# The Puzzle of Index Option Returns

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#### **Abstract**

We document that the leverage-adjusted returns on S&P 500 index calls and puts are decreasing in their strike-to-price ratio over 1986-2007, contrary to the prediction of the Black-Scholes-Merton model; and the leverage-unadjusted returns on S&P 500 index calls are decreasing in their strike-to-price ratio, contrary to the prediction and empirical results of Coval and Shumway (2001). Several factor models are tested and fail to explain the cross-section of option returns. Two option-specific factors, the change in monthly OTM put volume and the change in the VIX index, have some explanatory power when the factor premia are estimated from the universe of options but large alphas remain when the premia are estimated from equities. The three Fama-French factors leave large alphas even when the premia are estimated from options.

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Surprising little is known about the pattern of index option returns, even though call and put options on the S&P 500 index have high volume of trade on the Chicago Board Options Exchange since April 1986, are liquid, and the panel data on their transaction or closing prices is readily available. We find that the leverage-adjusted average returns on S&P 500 index call and put options are decreasing in their strike-to-price ratio over the period 1986-2007, contrary to the prediction of the Black and Scholes (1973) and Merton (1973) (hereafter "BSM") model that the leverage-adjusted returns on call and put options are invariant to their strike-to-price ratio. We also find that the leverage-unadjusted returns on S&P 500 index call options are decreasing in their strike-to-price ratio, contrary to the prediction of the theoretical model and empirical findings of Coval and Shumway (2001) that the leverage-unadjusted returns on call and put options are increasing in their strike-to-price ratio.<sup>1</sup>

A potential explanation of these results is that one or more priced factors are missing, such as stochastic volatility and jump risk which are omitted in the BSM model; and factors which have traditionally been applied to explain the cross-section of equity returns, such as value and size. We test a large number of plausible factor models, where the factor premia are estimated either from the universe of stocks or the universe of options. The most promising factor is the percentage change in the monthly OTM put option volume ( $\Delta$ OTM). The alphas are large when the factor premium is estimated from the universe of stocks but small when the premium is estimated from the universe of options. Another factor which shows promise is the percentage change in the VIX index at the end of the month relative to the beginning of the month ( $\Delta$ VIX). It is noteworthy that the two most promising factors are option specific. By contrast, the three Fama-French factors leave large alphas even when the premia are estimated from the universe of options. All models, including those with promising factors, are formally rejected. The results suggest that market sentiment, liquidity, and market segmentation are promising directions for future research.

A cross-section of index option returns of different moneyness presents a novel set of technical challenges. The first challenge is to obtain statistically significant variation in the cross-section of returns because, as Broadie, Chernov, and Johannes (2009) demonstrated, data errors and transaction costs may lead to the conclusion that even simple models can be consistent

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<sup>&</sup>lt;sup>1</sup> For related research on individual stock options see Chaudhuri and Schroder (2009), Şerban, Lehoczky, and Seppi (2008), and Ni (2007).

with the data. We address this issue by using portfolios of options with different moneyness as opposed to individual options. The second challenge is to generate portfolio returns which are stationary and not overly skewed. We address this issue by revising the portfolios daily in a way that the moneyness, maturity, and leverage of each portfolio remain fairly constant.<sup>2</sup> The third challenge stems from the occasional lack of price quotes when we wish to trade out of an options position which may lead to survivorship bias, look-ahead bias, and the revision of the portfolios at artificial prices. We address these problems and also demonstrate that the results are sensitive to the method of portfolio revision.

The paper is organized as follows. In Section I, we provide the theoretical motivation for our empirical investigation. We review the literature in Section II. In Section III, we describe the data sets, filters, and the formation of portfolios of options. In Section IV, we present our empirical results of tests the BSM model. In Section V, we present our empirical results of tests of factor pricing models. We conclude in Section VI. Technical descriptions are relegated to Appendices A-C.

#### I. Theoretical Motivation

The BSM option pricing model implies the following hypothesis:

 $H_0$ : The instantaneous expected rate of return of a leverage-adjusted option equals the expected rate of return of the underlying security and is independent of the option moneyness.

The leverage-adjusted expected rate of return on an option is defined as the expected rate of return on a portfolio of an option and the risk free rate, where the weight of the option in the portfolio equals the inverse of the elasticity of the option price with respect to the price of the underlying security. As we demonstrate below, this hypothesis also holds even when we relax the BSM assumption of constant volatility and allow the volatility to be a general function of the price of the underlying security and time.

<sup>2</sup> See Buraschi and Jackwerth (2001) for an early construction of portfolios addressing the first two challenges.

Specifically, let S be the price of the underlying security with dynamics  $dS/S = \mu_S dt + \sigma_S dB^S(t)$ , where  $\sigma_S = \sigma_S(S,t)$  is the volatility of the underlying security and  $B^S(t)$  is a Brownian motion. The option price, H(S,t), satisfies the partial differential equation

$$H_t + rSH_S + \sigma_S^2 S^2 H_{SS} / 2 = rH$$
 (1)

where r = r(S,t) is the risk free rate. The option elasticity is defined as  $\omega = SH_S/H$ . The leverage-adjusted instantaneous expected rate of return on the option is

$$\omega^{-1} \frac{1}{dt} E \left[ \frac{dH}{H} \right] + \left( 1 - \omega^{-1} \right) r = \left( \frac{SH_S}{H} \right)^{-1} \left\{ \frac{1}{dt} E \left[ \frac{dH}{H} \right] - r \right\} + r$$

$$= \left( \frac{SH_S}{H} \right)^{-1} \left\{ \frac{1}{H} \left( H_t + \mu_S SH_S + \sigma_S^2 S^2 H_{SS} / 2 \right) - r \right\} + r$$

$$= \left( \frac{SH_S}{H} \right)^{-1} \left\{ \frac{1}{H} \left( rH + \left( \mu_S - r \right) SH_S \right) - r \right\} + r$$

$$= \mu_S$$

$$(2)$$

and is independent of the option moneyness. In this paper, we test and reject this hypothesis.

In earlier related work, Coval and Shumway (2001) proposed and tested the following variant of hypothesis  $H_0$ :

 $H_{C-S}$ : The instantaneous expected rate of return of a leverage-unadjusted call or put option is increasing in the strike, if the stochastic discount factor is negatively correlated with the price of the underlying security over all ranges of the price of the underlying security and the risk premium on the underlying security is positive.

In the context of the BSM model, the leverage-unadjusted expected rate of return on an option is

$$\frac{1}{dt}E\left[\frac{dH}{H}\right] = \frac{1}{H}\left(H_t + \mu_S S H_S + \sigma_S^2 S^2 H_{SS} / 2\right)$$

$$= \frac{1}{H}\left(rH + (\mu_S - r)S H_S\right)$$

$$= r + \frac{SH_S}{H}(\mu_S - r)$$
(3)

and equals the risk free rate plus a premium for the risk of the underlying security. This premium is the product of the risk premium of the underlying security,  $\mu_S - r$ , and the option elasticity or "beta",  $SH_S/H$ , with respect to the underlying security. Coval and Shumway (2001) assumed that the risk premium on the underlying security is positive,  $\mu_S - r > 0$ . Since the BSM elasticity is increasing in the strike, the expected rate of return is increasing in the strike.

Coval and Shumway (2001) did not reject their hypothesis. By contrast, in this paper, we reject their hypothesis for calls on the S&P 500 index but not for puts. One possible reason for the conflicting results is that Coval and Shumway (2001) dropped certain options with missing future prices. However, Coval and Shumway (2001, page 991) stated "[i]n unreported robustness tests, we assign returns of -100 percent to all options for which we find a price on one day but no price on the next day. This leads to no discernible change in our results."

We then investigate an extended model that augments the single-factor model with additional priced factors. The extended model implies the following hypothesis:

 $H_A$ : The instantaneous expected rate of return of a leverage-adjusted option equals the expected rate of return of the underlying security plus a sum of premia; each premium is the product of the risk premium of a factor and the option beta with respect to that factor. The option betas may be either increasing or decreasing in moneyness and need not be monotone functions of moneyness.

The above hypothesis may be derived under a variety of assumptions, some of which allow the underlying security price to have discontinuous sample paths. Below, we illustrate the hypothesis when the sample paths are continuous and the state variable, x, is scalar with

dynamics  $dx = \mu_x(x,t)dt + \sigma_x(x,t)dB^x(t)$  and risk premium  $\lambda = \lambda(x,t) = \mu_x(x,t) - r(x,t)$ . The volatility of the underlying security is  $\sigma_S = \sigma_S(S,x,t)$ . The option price, H(S,x,t), satisfies the partial differential equation

$$H_{t} + rSH_{S} + (\mu_{x} - \lambda)H_{x} + \sigma_{S}^{2}S^{2}H_{SS} / 2 + \sigma_{Sx}SH_{Sx} + \sigma_{x}^{2}H_{xx} / 2 = rH$$
(4)

where  $\sigma_{Sx} \equiv \sigma_S \sigma_x E \left[ dB^S(t) dB^x(t) \right] / dt$ . The leverage-adjusted expected rate of return on the option is

$$\omega^{-1} \frac{1}{dt} E \left[ \frac{dH}{H} \right] + \left( 1 - \omega^{-1} \right) r = \left( \frac{SH_S}{H} \right)^{-1} \left\{ \frac{1}{dt} E \left[ \frac{dH}{H} \right] - r \right\} + r$$

$$= \left( \frac{SH_S}{H} \right)^{-1} \left\{ \frac{1}{H} \left( H_t + \mu_S SH_S + \mu_x H_x + \sigma_S^2 S^2 H_{SS} / 2 + \sigma_{Sx} SH_{Sx} + \sigma_x^2 H_{xx} / 2 \right) - r \right\} + r$$

$$= \left( \frac{SH_S}{H} \right)^{-1} \left\{ \frac{1}{H} \left( rH + \left( \mu_S - r \right) SH_S + \lambda H_x \right) - r \right\} + r$$

$$= \mu_S + \frac{H_x}{SH_S} \lambda.$$

$$(5)$$

The leverage-adjusted instantaneous expected rate of return on the option portfolio equals the expected rate of return on the underlying security plus a premium which is the product of the risk premium,  $\lambda$ , of the factor and the option "beta",  $H_x/SH_S$ , with respect to the factor. Without additional assumptions regarding the factor, it is impossible to determine whether the beta of a European or American call or put option is either increasing or decreasing in moneyness and whether it is a monotone function of moneyness. This is less of an issue in our empirical investigation because we directly measure the beta of each option from the time series of option returns.

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<sup>&</sup>lt;sup>3</sup> If we set  $\sigma_s(S, x, t) = \sqrt{x}$ , we obtain the class of stochastic volatility models such as Heston's (1993) model.

#### **Review of the Literature**

The first line of index options research tests the predictions of the BSM model. Rubinstein (1985) rejected the prediction of the model that the implied volatility of individual stock options is constant across strikes and Rubinstein (1994) rejected the corresponding prediction for index options. An equivalent prediction is that the risk-neutral stock price distribution is lognormal. Jackwerth and Rubinstein (1996), Ait-Sahalia and Lo (1998), Jackwerth (2000), and Ait-Sahalia and Duarte (2003) estimated the risk-neutral stock price distribution from the cross section of option prices.<sup>4</sup> Jackwerth and Rubinstein (1996) confirmed that, prior to the October 1987 crash, the risk-neutral stock price distribution is close to lognormal, consistent with a moderate implied volatility smile. Thereafter, the distribution is systematically skewed to the left, consistent with a more pronounced skew in implied volatilities. Rubinstein (1994) extended the complete-market no-arbitrage model by modeling volatility as a deterministic function of the price,  $\sigma = \sigma(S,t)$ . Dumas, Fleming, and Whaley (1998) rejected this model, pointing out that deterministic volatility models do not predict option prices well and that the parameters change widely across time.

Our first two tests fall under this line of research. Specifically, we test and reject the hypothesis  $H_0$  on S&P 500 index call and put returns; and we test and reject the hypothesis  $H_{C-S}$  on S&P 500 index call returns.

The second line of index options research drops the assumption of the BSM model that the market is complete but maintains the assumption that the stochastic discount factor is a (possibly non-linear) monotone function of just one state variable, the market. Ait-Sahalia and Lo (2000), Jackwerth (2000), and Rosenberg and Engle (2002) estimated the pricing kernel implied by the observed cross section of prices of S&P 500 index options as a function of wealth, where wealth is proxied by the S&P 500 index level. Jackwerth (2000) reported that the pricing kernel is everywhere decreasing during the pre-crash period 1986-1987, but widespread violations occur over the post-crash period 1987-1995. Ait-Sahalia and Lo (2000) examined the

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<sup>&</sup>lt;sup>4</sup> Jackwerth (2004) reviewed the parametric and non-parametric methods for estimating the risk-neutral distribution. Ait-Sahalia and Duarte (2003) estimated the implied risk neutral distribution from a sample of simultaneously-expiring European index option prices while constraining the option pricing function to be monotonic and convex.

year 1993 and reported violations; Rosenberg and Engle (2002) examined the period 1991-1995 and reported violations.<sup>5</sup> Thus, we are faced with an empirical pricing kernel puzzle.

In the same line of research, Constantinides, Jackwerth, and Perrakis (2009) found that S&P 500 index calls, as well as puts, are often overpriced and may be incorporated in portfolios that stochastically dominate portfolios that do not include such options. They rejected the hypothesis that the observed cross-sections of one-month S&P 500 index option prices are consistent with various economic models that explicitly allow for a dynamically incomplete market and also an imperfect market that recognizes trading costs and bid-ask spreads in the context of a one-factor model which is not necessarily linear. Constantinides, Czerwonko, Jackwerth, and Perrakis (2009) empirically verified that portfolios which incorporate S&P 500 index futures options indeed produce *out-of-sample* returns, net of trading costs and bid-ask spreads, which stochastically dominate the returns of portfolios which do not incorporate these options.

The third line of research recognizes that there may be priced factors over and above the market. Many of these models are critically discussed in Hull (2009), Jackwerth (2004), McDonald (2006), and Singleton (2006). Historically, the literature evolved via the stochastic jump model of Merton (1976) and the stochastic volatility models of Heston (1993) and Britten-Jones and Neuberger (2000) onto the modern combined stochastic volatility-stochastic jump models of Bates (1996), Eraker, Johannes, and Polson (2003), and Santa-Clara and Yan (2004).

Beyond stochastic volatility and jumps, a number of authors added other stochastic state variables. Chabi-Yo, Garcia, and Renault (2008) introduced latent state variables upon which then fundamental variables or preferences in turn might depend. Alternatively, Bates (2008) included the number of stock markets crashes as a state variable. Brennan, Liu, and Xia (2008) introduced the interest rate and the maximal Sharpe ratio as additional state variables. Christoffersen, Heston, and Jacobs (2006) added conditional skewness in a GARCH setting. Brown and Jackwerth (2004) suggested that the reported violations of the monotonicity of the pricing kernel may be an artifact of the maintained hypothesis that the pricing kernel is state independent but concluded that volatility cannot be the sole omitted state variable in the pricing kernel.

<sup>&</sup>lt;sup>5</sup> Rosenberg and Engle (2002) found violations when they used an orthogonal polynomial pricing kernel but not when they used a power pricing kernel which, by construction, is decreasing in wealth.

Others calibrated equilibrium models that generate a volatility smile pattern observed in option prices. David and Veronesi (2002) modeled the investors' learning about fundamentals, calibrated their model to earnings data, and provided a close fit to the panel of prices of S&P 500 options. Liu, Pan, and Wang (2005) investigated rare-event premia driven by uncertainty aversion in the context of a calibrated equilibrium model and demonstrated that the model generates volatility smiles similar to those observed in option prices. Benzoni, Collin-Dufresne, and Goldstein (2007) extended the above approach to show that uncertainty aversion is not a necessary ingredient of the model. Rather, they argued that a jump in dividends is unnecessary as long as there can be jumps in expected dividend growth rates. They also demonstrated that the model can generate the stark regime shift that occurred at the time of the 1987 crash. Drechsler and Yaron (2008) and Shaliastovitch (2008) generated the smile observed in the implied volatilities by modeling jumps in consumption growth.

Our main tests fall under this line of research. Specifically, we test the hypothesis  $H_A$  on S&P 500 index call and put returns and obtain largely negative results in that we only partially explain the observed option returns. In earlier related work, Jones (2006) reported that part of the abnormal returns on S&P 500 index options is attributable to a volatility premium, which, however, is insufficient to explain their magnitude, particularly for short term OTM puts.

There are a number of significant methodological differences between Jones' (2006) approach and ours. Jones empirically addressed the time variation of option betas driven by variation of the elasticity of the options, while we directly filter out the effect of the time variation of elasticity on the betas. Jones covered all maturities out to 128 days and all OTM options, while we concentrate on the more liquid ATM and ITM options and a fixed 45 days horizon. His sample spans 1986 to 2000 while we investigate 1986 to 2007. Jones' *daily* root mean squared mispricing error is 1.25% and is twice as large as our *monthly* root mean squared mispricing error of 0.57%. Jones' results may be affected by look-ahead bias, as he eliminated all options which could not be traded out on the following day. Jones used a parametric model for the dynamics of the factors while we simply measure them non-parametrically. Jones confined his attention to the three factors S&P, change in the log VIX, and change in the log 3-month interest rate, while we examine a broader set of factors. Jones estimated the risk premia from the set of options alone, while we report two sets of results, one with the risk premia estimated from the set of options and another with the risk premia estimated from the set of

equities. Thus our results allow us to address the extent to which the options and equities markets are integrated or segmented.

Other lines of research include buying pressure, suggested by Bollen and Whaley (2004), Garleanu, Pedersen, and Poteshman (2009), and Naranjo (2009); and behavioral explanations based on sentiment, suggested by Han (2008) and Shefrin (2005).

### II. Data Sets, Filters, and Portfolio Formation

We construct the return series of ten portfolios of S&P 500 European style options (SPX). Each portfolio is made up of either calls or puts with a targeted moneyness ratio, K/S. Our data starts in April 1986 and ends in June 2007. We initiate our main tests in the second half of 1988 in order to avoid both the incompleteness in the dataset and the destabilizing effect of the 1987 crash. We verify the robustness of our results by also analyzing the full sample period. Appendix A provides technical details on the data sets, filters, and portfolio formation.

## III.1 Data sets

We create our sample by appending two sets of data, the Berkeley Options database which extends to the end of 1995 and the OptionMetrics database which covers the remainder of our sample period. The Berkeley data contains intraday quotes on individual S&P 500 options. We obtain a single end-of-day cross-section of quotes in order to be consistent with the OptionMetrics data and also to avoid non-synchronous trading. Specifically, we look for the minute during the last trading hour with the broadest selection of simultaneous option quotes. We record the characteristics of each option as well as volume information in addition to the quoted bid and ask. We also record the realized dividend payments during the remaining life of each option and calculate a corresponding dividend yield by discounting with the relevant T-Bill rate.

OptionMetrics contains end-of-day quotes collected using a proprietary method similar to the one we outlined for the Berkeley dataset. As with Berkeley, we calculate a dividend yield for each option. Unlike Berkeley, OptionMetrics also provides the open interest of each option contract, and we collect that as well.

#### III.2 Filters

We apply several filters designed to minimize possible quoting errors. The first filter removes outliers. Specifically, we remove options with fewer than 14 or more than 180 days to expiration. The short maturity options often move erratically as they near expiration and the long options are thinly traded. Next, we remove all quotes with zero volume during the trading day because their quoted prices are not reliable. We also remove all options with implied volatility lower than 5% or higher than 100% as these values are unlikely, as well as options with moneyness below 0.8 or above 1.2, which are highly illiquid and have little option value above their intrinsic value. In some cases, we find quotes on options with the same characteristics yet different prices. Out of a set of such quotes, we keep the one with implied volatility closest to that of its moneyness neighbors.

Our next task is to construct a valid proxy for the risk free rate relevant for pricing the options in our data. We assume that put-call parity holds and infer an implied interest rate. We do this because applying the T-Bill rate often leads to violations of put-call parity. Specifically, out of all put-call pairs of a given maturity and moneyness between 0.95 and 1.05 on a given day, we take the median put-call parity-implied interest rate and apply it as the interest rate for all options of the same maturity. This interest rate and the implied volatilities derived from it are the ones used in all our tests.

Next, we remove implied volatility outliers to reduce the prevalence of apparent butterfly arbitrage opportunities. For each maturity on a given date, we fit a quadratic curve (separately to puts and calls) through the observed implied volatilities and then trim the 5% farthest outliers. In a related filter, we compare the interest rate implied by each put-call pair to the median interest rate for its maturity and again trim the 5% farthest outliers. These filters reduce the number of quotes in our data from close to 2 million to 320,000, 44% of which are calls and the rest are puts. On 6% of the trading days in our sample, no observations pass our filters. Most of the missing days are in the year of the October 1987 crash and so do not figure in our tests as we normally start our sample in July of 1988. On days with missing quotes we rebalance our option portfolios on the next date with available quotes, thereby avoiding look-ahead bias.

#### III.3 Portfolio formation

We use the filtered data to form the portfolios and calculate their returns. We trade in and out of portfolios daily to obtain monthly returns that are close to normal. We construct 10 portfolios, five made up of calls and five made up of puts, each with a target maturity of 45 days and moneyness in the discrete set 0.95, 0.98, 1.00, 1.02, and 1.05. We buy and sell options at their bid-ask midpoints.

To select which options go into our portfolios, on each day and for each portfolio, we calculate bivariate Gaussian weights centered on the target maturity and moneyness. Specifically, we set the maturity and moneyness bandwidths of the weighting kernel at 15 days and 0.02, respectively. Alternative bandwidth settings have minimal impact. We omit weights that are less than 1% to reduce the influence of far-out options and scale the resulting weights so that they sum to one. Applying these weights to the options surrounding the portfolio target, we can interpret them as fractions of a dollar invested in each option. We record an error measure that captures the typical distance of options in the portfolio from the target. Based on this measure, we find that options beyond our moneyness range of 0.95 to 1.05 are too noisy to be useful.

We revise each portfolio daily. If a held option has a quote on the following day, we use this quote as the trade-out price. If not, we hold it until it reappears or, if necessary, until the end of the month at which point we extrapolate its price based on a fitted implied volatility curve. When holding on to a missing option, we keep it on the books at the purchase price and rescale its weight, dividing it by the daily portfolio return to fix the original dollar investment in the option. When the option reappears, its new price reflects the cumulative return on the option throughout its time in the portfolio.

For example, if we invest 2 cents in a call and the value of our portfolio doubles from \$1 to \$2 while the call is not traded, the weight of the call becomes 0.01. If the call then comes back and its price too has doubled, its weight would be appropriately restored to 0.02, giving the correct cumulative portfolio return of 100%. In this way, we avoid any look-ahead bias and minimize the effect of missing options on the monthly portfolio return. Options that ultimately reappear do not introduce an error.

The problem of missing options arises both because of incomplete data and because of the application of our filters. On average, 24% of quotes go missing but are subsequently found, and 6% are extrapolated at the end of the month. However, the problem is much more severe in the Berkeley data where these numbers are 48% and 18%, respectively. We also try an alternative specification where we do not impose our filters on the trade-out side of the portfolio. This tends to increase the returns on OTM calls in the Berkeley data, somewhat reducing the steepness of the downward-sloping average return profile. There is almost no impact on puts or on the OptionMetrics data. We prefer using our filters because they avoid apparent arbitrage.

Since options that go missing are heavily concentrated in the Berkeley part of the sample, our use of extrapolation makes a difference to the portfolio returns during that period. In fact, if instead of extrapolating, we simply ignore options that go missing and remove them from the portfolio as if they were never in it, the resulting portfolio returns are increasing in moneyness in the Berkeley sample, whereas there is little change in the OptionMetrics period. We are thus able to replicate the results in Coval and Shumway (2001) when ignoring the options that go missing, as they did. This discrepancy is possibly the result of a tendency for options that go missing to be in the OTM range and to disappear when the index return is low.

In testing our hypothesis  $H_0$ , we construct leverage-adjusted portfolios. Rather than investing one dollar in the option, we invest  $\omega^{-1}$  dollars in the option and  $1-\omega^{-1}$  dollars in the risk free rate, where  $\omega$  is the BSM elasticity based on the implied volatility of the option. We combine the leverage-adjusted option returns into portfolio returns using their weights. This scaling ensures that our portfolios have index betas close to one so that any difference in average returns must be due to some other priced factor. In addition, the scaling mitigates the extreme volatility of each option and makes the portfolio returns close to normal. For example, the ATM monthly put returns on our daily-rebalanced portfolio exhibit skewness of -1.25 and kurtosis of 3.06. By contrast, Broadie, Chernov, and Johannes (2009, Table 2) reported skewness of 4.5 and kurtosis of 25.1 of the monthly put returns on their non-rebalanced portfolio.

Finally, we compound the daily portfolio returns into monthly returns. We calculate excess portfolio returns by subtracting the monthly return on 3-month T-Bills from CRSP, as is customary.

## III. Empirical Results for the Black-Scholes-Merton Model

The BSM model implies the hypothesis  $H_{C-S}$  of Coval and Shumway (2001) on leverage-unadjusted returns and our hypothesis  $H_0$  on leverage-adjusted returns.

#### [Table I about here]

First, we consider leverage-unadjusted portfolios. Surprisingly, we find that S&P 500 call option returns are decreasing in strike, contrary to the hypothesis  $H_{C-S}$ . This is the case in both the Berkeley and OptionMetrics data. On the other hand, put option returns are increasing in strike, consistent with the hypothesis  $H_{C-S}$ . Table I shows the average returns of five call and five put portfolios with leverage-unadjusted returns. In the top panel, we follow our outlined procedure for extrapolating the prices of options that are in a portfolio but then disappear from the data. In the bottom panel, we follow Coval and Shumway (2001) and ignore these options, removing them from the portfolio as if they were never in it. This method introduces look-ahead bias.

The pattern emerging from Table I is that, in the full sample, call portfolio returns are decreasing in leverage, whether missing options are extrapolated or ignored. This is because missing options are relatively rare in the OptionMetrics data and common in the Berkeley data. In the Berkeley subsample, ignoring missing options restores the increasing pattern reported by Coval and Shumway (2001). However, it is the decreasing pattern that is more robust, as it is evident in OptionMetrics and when the look-ahead bias of ignoring missing options is avoided. For this reason, we adopt our extrapolation technique which removes the look-ahead bias.

## [Tables II and III about here]

The next natural question is whether the decreasing pattern in call returns and the increasing pattern in put returns survive once leverage is taken into account. Our leverage-adjusted option portfolio returns are designed to answer this question. As Tables II and III show,

after removing leverage effects, both call and put option portfolio have returns that are decreasing in moneyness, contrary to our hypothesis  $H_0$ .

To further explore these results, we construct a third set of portfolios which we call "synthetic call" portfolios by converting the put prices into call prices via put-call parity. As Table IV shows, the average returns of the synthetic call portfolios are very similar to those of the actual call portfolios. We also construct a fourth set of portfolios which we call "synthetic put" portfolios by converting the call prices into put prices via put-call parity. As Table V shows, the average returns of the synthetic put portfolios are very similar to those of the actual put portfolios. These results suggest that our results are not an artifact of violations of put-call parity in our filtered sample.

## [Tables IV and V about here]

The differences in returns across moneyness are both economically and statistically significant. The last columns of Tables II-V show the average returns on a long-short strategy of buying the portfolio with moneyness 1.05 and selling the portfolio with moneyness 0.95. In each case, this long-short strategy has a statistically significant negative average return roughly commensurate with the overall level of portfolio returns. We conclude that neither hypothesis  $H_0$  nor, for call options, hypothesis  $H_{C-S}$  hold up in the data. In the following sections, we investigate whether factor premia explain these large and surprising patterns in average option returns.

## IV. Empirical Results for Factor Models

Given the rejection of the two hypotheses  $H_0$  and  $H_{C-S}$ , associated with the index being the only state variable, we now turn to a search for additional priced factors, consistent with hypothesis  $H_A$ . We design a linear asset pricing test to check whether proxies for various risk factors suggested by both the option and stock returns literatures account for the returns of our option portfolios. The standard practice in the cross-sectional pricing literature is to jointly estimate

factor premia and pricing errors on a given set of test assets. In our main tests in Section V.1, we eschew this approach for two reasons. First, there are at most two or three sizeable principal components in the covariance structure of the option portfolios and we may not reliably estimate factor premia from this cross-section. Second, guided by the literature on the cross-section of stock prices, we seek factors with premia consistent across asset classes, in this case equities and index options. Later on in Section V.2, we jointly estimate factor premia and pricing errors on a given set of options as test assets.

For our main tests in Section V.1, we modify the standard linear factor pricing model and use only stock portfolios for the estimation of risk premia. We then use these estimates to calculate the alphas of the option portfolios. Our leverage-adjusted option portfolios with daily rebalancing have portfolio returns closer to normal than is common in the option pricing literature. For this reason, we are comfortable with a linear pricing test. Our approach consists of several stages and may potentially introduce the problem of generated regressors. We deal with this issue by running all stages simultaneously in one GMM criterion, as suggested by Cochrane (2005). The methodology is described in Appendix B.

We test a broad spectrum of factor models, some suggested by the literature. Appendix C provides a listing of the factors and their description. We consider factors that have been shown to price equities such as the S&P 500 market factor, the three Fama-French factors, momentum, as well as innovations in the Sharpe ratio and the price-dividend ratio; we consider interest rate related factors such as the term spread, default, and innovations in the market interest rate; we consider factors suggested by the options literature, such as jumps, the innovation in option volume, the innovation in ATM implied volatility, the slope of the smile, and the spread between implied and realized volatility; finally, we introduce our own options-related factors, namely, the innovation in the VIX and in the OTM put volume.

We investigate 15 factor models that employ combinations of the factors described in Appendix C. The first model has only one factor, the return on the S&P 500 index. The second model has the three Fama-French factors: Market, SMB, and HML. All the other models are limited to two factors because the small size of the cross-section of option returns renders the returns collinear. In the two-factor models, the first factor is the return on the S&P 500 index.

In Table VI, we report the Fama-MacBeth betas associated with the factors of each model. In all the models, the S&P 500 (or, market) beta of each portfolio is close to one by construction: the option portfolios are leverage-adjusted to have index betas close to one so that any difference in average returns must be due to some other priced factor. Note also that the option portfolios have statistically significant betas with respect to several of the factors which we study: SMB, HML, Jump,  $\Delta$ IV, IVRV,  $\Delta$ VIX,  $\Delta$ Vol,  $\Delta$ OTM, and Slope; and insignificant betas with respect to Default, Term, Mom,  $\Delta$ R, Sharpe, and  $\Delta$ P/D. The next step is to determine which of these factors explain the cross-section of returns of the option portfolios.

## V.1 Factor pricing with premia estimated from equities

We estimate the factor premia from the set of 25 Fama-French size and book-to-market portfolios. These portfolios provide a rich cross-section, with large differences in average returns. We test whether a given set of factors and factor premia explain the cross-section of option returns. This joint test addresses the question whether any of the proposed factors can account for the spread in option returns with premia derived from equities. In Table VII, we report the factor premia for each of these models estimated from the 25 Fama-French portfolios. In Tables VIII-X, we report the pricing errors and test statistics for call portfolios, put portfolios, and pooled call and put portfolios, respectively.

## [Tables VII-X about here]

In all three Tables VIII-X, the S&P 500 single-factor model fails to explain the returns of option portfolios. As the pricing errors show in Table VIII, the call portfolios have returns that are too low given their S&P 500 betas (1.00 for the 0.95 call and 0.97 for the ATM call), and this is especially true for the 1.05 OTM call portfolios with a beta of only 0.87. The average pricing error of the call portfolios is 61 bps per month, and is much bigger than the actual return on any call portfolio. On the other hand, as the pricing errors show in Table IX, the put portfolios have returns that are too big given their S&P betas (0.97 for the 0.95 put, 1.02 for the ATM put, and 1.01 for the 1.05 put). Recall that puts are held short in these portfolios since their BSM

elasticities are negative. The typical pricing error for the put portfolios is 54 bps per month, with bigger pricing errors for the OTM put portfolios. When we use both calls and puts, Table X shows that the pricing error is 57 bps per month. In summary, the OTM calls and OTM puts are expensive based on the S&P 500 single-factor model. Below we investigate whether other factors explain this mispricing.

Of the remaining factors, the clear standout is the percentage change in the monthly OTM put option volume ( $\Delta$ OTM, model 8). This is not a price-based factor. We introduce this factor as a proxy for unusual demand for downside risk protection or market sentiment. When added to the market factor, this factor does a good job in pricing the put portfolios in Table IX with a pricing error of 22 bps, by far the lowest among the specifications in Table IX. Perhaps more surprisingly, the change in OTM put volume is also the best performer among calls with an average pricing error of just 10 bps in Table VII. When both calls and puts are used in Table X, the average pricing error is the average of these two or 17 bps. Despite the low pricing errors, this model is rejected among puts as well as when both calls and puts are used, although in the latter case the bootstrapped p-value has the marginal value of 0.02. While it does a good job in pricing the ITM calls and puts, this model is rejected on the OTM calls and, particularly, on the OTM puts.

The negative premium on the change in OTM put volume is consistent with its interpretation as market sentiment: assets that pay when fear is high tend to have negative risk premia because they provide insurance. It is also consistent with the alternative interpretation that an increase in OTM put volume signals a higher probability of a negative jump. Note also that this premium is estimated among equities so we can say that the ability of this factor to account for the returns of option portfolios is consistent with its premium in the cross-section of equities, even though the model is rejected among equities.

Among the other factors in Tables VIII-X, we note the marginal success of the factor that tracks the percentage change in the VIX index at the end of the month relative to the beginning of the month ( $\Delta$ VIX, model 6). This factor comes second to the change in OTM put volume in the horserace among all the factors based on bps mispricing, but is clearly unable to account for the returns of the option portfolios. We interpret this evidence as limited support of a stochastic volatility model of option prices.

Perhaps the most surprising take-away from the asset pricing tests is that none of the other option-based factors, in combination with the market factor, is able to price the cross-section of option portfolios. For example, we construct a slope factor equal to the difference in returns between our two highest and lowest performing portfolios, the OTM put and OTM call portfolios. This naïve factor fails to account for much of the difference in average returns between our test assets.

Other factors, including the Fama-French factors, momentum, the default and term spreads, and other factors related to changes in implied volatility are all unable to price the option portfolios. The jump factor, which is equal to the S&P return if it is below -5% and zero otherwise, obtains a negative premium among equities and is thus counter-productive in pricing the options.

## V.2 Factor pricing with premia estimated from options

We now relax the condition that the factor premia are estimated from the universe of equity returns and test whether a factor model explains the cross-section of option returns when the factor premia are estimated from the same cross-section of option returns. In Tables XI-XIII, we present results for the five call portfolios, the five put portfolios, and the pooled call and put portfolios, respectively.

#### [Tables XI-XIII about here]

In the combined pool of calls and puts in Table XIII, all nine models, except the model with the single S&P 500 factor and the model with the combined S&P 500 and HML factors, have low pricing errors. In seven out of the nine models in Table XIII, including the model with the promising factor  $\Delta$ Vol, the factor premia based on calls and puts are of the same sign as the factor premia based on equities. In the set of calls, the models with factors  $\Delta$ Vol and IVRV are not rejected with p-values 38% and 7%, respectively (Table XI). All other models in Tables XI-XIII are formally rejected.

Since we relax the condition that the factor premia are estimated from the universe of equity returns, it is not surprising that the pricing errors of all models in Tables XI-XIII are lower

than the corresponding pricing errors in Tables VIII-X. The take-away from these results is the large decrease in the pricing errors and this suggests that the equities and index options markets are segmented.

## V. Concluding Remarks

In this paper, we establish that both the leverage-adjusted and the leverage-unadjusted returns on S&P 500 index options exhibit patterns as functions of the strike which strongly reject the predictions of the Black-Scholes-Merton model. As a potential explanation of this puzzle, we consider a range of factor pricing theories, where the factor premia are estimated either from the universe of stocks or the universe of options. Whereas all models are formally rejected, two models with option specific factors are promising and provide directions for future research.

The best-performing factor in explain the cross-section of both the calls and puts is the percentage change in the monthly OTM put option volume ( $\Delta$ OTM) and this provides two clues to the puzzle. The negative premium on this factor is consistent with the interpretation as a factor for market sentiment: assets that pay when fear is high have negative risk premia because they provide insurance for down-side risk. This factor also has a different interpretation as a liquidity factor. The results suggest that market sentiment and liquidity are both promising directions for future research.

Among the other factors, the percentage change in the VIX index at the end of the month relative to the beginning of the month ( $\Delta$ VIX) comes second in the horserace among all the factors. The result suggests that stochastic volatility is a promising direction for future research.

Since we cannot find a common set of factors that explain the cross-section of both equity and index option returns, the results are open to the interpretation that the equities and index options markets are segmented due to market imperfections. This interpretation is consistent with equilibrium in segmented markets along the following lines. Mutual funds exert price pressure on OTM index puts because they buy them as insurance; and over-optimistic speculators exert price pressure on OTM calls because they buy them as a leveraged bet. Furthermore, Garleanu, Pedersen, and Poteshman (2008) argued that dealers inflate the prices of options. Large investors such as hedge funds who can potentially eliminate the overpricing do

not write these overpriced options in a large scale because, as Santa-Clara and Saretto (2009) pointed out, they face obstacles including margin calls and the lack of market depth.

#### **Appendix A: Data and Methodology**

#### A.1 Data

We construct returns on portfolios made up of S&P 500 European style call and put options, covering the period from April 2, 1986 to June 29, 2007. Specifically, we append two datasets, the Berkeley Options database that ends on December 29, 1995 and OptionMetrics starting on January 4, 1996. We start most of our tests on July 1, 1988 as the data up to that point suffer both from incompleteness and the destabilizing effect of the 1987 crash. We reintroduce the omitted period later to verify the robustness of our results.

The Berkeley Option Database consists of intraday quotes on individual options and we seek to extract a single end-of-day cross section of quotes, comparable to the quotes provided by OptionMetrics in the latter part of our sample. In addition, we seek to avoid the issue of non-synchronous trading. To that end, on each trading day, we find the minute between 3 and 4 PM Central Standard Time with the largest number of simultaneous quotes. We stop at 4PM because the market closes at 4:15 PM and we want to avoid contamination relating to last minute trading activity. We also record the intraday volume of each of our end-of-day options, as well as total daily call volume and total daily put volume. We also collect the present value of all realized dividend payments during the remaining life of each option, discounting with the relevant constant maturity T-bill rate from the H.15 statistical release of the Federal Reserve. We work out the associated continuously compounded dividend yield.

The OptionMetrics database is already in the form of end-of-day quotes, collected using a proprietary method similar to the one we outline for the Berkeley database. As with the Berkeley data, we compute a dividend yield based on the realized dividend payments over the remaining life of each option. OptionMetrics also includes the open interest of each option.

#### A.2 Data cleaning

We apply several filters designed to minimize possible quoting errors. In constructing our portfolios, we apply these filters on the trade-in side to make sure that we are buying into reliable quotes. Later when we seek to exit our position, if no quote is available in the filtered data, we look for a price in the raw data. Applying our filters on the buy side and only on the buy side minimizes the problem of having to make up trade-out prices for options that were bought but

cannot be sold due to missing observations. Our raw data contains 1,976,805 quotes to which we apply the following filters in sequence:

- a. Removing outliers. We first apply coarse filters designed to remove obvious outliers.
- (i) Days to maturity filter. We remove all options with fewer than 14 or more than 365 calendar days to expiration. The short maturity options tend to move erratically close to expiration and the long maturity options lack volume and open interest. This rule eliminates 508,478 quotes.
- (ii) Volume filter. We remove quotes on option contracts with zero volume during the day. Quotes for options that were not traded are less likely to be an accurate reflection of market conditions and suffer from illiquidity. In addition, we want our results to apply to options that were actually traded by investors. This filter eliminates 894,616 quotes.
- (iii) Extreme implied volatility filter. We remove all option quotes with implied volatilities lower than 5% or higher than 100%, computed using T-Bill interest rates. Such extreme values likely indicate quotation problems. This removes 20,935 quotes.
- (iv) Moneyness filter. We remove all option quotes with moneyness, the ratio of strike price to index price, below 0.7 or above 1.3. These options have little value beyond their intrinsic value and are also very thinly traded. This further reduces our observations by 15,591.
- (v) Duplicates filter. The OptionMetrics dataset appears to contain duplicate observations, defined as two or more quotes with identical option type, strike, expiration date, and price. In each such case, we eliminate all but one of the quotes, for a total of 91,611 deleted quotes.
- (vi) Different price duplicates filter. There are also a number of sets of quotes with identical terms (type, strike, and maturity) but different prices. When this occurs, we keep the quote whose T-Bill-based implied volatility is closest to that of its moneyness neighbors, and delete the others. This eliminates 91,754 quotes.
- b. Implied interest rate: When filtering outliers, we use T-Bill interest rates to compute implied volatilities. From the outlier-filtered data, we compute a put-call-parity implied interest rate and use it in the remainder of the paper and for further filters:
- (i) T-Bill interest rates are obtained from the Federal Reserve's H.15 release. We assign a T-Bill rate to each observation by assuming that we can use the next shortest rate if the time to expiration of the option is shorter than the shortest constant maturity rate. On 48 days, the T-Bill

interest rate was not provided but option prices were quoted. In these cases, we used the last observed interest rate which was recorded either one day earlier (for 11 days) or three days earlier (for 37 days).

(ii) Our goal is to obtain an interest rate that is as close as possible to the one faced by investors in the options market. It appears that the T-Bill rates are not the relevant ones when pricing these options. Specifically, when the T-Bill rates are used, put and call implied volatilities do not line up very well; for example the T-Bill rate tends to be too high for short maturity options, perhaps because no T-Bill has maturity of less than a month. To address these issues, we compute a put-call-parity-implied interest rate. Since we strongly believe that put-call parity holds reasonably well in this deep and liquid European options market, we use the put-call-parity implied interest rate as our interest rate.

To construct this rate, we take all put-call pairs of a given maturity and impose put-call parity using the bid-ask midpoint as the price, and allowing the interest rate to adjust. We remove 4,155 pairs with a negative implied interest rate. We then take the median implied interest rate across all remaining pairs of the same maturity with moneyness between 0.95 and 1.05 and assign it to all quotes with that maturity. We are able to directly assign an implied interest rate to 88% of our sample in this way. We fill in the gaps in two ways: by using the implied interest rate for the same maturity from one of the previous five trading days (this was done for 34,200 quotes), or if not possible, by taking a Gaussian weighted average of the implied interest rates for other maturities (this was done for 8,210 quotes). Together, these steps assign a put-call parity implied interest rate to 99% of our sample. We filter out the remaining observations. Our implied interest rate is on average 24 basis points below the T-Bill rate.

Next, we compute implied volatilities based on the put-call parity implied interest rate, and these tend to be only 0.007 away from the T-Bill implied volatilities. In the remainder of the paper, we work exclusively with these implied volatilities. There are 436,128 remaining quotes in our database at this stage.

c. Implied volatility filter: We remove implied volatility outliers to reduce the prevalence of apparent butterfly arbitrage. For each date and maturity, we fit a robust quadratic curve (separately to puts and calls) through the observed implied volatilities. About 18% (7,790) of all IV curves do not have enough degrees of freedom to fit a quadratic curve, in which case we fit a

linear one. We calibrate a confidence band around all curves using the entire sample. Combining the information from all days and maturities in the sample, we compute a typical (one standard deviation) relative distance in percent from the level of the fitted curve for different levels of moneyness (0.7,..., 0.75,..., 1.3). Thus, for each fitted IV curve, we compute the relative distance of all option IVs from the fitted IV curve and we calculate the standard deviation of these relative distances for each moneyness bin. In a second pass, we check for each option's IV, how many standard deviations it is apart from the fitted IV curve. These distances are tight in and around the money (about 3-4%) and wide in the out of the money range (around 5-9%). We eliminate 24,798 observations whose relative distance from the curve is larger than two standard deviations, which adds up to about 5.7% of the data. We are left with 411,330 observations.

d. Put-call parity filter. For every put-call pair with the same date, maturity, and moneyness, we insure that put-call parity holds and that violations are eliminated. Thus, for each put-call pair, we find the bid-ask midpoint put-call-parity implied interest rate. Next, we trim outliers in a similar way as with the IV filter. Specifically, we use the whole sample of relative distances of the put-call parity implied interest rates from the corresponding daily median implied interest rate to find the standard deviation of the corresponding relative distances. This distance is computed to be about 3% of the interest rate level. Finally, we remove 300 put-call pairs that deviate by more than two standard deviations from the corresponding daily average implied interest rate. Applying this final filter further reduces the number of observations to 411,030, 42% of which are calls and the rest puts.

### A.3 Portfolio formation

Having filtered the data, we are ready to form portfolios and compute their returns. In general we buy into a position using only the filtered quotes, and we sell out of that position using the raw data if necessary. This is done to minimize the number of options that disappear from the data after entering a portfolio. We form 10 portfolios, five made up of calls and five made up of puts, each with a time to maturity target of 45 days, and with moneyness targets of 0.95, 0.98, 1, 1.02, and 1.05, where moneyness is the ratio of strike price to index price. In constructing these portfolios, we adhere to the following procedure:

- a. At each date t, we use a bivariate Gaussian weighting kernel in moneyness and days to maturity to calculate weights for each portfolio. The weighting kernel has bandwidths of 15 days to maturity and 0.02 in moneyness, although alternative settings make little difference. The weights are scaled to sum up to one and have the interpretation of dollar amounts invested in each option.
- b. We record an error measure meant to capture the average distance between each option and the portfolio target. Specifically, we define the distance between option i and portfolio target j to be

$$dist(i,j) = \sqrt{\left(\frac{(K/S)_{i} - (K/S)_{j}}{h_{K/S}}\right)^{2} + \left(\frac{DTM_{i} - DTM_{j}}{h_{DTM}}\right)^{2}}$$

where  $h_{K/S}$  and  $h_{DTM}$  are the moneyness and maturity bandwidths of the weighting kernel. To get a portfolio-level error measure, we average these distances over all options for each target portfolio, where the averaging is done using the same Gaussian weights as in forming the portfolio. The average error measures are on average low near the money and increase towards the end of the moneyness range. For this reason, we restrict attention to the five midrange portfolios, those with moneyness between 0.95 and 1.05. We also note that the error measures are on average lower in the OptionMetrics part of the sample than in the Berkeley data.

c. For each option in a portfolio, we look for a quote on day t+1, including quotes from the unfiltered data in our search. If a quote is found, the quoted price is used to compute a return for the option. If not, we check if the option is about to expire in which case we use its expiration payoff to calculate a return. If expiry is not imminent, we hold the option in the portfolio until it reappears, or until the end of the month, whichever comes first. If the option fails to reappear by the end of the month, we compute an extrapolated price by fitting an implied volatility surface that is quadratic in maturity and log moneyness to the present filtered options and use the fitted implied volatility to deduce a price for the missing option. When holding on to an option in a portfolio because of a missing quote, we record a daily return of one and adjust its weight to

account for the returns on the remaining options. Whenever the option reappears, is exercised, or its price is extrapolated, we compute a cumulative return and use the rebalanced weight to calculate the resulting portfolio return. In this way, options that go missing and reappear before the end of the month do not introduce error.

When forming the portfolios, 96.2% of options bought are found and sold the next day, 2.8% reappear before the end of the month, 0.9% are sold using an extrapolated price, and 0.1% expire while in the portfolio.

d. Next, we scale option returns by their Black-Scholes elasticity  $\omega = \frac{\partial C}{\partial S} \times \frac{S}{C} = \Delta \times \frac{S}{C}$  for calls and analogously for puts. Specifically, rather than investing the allotted portfolio weight entirely into the option, we invest  $\omega^{-1}$  of that weight in the option and the remaining  $1-\omega^{-1}$  in the risk free asset.

This scaling allows us to tone down the extreme volatility of the individual options and get more precise estimates. The resulting portfolios have an index beta of one by design. Any differences in expected returns between them must be due to additional risk factors.

e. Finally, we compound the daily returns into monthly returns. For the asset pricing tests, we compute excess portfolio returns by subtracting the monthly T-Bill return.

## **Appendix B: Test Methodology of Factor Pricing Models**

Following Cochrane (2005), we run all stages of our regressions simultaneously in one GMM criterion. Specifically, we define the sample average of the vector of errors as

$$g_{T} = \frac{1}{T} \sum_{t=1}^{T} \begin{pmatrix} R_{t}^{S} - a^{S} - \beta^{S} F_{t} \\ (R_{t}^{S} - a^{S} - \beta^{S} F_{t}) \otimes F_{t} \\ R_{t}^{S} - \beta^{S} \lambda \\ R_{t}^{0} - a^{0} - \beta^{0} F_{t} \\ (R_{t}^{0} - a^{0} - \beta^{0} F_{t}) \otimes F \\ R_{t}^{0} - \beta^{0} \lambda \end{pmatrix}$$

where  $R_t^S \equiv \begin{bmatrix} R_{1,t}^S, R_{2,t}^S, ..., R_{N_S,t}^S \end{bmatrix}'$  is the vector of  $N_S$  stock excess returns at time t;  $R_t^O \equiv \begin{bmatrix} R_{1,t}^O, R_{2,t}^O, ..., R_{N_S,t}^S \end{bmatrix}'$  is the vector of  $N_O$  option portfolio excess returns at time t;  $a^S \equiv \begin{bmatrix} a_1^S, a_2^S, ..., a_{N_S}^S \end{bmatrix}'$  and  $a^O \equiv \begin{bmatrix} a_1^O, a_2^O, ..., a_{N_O}^O \end{bmatrix}'$  are time series intercepts;  $F_t \equiv \begin{bmatrix} F_{1,t}, F_{2,t}, ..., F_{N_F,t} \end{bmatrix}'$  is the vector of  $N_F$  factor innovations at time t;  $\beta_{1,1}^S = \begin{bmatrix} \beta_{1,1}^S & ... & \beta_{1,N_F}^S \\ \vdots & \ddots & \vdots \\ \beta_{N_S,1}^S & ... & \beta_{N_S,N_F}^S \end{bmatrix}$  and  $\beta^O \equiv \begin{bmatrix} \beta_{1,1}^O & ... & \beta_{N_O,N_F}^O \\ \vdots & \ddots & \vdots \\ \beta_{N_O,1}^O & ... & \beta_{N_O,N_F}^O \end{bmatrix}$  are factor loadings; and  $\beta^O \equiv \begin{bmatrix} \beta_{N_O,1}^O & ... & \beta_{N_O,N_F}^O \\ \vdots & \ddots & \vdots \\ \beta_{N_O,1}^O & ... & \beta_{N_O,N_F}^O \end{bmatrix}$  are factor loadings; and

 $\lambda \equiv \left[\lambda_1, \lambda_2, ..., \lambda_{N_F}\right]$  is a vector of factor premia. Note that we do not allow a free cross-sectional intercept. This increases statistical power and avoids a common problem where economically small variation in betas is amplified by large estimated premia together with a big cross-sectional intercept. Paradoxically, such estimates imply a non-zero excess return on the risk-free asset.

We estimate the model without using information from options to estimate premia and without allowing deviations from standard betas to better fit the cross-section: we use a selection matrix that ensures that the stock betas are obtained from the first two sets of moment restrictions, the factor premia are obtained from the third set of moment restrictions, and the

option betas are obtained from the fourth and fifth sets of moment restrictions, respectively. Specifically, we use the selection matrix

$$L \equiv \begin{pmatrix} I_{N_S} & 0 & 0 & 0 & 0 & 0 \\ 0 & I_{N_SN_F} & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta^{S'} & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{N_O} & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{N_ON_F} & 0 \end{pmatrix}$$

where  $I_N$  is the  $N \times N$  identity matrix where N can be a product term such as  $N_S N_F$ . In this way, we force OLS estimates of the factor premia.<sup>6,7</sup> In terms of the selection matrix L, our GMM estimator is the solution to the just-identified system of equations,  $Lg_T = 0$ .

The last moment of the GMM criterion corresponds to the pricing errors of the option portfolios. Since all parameters in that moment are estimated using the other moment restrictions (note that the last column of the selection matrix contains only zeros), the option pricing errors provide a set of  $N_o$  additional over-identified restrictions for testing the model (there are also  $N_S - N_F$  over-identified restrictions from the stock portfolios).

We apply GMM to this system, noting that the resulting standard errors take into account the covariances between the estimated parameters. Our test is otherwise similar to two-stage OLS, except for the fact that the option portfolios are not used in the estimation of the factor premia.

In addition to GMM standard errors, we report bootstrapped standard errors and test statistics in order to account for the small sample properties of our estimators. We bootstrap our

<sup>&</sup>lt;sup>6</sup> To see why, write the third element of  $Lg_T = 0$ :  $\beta^{s'}E(R_t^s - \beta^{s'}\lambda) = 0$ . Distributing, we obtain the OLS restriction  $\lambda = [\beta^{s'}\beta^s]^{-1}\beta^{s'}E[R_t^s]$ , which imposes OLS estimates for lambda. In the other restrictions, the identity matrices achieve the same because we have already multiplied by the factors  $F_t$  inside the criterion. In other words, we could have also written the criterion  $(R_t^s - \beta^{s'}\lambda)\beta^s$  and then used an identity matrix in the selection matrix L.

<sup>&</sup>lt;sup>7</sup> We do not iterate because we are not estimating efficient GMM. We are applying the *a priori* view that betas should be OLS betas and lambdas should be OLS lambdas. Efficient GMM here is problematic because some of the moments are collinear and because GMM distorts the betas to fit the cross-section. However, the whole content of our asset pricing test is that expected returns are linear in betas where beta is defined as the ratio of covariance to variance, which is OLS.

data by removing the estimated pricing errors from the returns on the test assets and reestimating the model under the null of zero pricing errors by randomly sampling our data while preserving its cross-sectional structure.

Furthermore, in additional computations used in Section V.2, we also estimate the premia based on the options only. Here, the selection matrix is

$$\begin{pmatrix} 0 & 0 & 0 & I_{N_O} & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{N_ON_F} & 0 \\ 0 & 0 & 0 & 0 & 0 & \boldsymbol{\beta}^{O'} \end{pmatrix}$$

## **Appendix C: Description of the Factors**

**S&P**: monthly return on the S&P 500 index

**Market**: monthly return on the CRSP value-weighted market index

**SMB** and **HML**: Fama and French SMB and HML monthly factor returns from http://mba.tuck.dartmouth.edu/pages/faculty/ken.french

**Jump**: excess monthly return on the S&P 500 if the net S&P monthly return is -5% or less, and zero otherwise

ΔIV: end-of-the-month ATM call IV divided by the beginning-of-the-month ATM call IV

**IVRV**: annualized implied volatility at the start of the month minus the realized volatility over the month

**ΔVIX**: percentage change in the VIX index at the end of the month relative to the beginning of the month

 $\Delta$ Vol: percentage change in the monthly option volume

**ΔΟΤΜ**: percentage change in the monthly 0.95 OTM put option volume

**Slope**: difference in the monthly return of the 1.05 OTM call portfolio and the 0.95 OTM put portfolio

**Default**: monthly premium of the BAA bond return over the AAA bond return

**Term**: monthly premium of the 10-year bond return over the 3-month T-Bill return

**Mom**: UMD momentum factor long stocks with high past return and short stocks with low past return

ΔR: innovation in the market interest rate as in Brennan, Wang, and Xia (2004)

**Sharpe**: innovation in the market Sharpe ratio as in Brennan, Wang, and Xia (2004)

ΔP/D: change in the price dividend ratio from http://www.econ.yale.edu/~shiller/data.htm

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**Table I.** Average leverage-unadjusted excess returns. In the top panel, portfolios are formed as in the rest of the paper, except returns are not scaled by option elasticities. Missing options are extrapolated if they do not reappear by the end of the month. In the bottom panel, missing options are ignored, i.e. an option that goes missing is erased from the portfolio holdings and thus introducing look-ahead bias. The full sample covers July 1988 through June 2007. The Berkeley sample covers July 1988 through December 1995. The OptionMetrics sample covers January 1996 through June 2007.

	Call portfolios						Put portfolios				
	0.95	0.98	1.00	1.02	1.05	0.95	0.98	1.00	1.02	1.05	
	Leverage-unadjusted returns										
Full sample	2.50	-0.37	-2.51	-9.08	-19.36	-53.59	-42.71	-32.25	-23.59	-16.48	
Berkeley	9.13	9.63	10.92	2.72	-14.55	-60.97	-47.51	-34.51	-24.18	-16.26	
OptionMetrics	-2.02	-7.19	-11.66	-17.12	-22.64	-48.56	-39.44	-30.72	-23.19	-16.62	
	Leverage-unadjusted returns, ignoring missing options										
Full sample	0.05	-1.42	-1.83	-3.31	-5.17	-49.13	-41.04	-31.84	-24.48	-20.38	
Berkeley	6.60	8.05	12.16	15.95	22.14	-48.84	-42.26	-32.13	-24.68	-22.30	
OptionMetrics	-4.41	-7.88	-11.36	-16.44	-23.80	-49.34	-40.21	-31.65	-24.35	-19.06	

**Table II.** Descriptive statistics for the leverage-adjusted excess returns of the five call option portfolios. J-B statistics are from Jarque-Bera normality and log-normality tests. The last column is a long-short between the 1.05 OTM portfolio and the 0.95 ITM portfolio. The sample covers July 1988 to June 2007.

			Call po	ortfolio mo	oneyness	
	0.95	0.98	1.00	1.02	1.05	OTM-ITM
_ <del>-</del>				Moments	S	
Mean	0.36	0.23	0.17	0.11	0.06	-0.30
(St. err.)	0.27	0.27	0.27	0.27	0.27	0.11
Sharpe	0.30	0.20	0.14	0.10	0.05	-0.66
St. Dev.	4.06	4.06	4.04	4.01	3.95	1.57
Skewness	-0.17	0.05	0.29	0.58	0.96	1.10
Kurtosis	0.53	0.83	1.47	2.32	3.36	3.05
			N	ormality t	test	
J-B stat.	4	6	23	62	138	131
(p-val.)	0.16	0.04	0.00	0.00	0.00	0.00
			Log	-normalit	y test	
J-B stat.	8	6	14	34	81	107
(p. val.)	0.02	0.04	0.00	0.00	0.00	0.00
S&P correl.	0.98	0.97	0.95	0.92	0.87	-0.34
S&P beta	1.00	0.99	0.97	0.94	0.87	-0.13

**Table III.** Descriptive statistics for the leverage-adjusted excess returns of the five put option portfolios. J-B statistics are from Jarque-Bera normality and log-normality tests. The last column is a long-short between the 1.05 ITM portfolio and the 0.95 OTM portfolio. The sample covers July 1988 to June 2007.

			Put por	rtfolio mon	eyness	
	0.95	0.98	1.00	1.02	1.05	ITM-OTM
				Moments		
Mean	1.79	1.42	1.15	0.97	0.81	-0.99
(St. err.)	0.31	0.31	0.31	0.30	0.29	0.11
Sharpe	1.33	1.08	0.87	0.75	0.64	-2.08
St. Dev.	4.67	4.59	4.56	4.48	4.37	1.64
Skewness	-1.63	-1.41	-1.25	-1.12	-0.99	1.00
Kurtosis	4.57	3.59	3.06	2.66	2.27	3.46
-			N	ormality te	st	
J-B stat.	291	193	145	112	84	148
(p-val.)	0.00	0.00	0.00	0.00	0.00	0.00
_			Log	-normality	test	
J-B stat.	466	307	238	191	147	120
(p. val.)	0.00	0.00	0.00	0.00	0.00	0.00
S&P correl.	0.82	0.86	0.89	0.91	0.92	0.11
S&P beta	0.97	1.00	1.02	1.03	1.01	0.04

**Table IV.** Descriptive statistics for the leverage-adjusted excess returns of the five synthetic call option portfolios. J-B statistics are from Jarque-Bera normality and lognormality tests. The last column is a long-short between the 1.05 ITM portfolio and the 0.95 OTM portfolio. The sample covers July 1988 to June 2007.

		S	ynthetic c	all portfol	io money	ness
	0.95	0.98	1.00	1.02	1.05	OTM-ITM
				Moment	:S	
Mean	0.29	0.20	0.18	0.16	0.32	0.03
(St. err.)	0.27	0.27	0.27	0.27	0.27	0.13
Sharpe	0.25	0.17	0.15	0.14	0.27	0.05
St. Dev.	4.06	4.05	4.01	3.99	4.03	1.90
Skewness	-0.11	0.02	0.21	0.47	1.04	1.21
Kurtosis	0.50	0.51	0.62	0.89	2.42	2.34
			N	Normality	test	
J-B stat.	3	2	5	15	95	105
(p-val.)	0.24	0.30	0.07	0.00	0.00	0.00
_			Log	g-normali	ty test	
J-B stat.	6	4	3	8	58	87
(p. val.)	0.04	0.17	0.21	0.02	0.00	0.00
S&P correl.	0.98	0.96	0.94	0.91	0.82	-0.35
S&P beta	1.00	0.98	0.96	0.92	0.83	-0.17

**Table V.** Descriptive statistics for the leverage-adjusted excess returns of the five synthetic put option portfolios. J-B statistics are from Jarque-Bera normality and log-normality tests. The last column is a long-short between the 1.05 ITM portfolio and the 0.95 OTM portfolio. The sample covers July 1988 to June 2007.

		5	Synthetic p	ut portfolio	o moneyne	ess
	0.95	0.98	1.00	1.02	1.05	ITM-OTM
				Moments		
Mean	1.48	1.33	1.19	1.02	0.85	-0.64
(St. err.)	0.31	0.30	0.29	0.29	0.28	0.15
Sharpe	1.12	1.05	0.96	0.82	0.70	-0.97
St. Dev.	4.59	4.40	4.31	4.28	4.18	2.28
Skewness	-1.44	-1.22	-1.03	-0.87	-0.75	1.69
Kurtosis	3.96	2.94	2.33	1.72	1.24	7.96
-			N	ormality to	est	
J-B stat.	222	135	90	55	35	692
(p-val.)	0.00	0.00	0.00	0.00	0.00	0.00
-			Log	g-normality	test /	
J-B stat.	369	222	153	97	62	472
(p. val.)	0.00	0.00	0.00	0.00	0.00	0.00
S&P correl.	0.81	0.88	0.94	0.96	0.97	0.15
S&P beta	0.94	0.98	1.02	1.03	1.02	0.09

**Table VI.** Estimated betas and associated t-statistics of the option portfolios for the factor models in Tables VII-IX. Factors are detailed in Appendix C. The sample covers July 1988 to June 2007.

		Call	portfolios				Put	portfolios		
K/S	0.95	0.98	1.00	1.02	1.05	0.95	0.98	1.00	1.02	1.05
S&P	1.00	0.99	0.97	0.94	0.87	0.97	1.00	1.02	1.03	1.01
t	69.54	55.00	45.28	35.92	26.23	21.37	25.35	28.81	31.88	34.53
Market	0.99	0.97	0.94	0.89	0.81	0.99	1.02	1.04	1.03	1.02
t	57.44	46.91	39.00	30.94	22.43	19.68	23.01	25.82	28.09	29.91
SMB	-0.28	-0.32	-0.34	-0.36	-0.39	-0.01	-0.06	-0.09	-0.13	-0.17
t	-14.94	-14.04	-12.85	-11.45	-9.97	-0.24	-1.20	-2.08	-3.17	-4.48
HML	-0.02	-0.05	-0.08	-0.12	-0.17	0.07	0.07	0.06	0.04	0.03
t	-1.06	-1.90	-2.56	-3.14	-3.38	0.97	1.09	1.08	0.85	0.60
S&P	1.05	1.07	1.07	1.06	1.01	0.79	0.86	0.91	0.95	0.96
t	55.85	46.83	39.43	31.77	23.79	13.44	16.66	19.50	22.07	24.42
Jump	-0.13	-0.21	-0.26	-0.30	-0.37	0.46	0.36	0.28	0.20	0.13
t	-4.00	-5.39	-5.55	-5.33	-4.99	4.52	4.07	3.48	2.77	1.88
S&P	1.05	1.06	1.05	1.03	0.98	0.80	0.85	0.89	0.92	0.94
t	70.53	57.76	48.63	38.75	28.66	17.38	21.37	24.63	27.28	29.49
$\Delta IV$	0.02	0.03	0.04	0.04	0.05	-0.08	-0.07	-0.06	-0.05	-0.03
t	6.92	7.59	7.84	7.42	6.90	-7.86	-8.06	-7.45	-6.28	-4.97
S&P	1.06	1.07	1.06	1.04	1.00	0.81	0.87	0.92	0.94	0.96
t	84.46	72.79	60.40	46.49	34.24	19.30	23.26	26.52	29.11	31.13
<b>IVRV</b>	-0.10	-0.14	-0.17	-0.19	-0.24	0.29	0.24	0.20	0.15	0.10
t	-11.29	-13.25	-13.04	-11.66	-10.95	9.39	8.53	7.64	6.32	4.39
S&P	1.07	1.08	1.10	1.09	1.05	0.72	0.79	0.85	0.88	0.91
t	63.23	52.43	44.62	36.74	27.51	13.82	16.96	19.63	22.03	23.94
$\Delta VIX$	0.03	0.04	0.05	0.06	0.07	-0.10	-0.09	-0.07	-0.06	-0.04
t	6.96	8.00	8.54	8.61	8.02	-8.35	-7.79	-7.02	-6.07	-4.63
S&P	1.01	1.00	0.99	0.95	0.89	0.95	0.98	1.01	1.01	1.00
t	71.83	57.53	47.47	37.81	27.67	21.07	25.11	28.57	31.56	34.09
ΔVol	0.01	0.01	0.01	0.02	0.02	-0.02	-0.02	-0.02	-0.01	-0.01
t	4.07	4.56	4.54	4.54	4.34	-3.20	-3.31	-3.12	-2.71	-2.05
S&P	1.00	0.99	0.97	0.94	0.87	0.97	1.00	1.02	1.03	1.01
t	73.30	58.30	47.60	37.45	27.03	22.99	27.21	30.65	33.53	35.77
ΔΟΤΜ	0.01	0.01	0.01	0.01	0.01	-0.03	-0.02	-0.02	-0.02	-0.01
t	4.96	5.23	4.84	4.40	3.73	-5.84	-5.75	-5.34	-4.80	-3.99
S&P	1.02	1.01	1.00	0.97	0.91	0.91	0.95	0.98	0.99	0.99
t	106.45	101.47	87.73	68.77	50.38	50.38	51.11	50.32	48.21	44.19
Slope	-0.16	-0.22	-0.26	-0.32	-0.40	0.60	0.50	0.43	0.36	0.28
t	-16.87	-22.45	-23.89	-23.28	-23.00	34.37	27.93	22.75	18.02	12.81
S&P	1.00	0.98	0.96	0.92	0.86	0.98	1.01	1.03	1.03	1.01
t	71.30	56.42	46.16	36.30	26.26	21.58	25.50	28.84	31.77	34.28
Default	0.03	0.03	0.04	0.04	0.04	-0.03	-0.03	-0.02	-0.01	0.00
t	4.23	4.37	4.10	3.65	3.20	-1.78	-1.56	-1.16	-0.66	0.11

S&P	1.00	0.99	0.97	0.94	0.87	0.97	1.00	1.02	1.02	1.01
t	70.17	55.37	45.47	35.96	26.23	21.44	25.39	28.85	31.91	34.54
Term	0.05	0.06	0.05	0.03	0.01	-0.13	-0.09	-0.07	-0.06	-0.04
t	1.96	1.67	1.30	0.68	0.19	-1.57	-1.19	-1.09	-0.99	-0.76
S&P	1.00	0.99	0.97	0.94	0.87	0.97	1.00	1.02	1.02	1.01
t	67.89	53.72	44.21	35.06	25.57	20.92	24.74	28.08	31.05	33.63
Mom	0.00	0.00	0.00	0.00	0.00	0.01	0.00	-0.01	-0.01	-0.01
t	0.11	0.16	0.08	0.01	-0.15	0.26	-0.01	-0.18	-0.32	-0.40
S&P	1.02	1.01	0.99	0.96	0.91	0.91	0.95	0.98	0.99	0.99
t	56.79	46.96	38.22	29.77	21.45	15.29	17.76	19.92	21.65	23.02
$\Delta R$	-0.30	-0.38	-0.48	-0.44	-0.34	0.78	0.68	0.61	0.55	0.48
t	-1.53	-1.58	-1.66	-1.23	-0.73	1.17	1.14	1.11	1.09	1.01
S&P	1.02	1.02	1.00	0.97	0.91	0.91	0.95	0.98	0.99	0.99
t	56.54	46.77	38.07	29.79	21.59	15.15	17.61	19.78	21.52	22.91
Sharpe	-0.01	-0.01	0.00	0.00	0.01	0.02	0.01	0.01	0.01	0.01
t	-0.62	-0.69	-0.27	0.01	0.49	0.56	0.52	0.51	0.60	0.63
S&P	0.99	0.97	0.95	0.91	0.84	0.96	0.99	1.02	1.02	1.00
t	57.23	45.04	36.93	29.23	21.19	17.64	20.95	23.93	26.41	28.43
$\Delta P/D$	0.04	0.05	0.05	0.05	0.06	0.01	0.01	0.00	0.01	0.02
t	1.68	1.84	1.70	1.33	1.22	0.22	0.21	0.02	0.17	0.52

**Table VII.** Factor premia estimated from the 25 Fama-French size and book-to-market portfolios. J-statistics are based on a joint test of the pricing errors. The tests account for errors in the variables. Factors are detailed in Appendix C. The sample covers July 1988 to June 2007.

Model	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
						Fac	ctor prem	nia (estima	ited from	stocks)					
1 <sup>st</sup> Factor	S&P	Market	S&P	S&P	S&P	S&P	S&P	S&P	S&P	S&P	S&P	S&P	S&P	S&P	S&P
	0.83	0.65	0.91	0.69	0.75	0.70	0.63	0.61	0.77	0.84	0.95	0.85	1.02	0.98	0.59
2 <sup>nd</sup> Factor		SMB	Jump	$\Delta IV$	<b>IVRV</b>	$\Delta VIX$	$\Delta Vol$	$\Delta$ OTM	Slope	Default	Term	Mom	$\Delta R$	Sharpe	$\Delta P/D$
		0.15	-0.22	-5.43	0.96	-5.72	-14.2	-27.1	0.41	0.42	0.68	-1.42	0.16	-5.1	-2.82
3 <sup>rd</sup> Factor		HML													
		0.40													
							Stock p	ortfolio p	ricing er	ors					
R.m.s. err.	0.36	0.22	0.35	0.35	0.36	0.38	0.34	0.33	0.36	0.36	0.34	0.35	0.27	0.26	0.24
J-stat	138	85	80	120	118	108	109	88	127	118	85	89	52	66	51
P-val.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Boot p-val.	0	0	0	0	0	0	0	0	0	0	0	0	0	0.01	0.08

**Table VIII.** Tests of call portfolios. The premia are estimated from the 25 Fama French size and book-to-market portfolios. J-statistics are based on a joint test of the pricing errors. The tests account for errors in the variables. Factors are detailed in Appendix C. The sample covers July 1988 to June 2007.

Model	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
1 <sup>st</sup> Factor	S&P	Market	S&P	S&P	S&P	S&P	S&P	S&P	S&P	S&P	S&P	S&P	S&P	S&P	S&P
2 <sup>nd</sup> Factor		SMB	Jump	$\Delta IV$	<b>IVRV</b>	$\Delta VIX$	$\Delta Vol$	$\Delta$ OTM	Slope	Default	Term	Mom	$\Delta R$	Sharpe	$\Delta P/D$
3 <sup>rd</sup> Factor		HML	-						•					-	
								Pricing er	rors						
K/S = 0.95	-0.47	-0.14	-0.63	-0.25	-0.33	-0.22	-0.15	-0.04	-0.36	-0.49	-0.64	-0.5	-0.47	-0.41	-0.13
S.e.	0.21	0.11	0.25	0.18	0.2	0.2	0.14	0.17	0.21	0.17	0.2	0.19	0.41	0.22	0.26
Boot s.e.	0.21	0.09	0.17	0.17	0.19	0.18	0.15	0.19	0.19	0.16	0.17	0.21	0.21	0.25	0.28
K/S = 1.00	-0.64	-0.27	-0.87	-0.36	-0.46	-0.31	-0.24	-0.12	-0.49	-0.65	-0.79	-0.66	-0.67	-0.58	-0.24
S.e.	0.23	0.13	0.37	0.24	0.26	0.27	0.19	0.21	0.3	0.19	0.21	0.2	0.42	0.25	0.29
Boot s.e.	0.22	0.11	0.22	0.22	0.24	0.25	0.18	0.21	0.27	0.18	0.19	0.23	0.23	0.27	0.31
K/S = 1.05	-0.66	-0.27	-0.95	-0.35	-0.46	-0.27	-0.2	-0.1	-0.48	-0.68	-0.78	-0.69	-0.62	-0.49	-0.27
S.e.	0.24	0.16	0.47	0.31	0.34	0.36	0.26	0.26	0.42	0.21	0.21	0.22	0.39	0.31	0.3
Boot s.e.	0.24	0.15	0.28	0.28	0.31	0.33	0.24	0.25	0.38	0.21	0.2	0.24	0.28	0.31	0.32
								Test statis	stics						
R.m.s. err.	0.61	0.24	0.84	0.34	0.43	0.28	0.21	0.10	0.46	0.62	0.76	0.63	0.61	0.53	0.23
J-stat	24	13	8	9	9	10	9	3	13	20	23	24	10	16	4
P-val.	0	0.02	0.15	0.11	0.13	0.08	0.11	0.71	0.03	0	0	0	0.08	0.01	0.51
Boot P-val.	0	0.01	0.04	0.07	0.1	0.05	0.11	0.68	0.04	0	0	0	0.02	0.01	0.52

**Table IX.** Tests of put portfolios. The premia are estimated from the 25 Fama French size and book-to-market portfolios. J-statistics are based on a joint test of the pricing errors. The tests account for errors in the variables. Factors are detailed in Appendix C. The sample covers July 1988 to June 2007.

Model	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
1 <sup>st</sup> Factor	S&P	Markt	S&P	S&P	S&P	S&P	S&P	S&P	S&P	S&P	S&P	S&P	S&P	S&P	S&P
2 <sup>nd</sup> Factor		SMB	Jump	$\Delta IV$	IVRV	$\Delta VIX$	$\Delta Vol$	$\Delta$ OTM	Slope	Default	Term	Mom	$\Delta R$	Sharpe	$\Delta P/D$
3 <sup>rd</sup> Factor		HML													
								Pricing err	ors						
K/S = 0.95	0.99	1.22	1.17	0.83	0.91	0.64	0.89	0.42	0.85	0.99	0.96	0.98	1.21	1.21	1.27
S.e.	0.22	0.22	0.52	0.39	0.34	0.41	0.3	0.54	0.55	0.22	0.21	0.23	0.41	0.3	0.3
Boot s.e.	0.22	0.21	0.34	0.38	0.35	0.4	0.28	0.57	0.6	0.23	0.21	0.24	0.3	0.3	0.31
K/S = 1.00	0.3	0.56	0.37	0.22	0.28	0.1	0.28	-0.04	0.22	0.29	0.22	0.27	0.47	0.47	0.55
S.e.	0.21	0.16	0.34	0.29	0.24	0.29	0.24	0.46	0.41	0.19	0.18	0.2	0.39	0.27	0.26
Boot s.e.	0.21	0.16	0.25	0.3	0.26	0.3	0.23	0.49	0.45	0.2	0.18	0.22	0.27	0.28	0.26
K/S = 1.05	-0.03	0.26	-0.04	-0.02	0	-0.09	0.05	-0.17	-0.06	-0.04	-0.13	-0.07	0.11	0.14	0.28
S.e.	0.21	0.15	0.22	0.19	0.16	0.2	0.17	0.39	0.29	0.17	0.17	0.19	0.37	0.26	0.23
Boot s.e.	0.21	0.13	0.19	0.21	0.18	0.22	0.19	0.43	0.34	0.18	0.17	0.21	0.25	0.27	0.25
								Test statist	ics						
R.m.s. err.	0.54	0.74	0.64	0.44	0.49	0.33	0.48	0.22	0.45	0.53	0.51	0.52	0.69	0.69	0.76
J-stat	88	77	27	82	57	64	98	52	52	109	80	139	65	64	118
P-val.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Boot P-val.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

**Table X.** Tests of pooled call and put portfolios. The premia are estimated from the 25 Fama French size and book-to-market portfolios. J-statistics are based on a joint test of the pricing errors. The tests account for errors in the variables. Factors are detailed in Appendix C. The sample covers July 1988 to June 2007.

Model	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
1 <sup>st</sup> Factor	S&P	Markt	S&P	S&P	S&P	S&P	S&P	S&P	S&P	S&P	S&P	S&P	S&P	S&P	S&P
2 <sup>nd</sup> Factor		SMB	Jump	$\Delta IV$	IVRV	$\Delta VIX$	$\Delta Vol$	$\Delta OTM$	Slope	Default	Term	Mom	$\Delta R$	Sharpe	$\Delta P/D$
3 <sup>rd</sup> Factor		HML													
							P	ricing erro	ors						
C K/S=0.95	-0.47	-0.14	-0.63	-0.25	-0.33	-0.22	-0.15	-0.04	-0.36	-0.49	-0.64	-0.50	-0.47	-0.41	-0.13
S.e.	0.21	0.11	0.38	0.17	0.19	0.19	0.13	0.17	0.21	0.17	0.20	0.21	1.53	0.26	0.60
Boot s.e.	0.21	0.09	0.17	0.16	0.18	0.18	0.16	0.19	0.20	0.16	0.17	0.21	0.21	0.25	0.28
C K/S=1.05	-0.66	-0.27	-0.95	-0.35	-0.46	-0.27	-0.20	-0.10	-0.48	-0.68	-0.78	-0.69	-0.62	-0.49	-0.27
S.e.	0.24	0.16	0.53	0.31	0.33	0.35	0.24	0.23	0.42	0.21	0.22	0.22	1.19	0.31	0.50
Boot s.e.	0.23	0.15	0.28	0.27	0.31	0.31	0.24	0.25	0.40	0.21	0.20	0.24	0.26	0.31	0.31
P K/S=0.95	0.99	1.22	1.17	0.83	0.91	0.64	0.89	0.42	0.85	0.99	0.96	0.98	1.21	1.21	1.27
S.e.	0.22	0.21	0.53	0.39	0.34	0.41	0.31	0.57	0.56	0.24	0.20	0.23	0.40	0.30	0.35
Boot s.e.	0.22	0.21	0.36	0.38	0.34	0.38	0.29	0.55	0.56	0.23	0.21	0.24	0.32	0.30	0.29
P K/S=1.05	-0.03	0.26	-0.04	-0.02	0.00	-0.09	0.05	-0.17	-0.06	-0.04	-0.13	-0.07	0.11	0.14	0.28
S.e.	0.21	0.16	0.26	0.20	0.17	0.20	0.20	0.46	0.31	0.22	0.18	0.20	0.37	0.28	0.34
Boot s.e.	0.20	0.14	0.20	0.21	0.18	0.21	0.19	0.43	0.31	0.18	0.17	0.22	0.25	0.27	0.25
							Г	est statist	ics						
R.m.s. err.	0.57	0.55	0.75	0.39	0.47	0.31	0.37	0.17	0.46	0.58	0.64	0.58	0.65	0.62	0.56
J-stat	103	106	53	103	79	85	127	85	76	106	117	178	72	91	163
P-val.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Boot P-val.	0	0	0	0	0	0	0	0.02	0	0	0	0	0	0	0

**Table XI.** Tests of call portfolios. The premia are estimated from the call portfolios. J-statistics are based on a joint test of the pricing errors. The tests account for errors in the variables. Factors are detailed in Appendix C. The sample covers July 1988 to June 2007.

Model	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
					Factor prei	mia			
1 <sup>st</sup> Factor	S&P	HML	Jump	$\Delta IV$	IVRV	$\Delta VIX$	$\Delta Vol$	$\Delta OTM$	Slope
	0.20	-0.37	0.20	-2.78	1.35	-1.86	-20.29	13.81	-0.61
					Pricing err	ors			
K/S = 0.95	0.16	0.17	0.14	0.12	0.07	0.13	0.03	0.24	0.32
(t-stat.)	3.42	3.67	2.58	1.80	0.66	2.07	0.28	2.47	1.66
K/S = 1.00	-0.02	-0.02	-0.03	-0.03	-0.03	-0.03	-0.03	0.00	0.06
(t-stat.)	1.84	1.50	2.12	2.29	1.11	2.26	0.38	0.20	0.93
K/S = 1.05	-0.12	-0.12	-0.10	-0.08	-0.03	-0.10	0.05	-0.14	-0.14
(t-stat.)	1.86	2.07	1.42	0.88	0.15	1.08	0.14	1.85	1.15
					Test statist	tics			
r.m.s. pricing err.	0.10	0.10	0.09	0.07	0.04	0.08	0.03	0.13	0.17
J-stat	25	25	15	15	9	19	4	11	18
p-val.	0.00	0.00	0.00	0.00	0.07	0.00	0.38	0.03	0.00

**Table XII.** Tests of put portfolios. The premia are estimated from the put portfolios. J-statistics are based on a joint test of the pricing errors. The tests account for errors in the variables. Factors are detailed in Appendix C. The sample covers July 1988 to June 2007.

Model	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
					Factor prei	mia			
1 <sup>st</sup> Factor	S&P	HML	Jump	$\Delta IV$	IVRV	$\Delta VIX$	$\Delta Vol$	$\Delta \text{OTM}$	Slope
	1.21	-2.29	0.94	-8.47	2.69	-6.20	-30.83	-60.27	2.39
					Pricing err	ors			
K/S = 0.95	0.62	0.55	0.51	0.45	0.36	0.44	0.40	0.09	0.16
(t-stat.)	7.70	7.60	6.90	6.33	4.30	6.00	3.96	0.37	1.15
K/S = 1.00	-0.09	-0.09	-0.11	-0.12	-0.12	-0.11	-0.13	0.08	0.09
(t-stat.)	6.72	6.26	7.49	7.47	6.22	7.32	4.10	1.11	2.80
K/S = 1.05	-0.42	-0.39	-0.36	-0.30	-0.23	-0.30	0.24	-0.04	-0.07
(t-stat.)	6.34	6.05	5.24	4.18	2.82	4.32	2.33	0.12	0.40
					Test statist	tics			
r.m.s. pricing err.	0.37	0.34	0.31	0.27	0.22	0.27	0.24	0.06	0.10
J-stat	79	79	85	72	46	79	21	6	28
p-val.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.17	0.00

**Table XIII.** Tests of pooled call and put portfolios. The premia are estimated from the combined call and put portfolios. J-statistics are based on a joint test of the pricing errors. The tests account for errors in the variables. Factors are detailed in Appendix C. The sample covers July 1988 to June 2007.

Model	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Factor premia								
1 <sup>st</sup> Factor	S&P 0.73	S&P 0.30	S&P 0.69	S&P 0.60	S&P 0.68	S&P 0.61	S&P 0.69	S&P 0.55	S&P 0.59
2 <sup>nd</sup> Factor		HML	Jump	$\Delta IV$	IVRV	$\Delta VIX$	$\Delta Vol$	ΔΟΤΜ	Slope
		-14.55	2.20	-12.84	3.27	-9.73	-34.15	-33.40	1.57
	Pricing errors								
C K/S = 0.95	-0.38	0.04	-0.08	0.01	-0.02	0.00	-0.04	0.06	0.00
(t-stat.)	4.11	0.05	0.34	0.13	0.47	0.05	0.37	0.35	0.03
C K/S = 1.05	-0.58	-0.93	0.16	0.12	0.15	0.13	0.17	0.04	0.15
(t-stat.)	4.21	1.24	0.97	2.05	2.21	1.99	1.73	0.29	2.76
P K/S = 0.95	1.08	0.58	0.25	0.33	0.29	0.29	0.39	0.30	0.32
(t-stat.)	6.80	1.44	1.52	6.69	4.40	5.76	3.52	2.02	5.34
P K/S = 1.05	0.07	0.25	-0.13	-0.19	-0.17	-0.17	-0.20	-0.19	-0.21
(t-stat.)	1.04	0.41	0.59	1.68	3.09	1.21	3.24	1.06	1.76
	Test statistics								
r.m.s. pricing err.	0.57	0.49	0.12	0.14	0.14	0.13	0.17	0.13	0.15
J-stat	104	727	90	118	61	90	149	97	60
p-val.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00