An error of collateral:
Why selling SPX put options may not be profitable[1]

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Abstract

Literature reports that selling put options on stock indices provides abnormal profits. We find that abnormal returns in some prior research may come from incorrectly set collateral amounts. Some researchers used *ex post* collateral choices instead of looking at the *ex ante* problem faced by investors and found abnormal profits where perhaps none exist. To see the effect of collateral choice, this paper investigates a subspace of put option selling strategies on the S&P 500 index (SPX). Collateral choice affects measured return in non-trivial quantitative and qualitative ways.

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Earning $1 on an investment in 30 days is a good return when the collateral required is $10 and a less good return when the collateral required is $10,000. While everyone knows that return depends on amount invested, some investments have *ex ante* unknown collateral requirements. For example, suppose a trader short sells a stock at $100 per share. When the stock price goes to $90, he closes the position for a $10 per share profit. What is the trader's return? Clearly, one needs to know the collateral requirements needed to open the short position, but one also needs to know the highest stock price during that time. If the stock price goes to $150 in the interim, the investor must provide at least $50 per share more in collateral, but if the stock price goes to $300 in the interim, the investor must provide at least $200 per share of additional collateral. If the investor does not provide adequate collateral, the position is liquidated for a loss. While actual return can be easily calculated *ex post* once the price path has been realized, the total amount of collateral an investor puts up *ex ante* does not have an obvious answer.

While investigating various stock index options trading strategies, we find that previous work may not have adequately accounted for *ex ante* versus *ex post* collateral issues. It may be the case that measured returns are overestimated in relation to what an *ex ante* investor would receive.

Taking account of collateral constraints in calculating returns seems a necessary requirement of empirical analysis and applies to all investments with *ex ante* uncertain collateral requirements. Day and Lewis [2004] describe how margin requirements on futures (NYMEX crude oil in their case) create a payoff structure similar to barrier options. Therefore, the question of measuring return on a futures contract may not be a well-formed question because the investment studied is implicitly an option with a strike price dependent on the margin requirements and collateral posted. It simply does not make sense to talk about return for the futures contract without referring to a specific amount of invested collateral.

These performance measurement issues are well-known in the industry, as described in Meyer [2003] p. 220, but it does not seem that a clear answer to the question exists. The question of investment performance in relation to collateral posted does not appear to have been well-addressed in some research such as stock index put selling—see our later discussion of Bollen and Whaley [2004].

Put selling returns

For a number of years, it has been thought that selling put options on stock indices is extremely profitable (*e.g.*, Jones [2006]). Benzoni, Collin-Dufresne, and Goldstein [2005] argue that this effect is due to particular risk preferences. Bollen and Whaley [2004] find excess returns to delta-hedged short put selling on a stock index. They claim buying pressure from investors cannot be fully hedged by arbitrageurs. Garleanu, Pedersen, and Poteshman [2005] claim that demand pressure causes option prices to be higher than they would be if there were more liquidity providers, so trading strategies that involve providing liquidity to the market (such as selling put options) appear to earn high returns.

Building on prior work on put option returns, Driessen and Maenhout [2007] claim that almost all investors ought to hold short-put positions in a stock index. Fostel and Geanakoplos [2010] advocate put selling with a model of endogenous leverage choices but do not directly deal with the issue of potentially unbounded collateral requirements. Their model offers an explanation of larger volatility premia for out-of-the-money puts.

In a work which gets at the problem we find, Santa-Clara and Saretto [2009] confirm abnormal returns to selling stock index puts but find that margin requirements reduce profitability. Our work echoes Santa-Clara and Saretto [2009] by showing that measurement of return is sensitive to actual collateral needs, but our paper goes further; we identify the *ex ante* investor's problem and look more closely at the effect of collateral choice on returns.

Options trading strategies and the VIX

For our empirical analysis, we look at trading strategies which sell put options based on the S&P 500 stock index. In addition to unconditioned strategies, we explore strategies conditioned on the level of an options volatility premium index. By looking at strategies which trade at different levels of options volatility, we compare different types of strategies (*e.g.,* momentum, contrarian) to see the effect collateral choice has on performance for different strategy styles.

The Chicago Board Options Exchange (CBOE) Volatility Index (VIX) is a measure of market expectations of near-term volatility as implied by S&P 500 stock index (SPX) option prices. Since its introduction in 1993, the VIX has been considered by many to be a barometer of investor sentiment (Whaley [2000]).

It has been well-documented that the VIX has had a negative correlation to the SPX, particularly in times of market stress. This negative correlation led Black [2006] and Szado [2009] to recommend using VIX futures for hedging stock market risk. The VIX also appears to predict future stock market returns (Giot [2005]), and future stock market volatility (Becker, Clements, and McClelland [2009]; Banerjee, Doran, and Peterson [2007]). However, some research, such as Arak and Mijid [2006], finds that trading stocks based on the absolute level of the VIX does not lead to outperformance.

This paper examines the profitability of various SPX option trading strategies which combine short-selling puts based on the behavior of the VIX. Taking margin requirements into account, we measure strategy performance using a number of standard tools: Sharpe ratio, Jensen's alpha, and delta-hedged return.

Since the trading strategies in this paper have uncertain and unbounded *ex ante* collateral requirements, a useful analysis of performance needs to offer some insight into the distribution of returns based on initial collateral. To that end, we provide two sets of estimated returns over 30-day periods. One set of returns uses the minimum collateral necessary to avoid forced liquidations as determined *ex post*. This set of data shows abnormally high performance for many put selling strategies using the described performance measures. A second set of returns uses a larger amount of collateral—the historical minimum necessary to avoid all margin calls during the time period studied. That set of data does not appear to show abnormally high profits.

We also notice that different types of strategies are top performers depending on the collateral employed. Collateral chosen *ex post* has a clear effect on improving performance with some strategy types benefiting more than others. We find that strategies which may be described as contrarian (*i.e.,* go long the stock market by selling puts when the VIX is high and presumably the SPX is down) outperform when collateral is chosen *ex post* for each 30 day period. However, when using the larger collateral, momentum strategies (*i.e.*, go long the stock market by selling puts when the VIX is low and presumably the SPX is up) become top performers on a risk-adjusted basis.

We do not advocate these collateral assumptions as being better than other possible assumptions and measures. The intent is to provide insight into the potential complexity collateral choice has on measured performance.

COLLATERAL AND RETURN

There exists a conceptual issue in calculating return rate on portfolios with *ex ante* uncertain and unbounded collateral requirements. Consider the following example. An investment yields cash return of y at the end of one period. It requires collateral of 1 to open the trade and requires collateral of $1+x$ (with $x>0$) during the course of the trade period. The investor is allowed to borrow x to maintain the trade collateral. The risk-free interest rate is r and the investor earns interest on collateral and borrows at that rate. This situation corresponds to a variety of trades—for instance, selling short a stock has potentially unlimited loss, and posting full collateral at the start of the trade is impossible.

There are infinite ways to calculate return rate R based on the amount borrowed to finance the investment. Suppose the amount of initial collateral is $1+z$. Then the return is given by

$$R = \frac{y + [(1+z) - I(x > z)(x - z)](e^r - 1)}{1 + z} \quad (1)$$

where the second expression in the numerator is the continuously compounded interest earned (or paid) on net collateral over one period.[2] It is clear that the choice of z has an effect on R, though there is no ambiguity in calculating return for any particular z and x.

Typically, brokerages do not allow unlimited borrowing (*i.e.*, margin loans) without demanding additional collateral—a margin call. Suppose that in the example above borrowing is not allowed at all and the investor earns a return of $w<y$ in case of margin call. Then (with $r=0$ for simplicity), expected return is

$$E[R] = \frac{1}{1+z}\{y\Pr(x \leq z) + wPr(x > z)\} \qquad (2)$$

For example, suppose $Pr(x > z) = e^{-z}$ and $y=1$ and $w=-1$, then $E[R] = \frac{1-2e^{-z}}{1+z}$ and it is maximized at $z \approx 2.1055$ with expected return of about 24.36%. The advantage of this approach is that it explicitly addresses the issue of collateral and return. Expected return cannot be calculated without assumptions about collateral and the probability distribution of collateral requirements. In many real-world situations the probability distribution is not known, and estimated distributions introduce another source of uncertainty to the expected return calculation.

We consider two choices for collateral assumptions in our return estimates. The first assumption appears reasonable at first, but in fact implicitly gives information *ex ante* which an investor would not have. The other assumption may provide a better sense of risk-control but does not have any claim to optimality. Without explicit assumptions

[2] The assumption is that all invested collateral earns interest r but, as the collateral requirement exceeds $1+z$, the trader is forced to borrow additional collateral x-z at rate r. The condition in the indicator function $I()$ is used to determine when borrowing occurs. Note that borrowed interest initially offsets earned interest.

about the stochastic process of collateral requirements, it seems difficult to put forward a satisfying assumption on collateral and thus calculate returns.

Perfectly chosen **ex post** *collateral*

The most straightforward assumption for collateral requirements may be to assume no *ex post* borrowing. Thus, for a particular time period, the maximum collateral requirements for the trade during that period are used as the amount invested, call it *m*. Investments of more than *m* lower the absolute value of excess *ex post* returns. However, returns on investments of less than *m* are ambiguous; returns (both *ex ante* and *ex post*) may be higher than returns for *m* collateral because liquidations resulting from margin calls could close the investor out of further losing positions or prevent the investor from taking losing positions. Therefore, it is not clear whether returns based on investments of *m* represent any sort of valid baseline return measurement especially since any investor who could accurately predict future collateral requirements would be rather rich. As we find in our empirical work, the use of collateral chosen *ex post* for each 30 day period not only increases the magnitude of return, but also benefits some types of trading strategies more than others.

Historical maximum collateral choice

Another possible choice of collateral is the maximum collateral requirement over the course of the entire dataset, call this *M.* Although this collateral choice is not necessarily and optimal choice for most investors, it may reflect a reasonable baseline choice for an investor who seeks to avoid margin calls based on long-term historical performance of the investment. Formally defined, $M = \max_\tau m_\tau$ where m_τ is the maximum collateral requirement for period τ.

Is there a problem with cash-secured put selling?

Since a short-put position has maximum loss limited to the strike price, it may appear the above comments do not apply to the study of short put strategies. An investor who sells an unhedged put has a loss limited to the strike price, and the return rate can be measured on that basis. But average return calculated on that basis measures a very particular situation: that of an investor who *ex ante* has unlimited funds so that he can open any position.[3] For example, selling a cash-secured at-the-money put tomorrow requires an unknown amount of collateral. A more general average return calculation would explicitly condition the average on a maximum amount of collateral available. Moreover, since the *ex ante* investor commits invested collateral before knowing the amount required, the return ought to be calculated on the basis of this *ex ante* committed amount.

DATA AND STRATEGY DESCRIPTIONS

The data include daily value ranges of the SPX and VIX indices[4] and SPX option end of day bid and offer prices. The time period spans from January 4, 1996 through October 30, 2009. During the time, the VIX ranged from a low of 9.89 to a high of 80.86 with a

[3] A trader who wishes to engage in a particular trading strategy for the next month may have *ex ante* unbounded collateral requirements. The manager of the trading desk sets limits on invested collateral and thereby affects expected return for the trader's strategy.

[4] The VIX calculation was modified on September 25, 2003. See Whaley [2009].

mean of 22.14 and a median of 20.82. The correlation coefficient between the VIX close and the percent change of SPX is 0.0127. However, the correlation coefficient between the percent change in the VIX close and the percent change in the SPX close is -0.738, demonstrating the negative-correlation which has earned the VIX the moniker “the investor fear gauge” (Whaley [2000]).

We look at strategies which short-sell put options. When portfolio short positions are not hedged, the portfolio gains value if the SPX index rises and loses money if the SPX drops.[5] We condition opening and closing positions on the day’s closing value of the VIX index. While a high VIX level implies that selling options provides more up-front cash to the seller, there may also be mean-reverting or momentum effects for SPX which may have an effect on our portfolios through the correlation between SPX and the VIX. To this end, we test four types of VIX-based short-selling put strategies. For each strategy type, we check a range of values of the VIX as thresholds at which to initiate or close positions. The sample of examined strategies is by no means exhaustive or representative of any specific theoretical model approach. Rather, these strategies serve as reference points within the space and provide insight on collateral choice and performance.

The four types of VIX based short-selling put strategies we consider are:

1. “VIX high”: Sell a put when the VIX rises above a certain value and close all positions (that is, buy back all short puts) when the VIX dips below another cut-off value.

[5] Many sources explain options trading, see *e.g.,* information at http://www.interactivebrokers.com

2. "VIX low": Sell a put when the VIX dips below a certain value and close all positions when the VIX rises above another cut-off value.
3. "VIX jump up": Sell a put when the daily VIX percent increase is greater than a certain threshold and hold positions until expiration.
4. "VIX jump down": Sell a put when the daily VIX percent decrease is above a certain threshold and hold positions until expiration.

"VIX high" and "VIX jump up" may be considered contrarian strategies because the strategy in effect goes long SPX when the VIX goes higher (and presumably, by historical negative correlation, SPX is going down). "VIX low" and "VIX jump down" strategies may be considered momentum strategies as these strategies go long SPX when the VIX goes lower (and presumably SPX is going up). All strategies may potentially benefit (or lose) from option premiums if there is some systemic mispricing correlated with VIX behavior.

Given particular parameters for strike price, and VIX values to open and close positions, the "VIX high" strategy operates as follows at the end of each trading day:

- Check whether the VIX is above the threshold to open a position.
- If yes, then find an SPX put option expiring within 45 days[6] which is closest to the desired strike price and sell it at the market bid price.
- Check whether the VIX is below the threshold to close.
- If yes, then buy back (close) all open short put option positions.

[6] This option expiration length follows Banerjee, Doran, and Peterson [2007] who find that it takes on average 44.1 days for the VIX to mean-revert.

The other strategy types operate similarly. We examined 1458 different strategies. Exhibit 1 summarizes the range of VIX level parameters for each type of strategy. As an example consider a "VIX high" strategy described as "Sell at strike 85% of SPX when VIX > 36, never cover." This strategy behaves as follows on October 30, 1997 when the VIX closed at 38.2 and SPX closed at 903.68: Sell the put option expiring November 22, 1997 with a strike price of 750 (closest strike price to 15% out-of-the-money and closest expiration date less than 45 days away). The strategy holds this option until expiration (*i.e.*, never covers the position). Since this particular option expires worthless (SPX never dipped below even 900 during that period), the strategy's portfolio earns the cash premium for selling the option.

Using the properly time-scaled interest rate on three-month U.S. Treasury bills as a proxy for the risk-free rate, we report the return premium and the Sharpe ratio.[7] Strategy performance is measured over 30-day periods.[8] We calculate Jensen's alpha using the 30-day returns of the SPX index (dividend adjusted). In order to separate out the portion of a strategy's performance due to the change in the underlying index, we calculate the strategy's daily delta-hedged performance. We report results of each performance measure for both *m*-collateral and *M*-collateral.

Transaction costs

[7] The Sharpe ratio is not necessarily the best measure of risk-adjusted performance because it relies on a normal distribution of returns, and we find that returns on the analyzed strategies are not generally normally distributed. Other measures have been invented—for instance, see Stutzer [2000] for a discussion and another suggested performance index.

[8] A 30-day period is often used in the options literature as a performance measurement period.

The profit calculation accounts for the cost of the bid-ask spread by using end of day bid (to sell) and offer (to buy) prices as the trade prices. The assumption is that an investor would be able to trade at these prices without any market impact. Commissions are assessed at $0.70 per contract traded plus a $1.00 trade fee.[9] Options expiration incurs no commission cost. The option value at expiration is the difference between the level of the SPX index on the close of the previous trading day and the strike price of the option or zero if this value is negative. Since SPX index options are cash settled, if the investor has sold the option then he must pay this cash value.

For the risk-free interest rate on cash, the profit calculation uses the then current rate on three month U.S. Treasury Bills. The annualized interest rates during this time period ranged from approximately zero to 6.24% with an average of 3.36%.[10] In our simulation, interest accrues daily to the cash in the portfolio.

Option margin requirements and return

Buying options incurs no margin costs—the investor pays the premium and owns the contingent security. Selling options requires the option writer to post collateral in order to be able to satisfy the exercise of the contingent security. We use the following rule (from Interactive Brokers LLC)[11] to calculate margin requirements on short puts on index options:

[9] The brokerage Interactive Brokers LLC charges commissions ranging from $0.70 to $0.15 per contract depending on the number of contracts traded in a month. Fee schedule at http://www.interactivebrokers.com/en/p.php?f=commission

[10] Interest rates on 3 month T-bills from http://research.stlouisfed.org/fred2/data/DTB3.txt

[11] Initial and maintenance margin is the same. Margin requirements at http://www.interactivebrokers.com/en/p.php?f=margin

$$Margin = PutPrice + Max(A_{OTM}, A_{STRIKE}) \tag{3}$$

where A_{OTM} *=15% × Index – Out-of-the-money amount* and A_{STRIKE}*=10%× Strike price*.

We find the maximum margin requirement for the options positions in the portfolio over the previous 30 day period.[12] We adjust this amount by any change in the cash position of the account due to trades and options expirations and count the adjusted total as the amount originally invested. Specifically, this amount is given by

$$m = \max_{\tau=1..T}\{G_\tau - C_{0,\tau}\} \tag{4}$$

where G_τ is the total margin requirement for the options positions in the portfolio at time τ and $C_{0,\tau}$ is the change in portfolio cash due to options trading and expiration from time *0* to time τ. The amount *m* represents the *m*-collateral value for the options trading strategy. The change in portfolio value from time *0* to time *T* is given by

$$R_T = V_T - V_0 + \sum_{t=0}^{T-1}(C_{t+1} - C_t)e^{r(T-t)} + m(e^{rT} - 1) \tag{5}$$

where V_t is the market value of the option positions (negative for short options), C_t is the cash in the portfolio at time *t*, and *r* is the risk-free interest rate.

Delta-hedging margin and return

The delta of an option can perhaps be described as the dollar change in option price due to a dollar increase in the price of the underlying. The delta of a European call option on a security which pays no dividends (*e.g.*, an SPX call option) is given by

[12] Maximum margin requirements use the lowest daily value of the SPX index for the time period.

$$\Delta_c = N\left[\frac{\ln\left(\frac{Se^{rT}}{X}\right) + 0.5\sigma^2 T}{\sigma\sqrt{T}}\right] \tag{6}$$

where S is the current price of the underlying, X is the strike price of the option, σ^2 is the variance in price of the underlying, T is the time until expiration, and r is the cash interest rate per period. The delta for a put option with the same strike price is simply the delta of the call minus one, so the delta of a long put ranges from negative one to zero. For the variance, we use the 30-day historical variance of SPX daily log returns.

For our simulated implementation of the daily delta-hedged return on an options selling strategy on SPX, we assume that the investor is able to trade at the end of each trading day in SPX futures. If the investor is hedging a positive (negative) delta option position, he takes short (long) SPX positions by selling (buying) SPX futures. We make the following simplifying assumptions:

1. Trade is costless—no bid-ask spread or trading fees.
2. Additional margin requirements for holding futures positions are zero.[13] Of course, the investor still needs to provide additional collateral to compensate for changes in the price of the underlying. Collateral requirements are based on the extreme prices (*i.e.* high or low) for the day rather than the closing price.

[13] Risk-based margin requirements (called portfolio margin) may reduce total required collateral for the hedged portfolio lower than the options positions collateral requirements. We discuss this later in the results section. Calculating portfolio margin is a difficult task, see *e.g.*, Interactive Brokers' Portfolio Margin at http://www.interactivebrokers.com/en/p.php?f=margin&p=pmar-default

Furthermore, since we use the maximum collateral requirement m to calculate *ex post* returns, there are no forced liquidations. From these assumptions, the total amount of collateral required to hedge a short position of put option p is given by

$$I_{\Delta,p} = \max_{\tau=1..T}\left\{\sum_{t=0}^{\tau-2}\Delta_{t,p}(S_{t+1}-S_t)+\Delta_{\tau+2,p}(\bar{S}_{\tau+2}-S_{\tau+1})\right\} \tag{7}$$

where $\Delta_{t,p}$ is the delta value on day t of the put option p, S_t is the closing price of SPX on day t, and $\bar{S}_\tau$ is the high price for SPX on day τ. Note that $\Delta_{t,p}>0$ for the short put and that hedging a short put requires shorting SPX. The invested amount $I_{\Delta,p}$ may be negative, in which case it offsets the increased margin requirements of the hedged option position.

The cash return to the daily delta-hedges on holding a short position of put option p is given by

$$R_{\Delta,p} = \sum_{t=0}^{T-1}\Delta_{t,p}(S_{t+1}-S_t)\,e^{r(T-t)}+I_{\Delta,p}(e^{rT}-1)-\sum_{t=0}^{T-1}\Delta_{t,p}S_t(e^{r}-1) \tag{8}$$

where r is the risk-free interest rate per day (assumed to be constant throughout the life of the trade). The first summation term consists of the daily delta-scaled price movements of the underlying and the interest earned (paid) on the resulting gains (losses) in collateral. The term represents the interest earned on cash collateral through the life of the trade based on risk-free interest rate r. The last summation represents the carry cost in the

SPX futures positions. Short futures positions in SPX have a negative carry cost (*i.e.*, gain interest implicitly), but our simulated returns ignore carry cost for simplicity.[14]

As made evident by the appendix on daily delta-hedging, daily delta-hedging does not fully hedge the options portfolio from daily price changes, especially so when the underlying has high volatility. So, daily delta-hedging may not provide an unbiased estimate of an option strategy's performance net of delta. A better estimate may be provided through delta-hedging based on percent value changes in the underlying. However, due to limitations of our dataset, we use daily delta-hedging.

In the case of a daily delta-hedged trading strategy, the calculation of *m* collateral for the portfolio during the analysis period becomes

$$I_\Delta = \max_{\tau=1..T} \left\{ \begin{matrix} \displaystyle\sum_{t=0}^{\tau-2} \left[\Delta_t (S_{t+1} - S_t) - C_{t+1,t} \right] \\ + \Delta_{\tau-1} (\bar{S}_\tau - S_{\tau-1}) - C_{\tau,\tau-1} + G_\tau \end{matrix} \right\} \tag{9}$$

where $C_{t+1,t}$ is the net change in portfolio cash from options trading between time *t+1* and time *t*, Δ_t is the delta of the whole options portfolio at time *t*, the *S* variables are the prices of SPX as described above, and G_τ is the margin requirement for the options positions in the portfolio at time τ. The cash return for the daily delta-hedged portfolio is

$$R_\Delta = \sum_{t=0}^{T-1} \Delta_t (S_{t+1} - S_t) + R_T \tag{10}$$

where R_T is the cash return to options trading as defined above—that is, it is the cash return from options trading plus the net cash from hedging.

[14] Although this omission is not likely a significant contributor to return, the omission biases our estimated returns downward as hedging short puts requires short selling SPX futures. Because our simulations also ignore trading costs in SPX futures, it may be that the two sins cancel somewhat.

RESULTS

For each tested strategy, we calculate excess return, Sharpe ratio, Jensen's alpha, and daily delta-hedged excess return. We find these descriptors for both *m* and *M* collateral. As a baseline, we look at unconditioned strategies—those which sell a put option regardless of the level of the VIX. Exhibit 2 shows performance by strike price. With *m*-collateral invested, it seems that selling puts with strikes 3% and 6% out-of-the-money does rather well on a risk-adjusted basis with delta-hedged 30-day returns of 0.489% and 0.445% respectively. However, as these returns rely on the *ex ante* investor picking collateral *ex post*, a more reasonable measure comes from the *M*-collateral returns. These returns do not show significant out-performance for any of these strategies and, in fact, show average negative returns for most strike prices. Although *M*-collateral invested may be overly conservative, it seems that SPX put selling may not be abnormally profitable when using a fixed, relatively large amount of collateral.

VIX conditioned strategies: m*-collateral*

In testing our sample of VIX-conditioned strategies, we are not surprised to find high abnormal profits for *m*-collateral. Although these returns may not be realizable by an *ex ante* investor, looking at *m*-collateral performance provides a reference point for comparison to *M*-collateral returns.

The highest excess return strategy for *m*-collateral invested is a "VIX jump up" strategy with a 4.926% average 30-day return. This strategy uses a strike price 6% in-the-money and opens positions when the VIX rises by more than 7%. Strategies with slight variations of those parameters perform comparably well. For instance, the 15th best performing strategy, with a 30-day excess return of 4.234%, has a strike 3% in-the-money and opens positions when the VIX rises by more than 4%. Ranking by Jensen's alpha on *m*-collateral returns finds a similar set of strategies as top performers with alpha values close to 4%. Looking at highest Sharpe ratios, we find that the best performer, with a ratio of 0.396, is a "VIX jump up" strategy with a 7% jump but with a strike price 9% out-of-the-money in contrast to the in-the-money top performers for returns which are not risk-adjusted. The lower strike price significantly reduces the standard deviation of returns. The set of top performers by Sharpe ratio are variations on this strategy's parameters.

The top delta-hedged performance with *m*-collateral comes from a "VIX jump up" strategy with a 7% jump and a strike 3% in-the-money. However, the other high performers include other strategy types such as "VIX high" strategies with strikes close to at-the-money. Even an unconditional put selling strategy, with strikes 3% out-of-the-money, comes in as the 7th best performer. Therefore, we cannot see an obvious common feature in best performers ranked this way, though many appear to be contrarian strategies.

Exhibit 3 lists performance characteristics of the described top performing strategies. With *m*-collateral invested to measure returns, these results appear to imply that contrarian strategies—going long the stock market via short puts when the VIX is high or moves higher—provide abnormal excess returns and better returns than momentum strategies. However, contrarian strategies may benefit more from *ex post* collateral choice if they are more risky than momentum strategies. *Ex post* collateral choice amplifies leverage before a favorable price move and reduces leverage before unfavorable moves.

Performance for M*-collateral*

Looking at *M*-collateral performance for the above (top *m*-collateral performing) strategies, we find, as expected, that increased collateral requirements reduce average returns. Exhibit 3 shows that, for *M*-collateral invested, these “VIX jump up” strategies have alpha and delta-hedged returns which are close to zero or slightly negative. Thus, it may be that without the benefit of hindsight, the *ex ante* investor must put up larger amounts of collateral for these contrarian strategies and consequently loses the supposed out-performance measured above.

We rank strategies by *M*-collateral performance. Highest excess returns accrue to a “VIX high” strategy, with average excess return of 0.650% over 30 days. The strategy sells at a strike price 3% in-the-money when the VIX goes above 26 and holds the short put until expiration. The top strategy by alpha (0.427%) is a “VIX high” strategy which sells at a strike price at-the-money when the VIX goes above 26 and holds until expiration. The other top performers by these metrics are “VIX high” strategies with slight variations of these parameters. So, contrarian strategies, even though different in substance between *m*- and *M*-collateral, appear to do well in excess return for both collateral measures. This outperformance may be due to mispricing of put options during times of market stress when the VIX is high.

Risk-adjusted performance for *M*-collateral, however, finds that contrarian strategies are more risky. Sharpe ratio rankings for *M*-collateral find a group of similar “VIX low” strategies performing best. The highest Sharpe ratio, 0.309, sells puts at a strike price 6% out-of-the money when the VIX goes below 16. Momentum strategies also do well in the risk-adjusted performance measure of delta-hedged returns. Highest returns for delta-hedged strategies with *M*-collateral invested accrue to "VIX low" strategies. The best performer is a strategy with strike prices 9% out-of-the-money which sells when the VIX dips below 24 and buys back the put when the VIX goes above 30. This strategy has a mean excess delta-hedged return of about 0.148%. In addition to statistics on the above top performing strategies, we include a "VIX jump down" strategy in Exhibit 3. This particular strategy is the 15th highest delta-hedged return for *M* collateral. The strategies ranked 2nd to 14th are slight variations on the described "VIX low" strategy.

For *M*-collateral, a better risk-adjusted return comes from momentum rather than contrarian strategies which do well for *m*-collateral. Risk control afforded by *ex post* collateral choice provides a large amount of collateral during market stress and therefore appears to reduce the variance of returns in addition to increasing average returns. Without this benefit, it appears momentum strategies are better. However, the delta-hedged returns for even these momentum strategies are not grossly abnormal.

CONCLUSION

In investigating excess returns to various put selling strategies on the S&P 500 stock index, we find that collateral choice is a central question which does not have an obvious answer. It appears that some prior work in the literature has not adequately addressed the collateral question and may have found excess returns where likely none exist for an actual *ex ante* investor. Our measurement of performance for unconditional put selling strategies using the minimum amount of collateral historically required to avoid forced liquidations finds near normal risk-adjusted returns.

In our performance measurement of put selling strategies which are conditioned on the behavior of the VIX, we find that collateral choice affects which type of strategy appears to be best. With collateral chosen *ex post* to provide the minimum invested amount necessary to avoid all margin calls, we find that contrarian strategies outperform. On the other hand, for a fixed, large amount of collateral conservative momentum strategies do better on our risk-adjusted performance metrics. This effect is likely due to the additional information provided by *ex post* collateral choice so that the amount of collateral chosen reduces leverage during market stress and amplifies it during rising markets thereby increasing returns while lowering risk. Overall, we do not find hugely abnormal risk-adjusted returns. We find, however, that collateral choice has a non-trivial effect on trading performance and collateral assumptions ought to be carefully considered in research on investment performance.

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Condition	Strategy type	Range of values
To open	VIX high	VIX > 20 to 40, increments of 2; that is VIX > 20, 22, 24, 28, etc.
	VIX low	VIX < 10 to 30, increments of 2; that is VIX < 10, 12, 14, 16, etc.
	VIX jump up/down	VIX % change > 1% to 19%, increments of 1%
To close	VIX high	VIX < open value to 20, increments of 3 (or let expire)
	VIX low	VIX > open value to 30, increments of 3 (or let expire)
	VIX jump up/down	Never close; that is, let expire

Exhibit 1: The range of VIX values used to initiate and close portfolio positions, listed by type of strategy. Positions open through short-selling a put and close by buying back all short-puts.

m collateral – 30 day returns				*Strike price (% of SPX)*	*M collateral – 30 day returns*			
Mean return (std. dev) (skewness) (kurtosis)	Sharpe ratio	Alpha	Delta-hedge return (std. dev) (skewness) (kurtosis)		Mean return (std. dev) (skewness) (kurtosis)	Sharpe ratio	Alpha	Delta-hedge return (std. dev) (skewness) (kurtosis)
1.834% (14.498%) (-0.029) (3.360)	0.126	1.023%	-0.541% (4.210%) (0.032) (7.504)	115%	-0.128% (7.017%) (-1.077) (6.190)	-0.018	-0.527%	-0.386% (1.949%) (-0.992) (9.031)
1.992% (15.233%) (-0.049) (3.310)	0.131	1.133%	-0.497% (4.506%) (0.039) (7.269)	112%	-0.103% (7.448%) (-1.092) (6.185)	-0.014	-0.527%	-0.387% (2.092%) (-1.031) (9.093)
2.315% (16.407%) (-0.082) (3.182)	0.144	1.402%	-0.315% (4.891%) (0.023) (6.719)	109%	-0.035% (7.871%) (-1.129) (6.214)	-0.004	-0.483%	-0.331% (2.264%) (-1.089) (8.991)
2.649% (16.395%) (-0.228) (3.069)	0.162	1.702%	-0.071% (5.146%) (-0.239) (6.268)	106%	0.064% (8.149%) (-1.267) (6.587)	0.008	-0.399%	-0.243% (2.408%) (-1.319) (9.430)
2.760% (15.331%) (-0.505) (3.471)	0.180	1.874%	0.183% (5.199%) (-0.486) (7.230)	103%	0.170% (7.864%) (-1.643) (8.285)	0.022	-0.266%	-0.130% (2.463%) (-1.696) (11.416)
2.622% (12.767%) (-0.921) (5.339)	0.205	1.922%	0.343% (5.109%) (-0.531) (9.838)	100%	0.281% (6.985%) (-2.383) (13.279)	0.040	-0.075%	-0.039% (2.436%) (-2.160) (15.595)
2.540% (10.084%) (-1.490) (9.499)	0.252	2.042%	0.489% (5.288%) (-0.465) (11.337)	97%	0.416% (5.898%) (-3.476) (23.117)	0.070	0.154%	0.034% (2.428%) (-2.618) (21.315)
2.153% (8.100%) (-1.875) (14.895)	0.266	1.806%	0.445% (5.242%) (-0.819) (11.883)	94%	0.396% (4.994%) (-4.536) (35.912)	0.079	0.207%	0.005% (2.439%) (-3.537) (30.069)
1.598% (6.401%) (-1.986) (21.260)	0.250	1.365%	0.283% (5.073%) (-0.964) (13.961)	91%	0.308% (4.020%) (-5.453) (51.263)	0.077	0.176%	-0.074% (2.390%) (-4.403) (39.668)
1.080% (5.054%) (-2.026) (29.318)	0.214	0.920%	0.072% (4.590%) (-1.065) (18.937)	88%	0.220% (3.323%) (-6.229) (70.188)	0.066	0.124%	-0.151% (2.254%) (-5.246) (50.644)
0.796% (3.733%) (-1.936) (43.890)	0.213	0.691%	-0.068% (3.707%) (-1.134) (23.969)	85%	0.203% (2.617%) (-7.007) (101.950)	0.077	0.135%	-0.176% (1.861%) (-6.169) (63.655)

Exhibit 2: Performance of unconditioned put selling strategies for overlapping 30-day time periods. All returns are in excess of the return on 3-month Treasury bills. Strategies differ in the strike price of the option, ranging from 15% in-the-money (115% of SPX) to 15% out-of-the-money (85% of SPX). The left side shows performance for *m* collateral invested; the right side shows the same strategy performance for *M* collateral invested.

m collateral – 30 day returns				*Strategy* (Strike of SPX) Trade conditions	*M collateral – 30 day returns*			
Mean return (std. dev) (skewness) (kurtosis)	Sharpe ratio	Alpha	Delta-hedge return (std. dev) (skewness) (kurtosis)		Mean return (std. dev) (skewness) (kurtosis)	Sharpe ratio	Alpha	Delta-hedge return (std. dev) (skewness) (kurtosis)
4.926% (15.045%) (-0.217) (3.319)	0.327	4.106%	0.458% (5.016%) (-0.397) (5.322)	(106%) when VIX % change > 7%, never cover **Highest excess return *m* collateral **Highest alpha *m* collateral	0.144% (4.618%) (-3.330) (27.746)	0.031	-0.068%	-0.255% (1.600%) (-4.311) (32.812)
1.932% (4.884%) (-1.665) (21.771)	0.396	1.782%	0.070% (4.287%) (-0.616) (13.867)	(91%) when VIX % change > 7%, never cover **Highest Sharpe ration *m* collateral	0.107% (2.663%) (-6.276) (94.739)	0.040	0.043%	-0.170% (1.669%) (-7.535) (81.344)
4.719% (13.308%) (-0.546) (4.001)	0.355	4.032%	0.603% (4.694%) (-0.880) (6.681)	(103%) when VIX % change > 7%, never cover **Highest delta-hedged return *m* collateral	0.166% (4.442%) (-3.922) (34.223)	0.037	-0.023%	-0.222% (1.641%) (-4.637) (37.384)
2.526% (11.397%) (-0.900) (7.516)	0.222	1.970%	0.591% (4.866%) (-0.791) (13.330)	(100%) when VIX > 22, never cover **2nd highest delta-hedged return *m* collateral	0.414% (5.913%) (-2.889) (22.180)	0.070	0.140%	0.029% (2.123%) (-2.994) (26.616)
3.216% (11.585%) (-0.362) (6.982)	0.271	2.674%	0.512% (4.557%) (-0.862) (14.290)	(103%) when VIX > 26, never cover **Highest excess return *M* collateral	0.650% (5.557%) (-1.661) (17.924)	0.116	0.422%	0.017% (2.130%) (-2.734) (30.818)
0.379% (1.049%) (2.539) (31.124)	0.361	0.371%	0.010% (1.075%) (1.238) (17.095)	(94%) when VIX < 16, never cover **Highest Sharpe ratio *M* collateral	0.183% (0.509%) (1.359) (39.716)	0.309	0.179%	-0.046% (0.588%) (-1.700) (17.977)
0.161% (3.492%) (-2.854) (18.272)	0.046	0.071%	0.307% (2.490%) (0.839) (14.601)	(91%) when VIX < 24, cover when VIX > 30 **Highest delta-hedge return *M* collateral	0.099% (1.805%) (-2.265) (31.291)	0.055	0.057%	0.148% (1.361%) (2.963) (40.291)
1.517% (8.609%) (-2.090) (15.631)	0.176	1.152%	0.426% (5.541%) (-0.688) (12.771)	(94%) when VIX % change < -5%, never cover **15th highest delta-hedge return *M* collateral	0.176% (3,970%) (-3.029) (25.677)	0.044	0.031%	0.081% (2.518%) (-1.517) (22.768)

Exhibit 3: Top performing strategies by various metrics. Top portion shows best performers for *m* collateral and bottom shows best performers for *M* collateral. All returns are in excess of the return on 3-month Treasury bills. The left side shows results for the strategy with *m* collateral invested. The right side shows results for the same strategy with *M* collateral.

APPENDIX: DAILY DELTA-HEDGING

Bollen and Whaley [2004] provide a formula to calculate abnormal risk-adjusted return for a delta-hedged call option selling strategy; they state that the formula for a delta-hedged put selling strategy is similar. Their approach is not completely correct in estimating the actual return because of an issue in determining the amount invested. Their abnormal return formula[15] (p. 745) is given as

$$ARET_c = \frac{-(c_T - c_0 e^{rT}) + \sum_{t=0}^{T-1} \Delta_t (S_{t+1} - S_t e^r + D_{t+1}) e^{r(T-t)}}{\Delta_0 S_0 - c_0} \tag{11}$$

where r is the risk-free interest rate, D_t is the dividend received on day t, Δ_t is the delta value on day t during the option's life, S_t is the closing price of the underlying asset on day t, and c_t is the call option price on day t. Each day during the life of the trade, the amount of the underlying asset in the portfolio is adjusted in order to delta-hedge the option. According to the formula above, the amount invested is simply the size of the initial delta-hedge asset position minus the cash premium received for selling the call. However, this amount[16] does not take into account additional moneys which need to be invested during the life of the trade in order to maintain the daily delta-hedge.[17]

As an extreme example to illustrate the point, suppose SPX is 1000 at $t=0$ and the strategy sells a near-the-money call option with a delta of 0.5, so the amount of SPX

[15] This formula is the corrected version on the *Journal of Finance* website at http://www.afajof.org/afa/all/jf-bollen-whaley-errata.pdf

[16] Furthermore, it is unclear what the initial investment ought to be in order to hedge a short put position. Hedging a short put requires selling short the underlying asset. One assumes that in the case of short-puts, the collateral used for the delta-hedge is the amount need to cash secure the put.

[17] Also, this collateral amount does not take advantage of portfolio margin which allows offsetting positions to reduce the overall required collateral. This effect may increase estimated returns.

necessary to hedge one contract is 500. Suppose further that SPX rises to 2000 in one day; the delta-hedge investment then has a value of 1000. Now consider the situation at $t=1$, the call option delta has increased to nearly 1, and so the amount required to delta-hedge in period $t=1$ is about 2000. Since the hedge investment has grown to only 1000, the investor needs to provide an additional 1000. Therefore, the total investment required (so far) for this strategy is 1500 – c_0 rather than the 500 – c_0 stated in the formula.

Daily delta-hedges do not adequately hedge the option position. In the example at $t=1$, the deep-in-the-money call option with a strike of 1000 has a value of approximately 1000 when SPX is at 2000, for a loss of about 1000 – c_0. The initial hedge provides protection against only 500 of this loss. The more extreme the daily price changes, the more likely it is that daily hedging deviates from full protection. A short put delta-hedge mirrors the above. Suppose, as before, SPX is at 1000 and the investor sells an at-the-money put; the put delta is -0.5, so the delta-hedge investment shorts 500 of SPX. SPX rises to 2000; the short hedge loses 500. At $t=1$, the put option has a delta of approximately 0, so the delta-hedge is effectively a zero position. If SPX goes back to 1000, the investor reestablishes the 500 SPX short. As SPX bounces between 1000 and 2000, the portfolio loses approximately 500 on every bounce. Thus, *ex ante* collateral requirements appear to be uncertain and unbounded for a daily delta-hedged short option position (calls and puts), even if the price of the underlying is known *ex ante*.