Exponential families

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These statistical models are of great importance in theoretical statistics.

1 Definition

An *exponential family* in k dimensions on a measurable space $(\mathcal{X}, \mathbb{E})$ is a parametrized statistical model $\{\nu_\theta | \theta \in \Theta\}$ with the following ingredients:

- A parameter space $\Theta \subseteq \mathbb{R}^k$ which is an open, convex set.
- A measurable function $t : \mathcal{X} \to \mathbb{R}^k$ known as the *canonical sample function*.
- A σ-finite measure μ on $(\mathcal{X}, \mathbb{E})$ known as the *base measure* for the family.

The probability measures in the family are then given by:

$$\forall A \in \mathbb{E}, \theta \in \Theta : \ \nu_\theta(A) = \frac{1}{c(\theta)} \int_A \exp\left[\theta^t t(x)\right] \ d\mu(x) \tag{1.1}$$

Here, $c(\theta)$ is a normalization constant given by:

$$c(\theta) = \int_A \exp\left[\theta^t t(x)\right] \ d\mu(x) \tag{1.2}$$

We will assume $c(\theta) < \infty$ for all $\theta \in \Theta$. The study of an exponential family often comes down to studying this function, as well as the image measure $t(\mu)$, known as the *structural measure* of the family.