Exercise for Discussion

1 Short Term Capital Management LLP

Let's create our own hedge fund, Short-Term Capital Management, (STCM). Our strategy is simple: each month we sell out-of-the-money put options. We invest the fund's assets along with the proceeds from selling the puts into riskless treasuries. One month later, the assets have grown by the riskless rate, and we buy back the put options to cover last month's position. We then repeat the process by selling fresh put options and investing everything back in the riskless rate.

Use the following data to test the strategy:

- Use the market index data in the **"S&P500"** tab.
- Assume a constant monthly riskless rate of $r_f = 0.0020$.
- Assume a constant implied volatility of $\sigma = 0.05$.
- Test the strategy assuming the fund begins with assets of $W_0 = \$50,000,000$.

The details of the strategy are as follows:

- At the end of the first month, we sell puts with maturity $\tau = 2$ and strike price of $K = 0.80 \times S$, where S is the current market price, P_t^m.
- Calculate the price you receive on the puts using the Black-Scholes formulas:

$$\begin{aligned} \text{put} = &\quad f_p\left(S, K, \sigma, \tau, r_f\right) \\ = &\quad \mathcal{N}\left(-d_2\right) K e^{-r_f \tau} - \mathcal{N}\left(-d_1\right) S \end{aligned}$$

$$d_1 \equiv \frac{\log \frac{S}{K} + \left(r_f + \frac{\sigma^2}{2}\right)\tau}{\sqrt{\tau}\sigma}, \qquad d_2 \equiv d_1 - \sigma\sqrt{\tau}$$

  where r_f is the constant riskless rate given above, σ is the constant implied volatility given above, and $\mathcal{N}(\cdot)$ is the standard normal cumulative distribution.

- We sell n_t of these put options, where n_t is calculated such that the premium we receive equals 3%of the fund's total assets, W_t,

$$n_t = \frac{0.03}{f_p\left(P_t^m, .80P_t^m, \sigma, 2, r_f\right)} W_t \tag{1}$$

  Thus the dollars received, G_t, from selling this number of puts will immediately increase the fund's assets by 3%.

$$G_t = n_t \times f_p\left(P_t^m, .80P_t^m, \sigma, 2, r_f\right) = 0.03W_t$$

- At the end of the following month, $t+1$, we must use L_{t+1} in assets to cover the puts from last month buy purchasing puts with $\tau = 1$ and $K = .80P_t^m$. We use the Black-Scholes formula above to calculate this amount:[1]

$$L_{t+1} = n_t \times f_p\left(P_{\boldsymbol{t+1}}^m, .80P_{\boldsymbol{t}}^m, \sigma, \mathbf{1}, r_f\right) \tag{2}$$

- Calculate the fund's gross (before-fees) assets, $\widetilde{W}$, at time $t+1$ after covering the puts:

$$\begin{aligned}\widetilde{W}_{t+1} &= (W_t + G_t)(1 + r_f) - L_{t+1} \\ &= (W_t + 0.03W_t)(1 + r_f) - L_{t+1}\end{aligned} \tag{3}$$

  where the first term is the past month's assets plus the 3%jump from issuing puts at the last month, all invested at the riskless rate until the end of this month, at which point we cover the position.

- Calculate the fund's gross (before fees) excess return from t to $t+1$

$$R_{t+1}^{\sharp stcm} = \frac{\widetilde{W}_{t+1} - W_t}{W_t} - r_f \tag{4}$$

- Calculate the management compensation at time t as follows:[2]

$$\pi_{t+1} = \max\left(\frac{.02}{12}W_t, 0\right) + \max\left(.20\, W_t\, R_{t+1}^{\sharp stcm}, 0\right) \tag{5}$$

- Get the fund's net capital by subtracting the managerial fee:

$$W_{t+1} = \widetilde{W}_{t+1} - \pi_{t+1} \tag{6}$$

- Let B_t be a true/false variable indicating whether the fund has gone bust. If $W_t < 0$, then assume the fund immediately re-opens with the initial asset level of $W_0 = \$50$ million.

$$\begin{aligned} \text{if} \quad & W_t < 0, & \\ & B_t = 1, & \text{and reset } W_t = W_0 = 50,000,000 \\ \text{else if} \quad & W_t \geq 0 & \\ & B_t = 0, & \text{and continue with } W_t \end{aligned} \tag{7}$$

- Finally, still at the end of month $t+1$, we repeat the whole process by selling n_{t+1} put options, so as to make the fund's net assets jump 3%immediately. Thus, we go back to equation (1) to calculate n_{t+1} as a function of W_{t+1} and P_{t+1}^m.

- Thus, equations (1) through (7) give a recursion for building a timeseries of the funds net assets, W_t, the managerial compensation, π_t, and the fund's gross excess returns, $R_t^{\sharp stcm}$.

[1] Notice that the put price depends on the time $t+1$ market price and the time t strike price. Also, its maturity is now just 1 month.

[2] Of course, it would be more realistic to calculate management's compensation at the end of each year—not at the end of each month. We do it this way to keep all calculations at the monthly frequency, thus making it a bit easier. But note that this simplification will overstate the management compensation.

1. Assets.

   (a) Plot the timeseries of the fund's net assets, W_t.

   (b) On which dates does the fund go bust? ie. On which dates is $B_t = 1$?

   (c) How does this compare to the lifespan of LTCM, which started in March 1994 and went bust in August 1998?

   (d) The subsamples of time between the fund going bust will be referred to as the various lifespans of the fund.[3]

   How high do the net assets of the fund become in each lifespan?

   (e) What is the maximum value of net assets, W_t across the entire sample? At which date does this maximum occur?

Solution:

(a) See the second panel of Figure 1.

(b) The fund goes bust at the end of August 1998 and the end of October 2008.

(c) LTCM went under the same month that our fund first goes bust.

(d) See Table 1.

Table 1: STCM dollar-figures ($ millions)

start date	Apr 1994	Sep 1998	Nov 2008
death date	Aug 1998	Oct 2008	*Sep 2015
max net assets	166.56	198.72	110.29
cumulative compensation	37.17	78.10	30.12
gross assets at death	-581.90	-2,022.20	—

*Fund is ongoing as of Sep 2015

(e) The fund acheives a maximum of net assets equal to $198.72 million. This high-water mark is achieved at the end of Oct 2008.

[3] When it goes bust at $B_t = 1$, the fund dies and is then reborn with $W_t = \$50$ million.

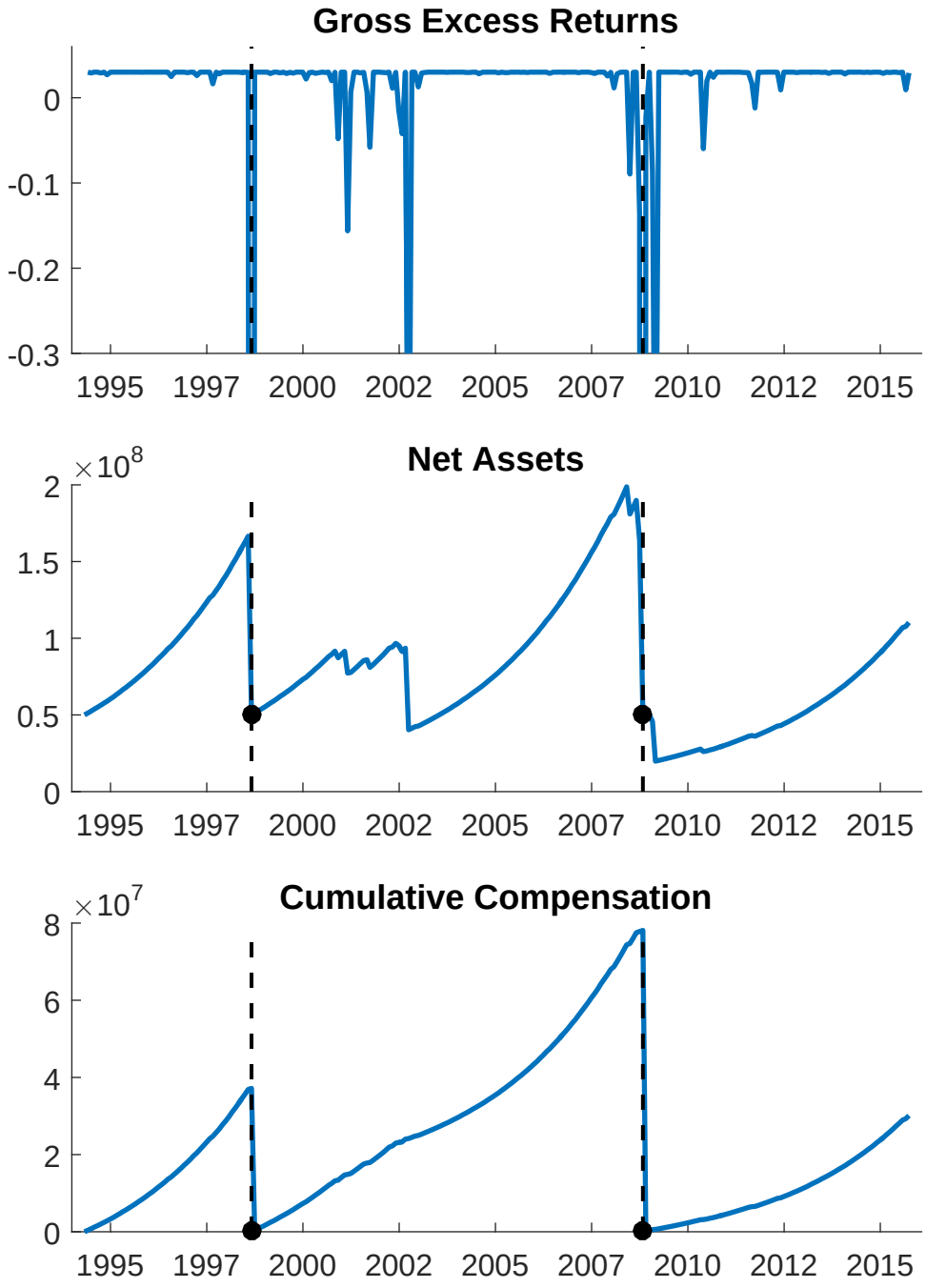

Figure 1: Top panel: Gross excess returns of STCM. Middle panel: Net assets. Bottom panel: Cumulative compensation.

2. Compensation

   (a) Plot the cumulative compensation to management. At each point the fund goes bust, reset the plot to zero so that it is easy to visualize how much total compensation was earned in each lifespan.

   (b) What is the total compensation earned in each lifespan? Which of these lifespans of the fund is most lucrative for management?

Solution:

   (a) See the bottom panel of Figure 1.

   (b) See Table 6. Note that the most lucrative lifespan for management is Sep 1998 - Oct 2008. Though this period does not accumulate as much in net assets as the first period, management ends up with more fees.

3. Performance.

   (a) Plot the time-series of the fund's excess returns, $R^{\sharp stcm}$.

   (b) Report the following stats of the fund's excess return, $R^{\sharp stcm}$. In calculating the stats, EXCLUDE dates in the sample where the fund goes bust, $B_t = 1$.[4]

   - mean
   - volatility
   - Sharpe ratio
   - skewness
   - kurtosis

   Scale the mean by 12, the volatility by $\sqrt{12}$, and the Sharpe ratio by $\sqrt{12}$ to get annualized quotes for these stats.

   (c) Are any of these stats particularly large or small relative to what we typically see?

   (d) Report the exact same stats, but this time base the calculation on the entire sample. That is, include the dates where $B_t = 1$.

   (e) Does including the handful of "bust" dates where $B_t = 1$ have a large impact on the stats?

   (f) What lesson do you take away regarding how investors should judge a hedge-fund's performance?

Solution:

(a) See the top panel of Figure 1.

(b) See Table 2.

Table 2: STCM Excess Return Summary Stats (annualized)

	Conditional	Unconditional
mean	24.20%	-59.31%
volatility	19.66%	305.19%
Sharpe ratio	123.13%	-19.43%
skewness	-9.06	-13.97
kurtosis	91.76	206.20

(c) For the conditional stats, the mean is quite high and the Sharpe ratio is fantastic. However, the returns are very negatively skewed and massively kurtotic. Thus, even conditional on surviving it is clear the fund has substantial nonlinear risk.

(d) As seen in Table 2, the return stats change enormously. The mean return is now negative and the volatility is massive. Furthermore, the skewness and kurtosis were already extreme but now even more so.

(e) Investors in nonlinear strategies need to be very careful that they evaluate the fund performance on a large enough data sample so as to see the true distribution of the returns.

[4] In formal terms, these stats are *conditional* on periods the fund survives.