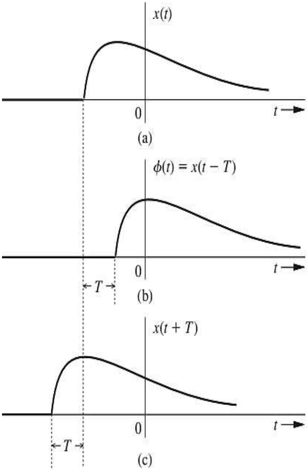
Lecture 2

- Some Useful Signal Operations
- Even and Odd Functions

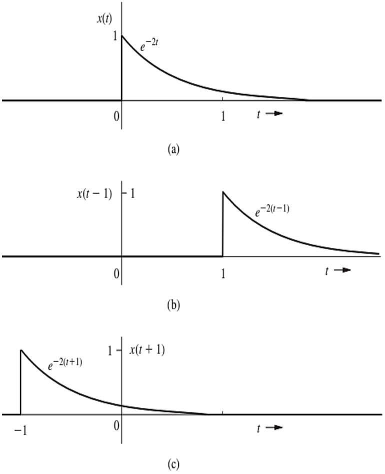
Some Useful Signal Operations

- Time Shifting
- Time Scaling
- Time Reversal

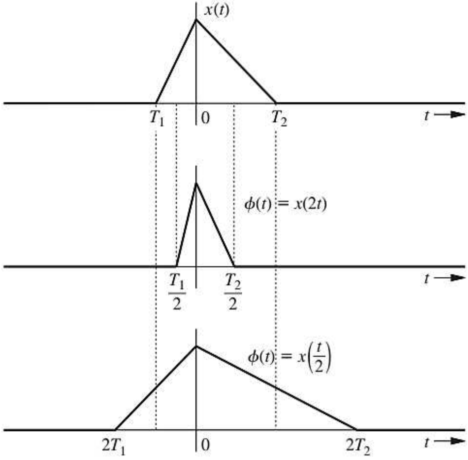
Time Shifting

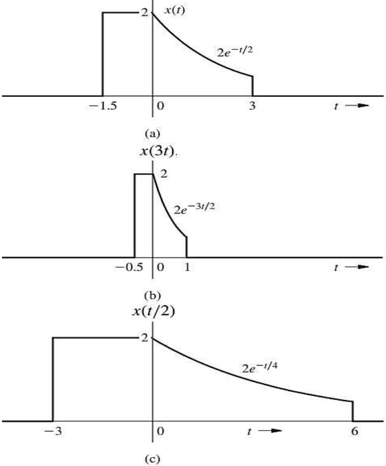


Time-shifting a signal.

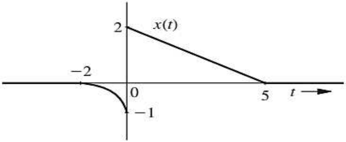


Time Scaling

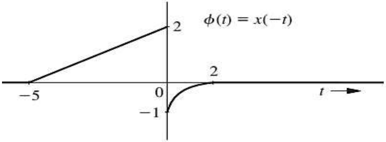




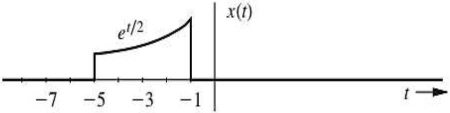
Time Reversal



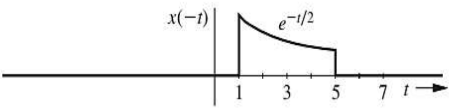
(a)



(b)



(a)

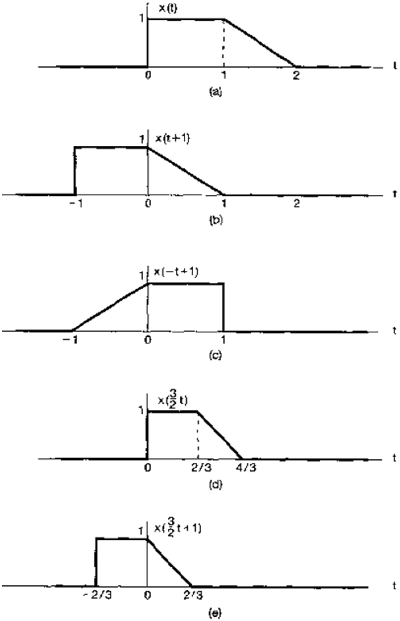


(b)

Combined Operation

How to realize $x(at-b)$?

- Time-shift $x(t)$ by b to obtain $x(t-b)$. Now time-scale the shifted signal $x(t-b)$ by a [i.e., replace t with at] to obtain $x(at-b)$.
- Time-scale $x(t)$ by a to obtain $x(at)$. Now time-shift $x(at)$ by b/a [i.e., replace t with $t-(b/a)$] to obtain $x[a(t-b/a)] = x(at-b)$. In either case, if a is negative, time scaling involves time reversal.


x(t)
1
0
1
2
t
(a)
x(t+1)
1
-1
0
1
2
t
(b)
x(-t+1)
1
-1
0
1
t
(c)
x(3/2 t)
1
0
2/3
4/3
t
(d)
x(3/2 t + 1)
1
-2/3
0
2/3
t
(e)

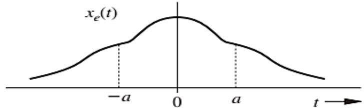
Even and Odd Functions

- A function $x_e(t)$ is said to be an even function of t if it is symmetrical about the vertical axis:

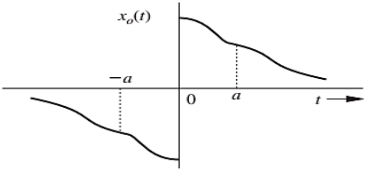
$$x_e(t) = x_e(-t)$$

- A function $x_o(t)$ is said to be an odd function of t if it is anti-symmetrical about the vertical axis:

$$x_o(t) = -x_o(-t)$$



(a)



(b)

Some Properties of Even and Odd Functions

even function $\times$ odd function $=$ odd function
odd function $\times$ odd function $=$ even function
even function $\times$ even function $=$ even function

AREA

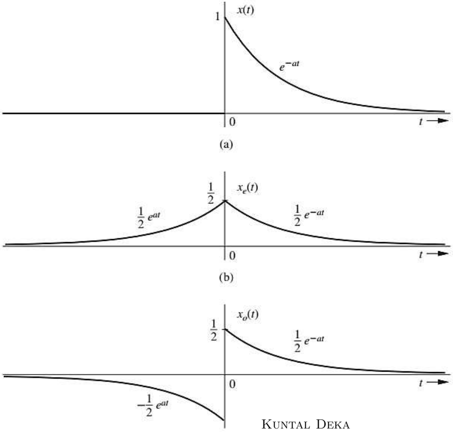
Because of the symmetries of even and odd functions about the vertical axis, we have

$$\int_{-a}^{a} x_e(t)dt = 2\int_{0}^{a} x_e(t)dt$$
$$\int_{-a}^{a} x_o(t)dt = 0$$

Even an Odd Components of a Signal

Every signal $x(t)$ can be expressed as a sum of even and odd components because

$$x(t) = \underbrace{\frac{1}{2}\left[x(t) + x(-t)\right]}_{\text{even part}} + \underbrace{\frac{1}{2}\left[x(t) - x(-t)\right]}_{\text{odd}}$$



Finding the Even and Odd Components of a Complex Signal

Find the even and odd components of e^{jt}.

$$e^{jt} = x_e(t) + x_o(t)$$

where

$$x_e(t) = \frac{1}{2}[e^{jt} + e^{-jt}] = \cos t \qquad \text{and} \qquad x_o(t) = \frac{1}{2}[e^{jt} - e^{-jt}] = j \sin t$$

A MODIFICATION FOR COMPLEX SIGNALS

- A complex signal $x(t)$ is said to be conjugate-symmetric if $x(t) = x^*(-t)$.
- A conjugate-symmetric signal is even in the real part and odd in the imaginary part.
- A signal is conjugate-antisymmetric if $x(t) = -x^*(-t)$.
- A conjugate-antisymmetric signal is odd in the real part and even in the imaginary part.
- Any signal $x(t)$ can be decomposed into a conjugate-symmetric portion $x_{cs}(t)$ plus a conjugate-antisymmetric portion $x_{ca}(t)$. That is,

$$x(t) = x_{cs}(t) + x_{ca}(t)$$

where

$$x_{cs}(t) = \frac{x(t)+x^*(-t)}{2} \qquad \text{and} \qquad x_{ca}(t) = \frac{x(t)-x^*(-t)}{2}$$