Restricted Boltzmann Machines

Kristian Wichmann

April 4, 2017

1 Boltzmann machines

A *Boltzmann machine* is a collection of units divided into two parts: A *visible layer* and a *hidden layer*. The *state* of unit i is represented by a number s_i. According to the type of machine considered, these may take one different ranges of values. All units are connected to every other unit, with *weights* w_{ij} being the connection between units i and j. Figure 1 shows an example of such a atructure. In addition, unit i has a *bias* θ_i.

The *energy* of the machine is:

$$E = -\left(\sum_{i<j} w_{ij} s_i s_j + \sum_i \theta_i s_i\right) \tag{1.1}$$

2 Restricted Boltmann machines

A *restricted Boltzmann machine* (RBM) is a Boltzmann machine where there's no connections between units in the two layers. Assuming there's n visible, and m hidden units, this means that we may write the energy:

$$E = -\left(\sum_{i=1}^{n} a_i v_i + \sum_{j=1}^{m} b_j h_j + \sum_{i=1}^{n}\sum_{j=1}^{m} v_i w_{ij} h_j\right) \tag{2.1}$$

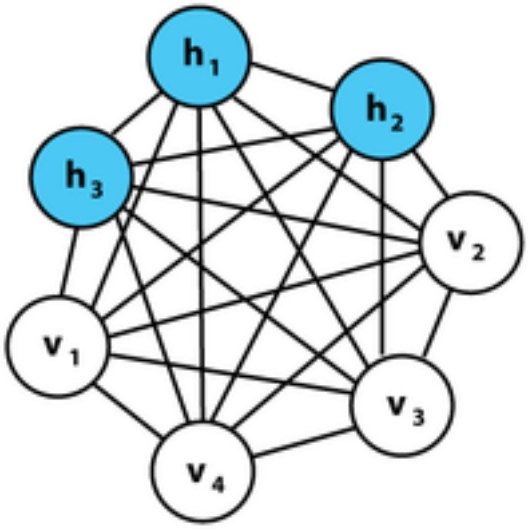

Figure 1: The structure of a general Boltzmann machine with 4 visible and 3 hidden units.

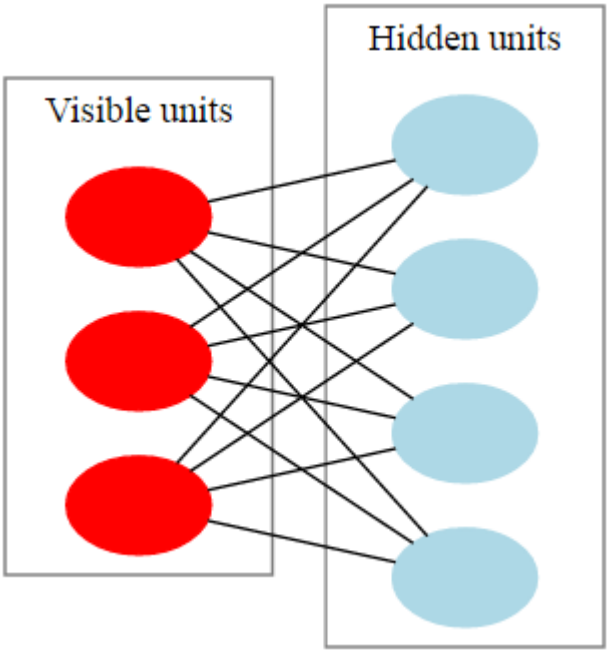

Figure 2: The structure of a restricted Boltzmann machine with 3 visible and 4 hidden units.