Assignment 1

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FINM 36702: Portfolio Credit Risk: Modeling and Estimation

Due: 18:00 (CT) March 30th 2023

1: Correlation and Default Correlation

$\rho_{1,2}$	$\rho_{1,3}$	$\rho_{2,3}$
0.60	0.43	-6.5 $\times 10^{-9}$

$Corr[D_1, D_2]$	$Corr[D_1, D_3]$	$Corr[D_2, D_3]$
0.33	0.22	0.00

2: PDJ with Gauss Copula Assumption

(i: PDJ)

$PDJ_{1,2}$	$PDJ_{1,3}$	$PDJ_{2,3}$
0.027	0.032	0.039

(ii: Range)

We know that

$$\mathbb{P}\{D_i \cap D_j\} = PDJ_{i,j}$$

Moreover, due to basic set theory, $\forall(i,j)$:

$$\mathbb{P}\{D_1 \cap D_2 \cap D_3\} \leq \mathbb{P}\{D_i \cap D_j\}$$

$$\therefore \mathbb{P}\{D_1 \cap D_2 \cap D_3\} \leq \min_{(i,j)} \mathbb{P}\{D_i \cap D_j\} = PDJ_{1,2} \approx 0.027$$

Therefore, the probability that all three firms default ranges from 0 to 0.027

(iii: All Default)

Under Gauss copula, the probability that all three default is ≈ 0.016

3: Firm "A" and Firm "B"

Firm "B"				
	A	0.02	0.16	0.32
	B	0.05	0.19	0.16
	D	0.03	0.05	0.02
		D	**B**	**A**
			Firm "A"	

4: Consistency with Gauss copula

The given situation yields a correlation matrix that is approximately:

$$\begin{bmatrix} 1 & 0.31 & 0.24 & 0.18 \\ 0.31 & 1 & 0.10 & 0.044 \\ 0.24 & 0.10 & 1 & -0.036 \\ 0.18 & 0.044 & -0.036 & 1 \end{bmatrix}$$

The eigen-vectors and eigen-values decomposition from python tells us below.

First, for the eigen-values:

λ_1	λ_2	λ_3	λ_4
1.5	0.61	1.0	0.87

The corresponding eigen-vectors are:

$$v_1 = \begin{pmatrix} -0.67 \\ -0.74 \\ 0.060 \\ 0.031 \end{pmatrix}, \ v_2 = \begin{pmatrix} -0.55 \\ 0.46 \\ -0.047 \\ -0.70 \end{pmatrix}, \ v_3 = \begin{pmatrix} -0.42 \\ 0.36 \\ -0.56 \\ 0.61 \end{pmatrix}, \ v_4 = \begin{pmatrix} -0.27 \\ 0.33 \\ 0.82 \\ 0.38 \end{pmatrix}$$

The eigen decomposition tells us that the correlation matrix has 4 ranks and thus non-singular.

Moreover, the eigen-values are non-negative.

Therefore, we may rule that the correlation matrix is positive semi-definite.

Together, the connection between the defaults of the four firms is consistent with a Gauss copula.