# Convolutions Neural Networks

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## 1 Mathematical convolution

In mathematics, the *convolution* of two functions $f$ and $g$ is defined as:

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)\ d\tau \tag{1.1}$$

By a change of variables $\sigma = t - \tau$, we have $\tau = t - \sigma$, and therefore $d\tau/d\sigma = -1$. Therefore:

$$(f*g)(t) = \int_{\infty}^{-\infty} f(t-\sigma)g(\sigma)(-d\sigma) = \int_{-\infty}^{\infty} f(t-\sigma)g(\sigma)\ d\sigma = (g*f)(t) \tag{1.2}$$

### 1.1 Discrete convolution

If $f$ and $g$ are only defined on evenly spaced lattice points the integral in the convolution turns into a sum. Without loss of generality, we can assume these lattice points to be the integers:

$$(f * g)(x) = \sum_{\Delta x \in \mathbb{Z}} f(\Delta x)g(x - \Delta x), \quad x \in \mathbb{Z} \tag{1.3}$$

## 2 Edge detection