## 1 The linear model

The *linear model* is a theoretical framework that unifies a number of statistical concepts, like ANOVA and regression.

## 1.1 Least squares estimation

## 1.1.1 Statement of the problem

The general problem is this: We wish to model a linear relationship between a response variables Y and p predictor variables  $X_1, X_2, \dots X_p$ . In other words:

$$Y = \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_p X_p \tag{1}$$

Here, the  $\beta$ 's are the coefficients corresponding to the X's. Now, assume that we have n data points, so that  $y_i$  corresponds to  $x_{i1}, x_{i2}, \dots x_{ip}$ . In matrix form equation (1) now becomes:

$$y = X\beta \tag{2}$$

Here,  $y \in \mathbb{R}^{n \times 1}, X \in \mathbb{R}^{n \times p}$  and  $\beta \in \mathbb{R}^{p \times 1}$ . Given y and X, we seek the best fit for  $\beta$ .