Chapter 8 - Graphical models

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1 Exercise 8.1

The parent node factorization for a Bayesian network is:

$$p(\mathbf{x}) = \prod_i p(x_i|\mathrm{pa}_i) \tag{1.1}$$

Show that if each of the conditional distributions are normalized, then so is the joint distribution.

1.1 Solution

The assumption is, that for any combination of values of pa_i, we have:

$$\int p(x_i|\mathrm{pa}_i)\ d\mu_i(x_i) = 1 \tag{1.2}$$

Here μ_i is the relevant dominating measure for the probability density function for x_i.

We wish to calculate:

$$\int p(\mathbf{x})\ d\mu_1(x_1)\cdots d\mu_K(x_K) \tag{1.3}$$

Using equation 1.1 this can be written:

$$\int p(x_K|\mathrm{pa}_K)\cdots p(x_1|\mathrm{pa}_1)d\mu_1(x_1)\cdots d\mu_K(x_K) \tag{1.4}$$

Now using the assumption from equation 1.2 for $i = 1$, the inner innermost integration turns to one. Doing so for $i = 2$ and so on up to $i = K$, we get that the entire integral is equal to 1 as desired.

2 Exercise 8.2

Show that the property of there being no directed cycles in a directed graphs follows from the statement that there exists an ordering of the nodes, such that there are no links to a lower-numbered node.

2.1 Solution

Let $x_1, x_2, \ldots, x_K$ be such an ordering. Assume that a directed cycle exists

$$x_{i_1} \to x_{i_2} \to \cdots \to x_{i_N} = x_{i_1} \tag{2.1}$$

By the ordering assumption we must have:

$$i_1 < i_2 < \cdots < i_n \tag{2.2}$$

But since $i_1 = i_n$ we have $i_1 < i_1$, which is a contradiction.