SEG DENVER
26-31 OCTOBER 2014

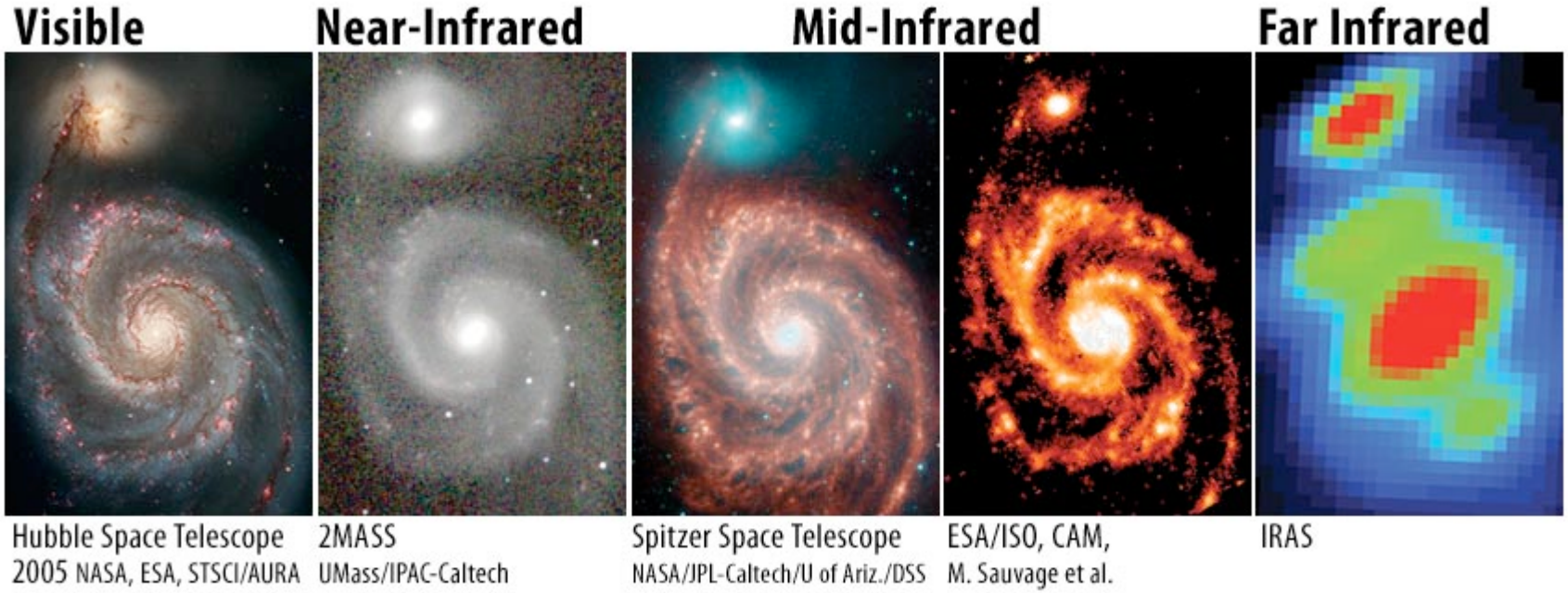
Agile*

timefreak

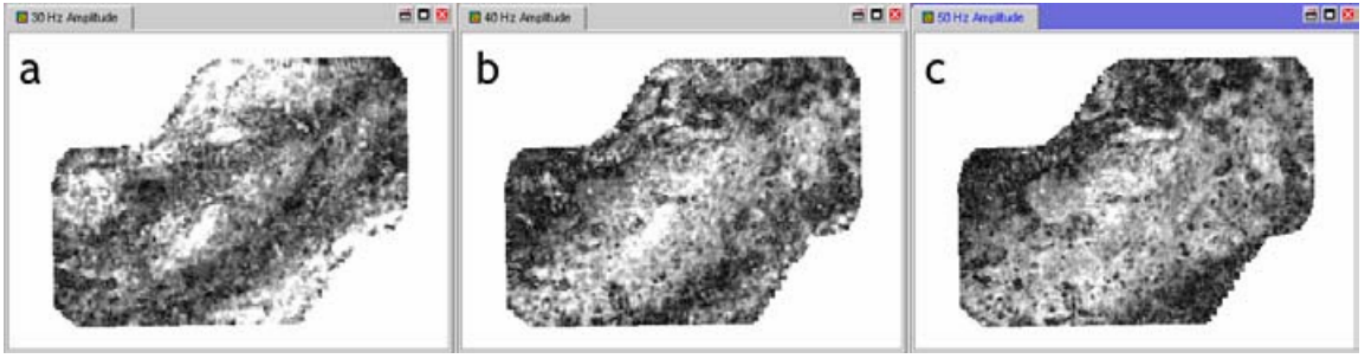
the fidelity of time-frequency representations

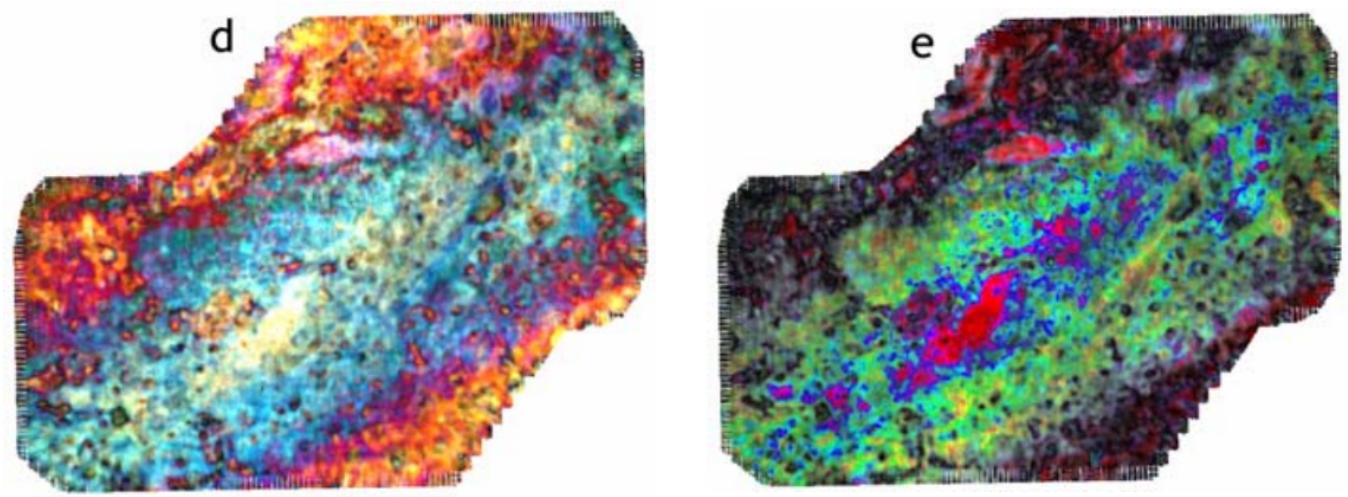
matt hall
matt@agilegeoscience.com

- Geological and pedagogical motives
- Gabor's uncertainty principle
- What is a good t-f representation?
- Graphical representations
- A synthetic benchmark


Visible
Near-Infrared
Mid-Infrared
Far Infrared
Hubble Space Telescope
2005 NASA, ESA, STSCI/AURA
2MASS
UMass/IPAC-Caltech
Spitzer Space Telescope
NASA/JPL-Caltech/U of Ariz./DSS
ESA/ISO, CAM,
M. Sauvage et al.
IRAS

M51A, M51B



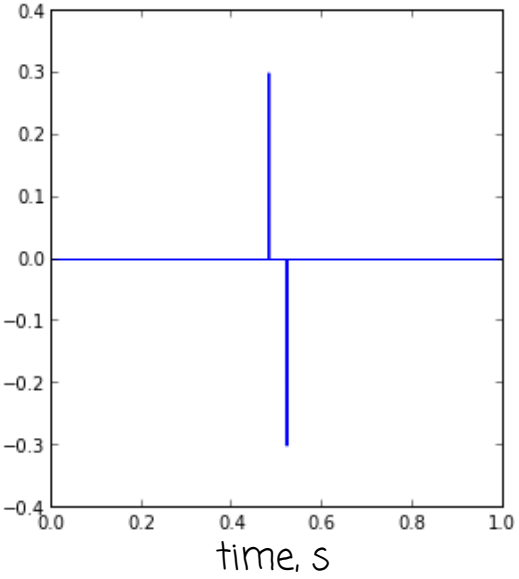


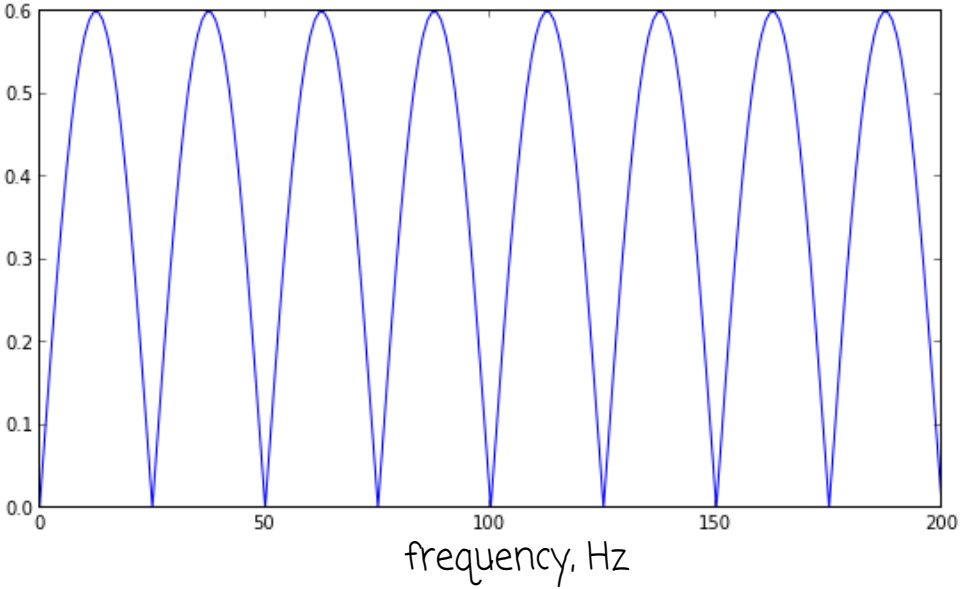
Hall & Trouillot, CSEG Convention, 2003

30 Hz, 40 Hz, 50 Hz

cepstrum.ipynb

Thin bed response





```
rgb_arr = np.array(timesteps)
```

In [48]:
```
rgb_arr.shape
```

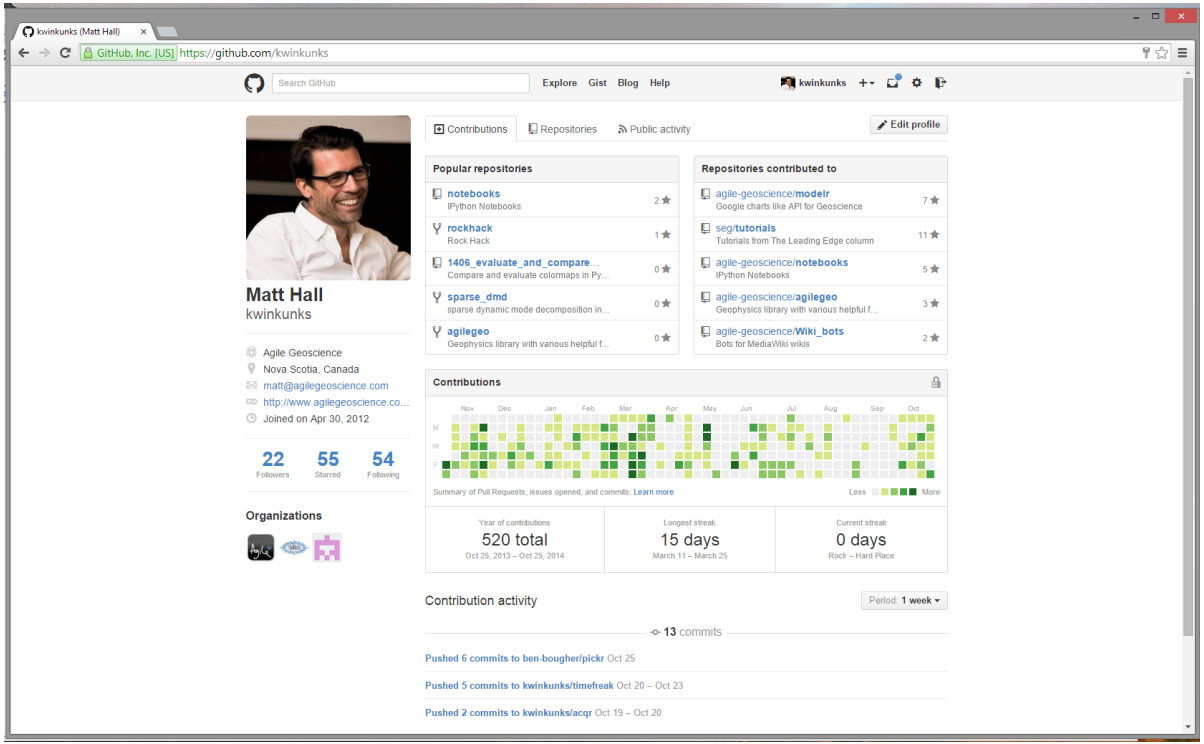
Out[48]: (102, 1012, 3)

The `matplotlib` function `imshow` interprets arrays like this as 3-channel colour images, so we can just display this array directly. We'll chop off the negative frequencies again.

In [53]:
```
plt.figure(figsize=(12,4))
plt.imshow(rgb_arr[:rgb_arr.shape[0]/2.,...], aspect="auto", origin="lower", interpolation="none")
plt.show()
```

kwinkunks

github.com/kwinkunks/timefreak



Gabor uncertainty…

Heisenberg, Donoho & Stark, Robertson,…

It's important to note that no signal processing tricks can do away with the **fundamental tradeoff** in time-frequency resolution. — Dan Meliza

…through which high frequencies (which are **inherently low resolution**) are coarsely sampled and low frequencies (which are inherently high resolution) are finely sampled… — Mostafa Naghizadeh & Kris Innanen

…in signal processing, [the uncertainty principle] establishes limits on the extent to which the "instantaneous frequency" of a signal **can be measured**. — David Donoho & Philip Stark

...what Gabor uncertainty?

All these methods are bound by the Heisenberg/Gabor uncertainty principle with a trade-off between time and frequency resolutions. Also, signal windowing leads to smearing, which correspond to the widening of the main lobe around its central frequency, and side-lobe leakage. Lately, various new transforms have been developed to **circumvent these issues** such as Empirical Mode Decomposition (EMD),... – Tary et al. 2014

The synchrosqueezing is a post-processing method which **circumvents the uncertainty relation** inherent to these linear transforms... – Oberlin et al., 10.1109/ICASSP.2014.6853609

How shall we write it?

Donoho & Stark 1988,
Gabor 1946

$$\Delta t \cdot \Delta f \geq 1$$

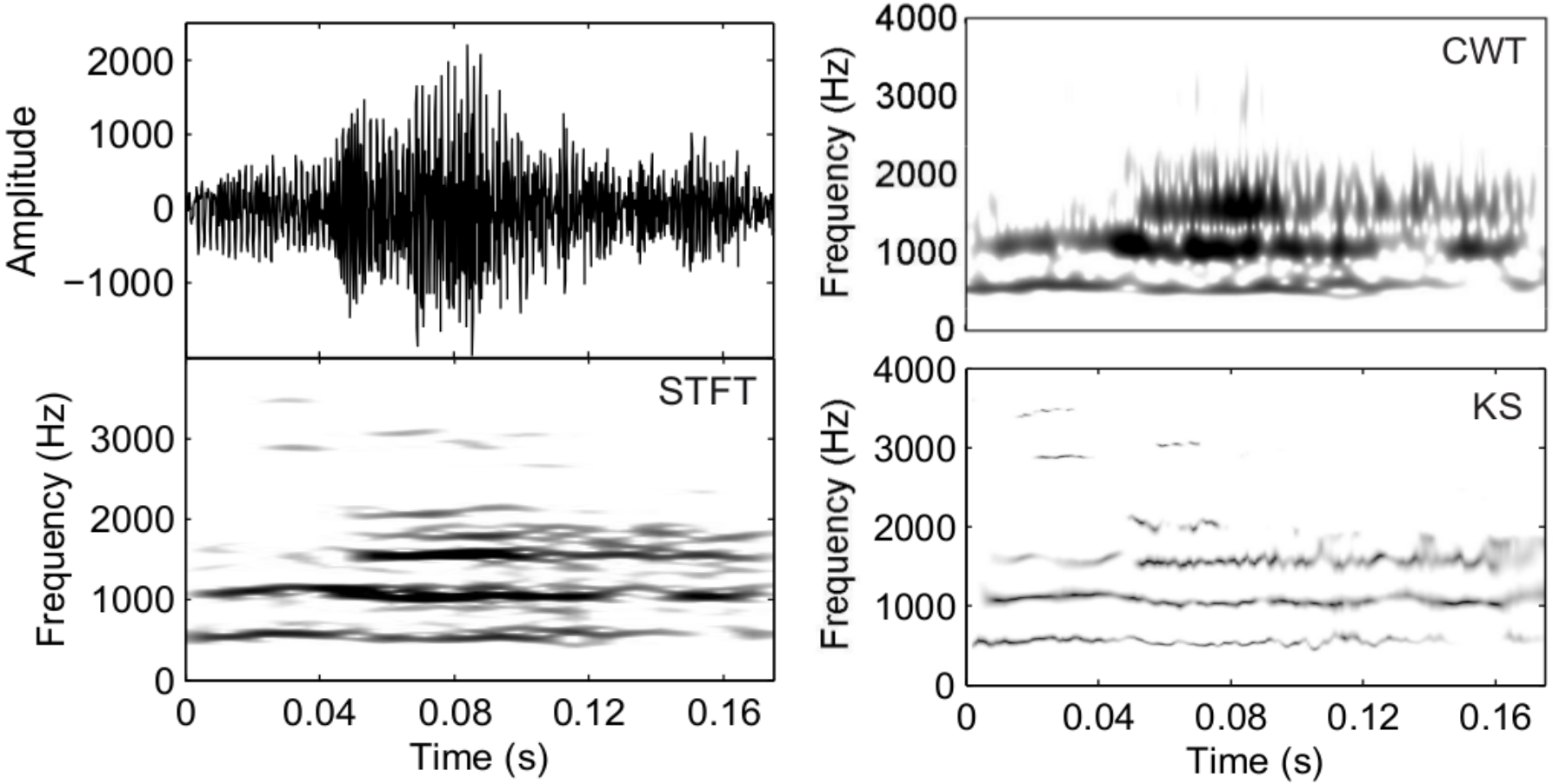
Dirk Lorenz's blog

$$\mathrm{Var}_g \cdot \mathrm{Var}_{\hat{g}} \geq \frac{1}{4}$$

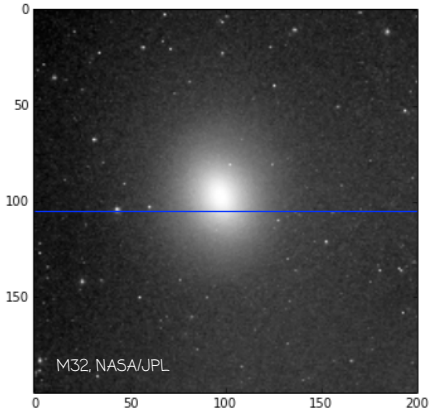
Heisenberg

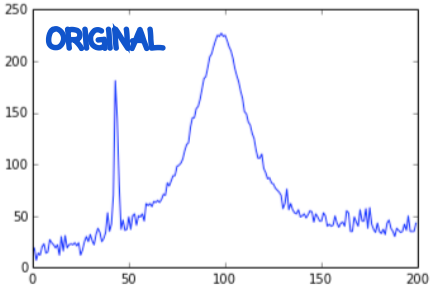
$$\sigma_t \cdot \sigma_f \geq \frac{1}{4\pi}$$

Tary et al. 2014

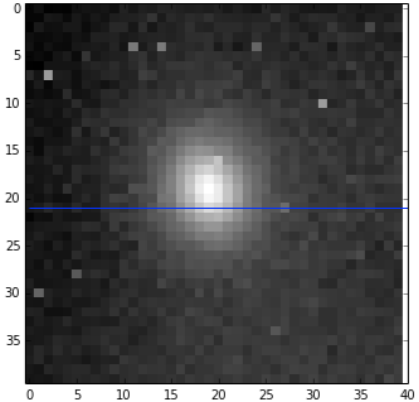


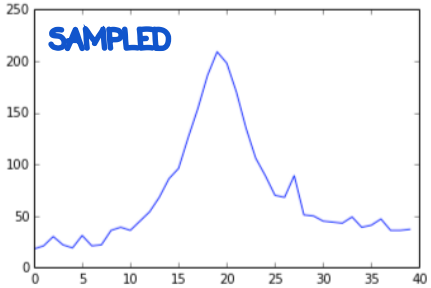
High resolution
High localization
High precision



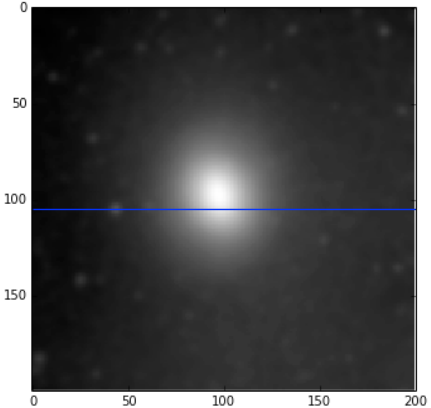


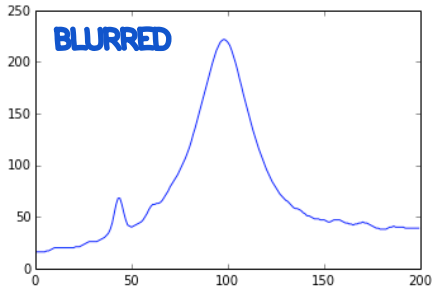
Low resolution
High localization
High precision



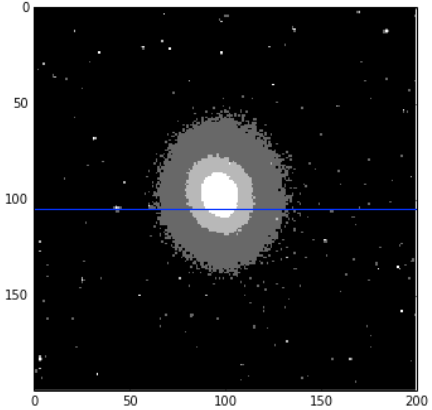


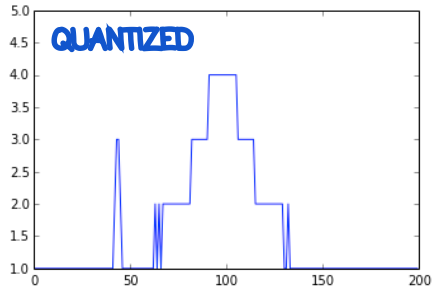
High resolution
Low localization
High precision



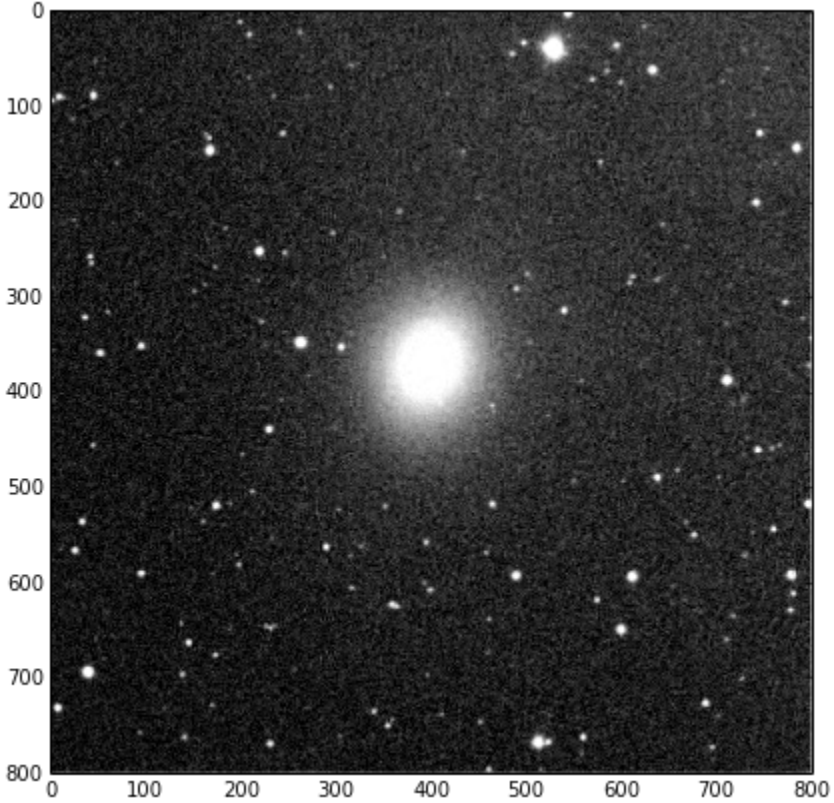


High resolution
High localization
Low precision

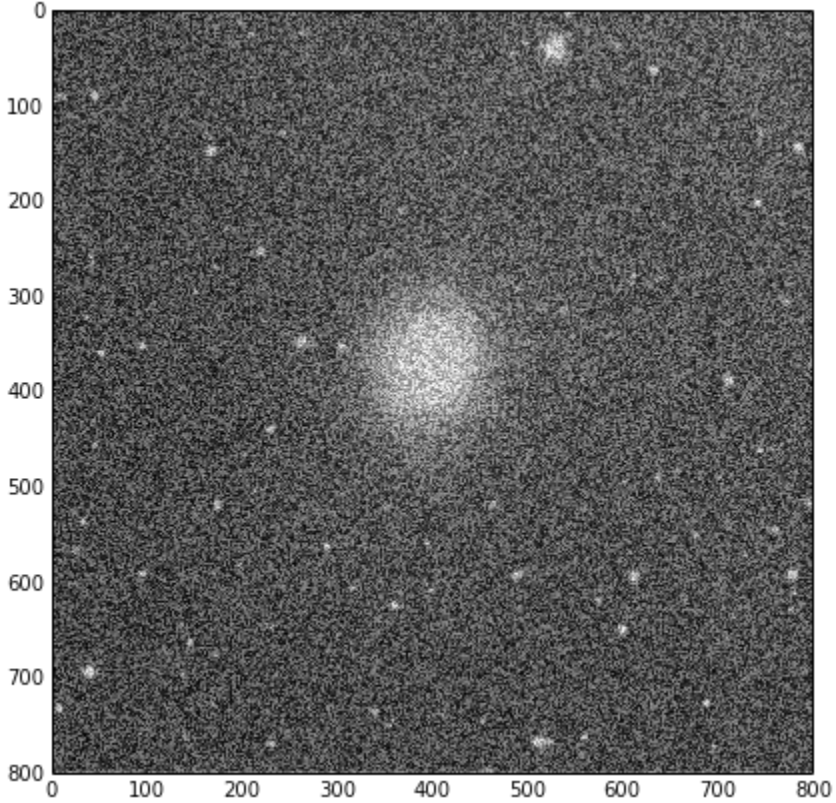




High signal:noise

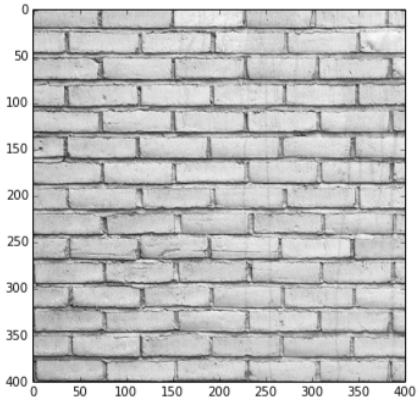


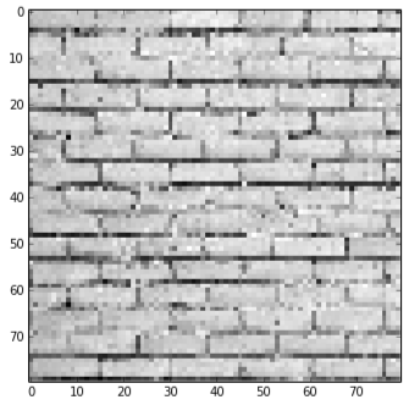
Low signal:noise

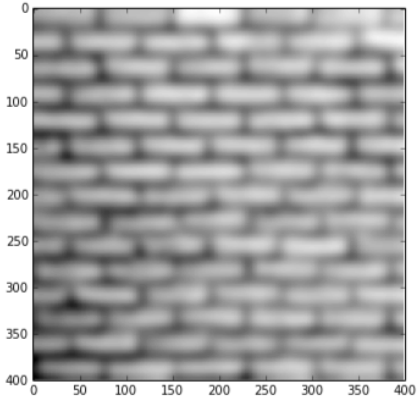


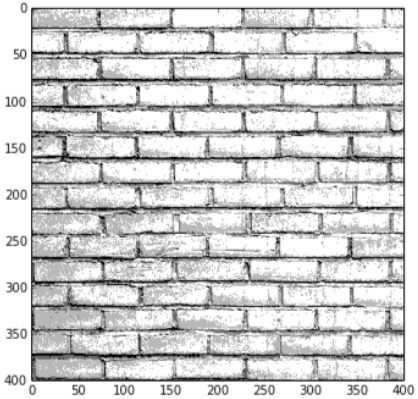
images.ipynb

Few-value, few-scale



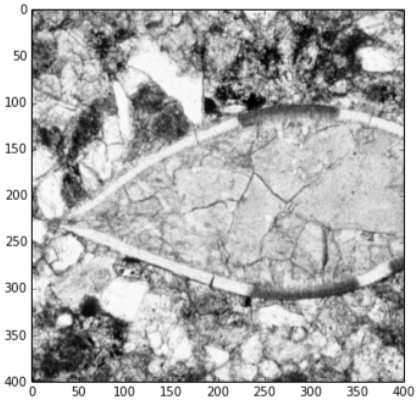


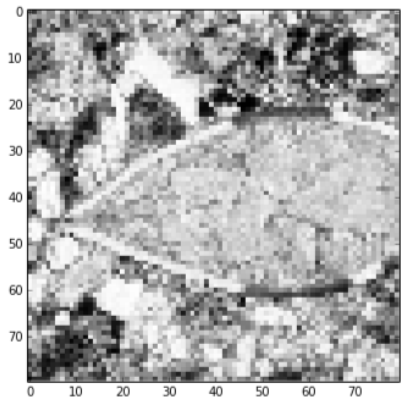


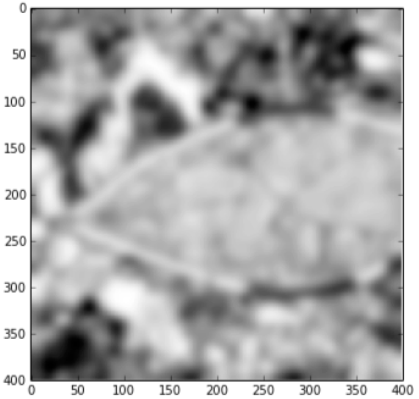


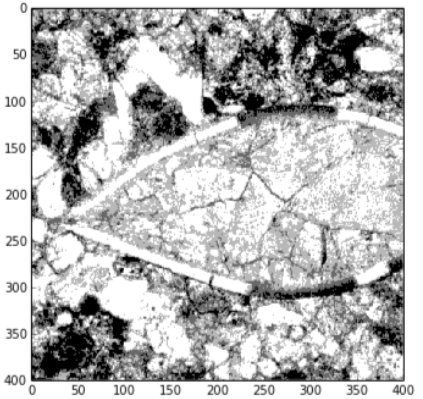
Bricks, flickr/Grzesiek CC-BY

Multi-value, multi-scale







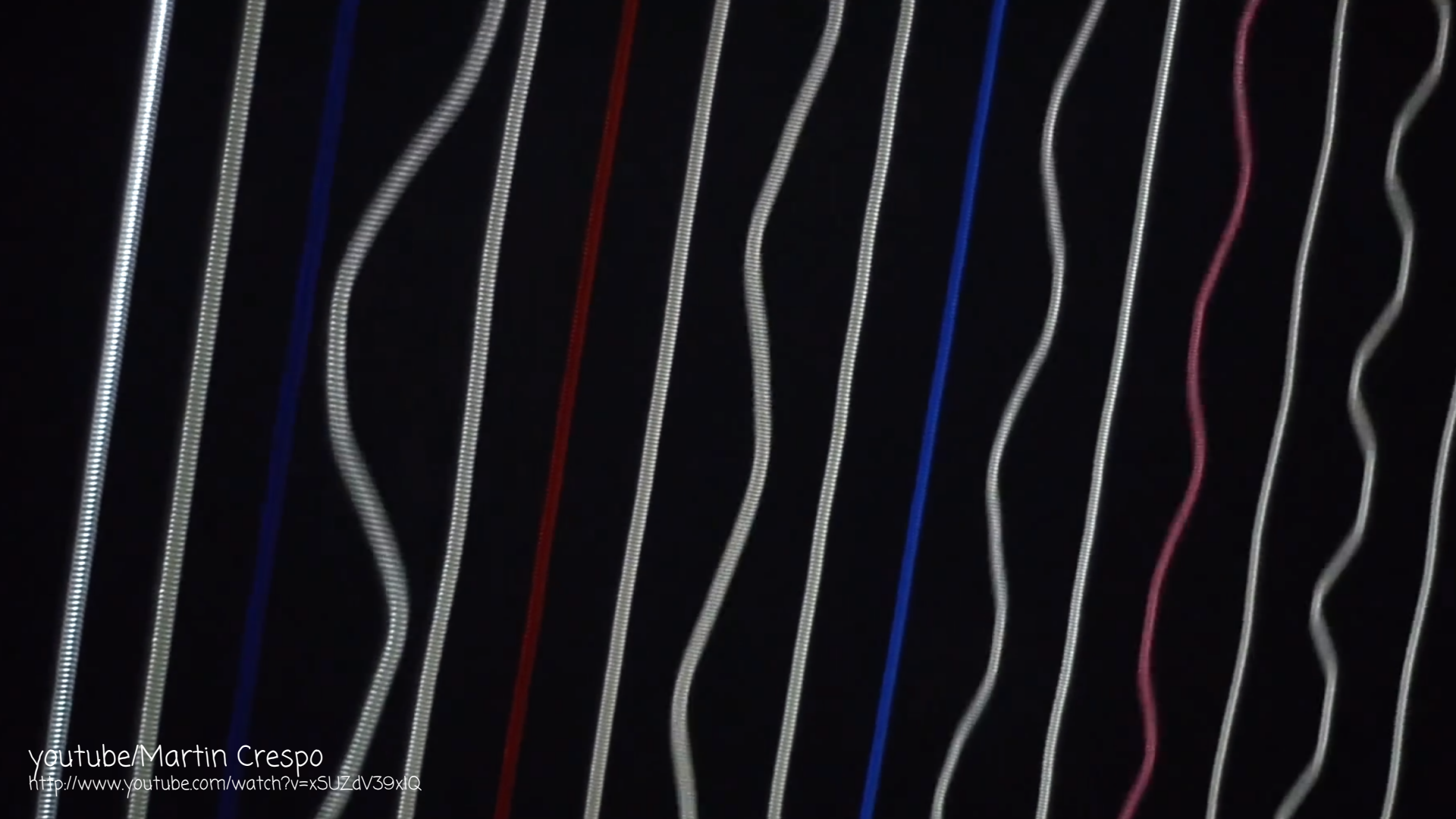


Matt Hall CC-BY

Book Cliffs, Utah

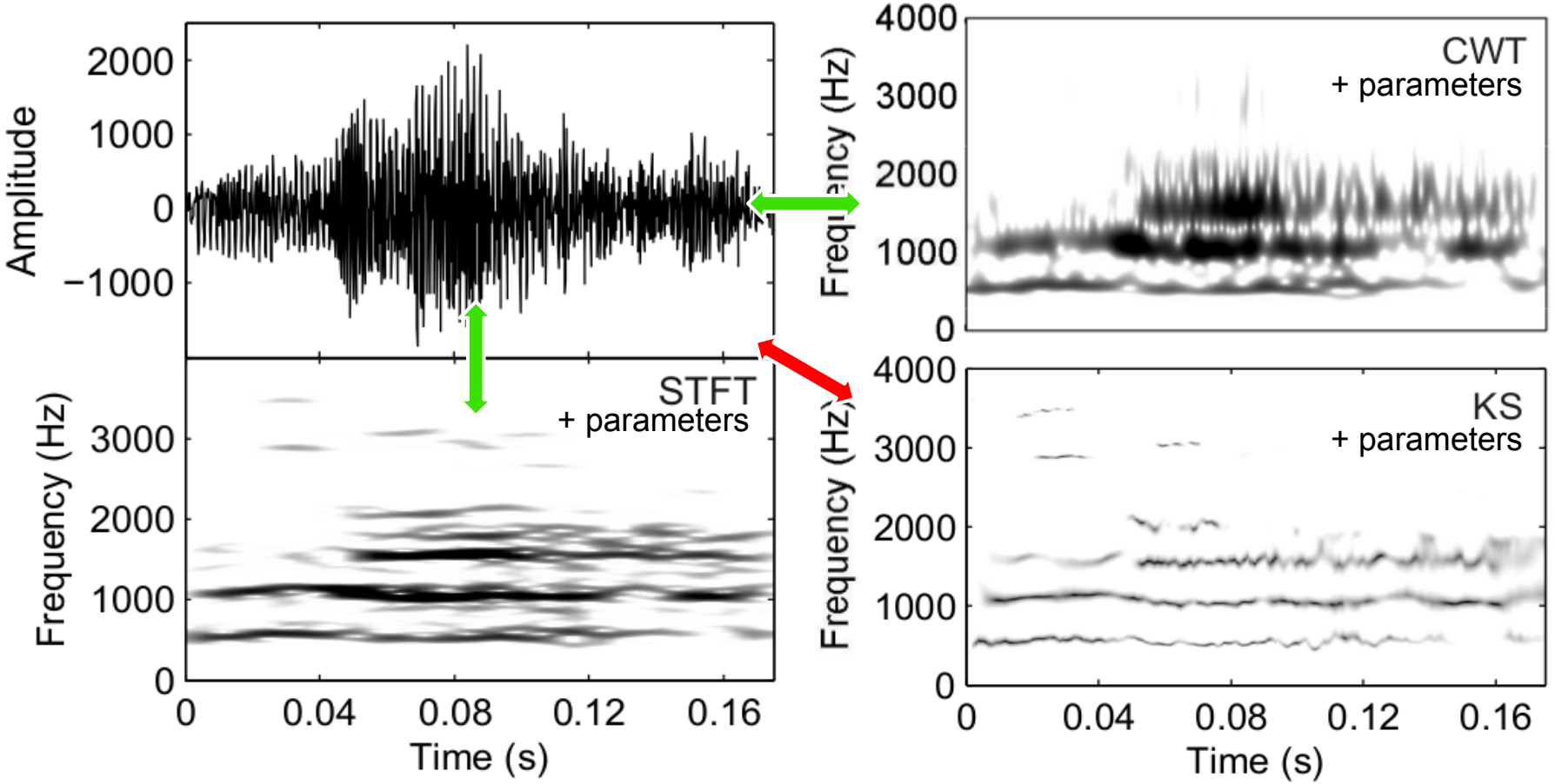
flickr/David Herrera CC-BY

http://www.flickr.com/photos/dph1110/10550053375


youtube/Martin Crespo
http://www.youtube.com/watch?v=x5UZdV39xIQ

What is 'good'?

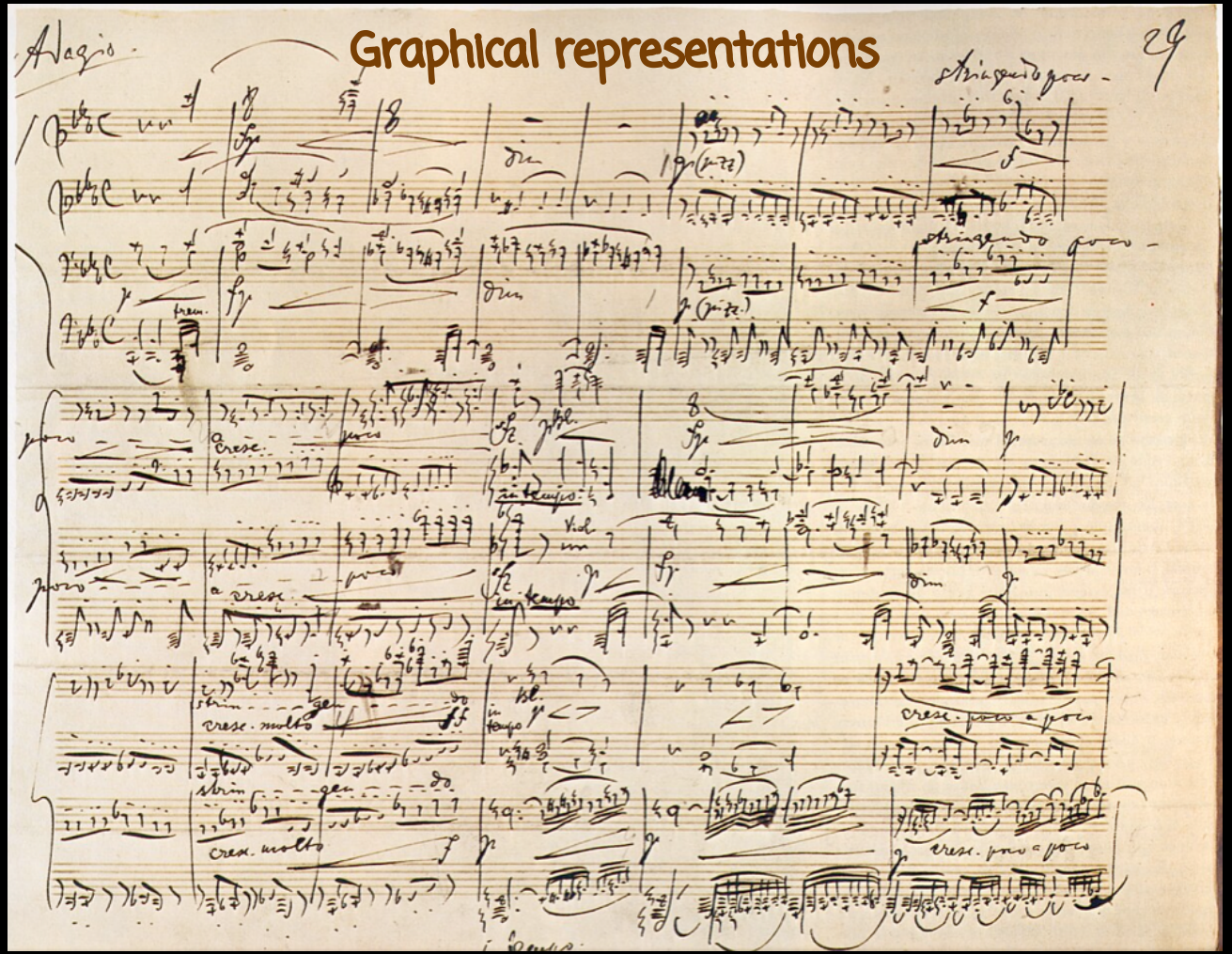
Tary et al. 2014



subsurfwiki.org/wiki/Time-frequency_transforms

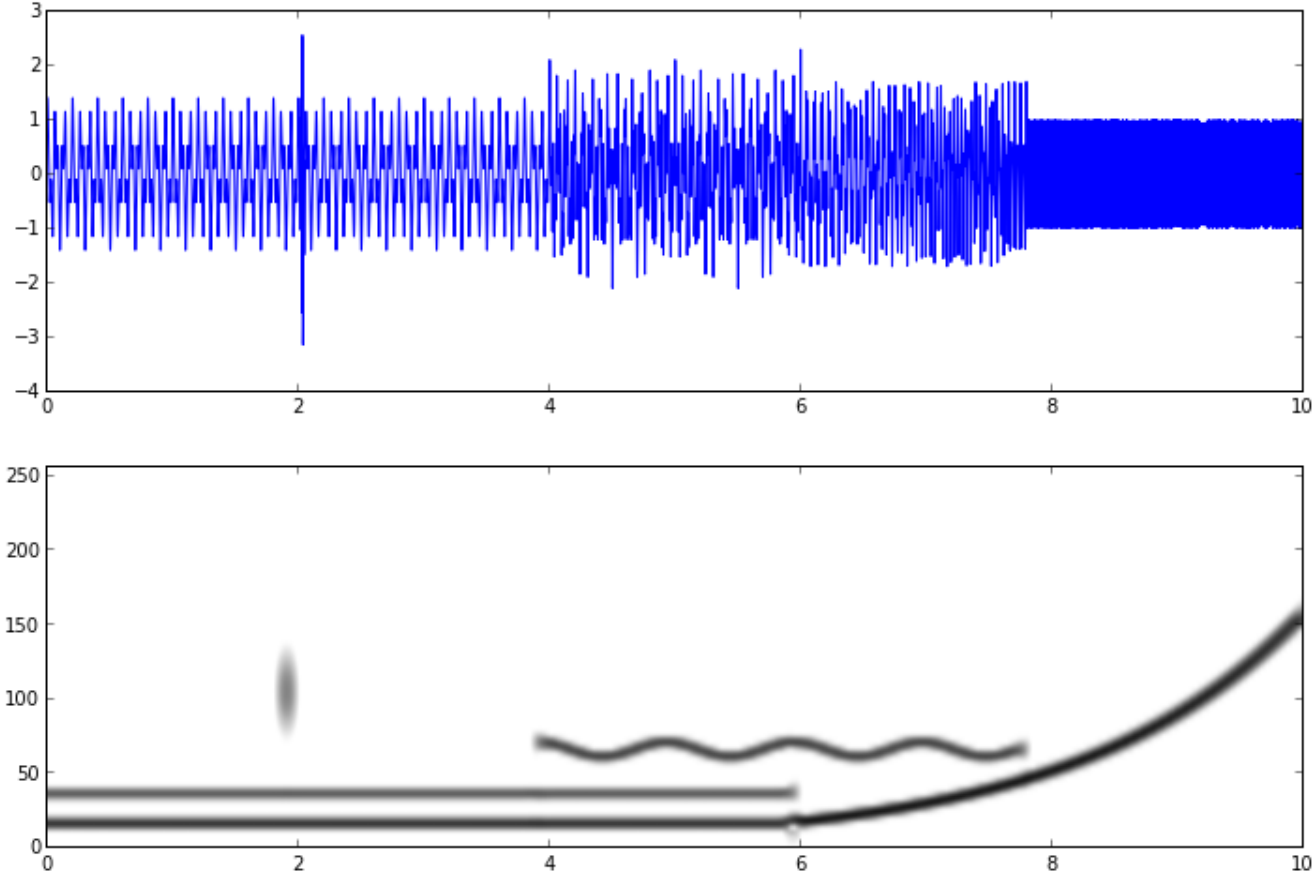
	Name	Note	Python module	Key parameters	Ease of use	Cost	Synthetic	Natural
FFT	Fourier transform	Fast or discrete Fourier transform	`numpy.fft`, `scipy.fftpack.fft`	None				
STFT	Short-time Fourier transform	aka Gabor transform, short window FT	`pyplot.specgram`	Length, step, taper				
ST	S-transform[1]		`pygft`[2]	None	Easy	Fast		
CWT	Continuous wavelet transform		`scipy.cwt`					
DWT	Discrete wavelet transform		`pywt`, `wavelets.py`					
SST	Synchro-squeezing transform	Includes reassignment step						
MP	Matching pursuit		`py-pursuit`[3],					
BP	Basis pursuit							
DMD	Dynamic mode decomposition							
AR	Autoregressive method							
ST-AR	Short-time autoregressive method							
KS	Kalman smoother							
EEMD	Ensemble empirical mode decomposition							
CEEMD	Complete EEMD							

Graphical representations

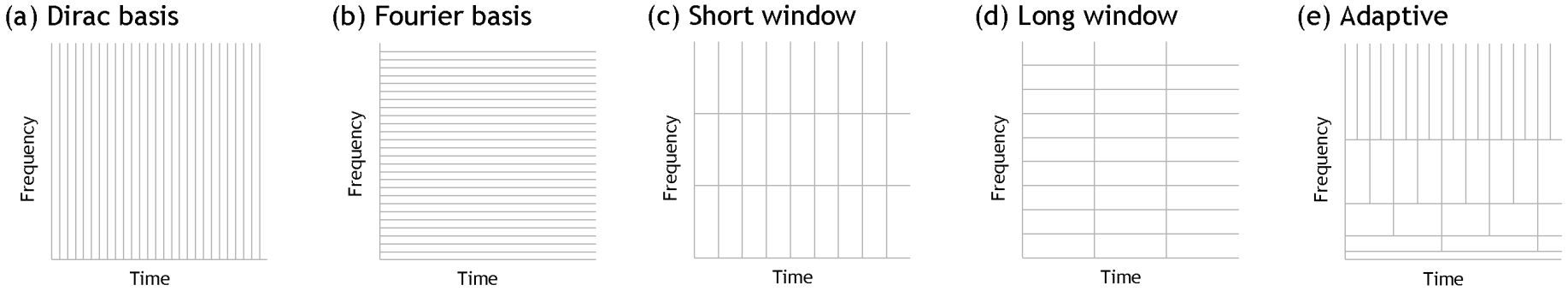


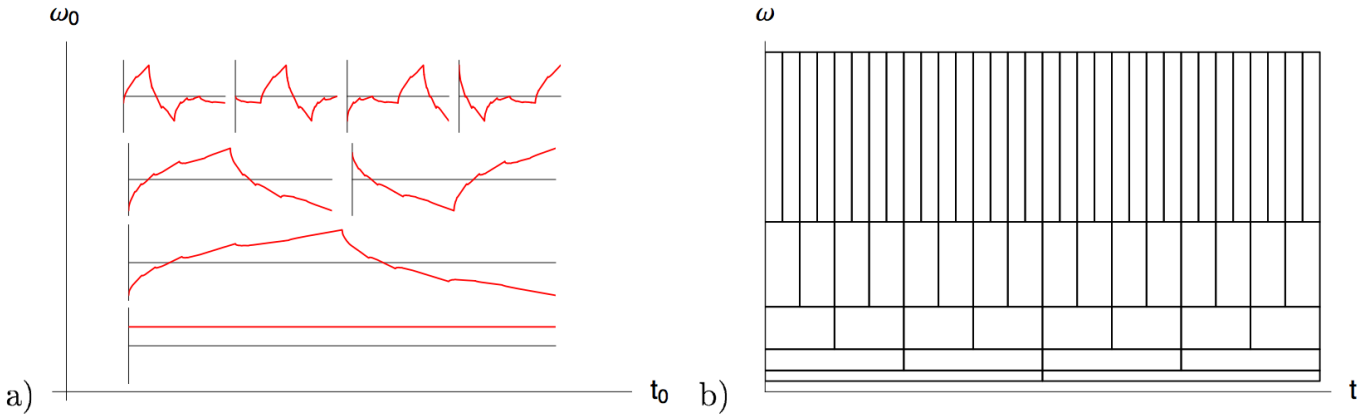
Brahms, ca. 1855
Symphony No. 1

stft.ipynb



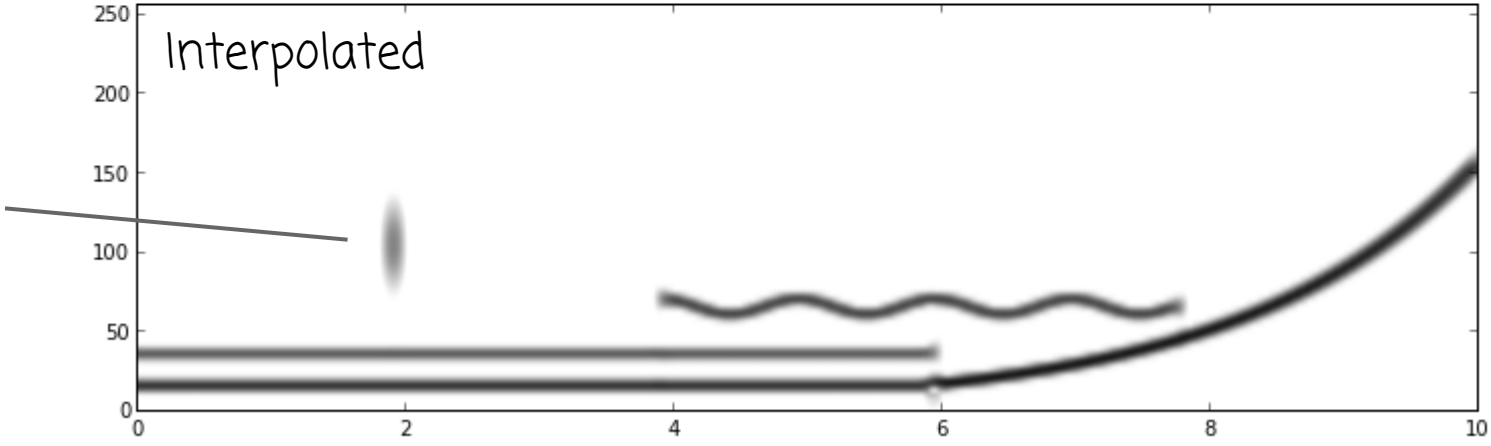
Hall 2005, The resolution of spectral decomposition

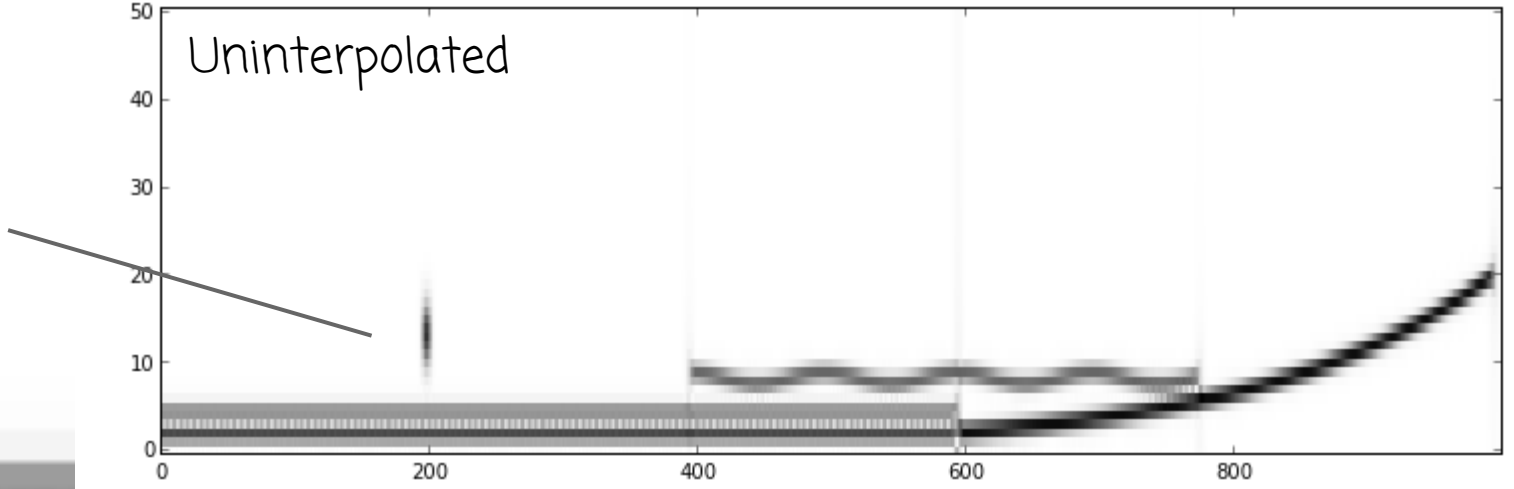


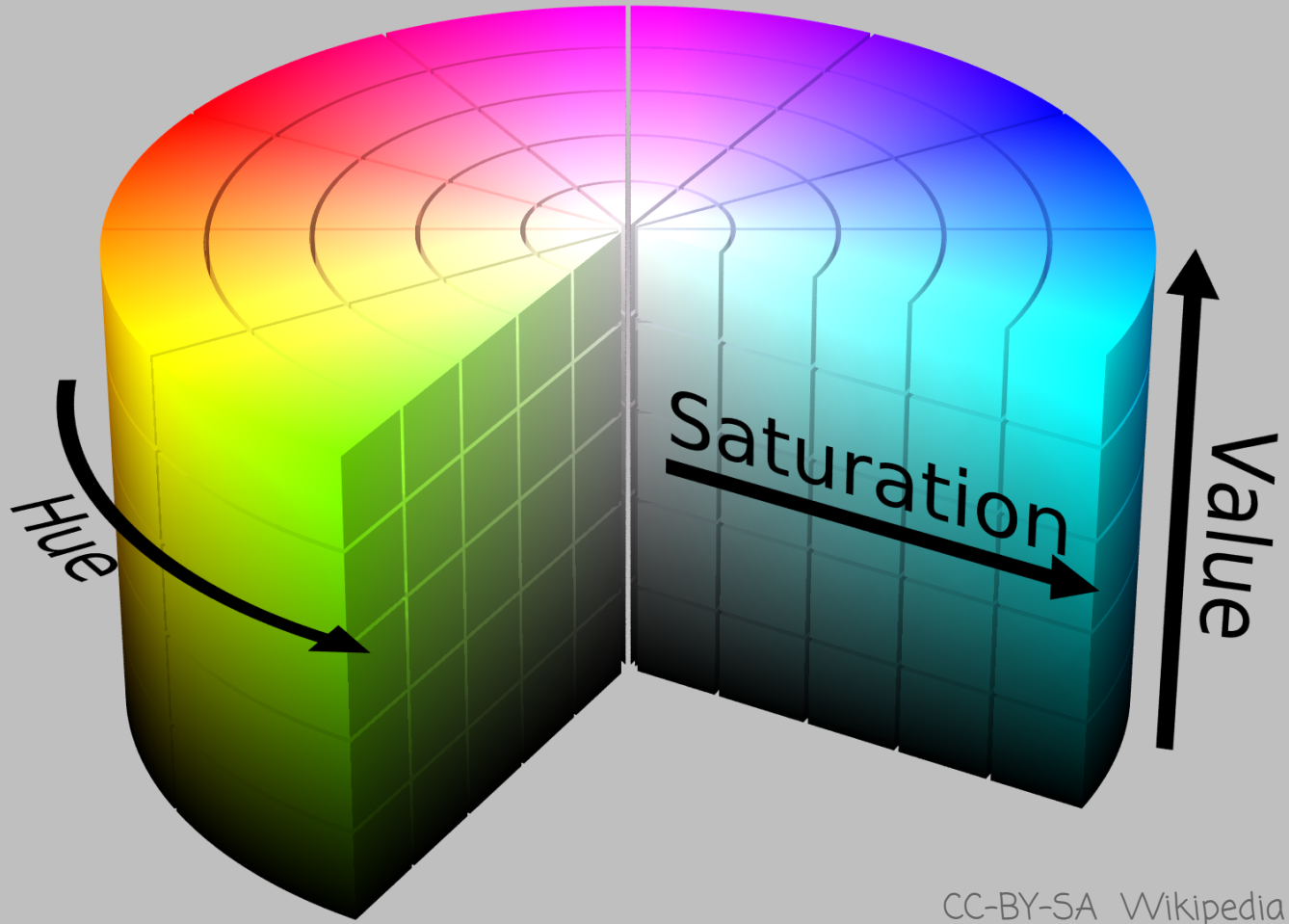


Thesis of Jon Harrop http://www.ffconsultancy.com/free/thesis.pdf

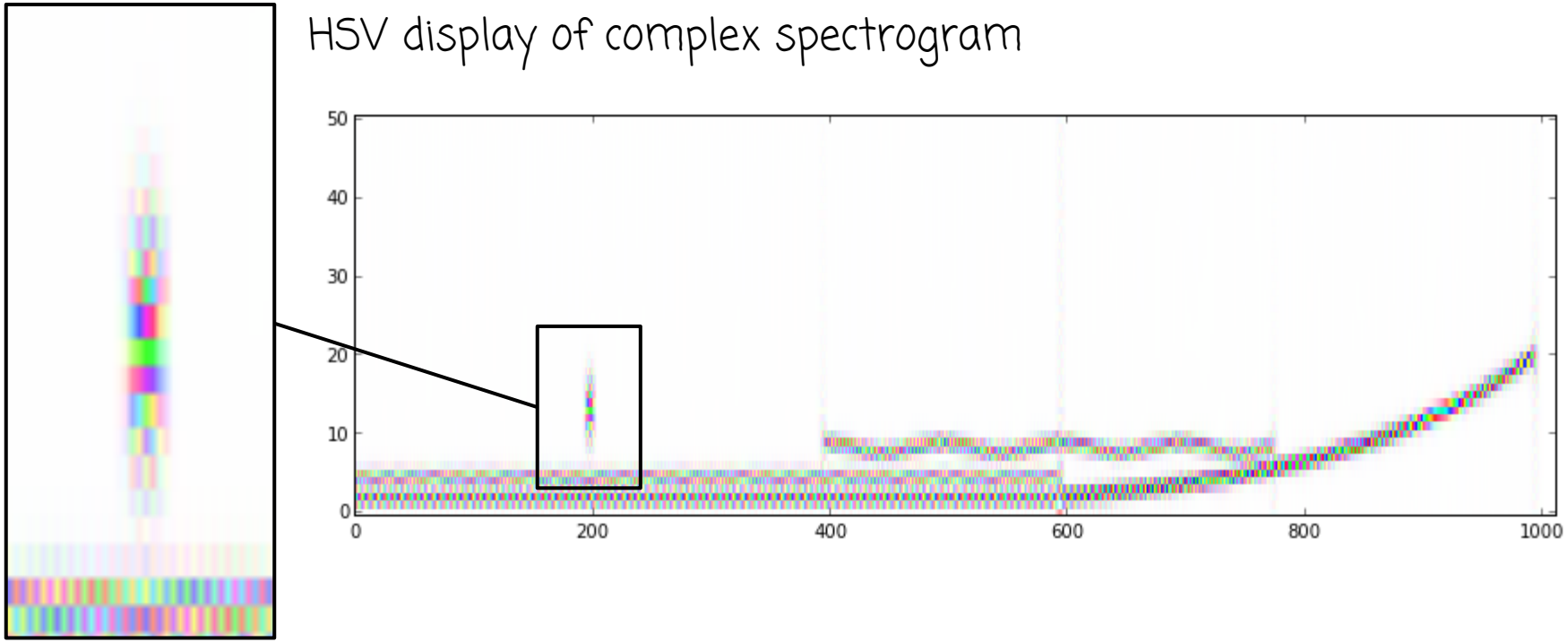
stft.ipynb


Interpolated
250
200
150
100
50
0
0
2
4
6
8
10

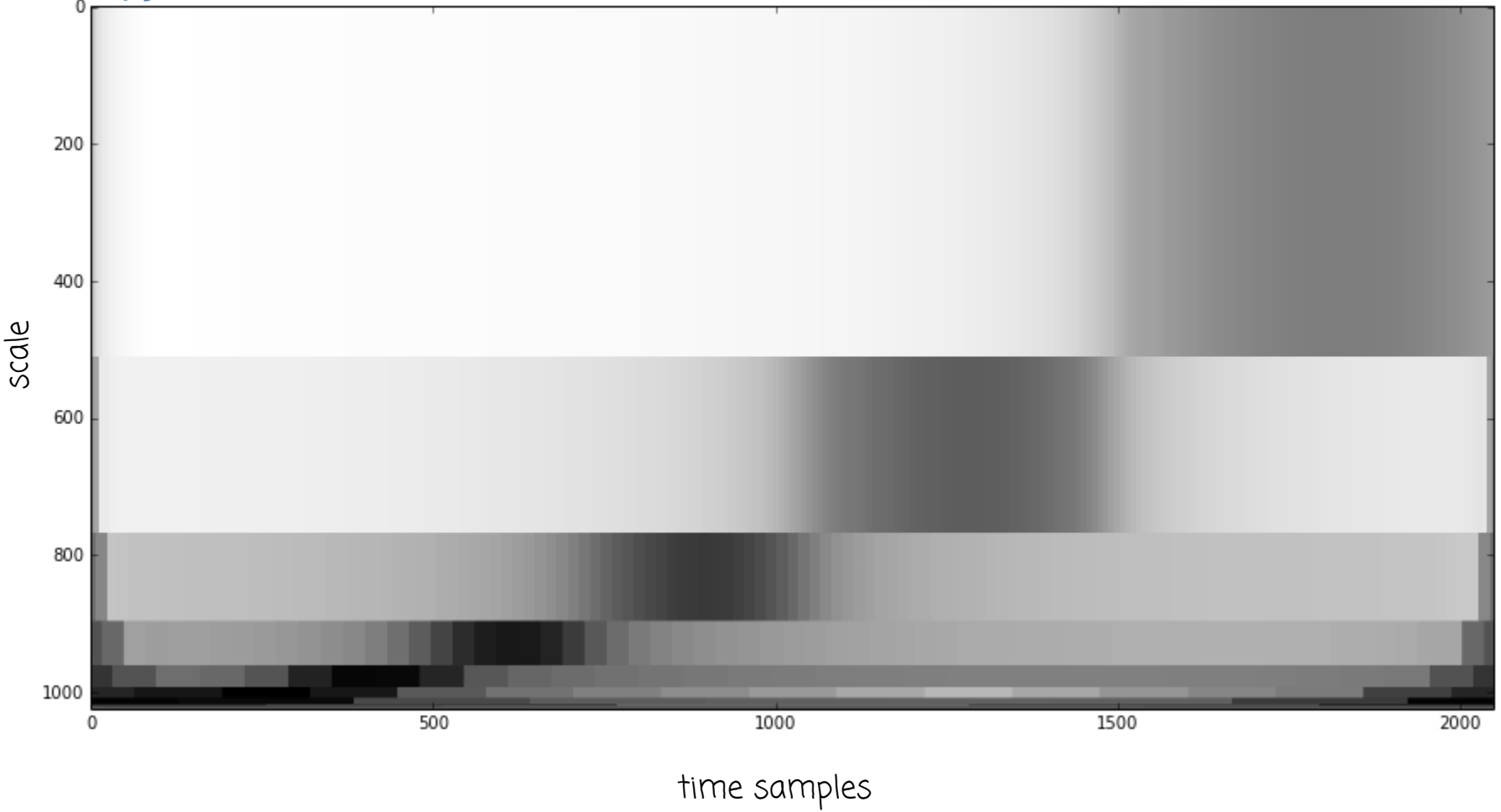

Uninterpolated
50
40
30
20
10
0
0
200
400
600
800



stft.ipynb


HSV display of complex spectrogram
50
40
30
20
10
0
0
200
400
600
800
1000

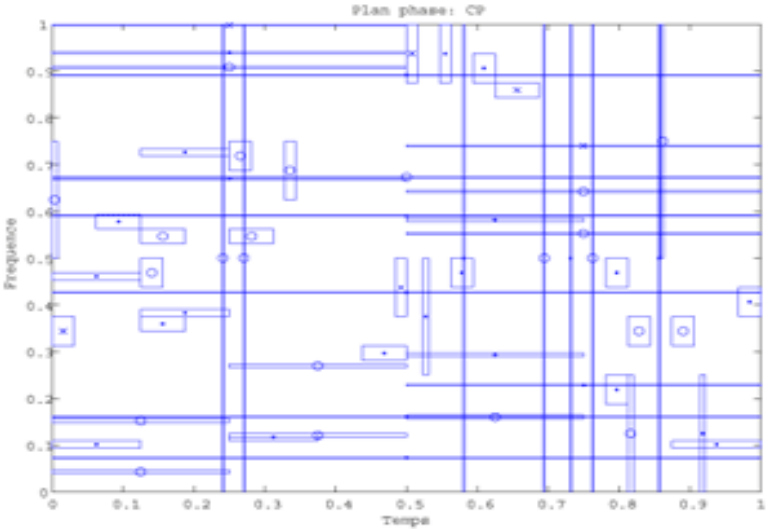
stft.ipynb



Heisenberg boxes or Gabor logons of matching pursuit atoms

time

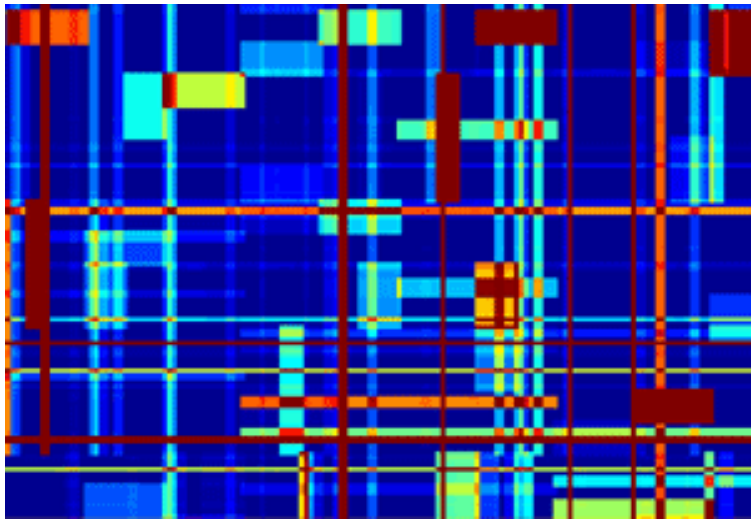
frequency



http://www.isnld.com/indexD7.html

time

frequency

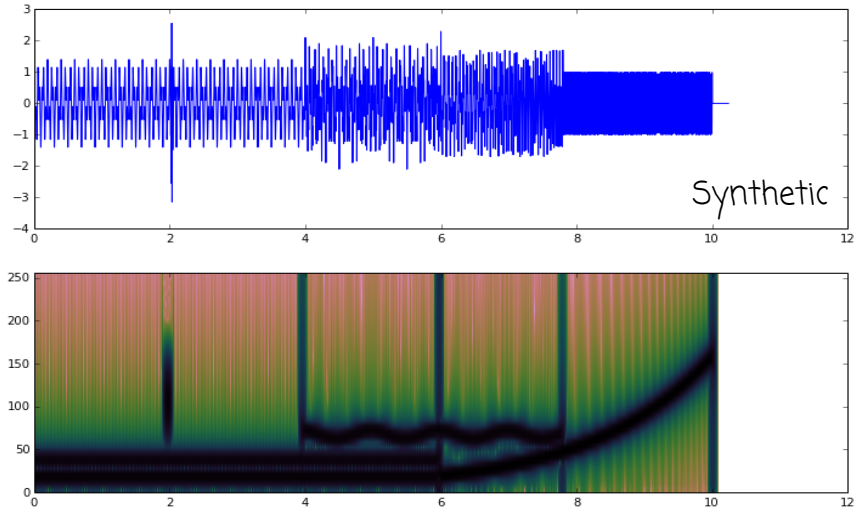


http://www.isnld.com/indexD7.html

Features of a geologically useful synthetic

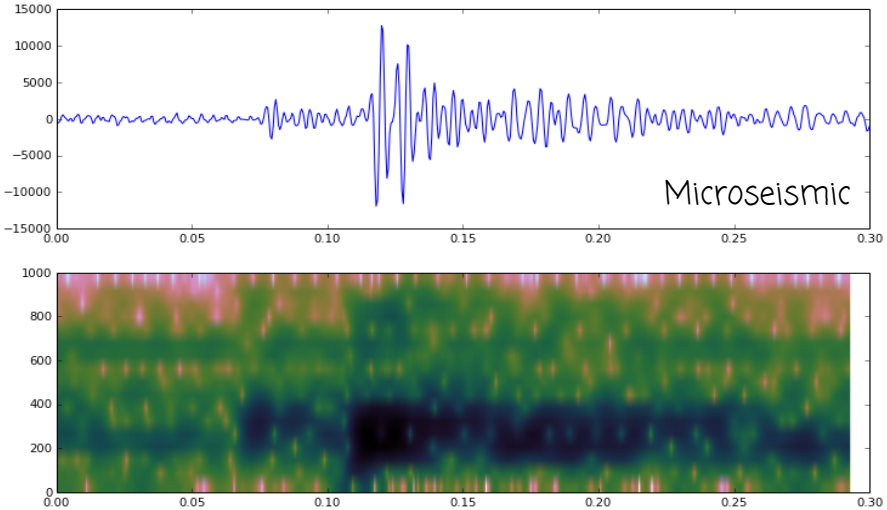
spectrograms.ipynb

Not natural



1. Naturalistic
2. Complex
3. Multi-scale

Not known



1. Natural wavelets
2. Natural incoherent noise
3. Natural coherent noise

- Geological and pedagogical motives
- Gabor's uncertainty principle
- What is a good t-f representation?
- Graphical representations: components and pixels
- A synthetic benchmark

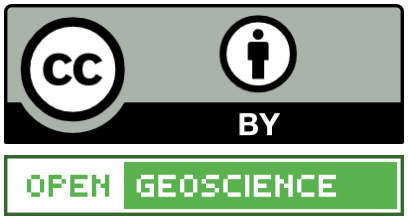
SEG DENVER
26-31 OCTOBER 2014

Agile*

time-freak

matt hall

matt@agilegeoscience.com



http://nbviewer.ipython.org/github/unpingco/Python-for-Signal-Processing/blob/master/Windowing_Part2.ipynb
http://tomwhipple.com/projects/explorations/basic-signal-processing-with-ipython-notebook.html

http://inst.eecs.berkeley.edu/~ee123/fa12/Notes/Lecture10.pdf
http://www.isnld.com/indexD9.html

http://dsp.stackexchange.com/questions/14947/heisenberg-boxes-image-for-matching-pursuit
More on tilings... http://research.microsoft.com/en-us/um/people/cormac/papers/tiling.pdf

http://perso.ens-lyon.fr/patrick.flandrin/1569582263.pdf

http://perso.ens-lyon.fr/patrick.flandrin/SPIE01_PF.pdf
http://statweb.stanford.edu/~donoho/Reports/Oldies/UPSR.pdf

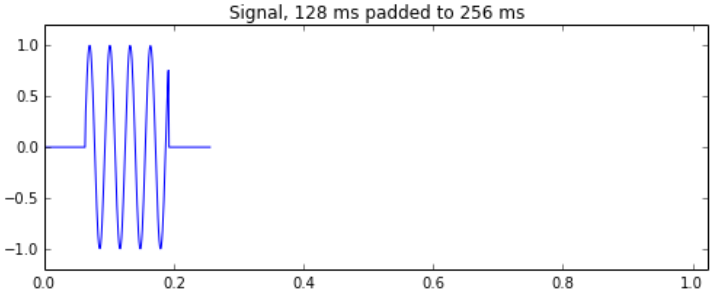
http://www.youtube.com/watch?v=2wbNaEXmyrw
http://en.wikipedia.org/wiki/A_Hard_Day%27s_Night_%28song%29#Opening_chord

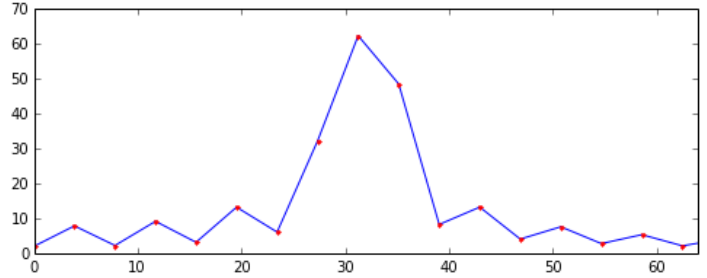
http://scipy-lectures.github.io/advanced/image_processing/
potentially intertesting method... http://arxiv.org/ftp/arxiv/papers/1401/1401.5444.pdf

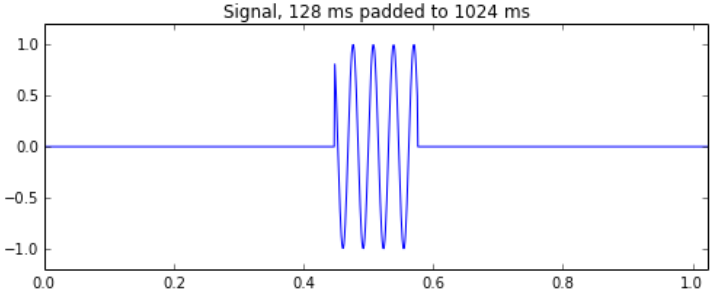
Frequency for WT
http://dsp.stackexchange.com/questions/10127/how-can-i-plot-frequency-vs-magnitude-of-wavelet-transform

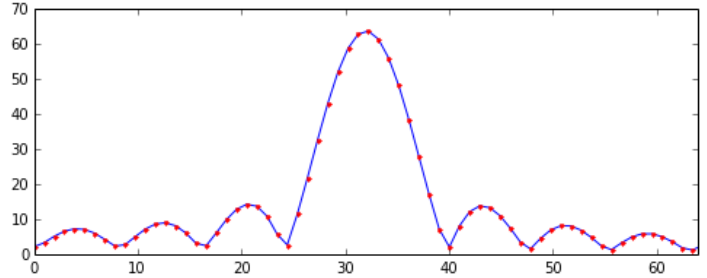
Short **signals**, not windows, have poor frequency localization

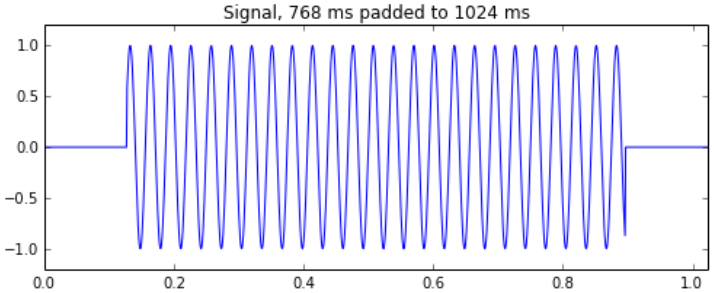
basics.ipynb

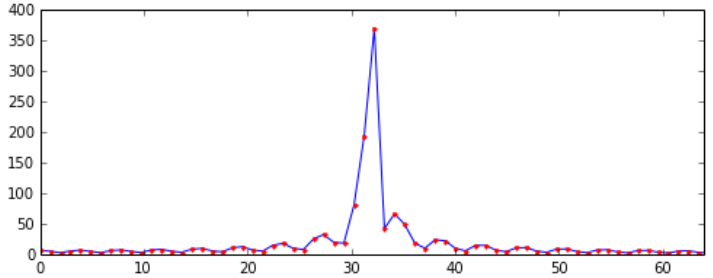












Tapers suppress sidelobes but reduce magnitude & localization

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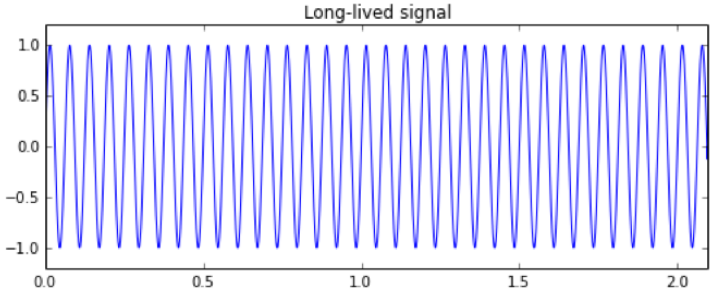

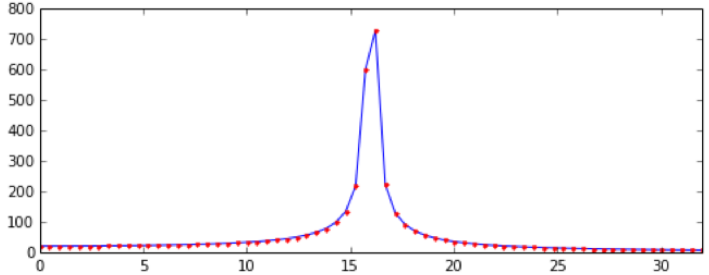

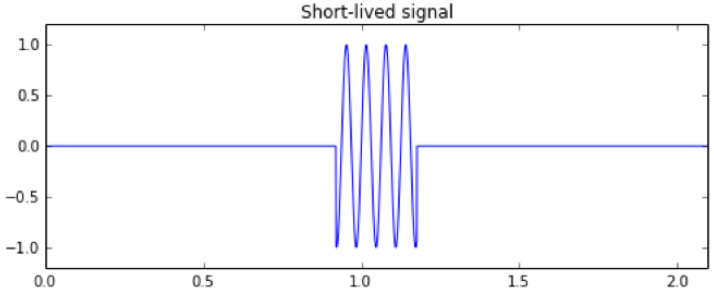

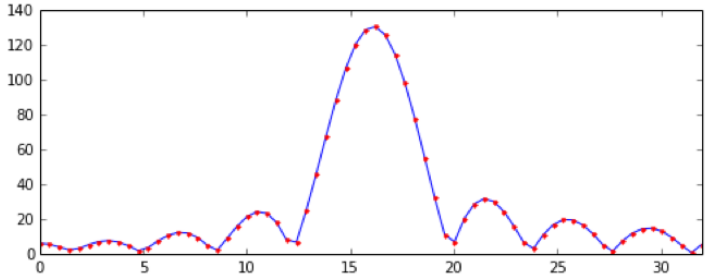

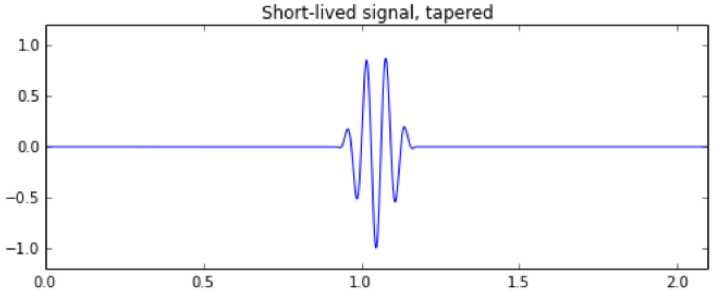

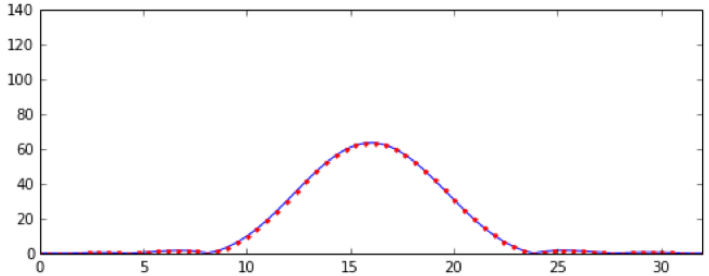

Interactive widgets help tune intuition

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```
In [207]: widgets.interactive(slidy_signal, T=4, support=2.5, noise=0.0)
```

support 2.5

noise 0

T 4