Binary classification

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1 Basic definitions

Binary classification is a situation where there's two distinct outcomes which we are trying to predict. Associated predictors are known as *binary predictors.*

1.1 Positives and negatives

The two outcomes are denoted *positives* and *negatives*, respectively. Given a data point, and the corresponding prediction of a binary predictor, there's four different possibilities:

- *True positive*: Both the actual outcome and the prediction is positive.
- *False positive*: The prediction is positive, but the actual outcome is negative. This is also known as a *type I error*.
- *False negative*: The prediction is negative, but the actual outcome is positive. This is also known as a *type II error*.
- *True negative*: Both the actual outcome and the prediction is negative.

1.2 The confusion matrix

The frequency of the four above-mentioned events are usually presented in matrix form, in what is known as the *confusion matrix*. It is shown in table 1.2, where TP means 'True Positive' and so on.

The entries can either be specified absolutely or relatively.

Actual/Predicted	Positive	Negative
Positive	TP	FN
Negative	FP	TN

Table 1: The confusion matrix.

2 Sensitivity, specificity and prevalence

A common way of describing a predictor is through *sensitivity* and *specificity*:

- Sensitivity - also known as *recall* or *true positive rate* (TPR) - is the rate of actual positives that are classified as such. It can be expressed as:
$$\mathrm{TPR} = \frac{\mathrm{TP}}{\mathrm{TP} + \mathrm{FN}} \tag{2.1}$$
- Specificity - also known as *true negative rate* (TNR) - is the rate of actual negatives that are classified as such. It can be expressed as:
$$\mathrm{TNR} = \frac{\mathrm{TN}}{\mathrm{TN} + \mathrm{FP}} \tag{2.2}$$

One might think, that if a predictor has a high specificity and sensitivity, then it is useful. It turns out that this is not automatically true: *prevalence*, i.e. the overall occurence rate of positives also plays a role. The prevalence is $\mathrm{TP} + \mathrm{FN}$.

2.1 Example

Assume that a disease has a prevalence of 0.1% in a given population. A screening procedure with a specificity of 99% and a sensitivity of 95%. At first glance, this looks like a good predictor. But in practice, it is less impressive:

Since only $0.1\% = 0.001$ of the population actually has the disease (positive), 99.9% has not (negative). Of that 0.1%, 99% are true positives:

$$\mathrm{TP} = 0.001 \cdot 0.99 = 0.00099 \tag{2.3}$$

The final 1% of the actual positives are false negatives:

$$\mathrm{TP} = 0.001 \cdot 0.01 = 0.00001 \tag{2.4}$$

Out of the actual positives, 95% are true negatives:

$$\mathrm{TN} = 0.999 \cdot 0.95 = 0.94905 \tag{2.5}$$

Actual/Predicted	Positive	Negative
Positive	0.00099	0.00001
Negative	0.04995	0.94905

Table 2: Example confusion matrix.

The 5% of the actual negatives are false positives:

$$\mathrm{FP} = 0.999 \cdot 0.05 = 0.04995 \tag{2.6}$$

The corresponding confusion matrix is shown in table 2.1.

3 Positive and negative predictive value

In practice, the following statistics are often of great importance:

- The *positive predictive value* (PPV) is the conditional probability that a sample is actually positive given that the prediction is positive. It can be expressed:

$$\mathrm{PPV} = \frac{\mathrm{TP}}{\mathrm{TP} + \mathrm{FP}} \tag{3.1}$$

- The *negative predictive value* (NPV) is the conditional probability that a sample is actually negative given that the prediction is negative. It can be expressed:

$$\mathrm{NPV} = \frac{\mathrm{TN}}{\mathrm{TN} + \mathrm{FN}} \tag{3.2}$$

3.1 Example

In the example above, the positive predictive value is:

$$\mathrm{PPV} = \frac{0.00099}{0.00099 + 0.04995} \approx 0.019 \tag{3.3}$$

The negative predictive value is:

$$\mathrm{NPV} = \frac{0.94905}{0.94905 + 0.00001} \approx 1.000 \tag{3.4}$$

So, the screening is very good at actually predicting negatives: If the test is negative, you're almost certain not to have the disease. However, if the test is positive, there's only a 1.9% chance that you actually have the disease! This is not very reassuring! The deeper reason for this low number, is that since the prevalence is low, there ends up being a comparatively large number of false positives, even if the specificity is high.

		Predicted condition			
	Total population	Predicted Condition positive	Predicted Condition negative	Prevalence = $\frac{\Sigma \text{ Condition positive}}{\Sigma \text{ Total population}}$	
True condition	condition positive	**True positive**	**False Negative** (Type II error)	True positive rate (TPR), Sensitivity, Recall, probability of detection = $\frac{\Sigma \text{ True positive}}{\Sigma \text{ Condition positive}}$	False negative rate (FNR), Miss rate = $\frac{\Sigma \text{ False negative}}{\Sigma \text{ Condition positive}}$
	condition negative	**False Positive** (Type I error)	**True negative**	False positive rate (FPR), Fall-out, probability of false alarm = $\frac{\Sigma \text{ False positive}}{\Sigma \text{ Condition negative}}$	True negative rate (TNR), Specificity (SPC) = $\frac{\Sigma \text{ True negative}}{\Sigma \text{ Condition negative}}$
	Accuracy (ACC) = $\frac{\Sigma \text{ True positive} + \Sigma \text{ True negative}}{\Sigma \text{ Total population}}$	Positive predictive value (PPV), Precision = $\frac{\Sigma \text{ True positive}}{\Sigma \text{ Test outcome positive}}$	False omission rate (FOR) = $\frac{\Sigma \text{ False negative}}{\Sigma \text{ Test outcome negative}}$	Positive likelihood ratio (LR+) = $\frac{\text{TPR}}{\text{FPR}}$	Diagnostic odds ratio (DOR) = $\frac{\text{LR+}}{\text{LR−}}$
		False discovery rate (FDR) = $\frac{\Sigma \text{ False positive}}{\Sigma \text{ Test outcome positive}}$	Negative predictive value (NPV) = $\frac{\Sigma \text{ True negative}}{\Sigma \text{ Test outcome negative}}$	Negative likelihood ratio (LR−) = $\frac{\text{FNR}}{\text{TNR}}$	

Figure 1: Confusion matrix and derived statistics. Source: Wikipedia.