Assignment 4

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FINM 36702: Portfolio Credit Risk: Modeling and Estimation

Due: 18:00 (CT) April 20 2023

1: EL, ELGD, EcLGD

- EL:

$$
\begin{aligned}
\mathbb{E}[Loss] &= \sum_{states} \mathbb{P}\{state\} \times cPD_{state} \times cLGD_{state} \\
&= 0.40 \times 0.02 \times 0.10 \\
&+0.30 \times 0.04 \times 0.30 \\
&+0.20 \times 0.06 \times 0.50 \\
&+0.10 \times 0.08 \times 0.70 \\
&= 0.016
\end{aligned}
$$

- ELGD:

$$ELGD = \frac{EL}{PD}$$

Here,

$$
\begin{aligned}
\mathbb{P}\{D\} &= \sum_{states} \mathbb{P}\{D \mid state\} \times \mathbb{P}\{state\} \\
&= 0.02 \times 0.40 + 0.04 \times 0.30 + 0.06 \times 0.20 + 0.08 \times 0.10 \\
&= 0.04
\end{aligned}
$$

Therefore,

$$ELGD = \frac{EL}{PD} = \frac{0.016}{0.04} = 0.4$$

- EcLGD:

$$
\begin{aligned}
\mathbb{E}[Loss \mid D] &= \sum_{states} \mathbb{P}\{state\} \times cLGD_{state} \\
&= 0.40 \times 0.10 + 0.30 \times 0.30 + 0.20 \times 0.50 + 0.10 \times 0.70 \\
&= 0.30
\end{aligned}
$$

2: "Variant A" LGD function

Given the LGD function:

$$f_{cLGD}(cPD \mid a, PD, EL, ELGD, \rho) = ELGD^a \times \frac{\Phi\left[\Phi^{-1}[cPD] - \frac{\Phi^{-1}[PD]-\Phi^{-1}[\frac{EL}{ELGD^a}]}{\sqrt{1-\rho}}\right]}{cPD}$$

Here, across various a, PD was given as 0.05; $ELGD$ was given as 0.3; ρ was given as 0.15.

EL may appear missing; however, it may be calculated using the relation between PD and $ELGD$ as:

$$EL = PD \times ELGD = 0.015$$

Plotting the function above with cPD on the x-axis for $a \in \{-1, 0, 1, 2\}$ is shown as below:

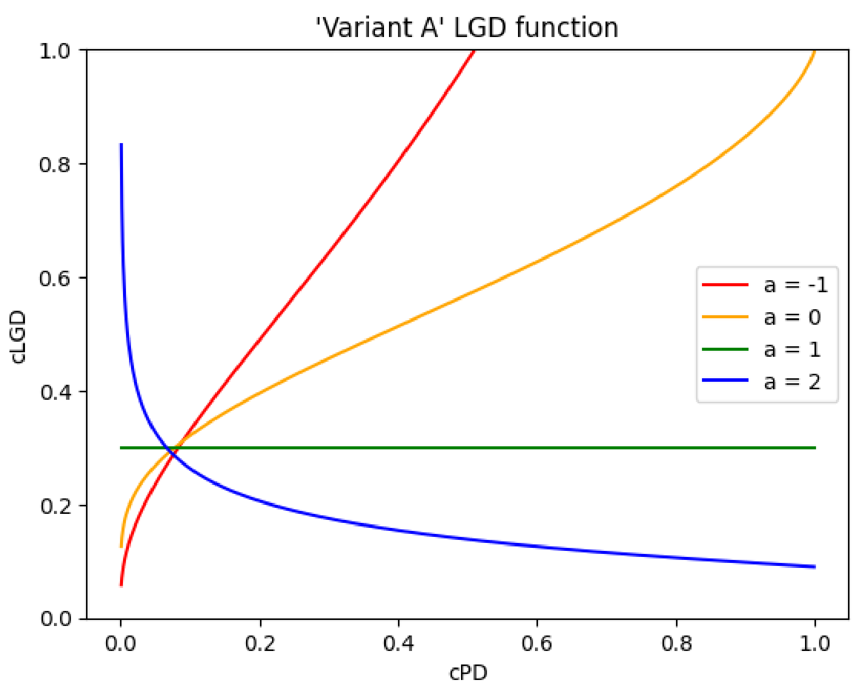

Figure 1: "Variant A" LGD functions

3: Co-monotonic cPD and cLoss

Co-monotonic cPD and $cLoss$ that follows the same distribution family means that the quantiles of the two values should match precisely.

Therefore, if we get the CDF values of cPD from the $Vasicek[PD = 0.02, \rho = 0.10]$ distribution and plug the quantile back into the inverse-CDF function prescribing $cLoss$ from the $Vasicek[El = 0.01, \rho]$ for each of the ρ values, we would be able to get $cLoss$ values from the cPD values.

Then, we can create a LGD function plot using the relationship:

$$cLGD = \frac{cLoss}{cPD}$$

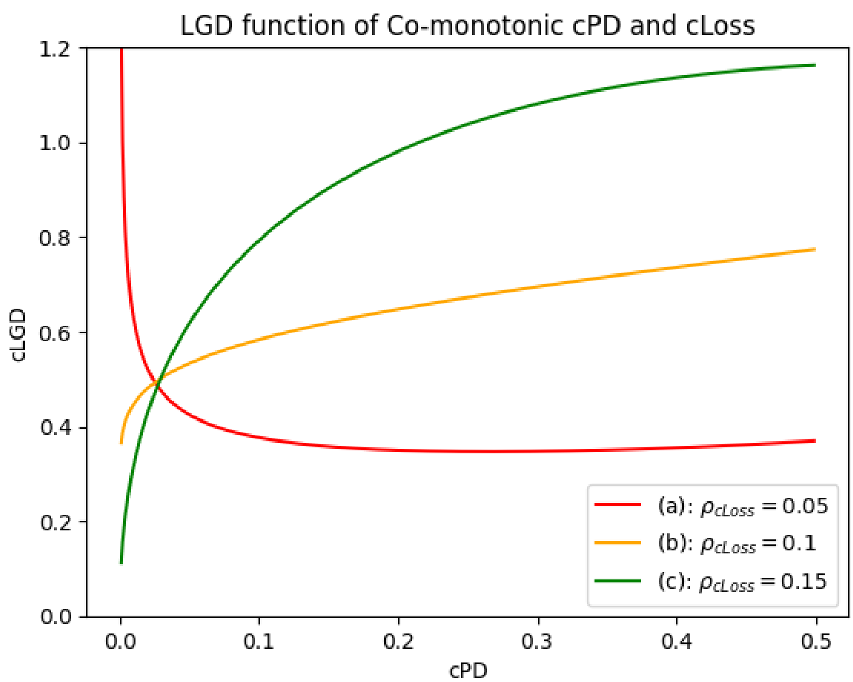

Figure 2: Co-monotonic Vasicek cPD and cLoss

- (a):

  When $\rho = 0.05$, the $cLGD$ and cPD does not resemble co-monotonicity. Moreover, the $cLGD$ function gets rather flat as cPD increases. It would be hard to find situations where this LGD function is useful since, generally, $cLGD$ increases as default rate increases.

- (b):

  The $cLGD$ increases monotonically with cPD values, as expected from real-life cases. This LGD-function seems to be the most useful

- (c):

  The $cLGD$ increases monotonically yet more rapidly than in (b). Such rapid slope would be useful for extremely turbulent times. However, this LGD function has a critical flaw since the $cLGD$ values exceed 1.0, which, in context, is nonsensical. Therefore, overall, the LGD function would not be useful.

4: ELGD, EL, PD

The relevant assumptions of Question 3(b) is:

$$PD = 0.02$$
$$EL = 0.01$$

Now using the relationship between $ELGD$, EL, and PD, the $ELGD$ value can be computed as:

$$ELGD = \frac{EL}{PD} = \frac{0.01}{0.02} = 0.5$$