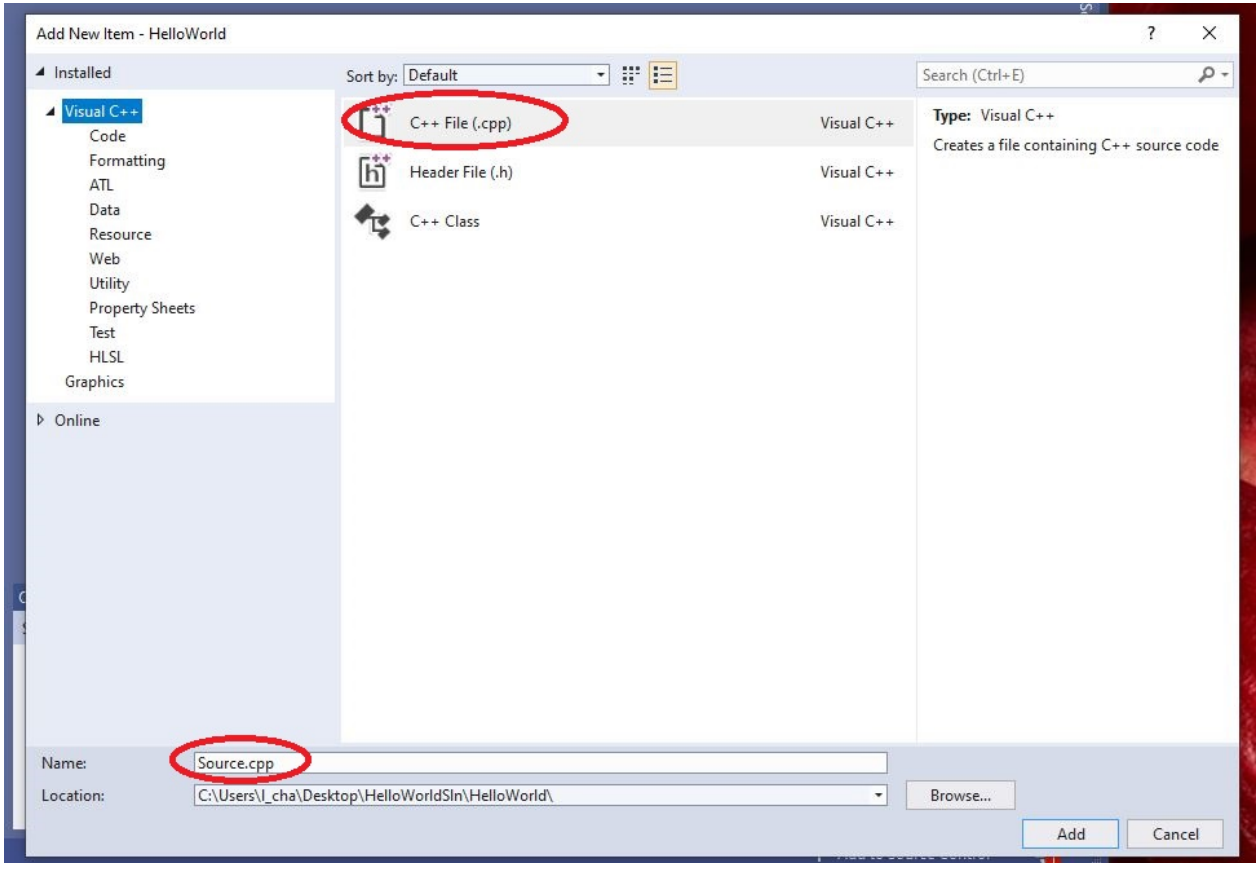
- We have a new Solution with 1 project.



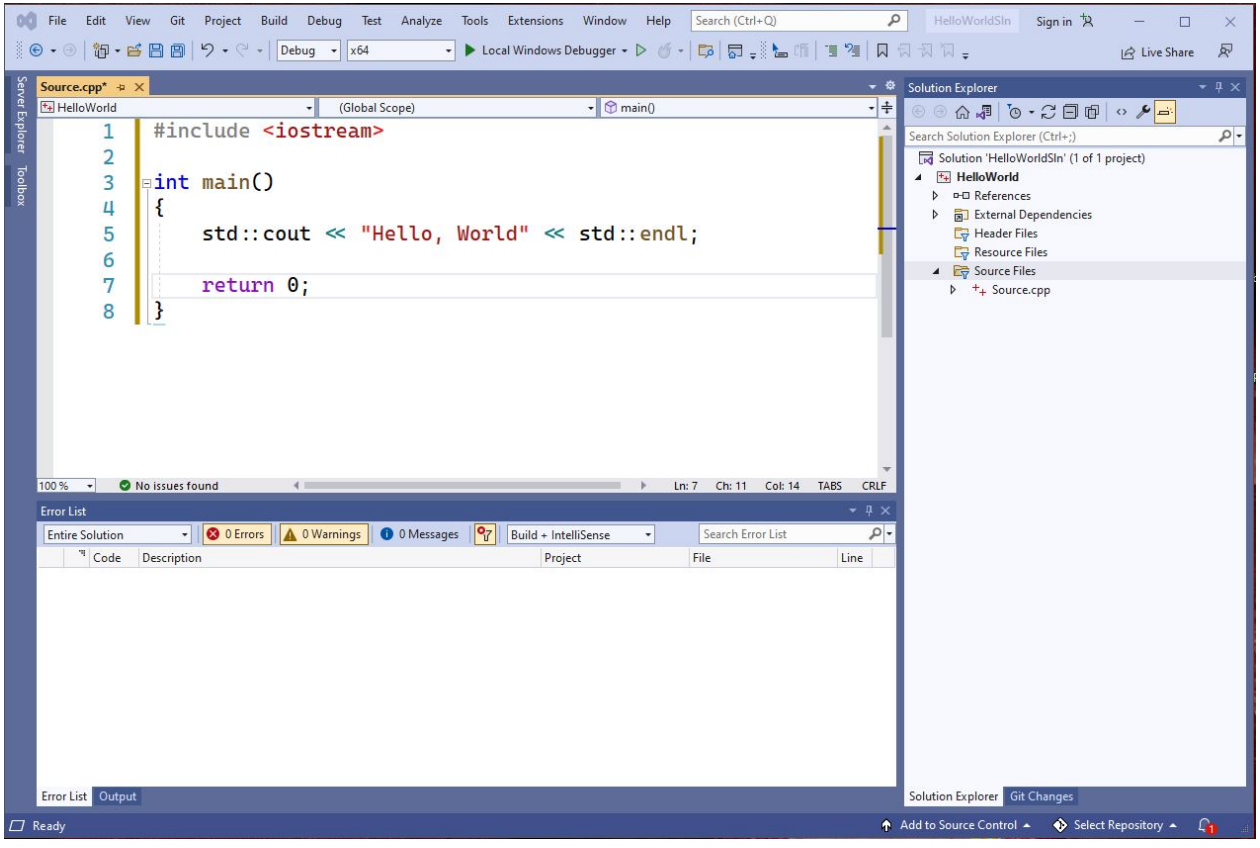
- Let's add a new source file to the project so we can write code.
- Right click on the "Source Files" folder or the Project name on the Solutions Explorer.
- Select, Add -> New Item



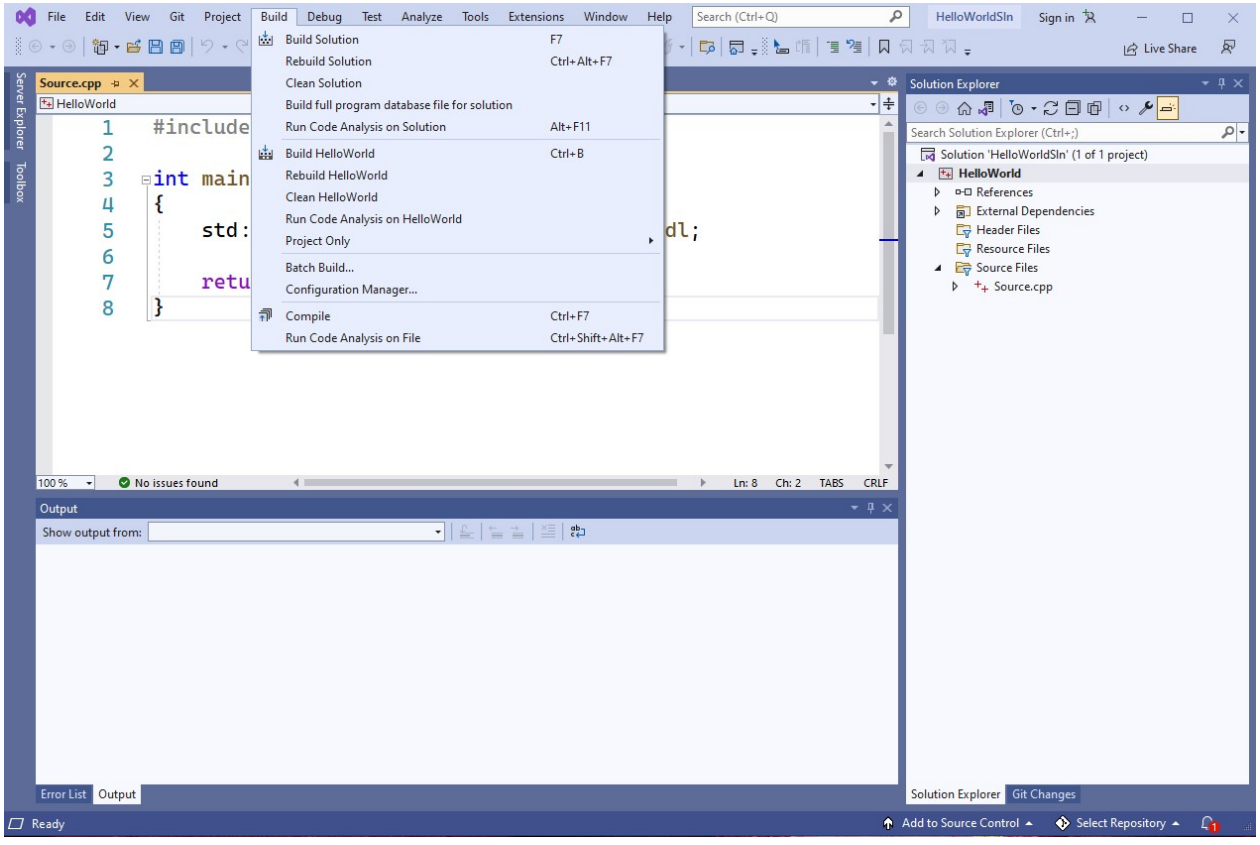
- Select **C++ File**
- In large projects we name the source files appropriately (more on this later). For this tutorial the default of Source.cpp is just fine. Click *Add* button.
- Now you have a empty cpp file.



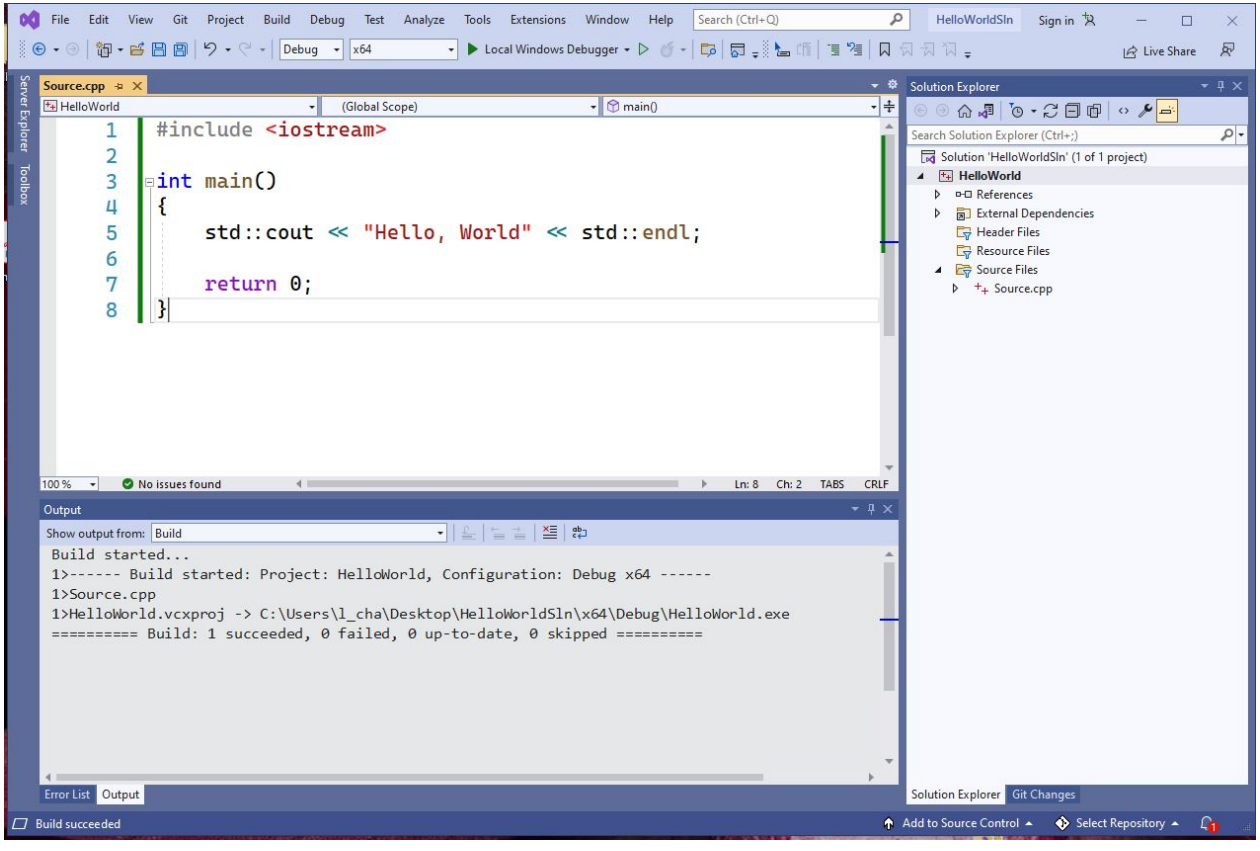
- You may type code, or copy and paste code below:
- If you type, note that C++ is a case sensitive language.



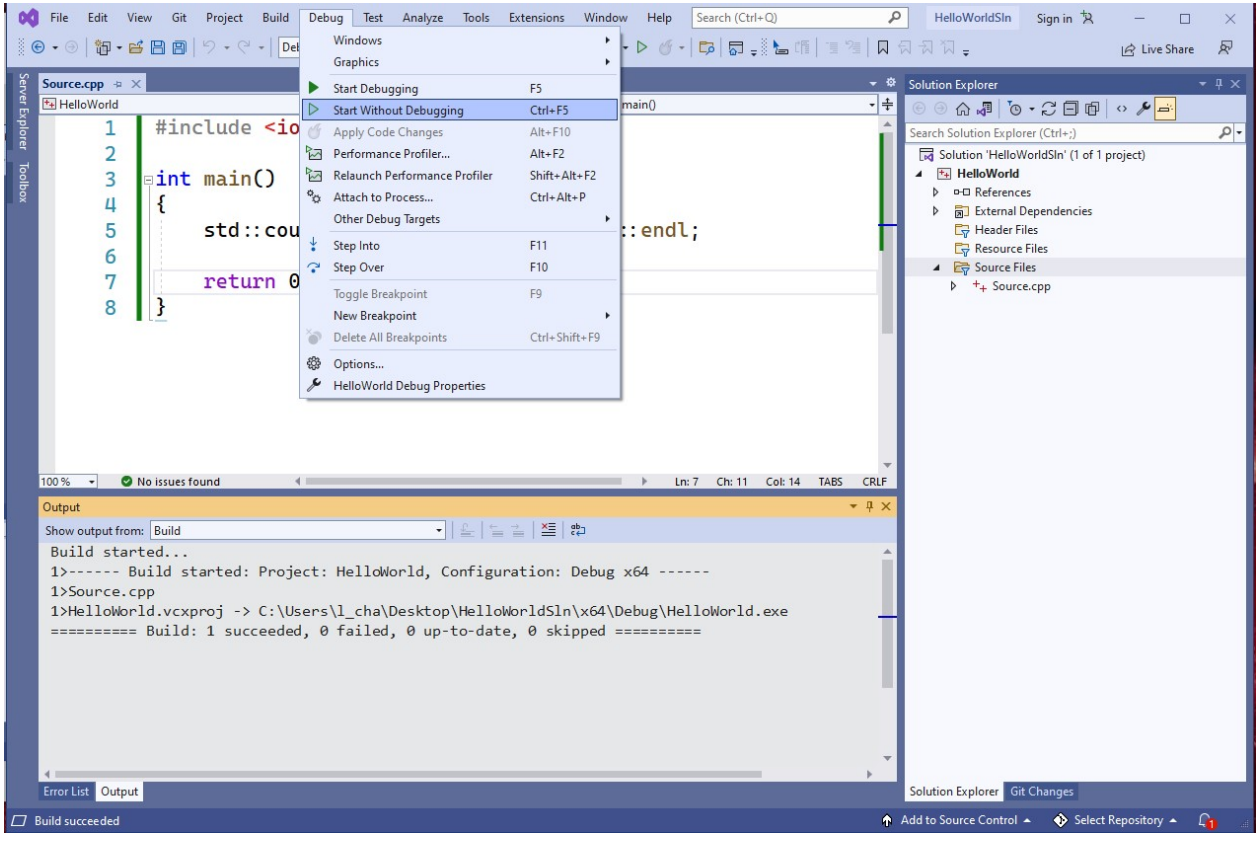
- Next step is to build the project. Select, *Build* -> *Build Solution*.



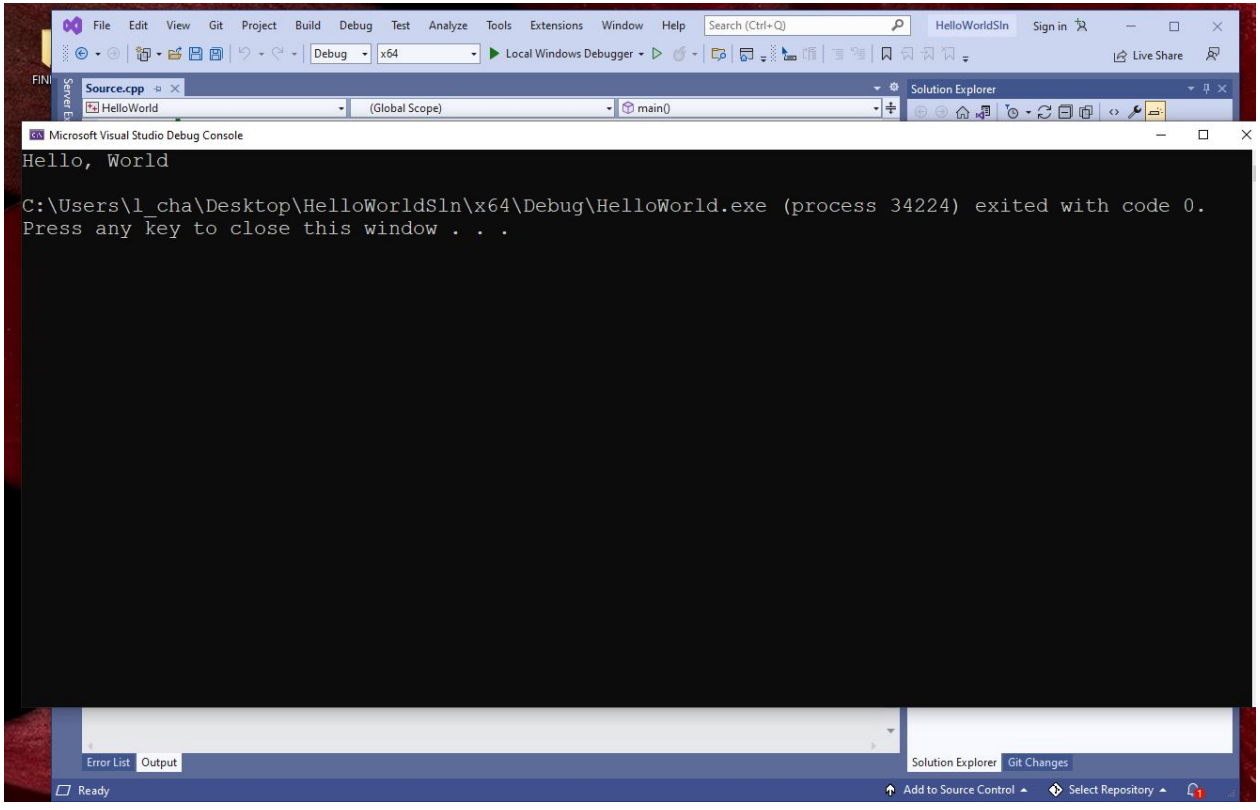
- Make sure the program builds successfully.



- To run, select *Debug* -> *Start Without Debugging*

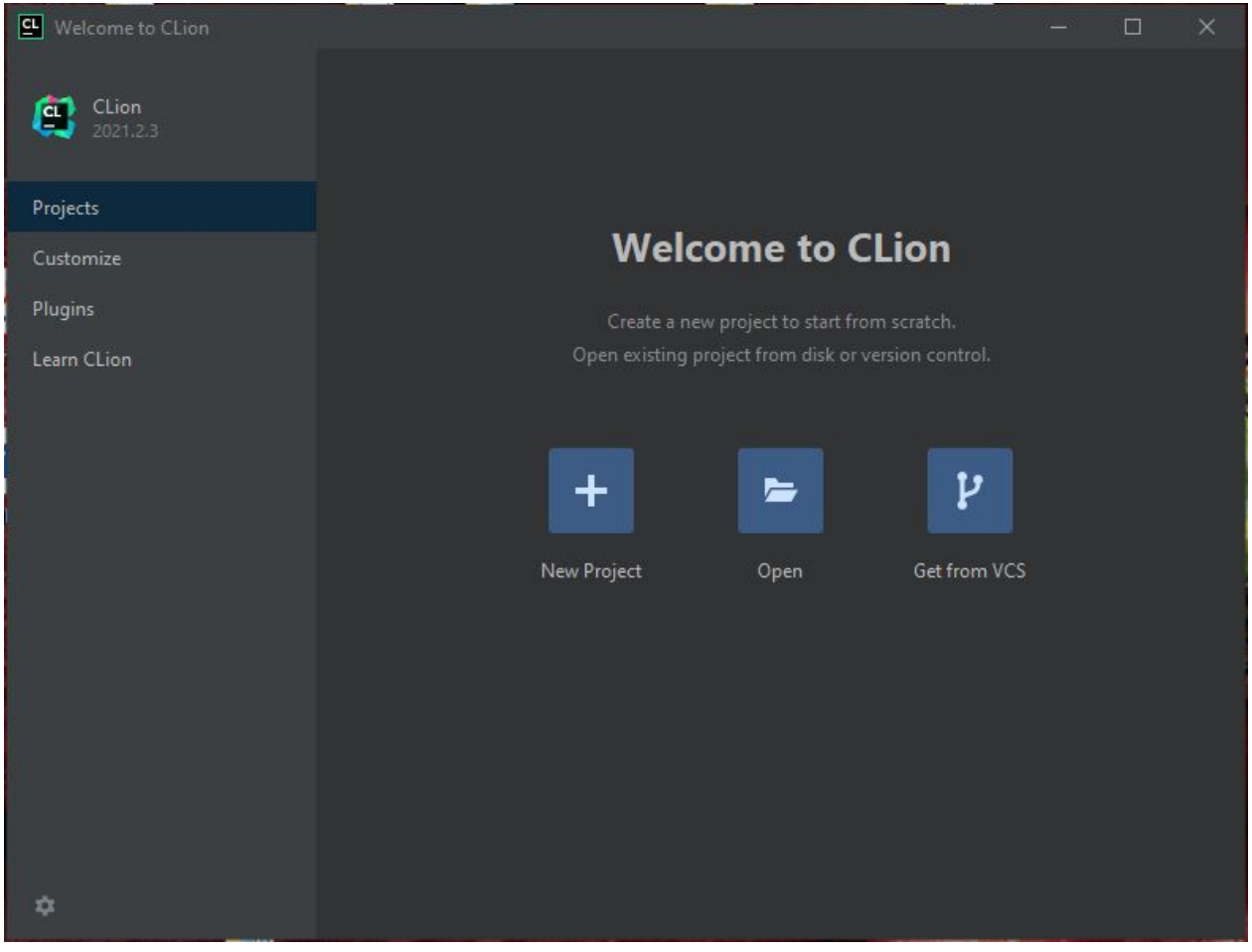


- You'll see the greeting message on Console.

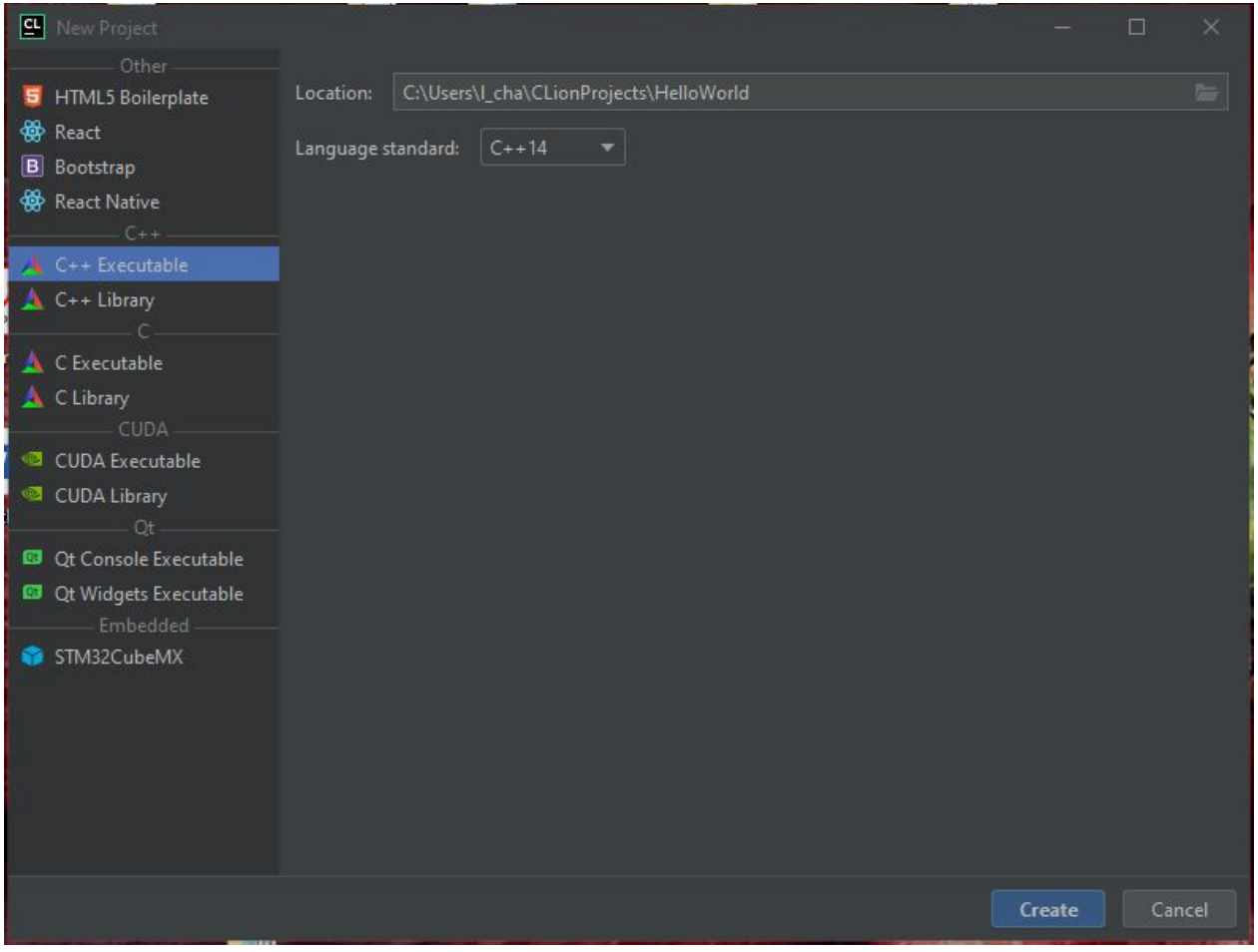


Appendix B: Getting Started with CLion

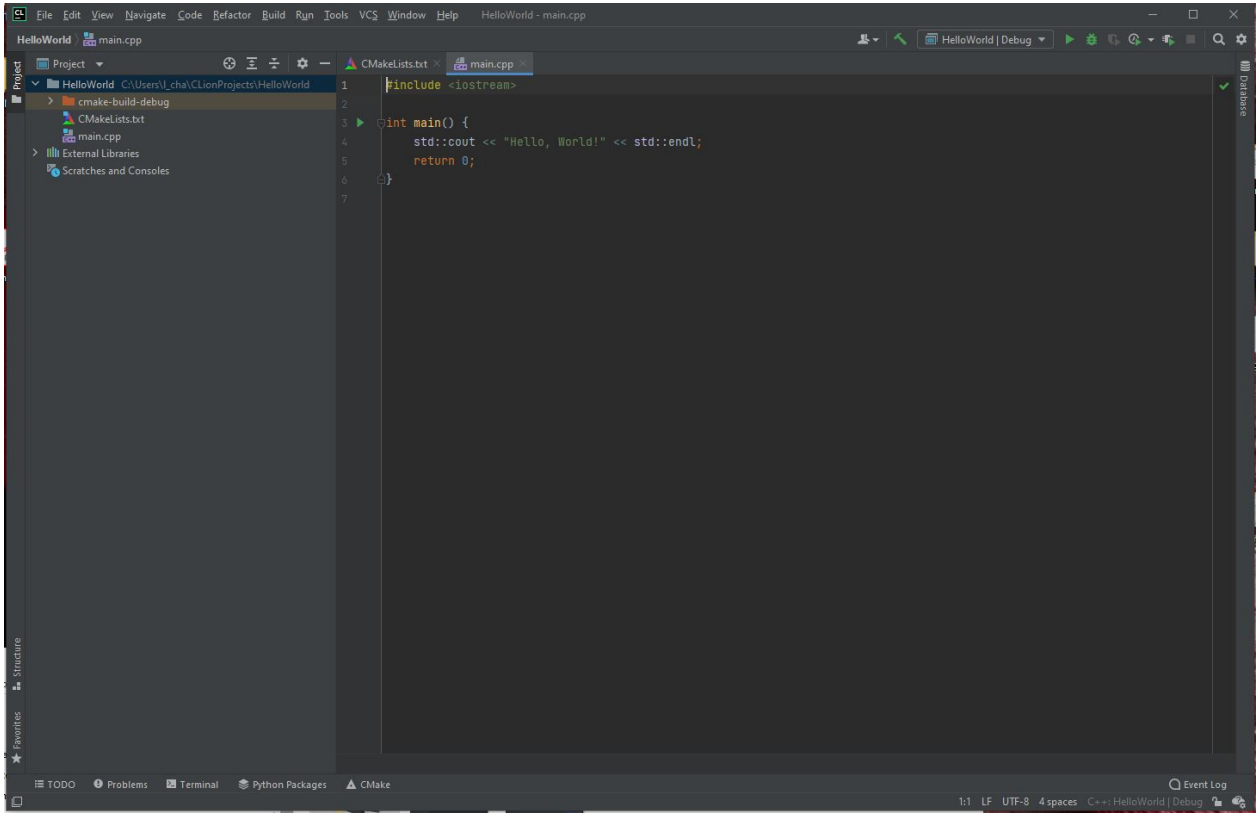
- Open CLion IDE.



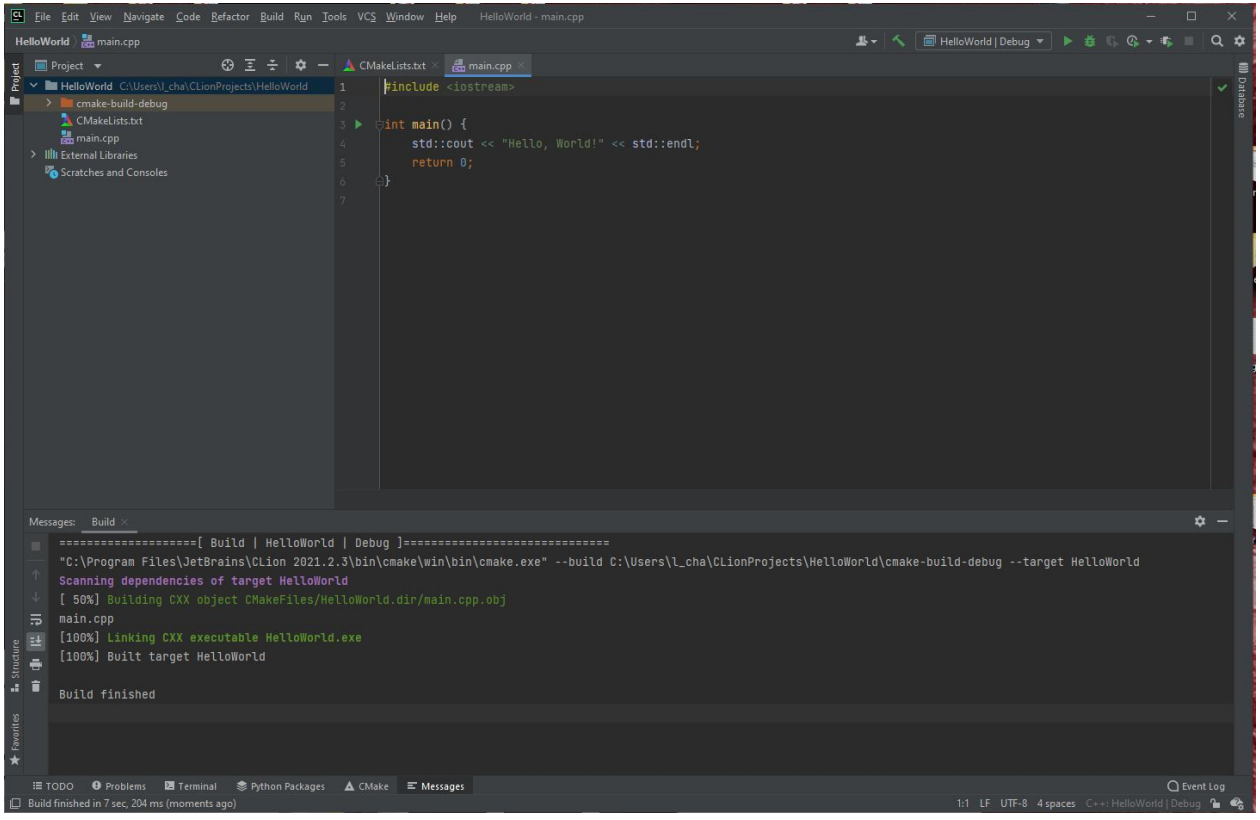
- Select “C++ Executable” project type.
- Select a location for the project.



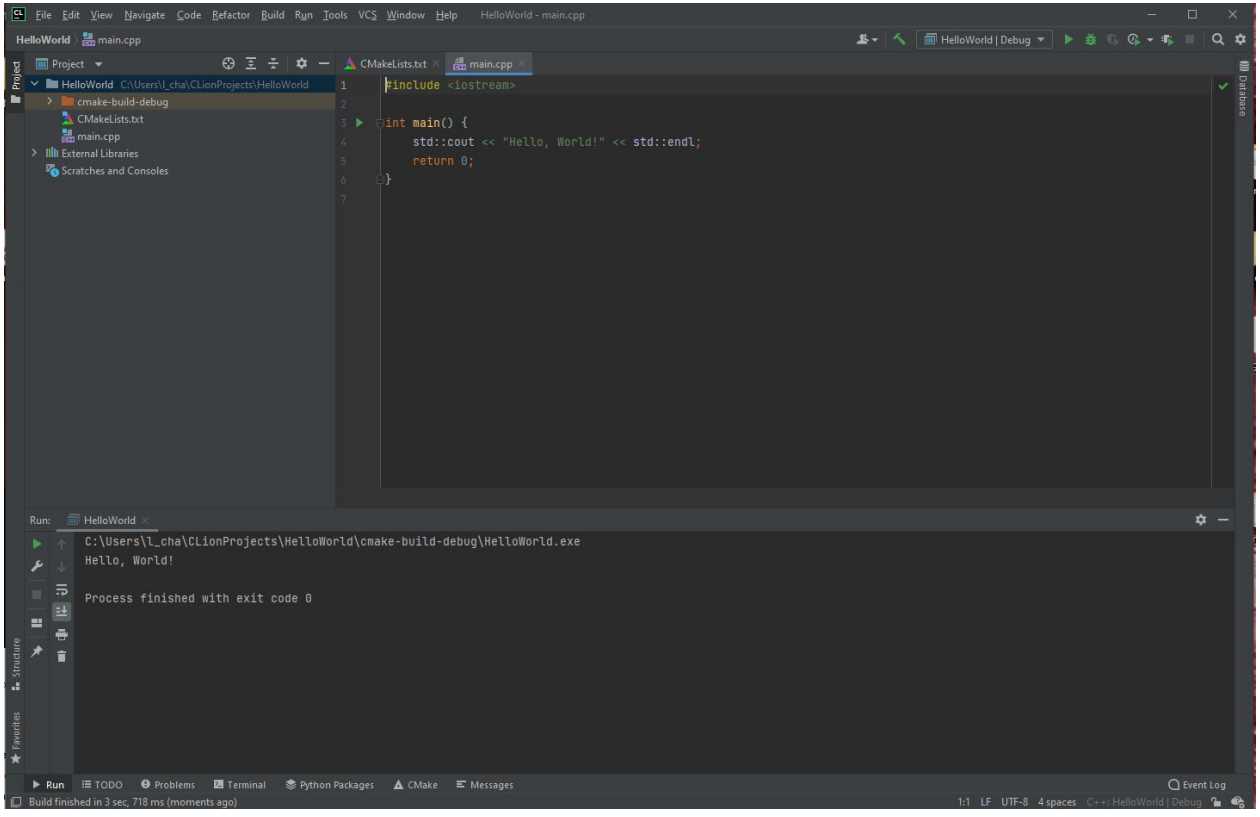
- CLion by default includes code for Hello-World example.



- Build the project. Make sure it builds.



- And, run it.
- You'll see the greeting message on Console.



- Steps above illustrate steps to get started.
- I will illustrate more advanced examples later but the amount of time we spend on CLion is limited.
- For more info, use: `https://www.jetbrains.com/help/clion/clion-quick-start-guide.html`

Appendix C: Binary Representation

Storing Values in Computer Memory

- We use bits store data; a bit stores a binary (0/1) value.
- A computer memory is organized as units of bytes.
- One byte is 8 bits.
- Suppose we have 3 bits and we want to store an integer value. We can store: 000, 001, 010, 011, 100, 101, 110, 111.
- First, let's look at a familiar example using decimal representation: $147 = 1 * 10^2 + 4 * 10^1 + 7 * 10^0$
- Similarly, we can use the binary representation:

Binary	Decimal Value
000	$0 * 2^2 + 0 * 2^1 + 0 * 2^0 = 0$
001	$0 * 2^2 + 0 * 2^1 + 1 * 2^0 = 1$
010	$0 * 2^2 + 1 * 2^1 + 0 * 2^0 = 2$
011	$0 * 2^2 + 1 * 2^1 + 1 * 2^0 = 3$
100	$1 * 2^2 + 0 * 2^1 + 0 * 2^0 = 4$
101	$1 * 2^2 + 0 * 2^1 + 1 * 2^0 = 5$
110	$1 * 2^2 + 1 * 2^1 + 0 * 2^0 = 6$
111	$1 * 2^2 + 1 * 2^1 + 1 * 2^0 = 7$

- How do we store a negative value?
- We use the leftmost bit to store the sign.
- This reduces the maximum value we can store.
- Usually signed values are stored using what's known as *twos-complement* method[4].

[4] `http://en.wikipedia.org/wiki/Two's_complement`

- How do we store fractions?
- Again, let's look at a familiar example:
  $0.147 = 1 * 10^{-1} + 4 * 10^{-2} + 7 * 10^{-3}$
- Similarly, we can use the binary representation: Let's see what happens if we were to use 2 bits to store the fractional part

Binary	Decimal Value
00	$0 * 2^{-1} + 0 * 2^{-2} = 0$
01	$0 * 2^{-1} + 1 * 2^{-2} = 0.25$
10	$1 * 2^{-1} + 0 * 2^{-2} = 0.50$
11	$1 * 2^{-1} + 1 * 2^{-2} = 0.75$

- Suppose we have 3 bits:

Binary	Decimal Value
000	0
001	0.125
010	0.25
011	0.375
100	0.5
...	...

- Homework: Complete the table above.

- Observations:
  1. Very few values can be exactly represented as a real value
  2. How close can we get depends on the number of bits
- Note: Real values are stored using an exponent and a mantissa:
  - value $= \mathit{mantissa} * 2^{\mathit{exponent}}$
  - how many bits are reserved for the mantissa part and exponent part are defined (by the IEEE standard).