

Binary classification

Kristian Wichmann

January 21, 2017

1 Basic definitions

Binary classification is a situation where there's two distinct outcomes which we are trying to predict. Associated predictors are known as *binary predictors*.

1.1 Positives and negatives

The two outcomes are denoted *positives* and *negatives*, respectively. Given a data point, and the corresponding prediction of a binary predictor, there's four different possibilities:

- *True positive*: Both the actual outcome and the prediction is positive.
- *False positive*: The prediction is positive, but the actual outcome is negative. This is also known as a *type I error*.
- *False negative*: The prediction is negative, but the actual outcome is positive. This is also known as a *type II error*.
- *True negative*: Both the actual outcome and the prediction is negative.

1.2 The confusion matrix

The frequency of the four above-mentioned events are usually presented in matrix form, in what is known as the *confusion matrix*. It is shown in table 1.2, where TP means 'True Positive' and so on.

The entries can either be specified absolutely or relatively.

Actual/Predicted	Positive	Negative
Positive	TP	FN
Negative	FP	TN

Table 1: The confusion matrix.

2 Sensitivity, specificity and prevalence

A common way of describing a predictor is through *sensitivity* and *specificity*:

- Sensitivity - also known as *recall* or *true positive rate* (TPR) - is the rate of actual positives that are classified as such. It can be expressed as:

$$\text{TPR} = \frac{\text{TP}}{\text{TP} + \text{FN}} \quad (2.1)$$

- Specificity - also known as *true negative rate* (TNR) - is the rate of actual negatives that are classified as such. It can be expressed as:

$$\text{TNR} = \frac{\text{TN}}{\text{TN} + \text{FP}} \quad (2.2)$$

One might think, that if a predictor has a high specificity and sensitivity, then it is useful. It turns out that this is not automatically true: *prevalence*, i.e. the overall occurrence rate of positives also plays a role. The prevalence is $\text{TP} + \text{FN}$.

2.1 Example

Assume that a disease has a prevalence of 0.1% in a given population. A screening procedure with a specificity of 99% and a sensitivity of 95%. At first glance, this looks like a good predictor. But in practice, it is less impressive:

Since only $0.1\% = 0.001$ of the population actually has the disease (positive), 99.9% has not (negative). Of that 0.1%, 99% are true positives:

$$\text{TP} = 0.001 \cdot 0.99 = 0.00099 \quad (2.3)$$

The final 1% of the actual positives are false negatives:

$$\text{FN} = 0.001 \cdot 0.01 = 0.00001 \quad (2.4)$$

Out of the actual positives, 95% are true negatives:

$$\text{TN} = 0.999 \cdot 0.95 = 0.94905 \quad (2.5)$$

Actual/Predicted	Positive	Negative
Positive	0.00099	0.00001
Negative	0.04995	0.94905

Table 2: Example confusion matrix.

The 5% of the actual negatives are false positives:

$$\text{FP} = 0.999 \cdot 0.05 = 0.04995 \quad (2.6)$$

The corresponding confusion matrix is shown in table 2.1.

3 Positive and negative predictive value

In practice, the following statistics are often of great importance:

- The *positive predictive value* (PPV) is the conditional probability that a sample is actually positive given that the prediction is positive. It can be expressed:

$$\text{PPV} = \frac{\text{TP}}{\text{TP} + \text{FP}} \quad (3.1)$$

- The *negative predictive value* (NPV) is the conditional probability that a sample is actually negative given that the prediction is negative. It can be expressed:

$$\text{NPV} = \frac{\text{TN}}{\text{TN} + \text{FN}} \quad (3.2)$$

3.1 Example

In the example above, the positive predictive value is:

$$\text{PPV} = \frac{0.00099}{0.00099 + 0.04995} \approx 0.019 \quad (3.3)$$

The negative predictive value is:

$$\text{NPV} = \frac{0.94905}{0.94905 + 0.00001} \approx 1.000 \quad (3.4)$$

So, the screening is very good at actually predicting negatives: If the test is negative, you're almost certain not to have the disease. However, if the test is positive, there's only a 1.9% chance that you actually have the disease! This is not very reassuring! The deeper reason for this low number, is that since the prevalence is low, there ends up being a comparatively large number of false positives, even if the specificity is high.

		Predicted condition			
		Total population	Predicted Condition positive	Predicted Condition negative	Prevalence = $\frac{\Sigma \text{ Condition positive}}{\Sigma \text{ Total population}}$
True condition	condition positive	True positive	False Negative (Type II error)	True positive rate (TPR), Sensitivity, Recall, probability of detection = $\frac{\Sigma \text{ True positive}}{\Sigma \text{ Condition positive}}$	False negative rate (FNR), Miss rate = $\frac{\Sigma \text{ False negative}}{\Sigma \text{ Condition positive}}$
	condition negative	False Positive (Type I error)	True negative	False positive rate (FPR), Fall-out, probability of false alarm = $\frac{\Sigma \text{ False positive}}{\Sigma \text{ Condition negative}}$	True negative rate (TNR), Specificity (SPC) = $\frac{\Sigma \text{ True negative}}{\Sigma \text{ Condition negative}}$
	Accuracy (ACC) = $\frac{\Sigma \text{ True positive} + \Sigma \text{ True negative}}{\Sigma \text{ Total population}}$	Positive predictive value (PPV), Precision = $\frac{\Sigma \text{ True positive}}{\Sigma \text{ Test outcome positive}}$ False discovery rate (FDR) = $\frac{\Sigma \text{ False positive}}{\Sigma \text{ Test outcome positive}}$	False omission rate (FOR) = $\frac{\Sigma \text{ False negative}}{\Sigma \text{ Test outcome negative}}$ Negative predictive value (NPV) = $\frac{\Sigma \text{ True negative}}{\Sigma \text{ Test outcome negative}}$	Positive likelihood ratio (LR+) = $\frac{\text{TPR}}{\text{FPR}}$ Negative likelihood ratio (LR-) = $\frac{\text{FNR}}{\text{TNR}}$	Diagnostic odds ratio (DOR) = $\frac{\text{LR+}}{\text{LR-}}$

Figure 1: Confusion matrix and derived statistics. Source: Wikipedia.