



Agile*

timefreak

the fidelity of time-frequency representations

matt hall

matt@agilegeoscience.com



- Geological and pedagogical motives
- Gabor's uncertainty principle
- What is a good t - f representation?
- Graphical representations
- A synthetic benchmark

Visible



Hubble Space Telescope
2005 NASA, ESA, STSCI/AURA

Near-Infrared



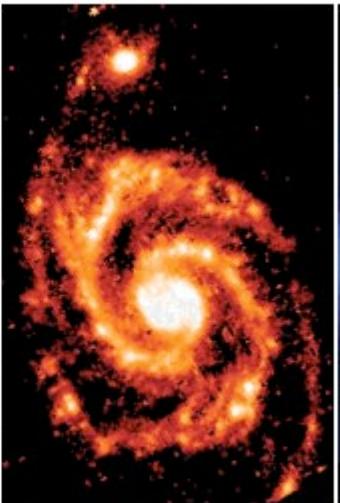
2MASS
UMass/IPAC-Caltech

Mid-Infrared

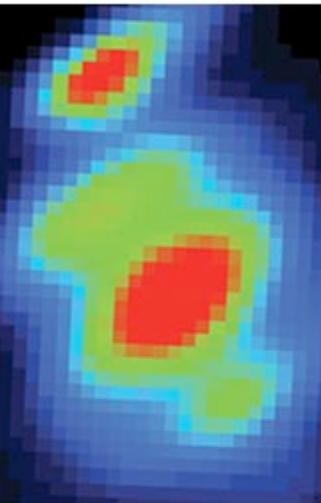


Spitzer Space Telescope
NASA/JPL-Caltech/U of Ariz./DSS

Far Infrared

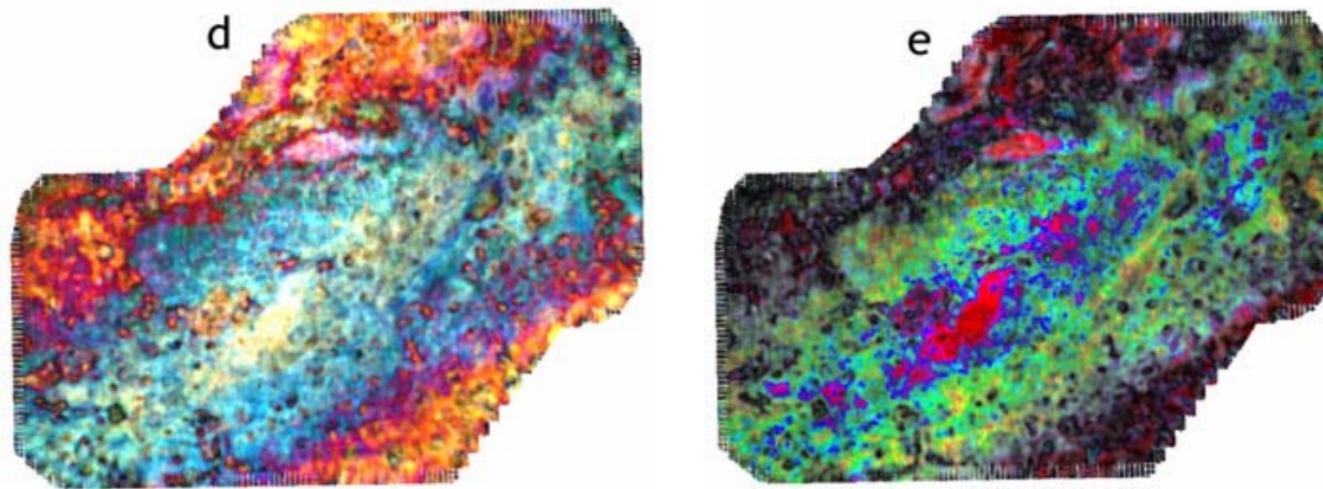
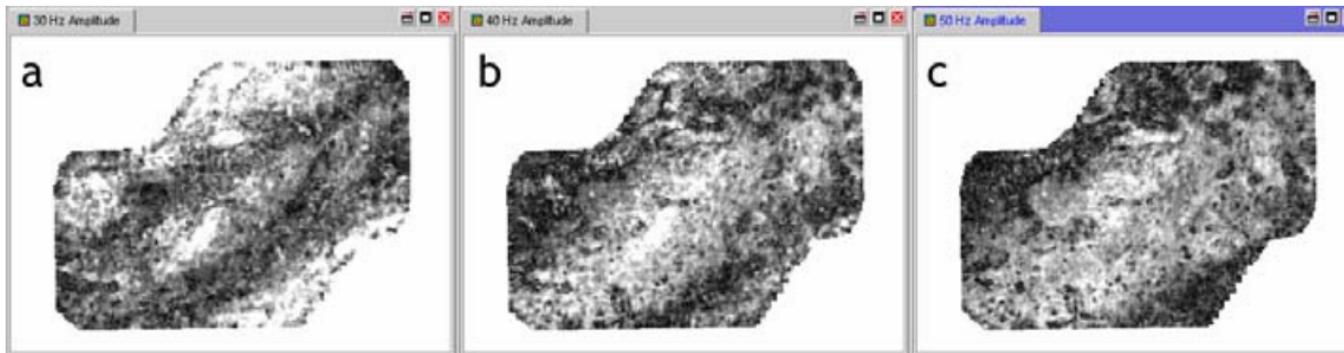


ESA/ISO, CAM,
M. Sauvage et al.



IRAS

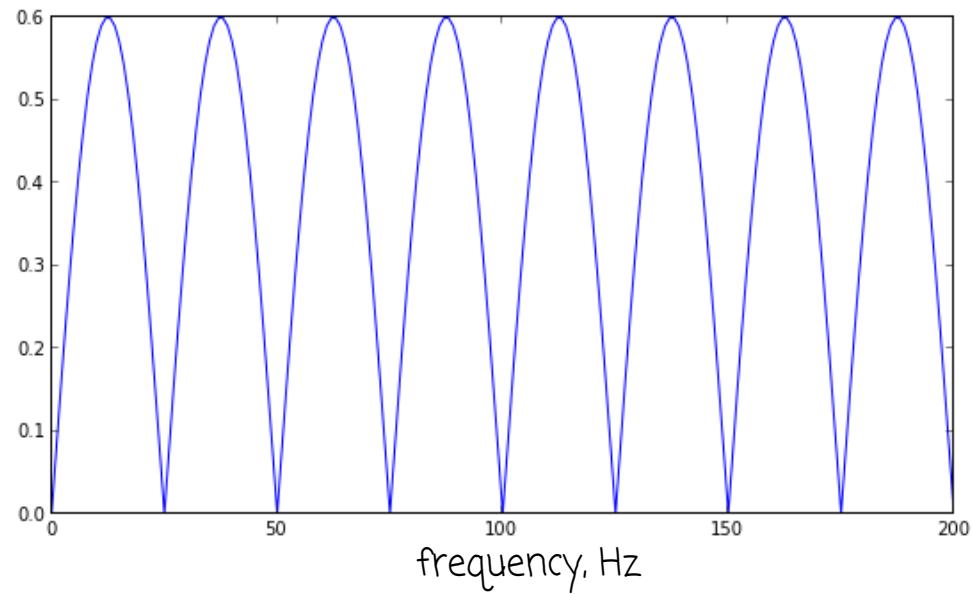
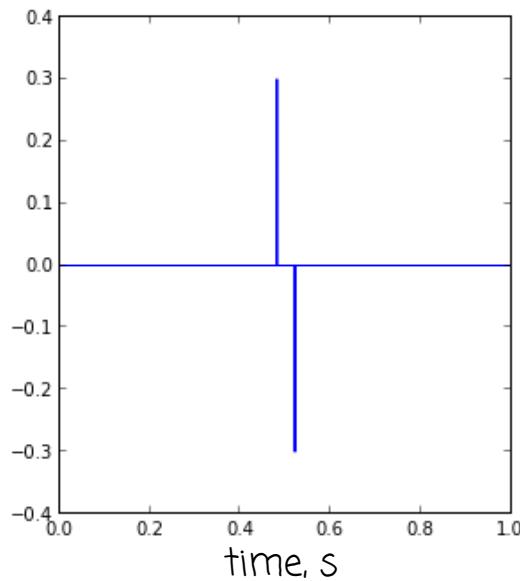
M51A, M51B



Hall & Trouillot, CSEG Convention, 2003

30 Hz, 40 Hz, 50 Hz

Thin bed response



IP[y]: Notebook

stft Last Checkpoint: Oct 28 09:35 (autosaved)

File Edit View Insert Cell Kernel Help



Cell Toolbar: None

rgb_arr = np.array(timesteps)

In [48]: rgb_arr.shape

Out[48]: (102, 1012, 3)

The matplotlib function `imshow` interprets arrays like this as 3-channel colour images, so we can just display this array directly. We'll chop off the negative frequencies again.

In [53]:

```
plt.figure(figsize=(12,4))
plt.imshow(rgb_arr[:rgb_arr.shape[0]/2,...], aspect="auto", origin="lower", interpolation="none")
plt.show()
```





kwinkunks



github.com/kwinkunks/timefreak

A screenshot of Matt Hall's GitHub profile page. The profile picture shows a smiling man with glasses. The bio section includes his name, location (Nova Scotia, Canada), email, website, and the date he joined (April 30, 2012). Key statistics are displayed: 22 Followers, 55 Stars, and 54 Following. The 'Contributions' tab is selected, showing a heatmap of activity over time and a summary of contributions: 520 total, 15 days longest streak (from March 11 to March 25), and 0 days current streak (Rock - Hard Place). The 'Repositories' tab shows popular repos like notebooks, rockhack, 1406_evaluate_and_compare..., sparse_dmd, and agilegeo. The 'Contributed To' section lists repositories such as agile-geoscience/modeler, seg/tutorials, 1406_notebooks, agile-geoscience/agilegeo, and agile-geoscience/Wiki_bots. The 'Blog' tab is also visible at the top.

Gabor uncertainty...

Heisenberg, Donoho & Stark, Robertson....

It's important to note that no signal processing tricks can do away with the **fundamental tradeoff** in time-frequency resolution. – Dan Meliza

...through which high frequencies (which are **inherently low resolution**) are coarsely sampled and low frequencies (which are inherently high resolution) are finely sampled... – Mostafa Naghizadeh & Kris Innanen

...in signal processing, [the uncertainty principle] establishes limits on the extent to which the "instantaneous frequency" of a signal **can be measured**.
– David Donoho & Philip Stark

...what Gabor uncertainty?

All these methods are bound by the Heisenberg/Gabor uncertainty principle with a trade-off between time and frequency resolutions. Also, signal windowing leads to smearing, which correspond to the widening of the main lobe around its central frequency, and side-lobe leakage. Lately, various new transforms have been developed to **circumvent these issues** such as Empirical Mode Decomposition (EMD),... – Tary et al. 2014

The synchrosqueezing is a post-processing method which **circumvents the uncertainty relation** inherent to these linear transforms... – Oberlin et al.,
[10.1109/ICASSP.2014.6853609](https://doi.org/10.1109/ICASSP.2014.6853609)

How shall we write it?

Donoho & Stark 1988,
Gabor 1946

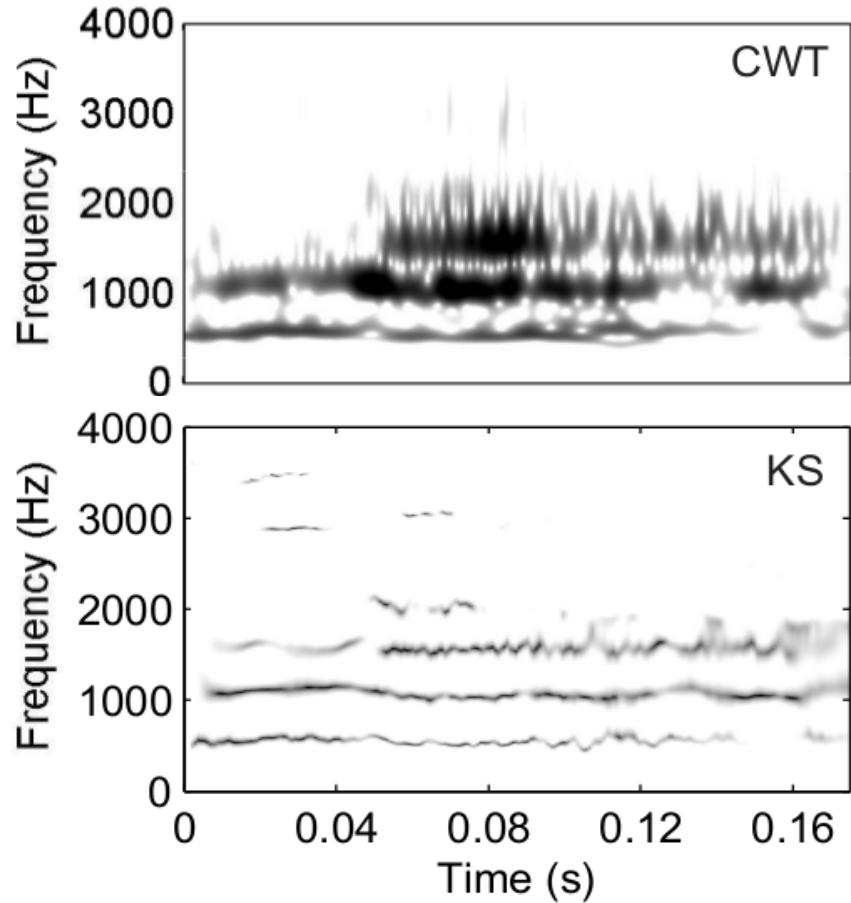
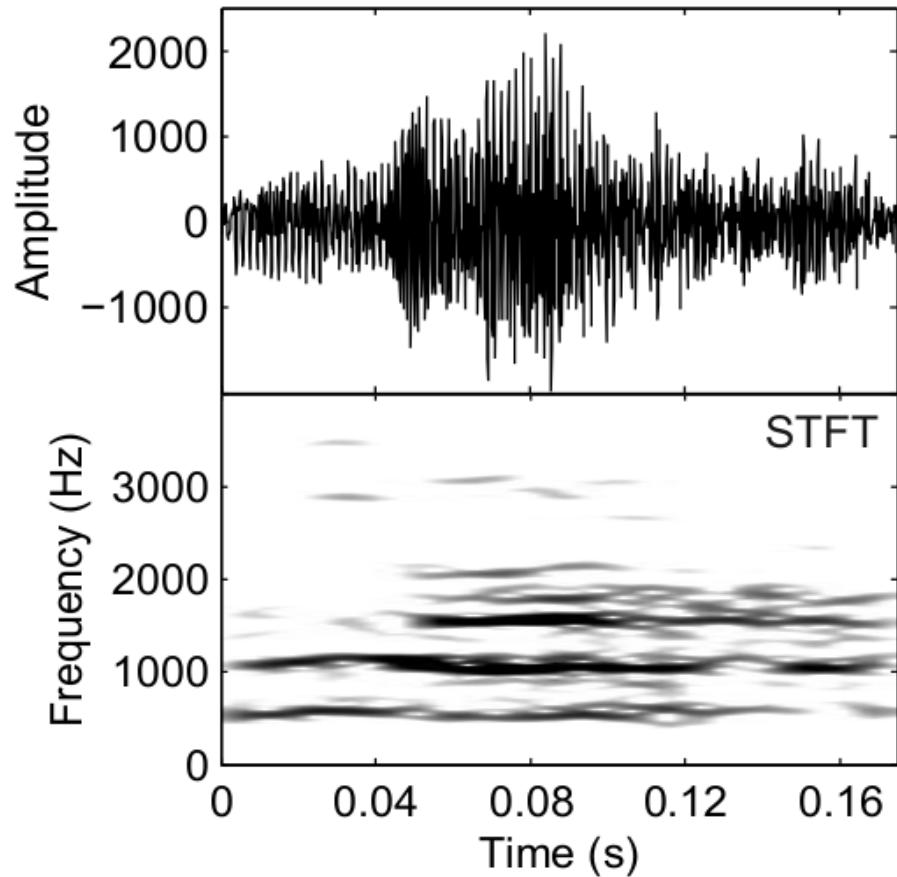
$$\Delta t \cdot \Delta f \geq 1$$

Dirk Lorenz's blog

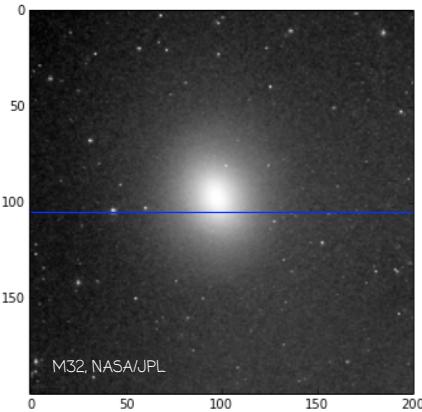
$$\text{Var}_g \cdot \text{Var}_{\hat{g}} \geq \frac{1}{4}$$

Heisenberg

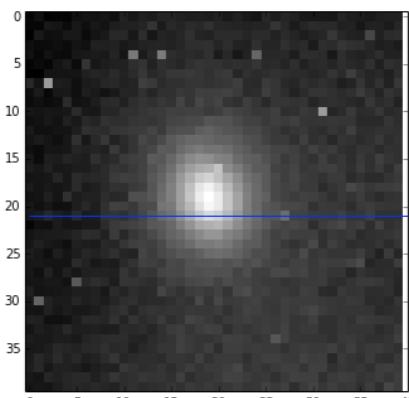
$$\sigma_t \cdot \sigma_f \geq \frac{1}{4\pi}$$



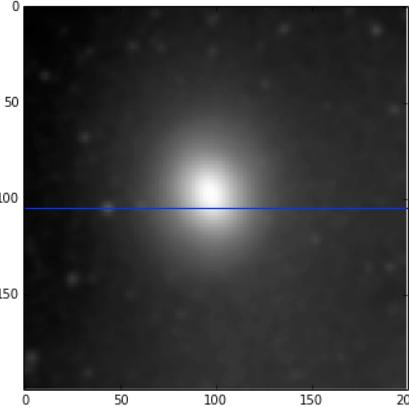
High resolution
High localization
High precision



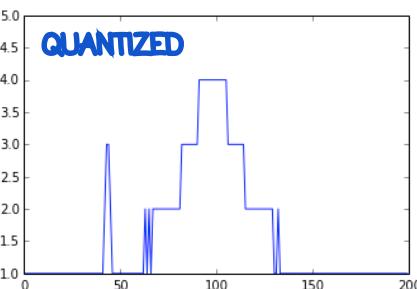
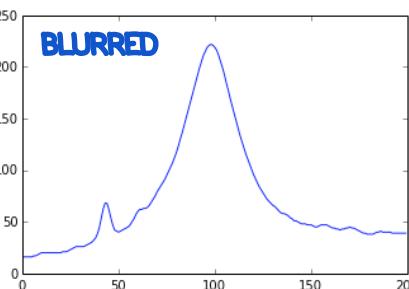
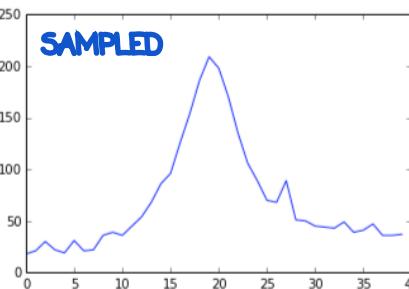
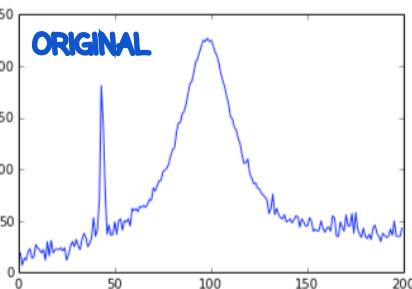
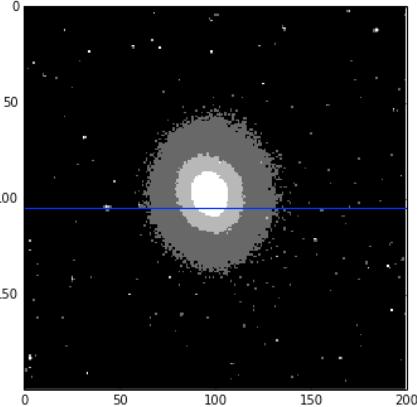
Low resolution
High localization
High precision



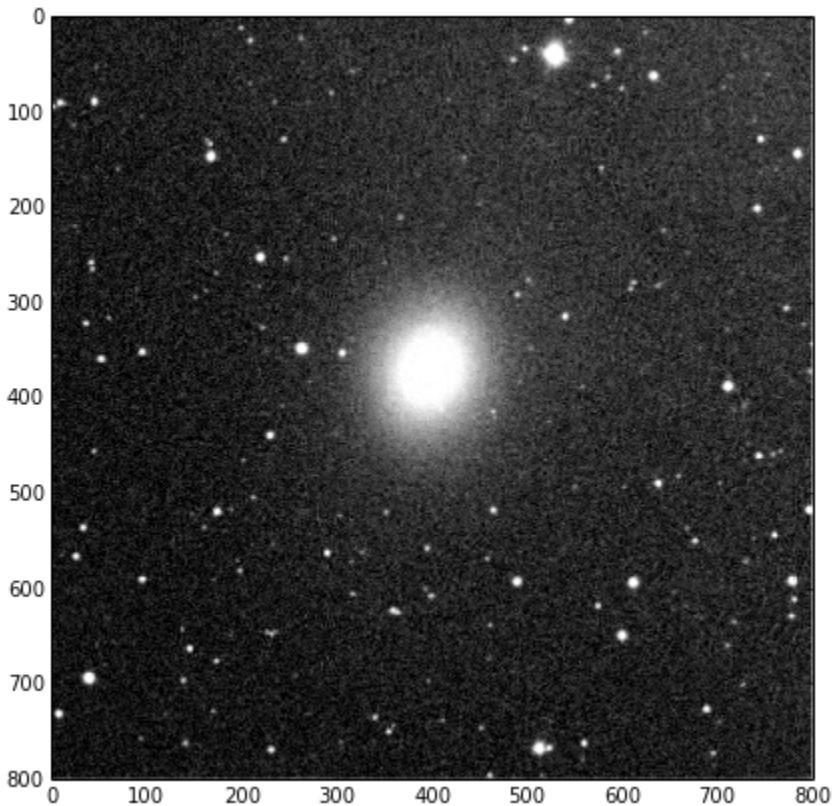
High resolution
Low localization
High precision



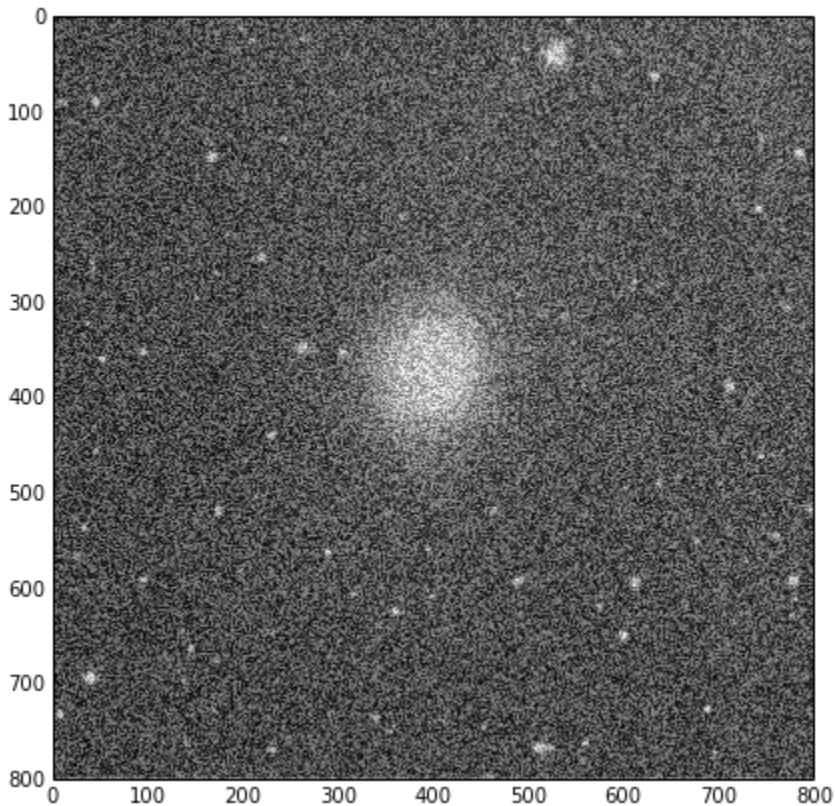
High resolution
High localization
Low precision



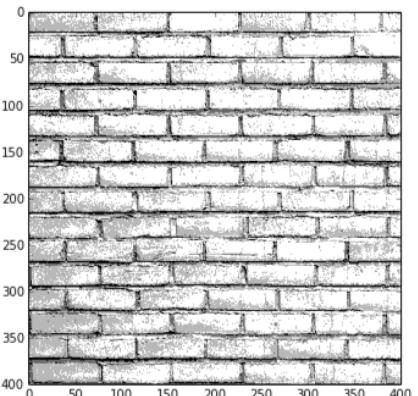
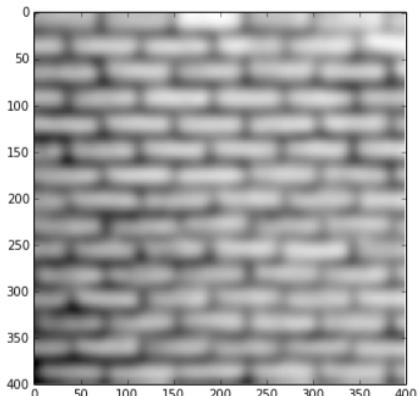
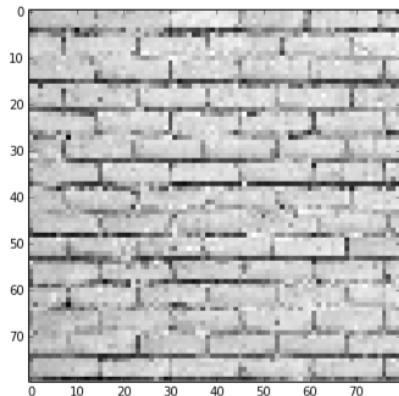
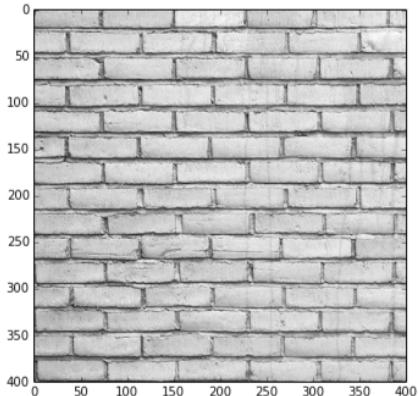
High signal:noise



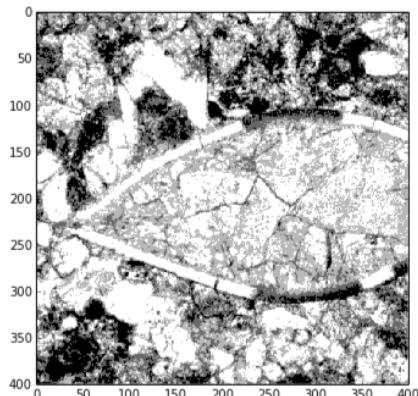
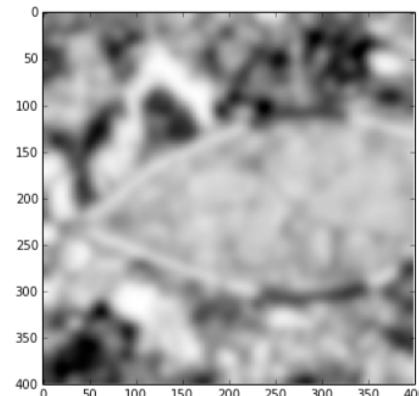
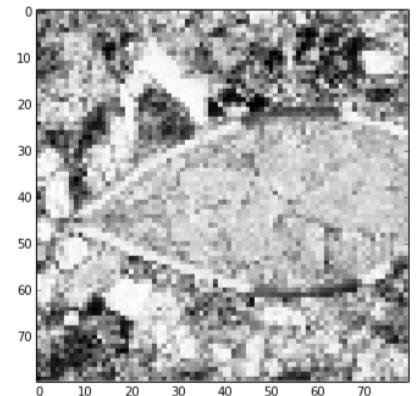
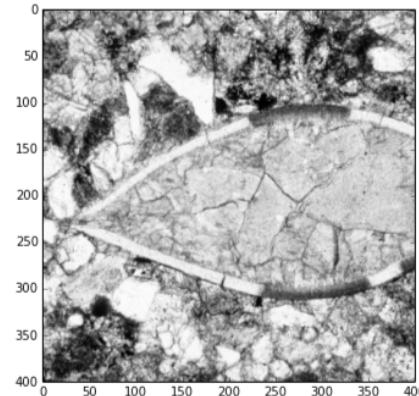
Low signal:noise



Few-value, few-scale



Multi-value, multi-scale



Bricks, flickr/Grzesiek CC-BY

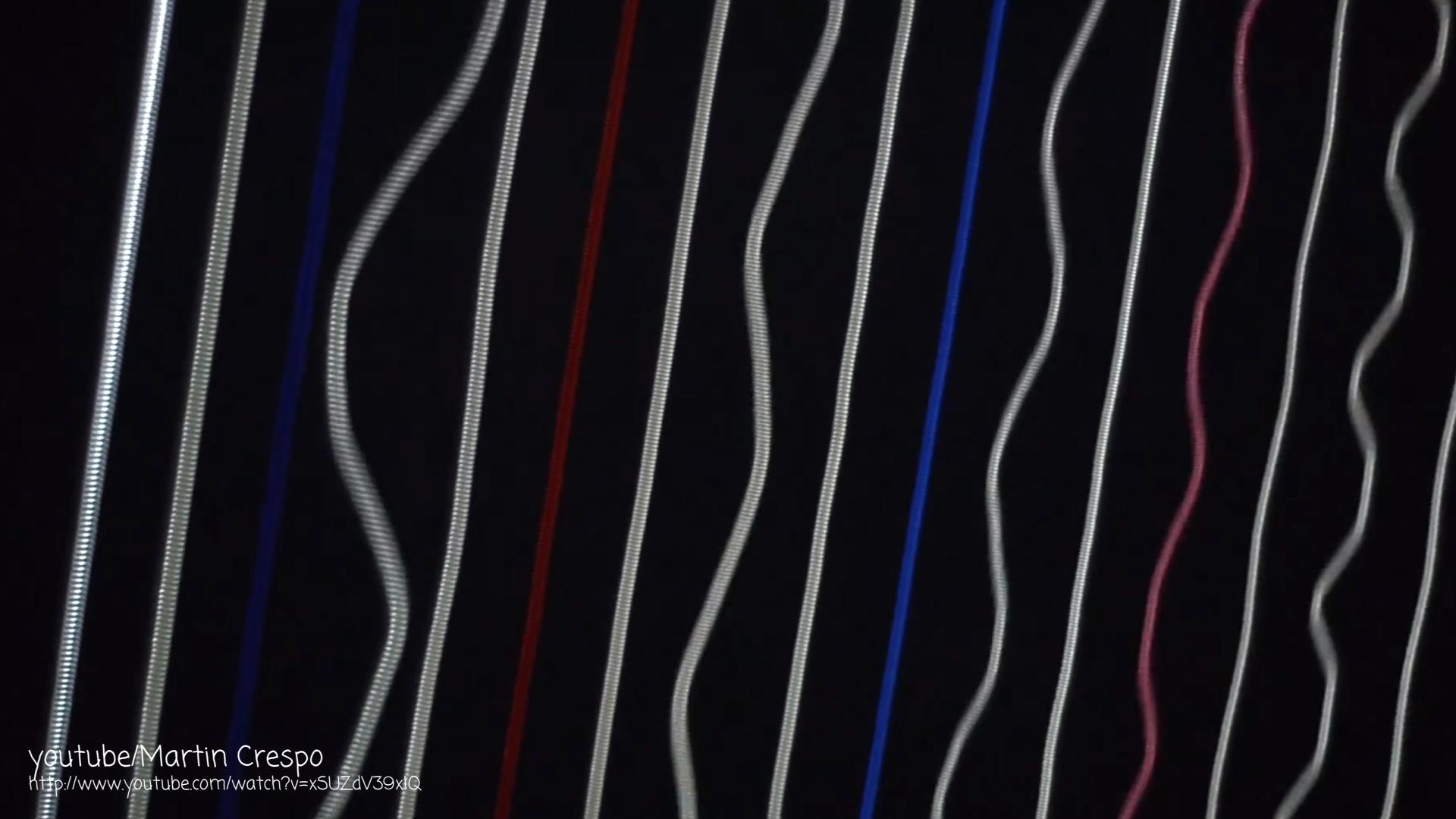
Matt Hall CC-BY



Book Cliffs, Utah

flickr/David Herrera CC-BY

<http://www.flickr.com/photos/dph110/10550053375>

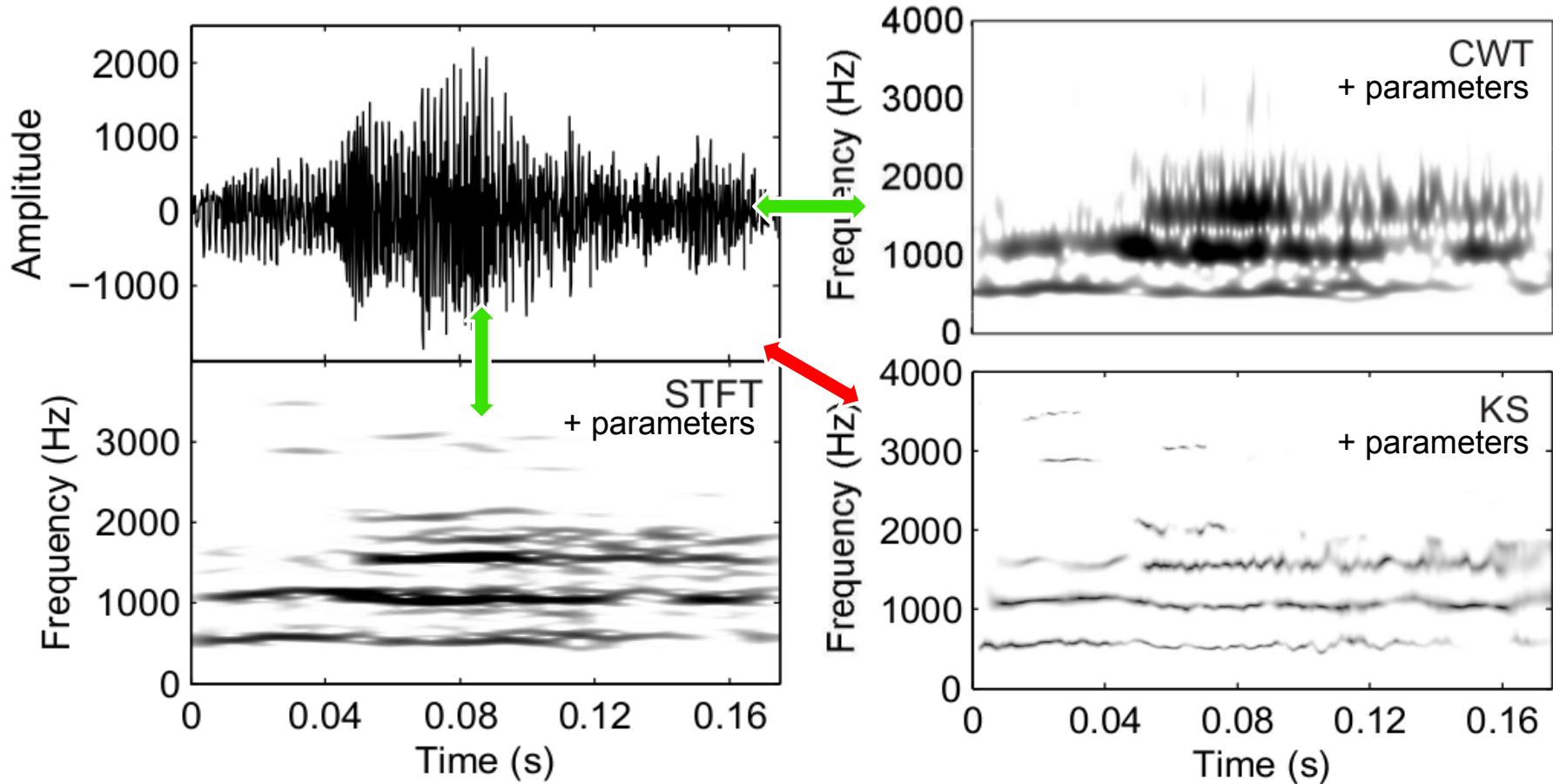
The background of the image consists of a series of vertical, wavy lines. These lines are composed of small, light-colored dots and are set against a dark, solid background. The lines are slightly curved, creating a sense of depth and movement. They are evenly spaced and extend from the top to the bottom of the frame.

youtube/Martin Crespo

<http://www.youtube.com/watch?v=xSUZdV39xIQ>

What is 'good'?

Tary et al. 2014



subsurfwiki.org/wiki/Time-frequency_transforms

| | Name | Note | Python module | Key parameters | Ease of use | Cost | Synthetic | Natural |
|-------|---------------------------------------|--------------------------------------|--|---------------------|-------------|------|-----------|---------|
| FFT | Fourier transform | Fast or discrete Fourier transform | <code>numpy.fft</code> , <code>scipy.fftpack.fft</code> | None | | | | |
| STFT | Short-time Fourier transform | aka Gabor transform, short window FT | <code>pyplot.specgram</code> | Length, step, taper | | | | |
| ST | S-transform ^[1] | | <code>pygft^[2]</code> | None | Easy | Fast | | |
| CWT | Continuous wavelet transform | | <code>scipy.cwt</code> | | | | | |
| DWT | Discrete wavelet transform | | <code>pywt.wavelets.py</code> | | | | | |
| SST | Synchro-squeezing transform | Includes reassignment step | | | | | | |
| MP | Matching pursuit | | <code>py-pursuit^[3]</code> , | | | | | |
| BP | Basis pursuit | | | | | | | |
| DMD | Dynamic mode decomposition | | | | | | | |
| AR | Autoregressive method | | | | | | | |
| ST-AR | Short-time autoregressive method | | | | | | | |
| KS | Kalman smoother | | | | | | | |
| EEMD | Ensemble empirical mode decomposition | | | | | | | |
| CEEMD | Complete EEMD | | | | | | | |

Adagio

Graphical representations

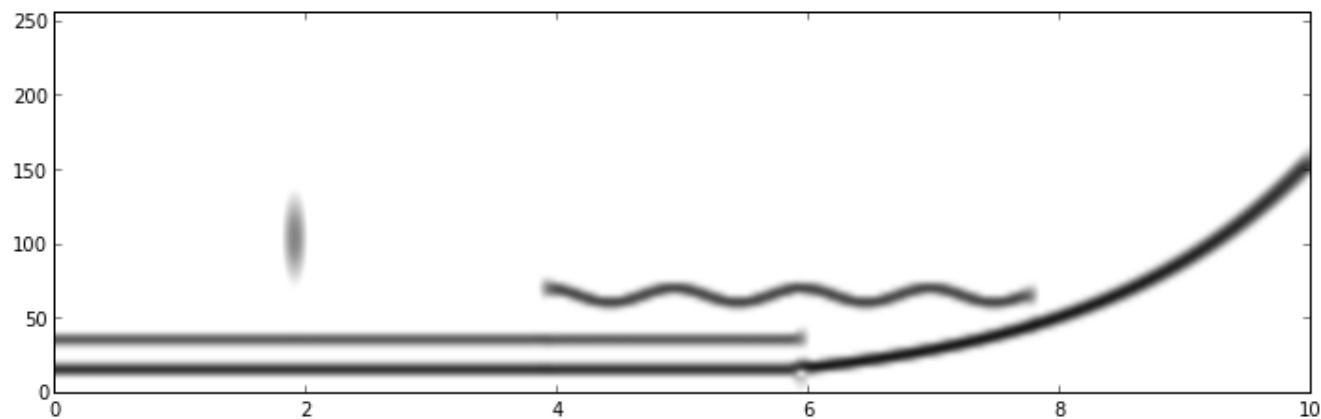
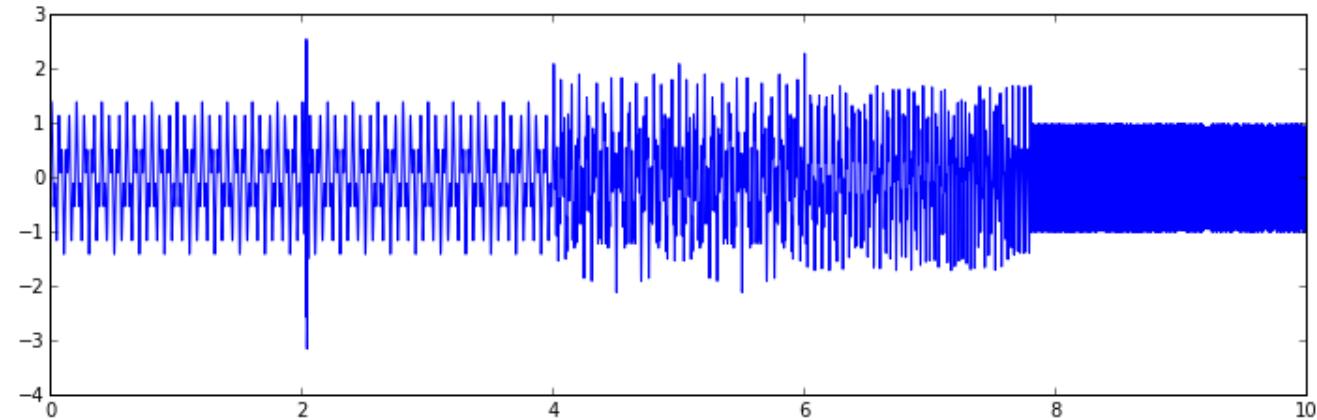
Stringendo poco

29

The image shows a page from Brahms' manuscript of Symphony No. 1, Movement 1. The page is filled with musical notation for multiple instruments, including strings and woodwinds. The notation is highly graphical, using various symbols like vertical dashes for silence, horizontal dashes for sustained notes, and different patterns of dots and dashes for rhythmic values. There are also many dynamic markings such as 'f', 'ff', 'p', 'pp', and 'dim'. The manuscript is dated 'ca. 1855'.

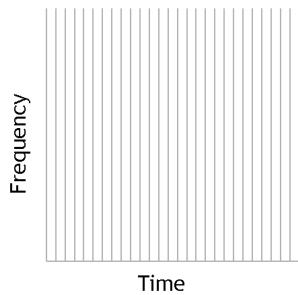
Brahms, ca. 1855
Symphony No. 1

stft.ipynb

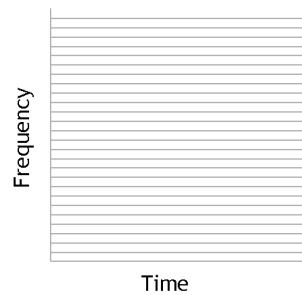


Hall 2005, The resolution of spectral decomposition

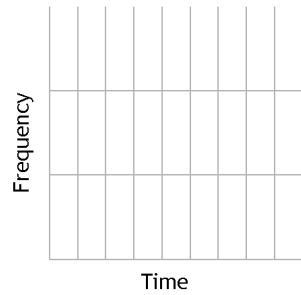
(a) Dirac basis



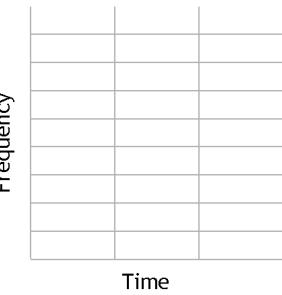
(b) Fourier basis



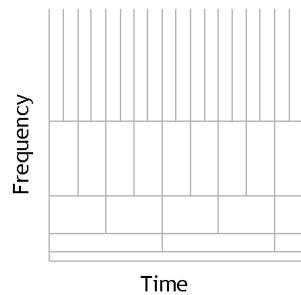
(c) Short window



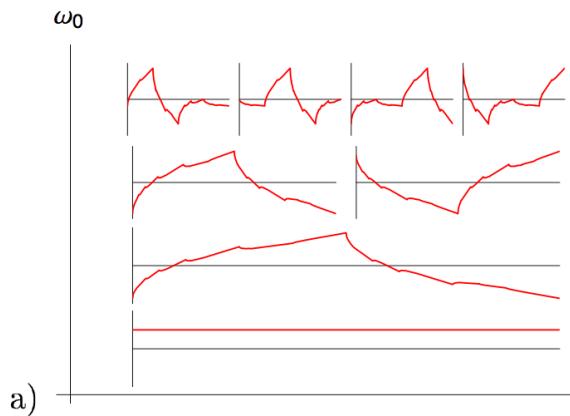
(d) Long window



(e) Adaptive



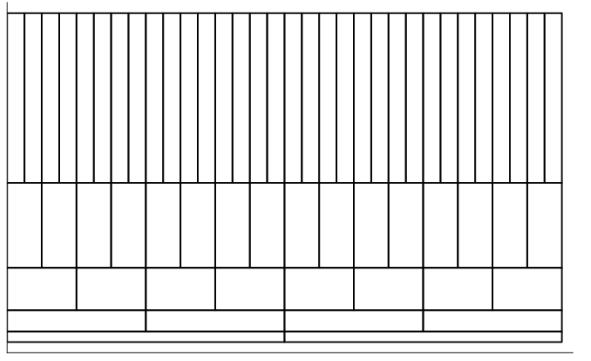
ω_0



a)

t_0 b)

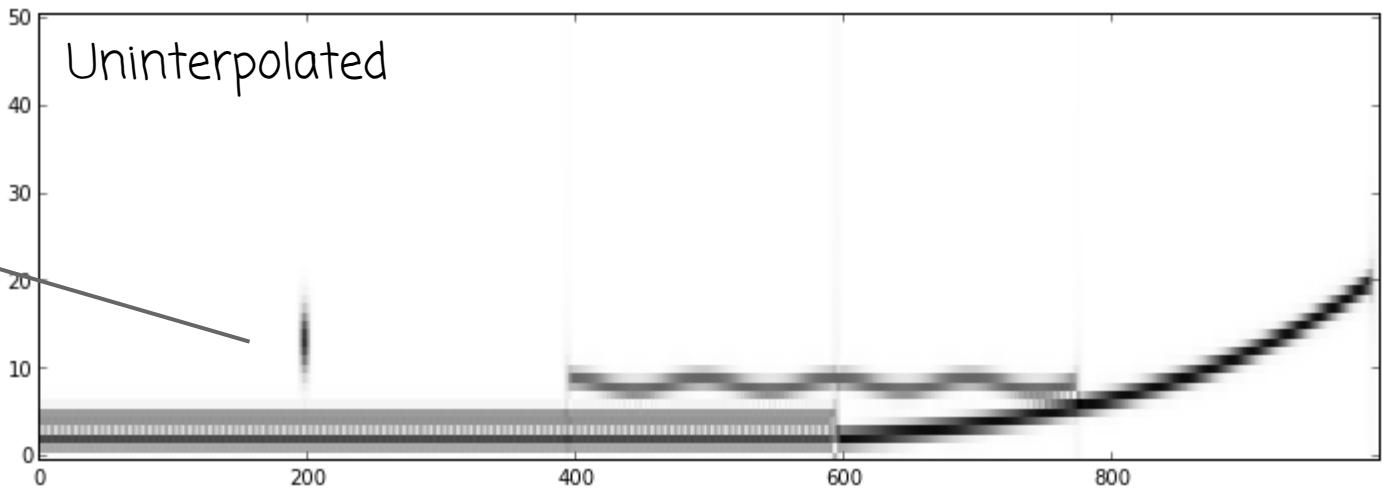
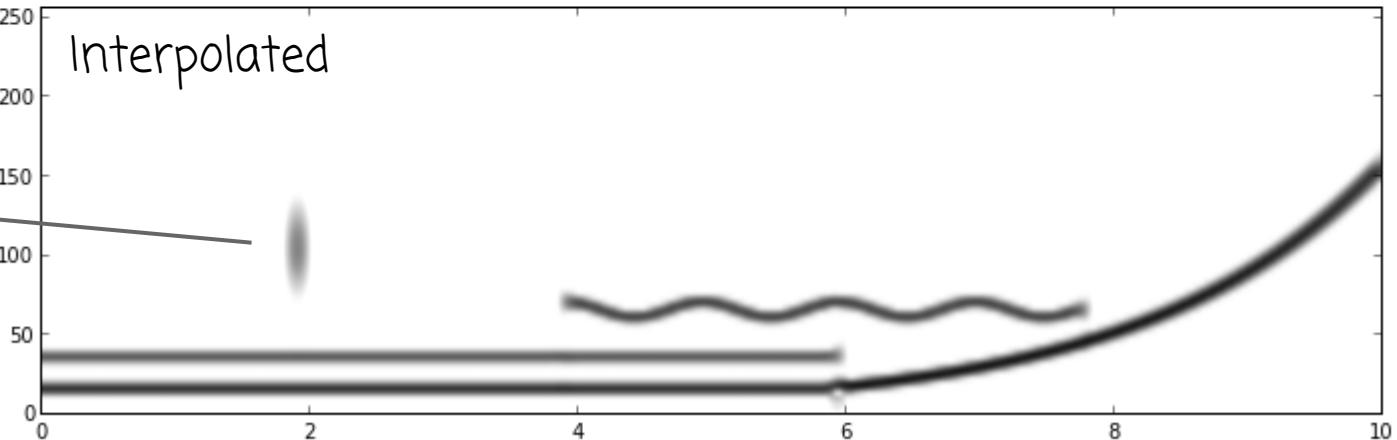
ω

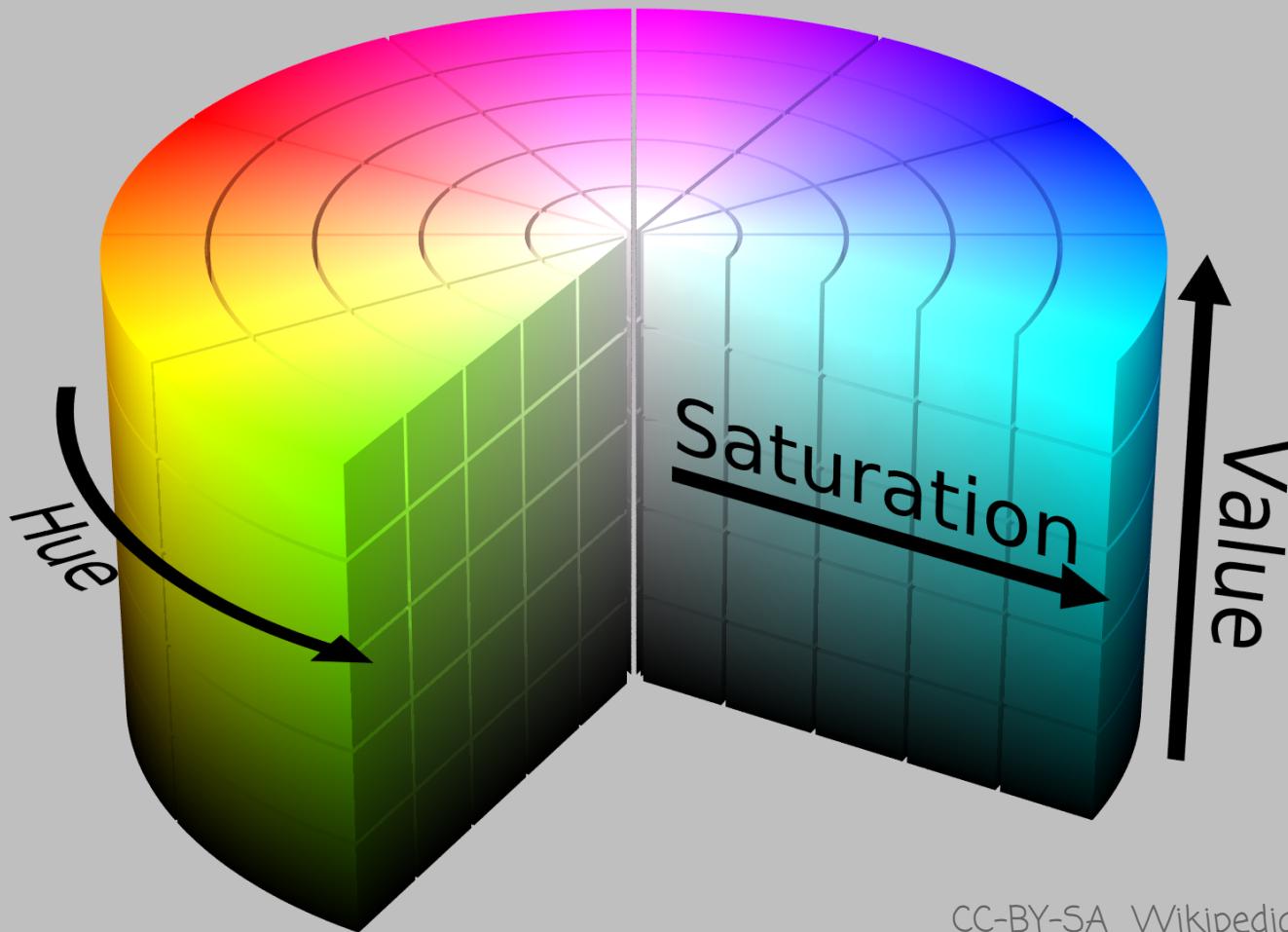


t

Thesis of Jon Harrop <http://www.ffconsultancy.com/free/thesis.pdf>

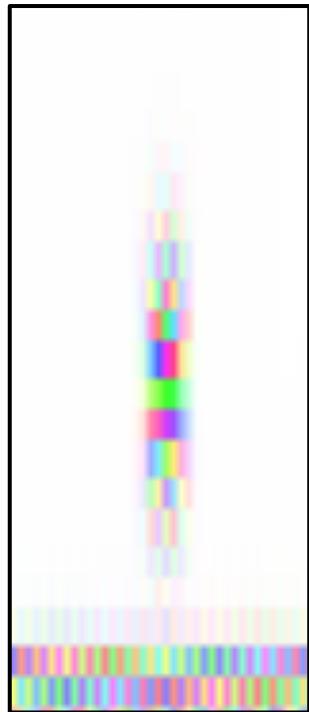
stft.ipynb



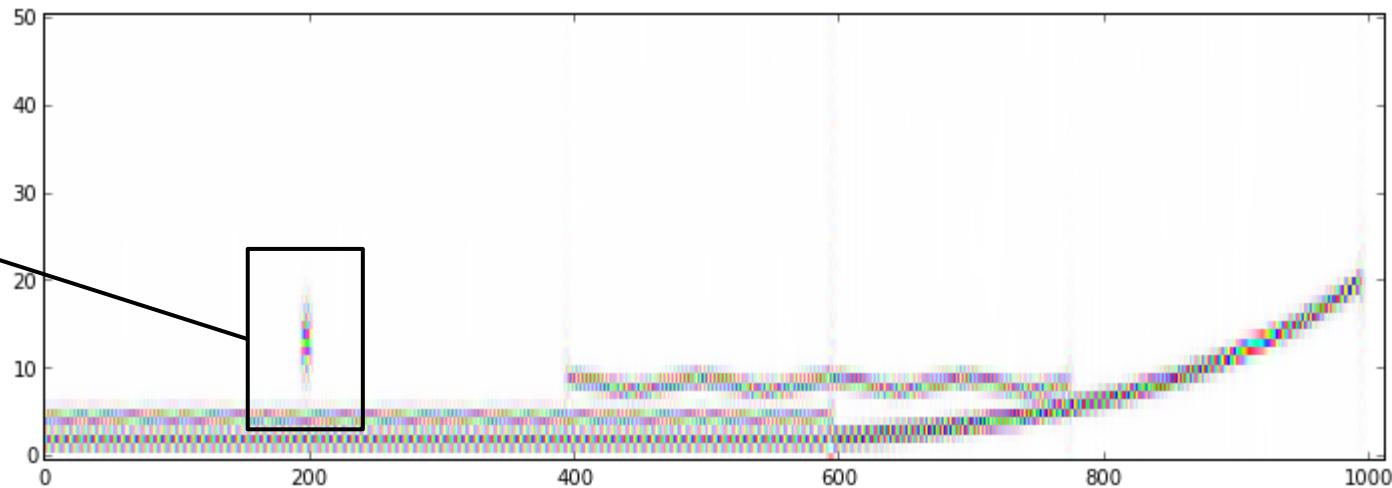


CC-BY-SA Wikipedia user SharkD

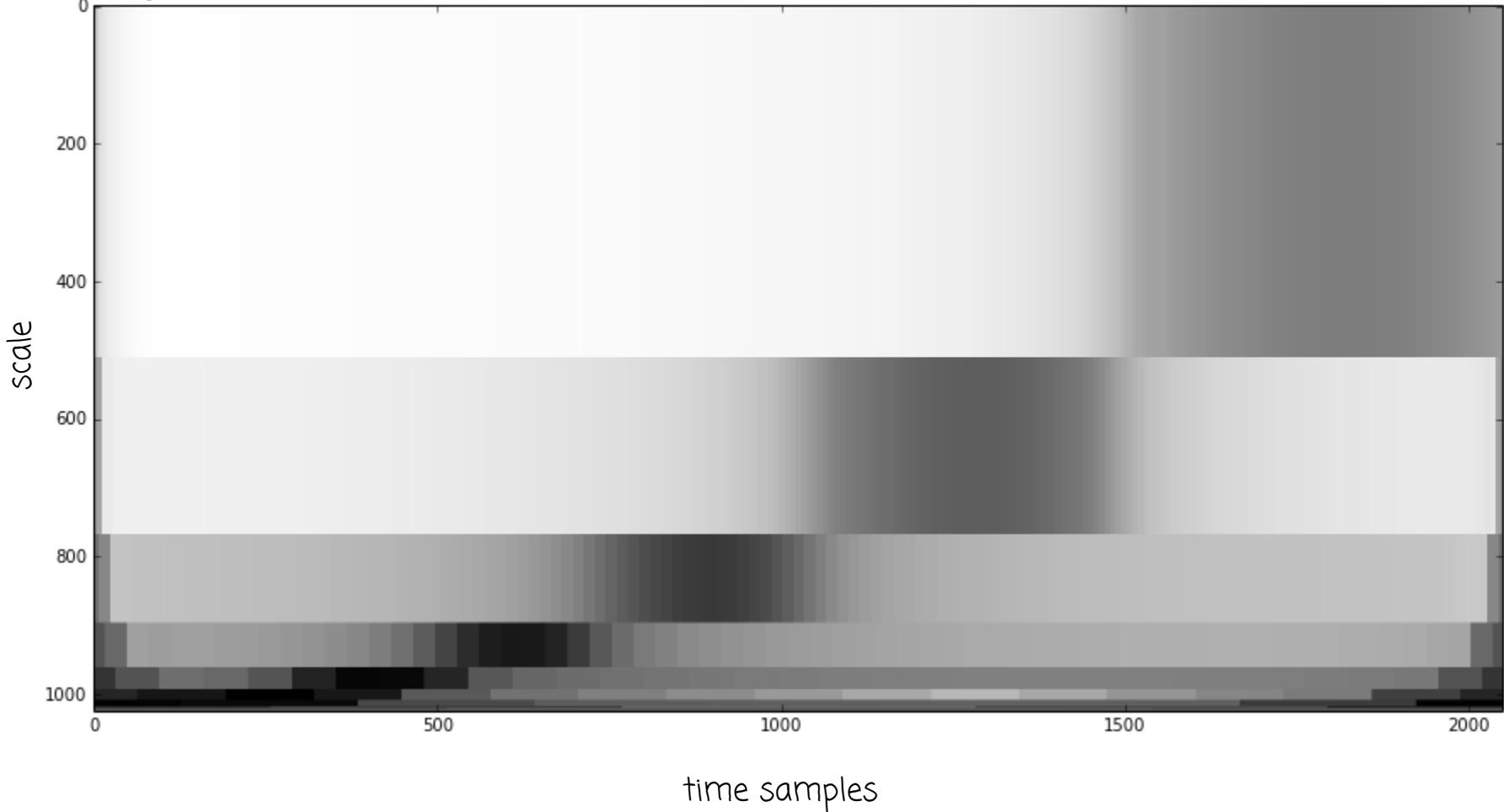
stft.ipynb



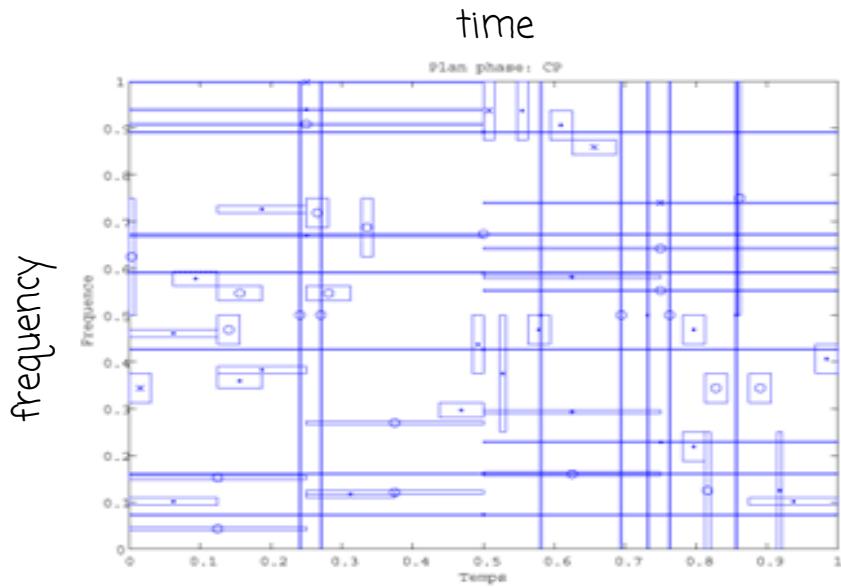
HSV display of complex spectrogram



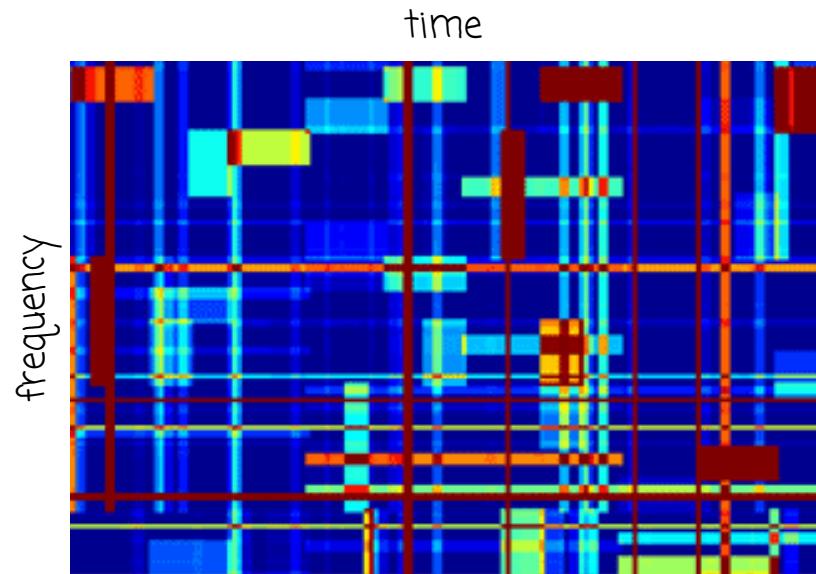
stft.ipynb



Heisenberg boxes or Gabor logons of matching pursuit atoms



<http://www.isnld.com/indexD7.html>

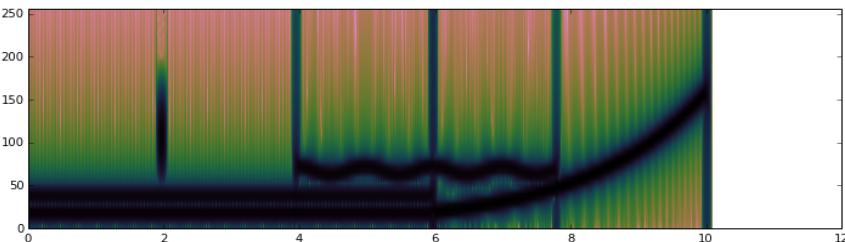
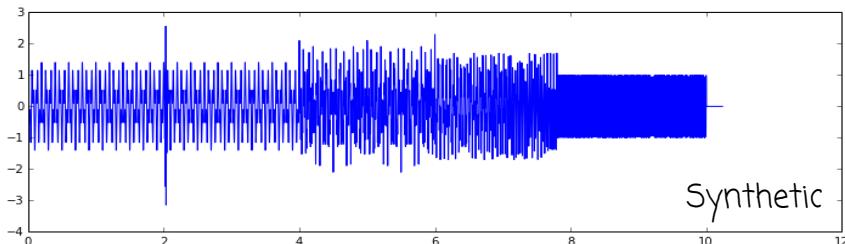


<http://www.isnld.com/indexD7.html>

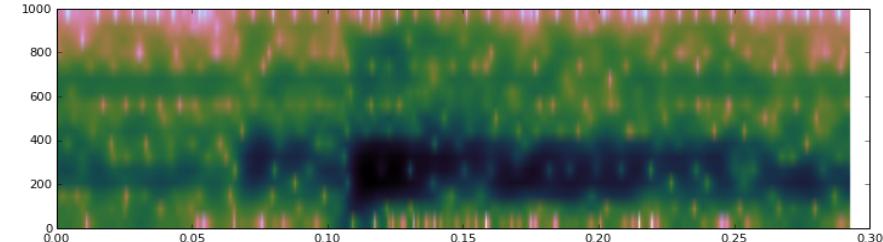
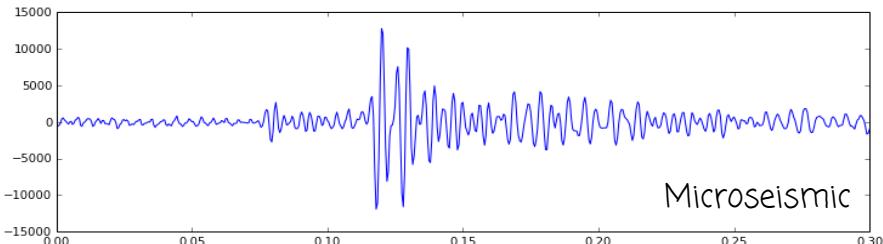
Features of a geologically useful synthetic

spectrograms.ipynb

Not natural



Not known



1. Naturalistic
2. Complex
3. Multi-scale

1. Natural wavelets
2. Natural incoherent noise
3. Natural coherent noise

- Geological and pedagogical motives
- Gabor's uncertainty principle
- What is a good $t-f$ representation?
- Graphical representations: components and pixels
- A synthetic benchmark



Agile*

time-freak

matt hall

matt@agilegeoscience.com



http://nbviewer.jupyter.org/github/unpingco/Python-for-Signal-Processing/blob/master/Windowing_Part2.ipynb

<http://tomwhipple.com/projects/explorations/basic-signal-processing-with-ipython-notebook.html>

<http://inst.eecs.berkeley.edu/~ee123/fa12/Notes/Lecture10.pdf>

<http://www.isnld.com/indexD9.html>

<http://dsp.stackexchange.com/questions/14947/heisenberg-boxes-image-for-matching-pursuit>

More on tilings... <http://research.microsoft.com/en-us/um/people/cormac/papers/tiling.pdf>

<http://perso.ens-lyon.fr/patrick.fladrin/1569582263.pdf>

http://perso.ens-lyon.fr/patrick.fladrin/SPIE01_PF.pdf

<http://statweb.stanford.edu/~donoho/Reports/Oldies/UPSR.pdf>

<http://www.youtube.com/watch?v=2wbNaEXmyrw>

http://en.wikipedia.org/wiki/A_Hard_Day%27s_Night_%28song%29#Opening_chord

http://scipy-lectures.github.io/advanced/image_processing/

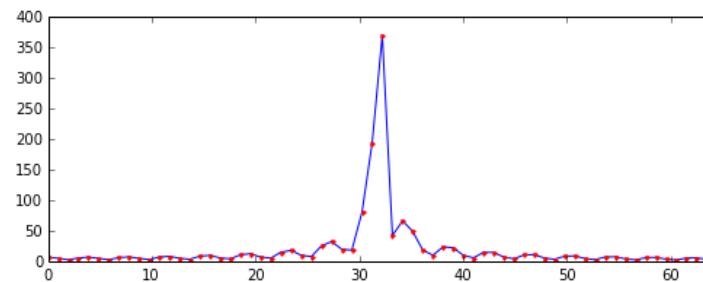
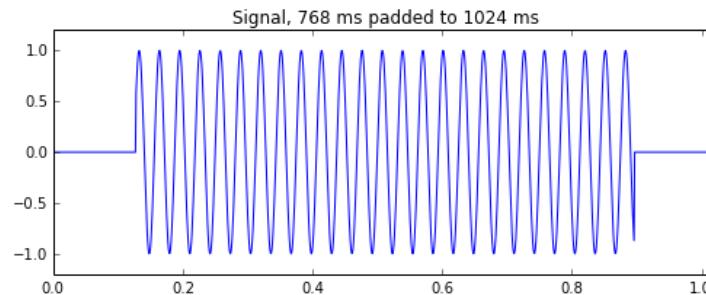
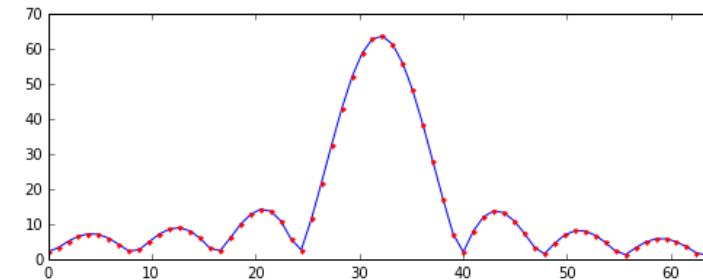
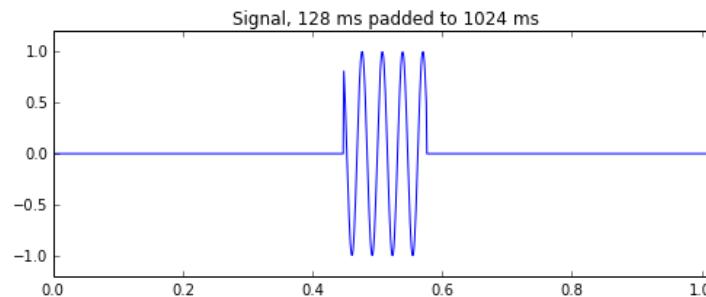
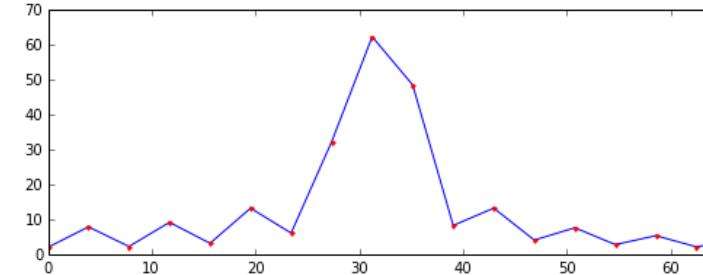
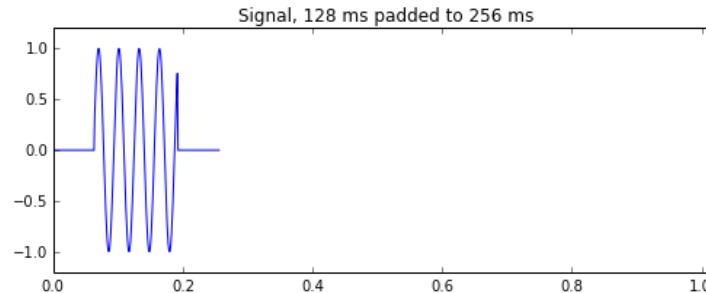
Potentially interesting method... <http://arxiv.org/ftp/arxiv/papers/1401/1401.5444.pdf>

Frequency for WT

<http://dsp.stackexchange.com/questions/10127/how-can-i-plot-frequency-vs-magnitude-of-wavelet-transform>

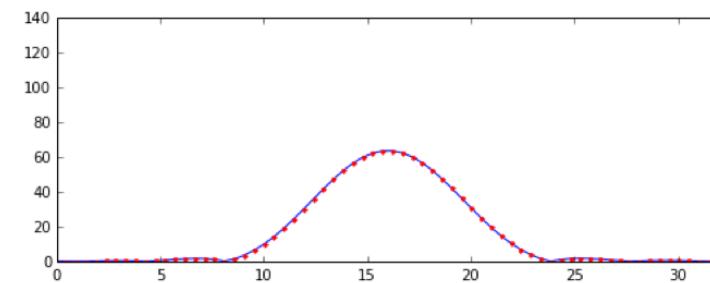
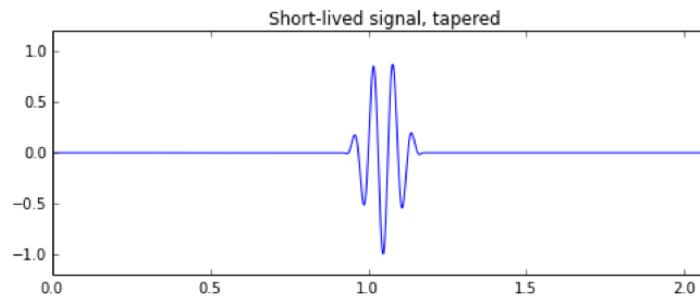
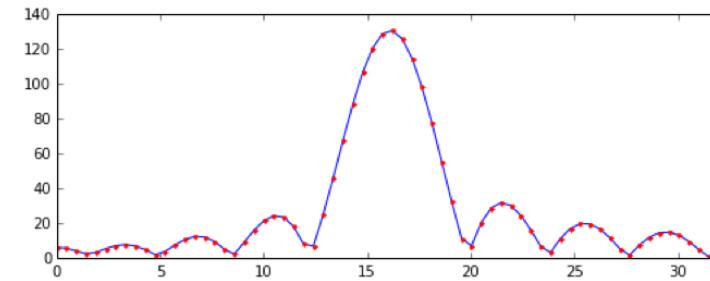
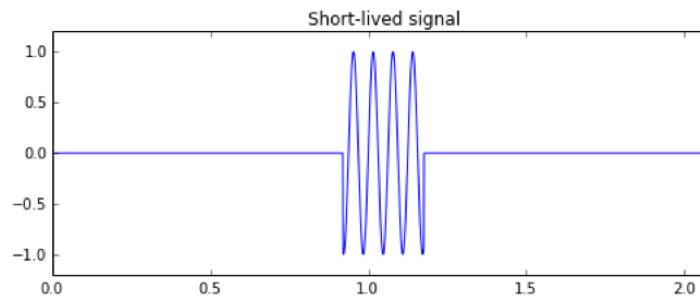
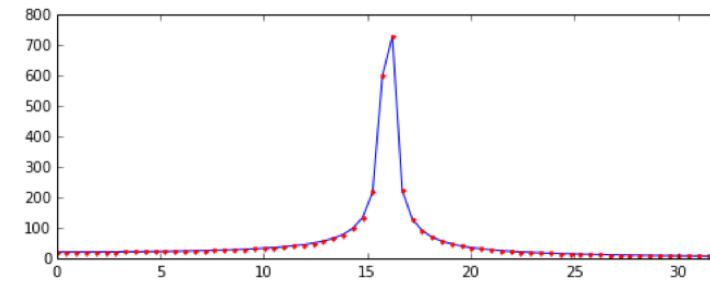
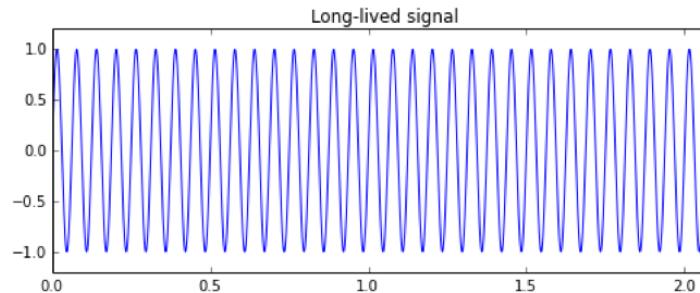
Short **signals**, not windows, have poor frequency localization

[basics.ipynb](#)



Tapers suppress sidelobes but reduce magnitude & localization

basics.ipynb



Interactive widgets help tune intuition

basics.ipynb

```
In [207]: widgets.interactive(slidy_signal, T=4, support=2.5, noise=0.0)
```

X

support

noise

T

The image shows three horizontal sliders. The first slider is labeled 'support' and has a value of 2.5. The second slider is labeled 'noise' and has a value of 0. The third slider is labeled 'T' and has a value of 4.

