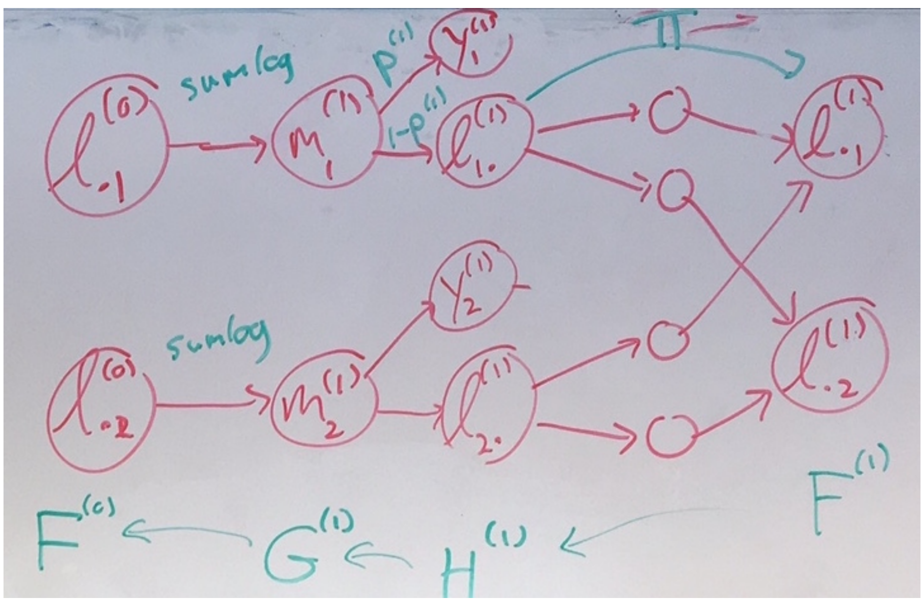
Poisson-gamma notes

January 3, 2017

1 Poisson-gamma dynamical systems



Let $F^{(t)}, G^{(t)}, H_y^{(t)}, H_l^{(t)}, g$ be the PGFs of $\mathbf{l}_{\cdot}^{(t)}, \mathbf{m}^{(t)}, \mathbf{y}^{(t)}, (l_{k\cdot}^{(t)})_{k=1}^{K}, x_{ki}$, respectively. Let $\mathbf{l}_{\cdot k}^{(0)} \sim Poisson(\lambda_k)$ and $m_k = \sum_i x_{ki} \sim SumLog$.

$$
\begin{aligned}
F^{(0)}(\mathbf{s}) &= \sum_{\mathbf{l}_\cdot} \prod_k s_k^{l_{\cdot k}} p(\mathbf{s}) \\
&= \prod_k e^{\lambda_k (s_k - 1)}, \text{ since independent} \\
&= \prod_k s_k^{l_{\cdot k}^{(0)}}, \text{ if observed}
\end{aligned}
$$

$$G^{(1)}(\mathbf{t}) = F^{(0)}(g(t_1), \ldots, g(t_K))$$

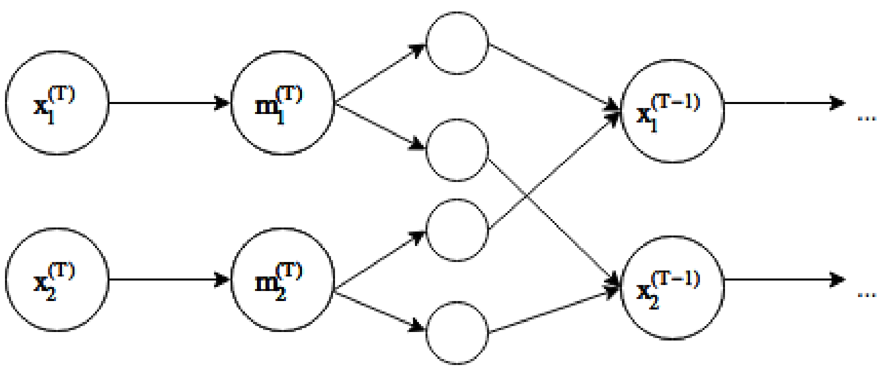
$$H^{(1)}(u_1, v_1, u_2, v_2) = G^{(1)}(u_1 p + v_1(1-p), u_2 p + v_2(1-p))$$

$$\begin{aligned} H_y^{(1)}(u_1, u_2) &= H^{(1)}(u_1, 1, u_2, 1) \\ &= G^{(1)}(u_1 p + (1-p), u_2 p + (1-p)) \end{aligned}$$

$$\begin{aligned} H_l^{(1)}(v_1, v_2) &= H^{(1)}(1, v_1, 1, v_2) \\ &= G^{(1)}(p + v_1(1-p), p + v_2(1-p)) \end{aligned}$$

$$\begin{aligned} F^{(1)}(\mathbf{s}^{(1)}) &= H_l^{(1)}(\mathbf{s}) \\ &= G^{(1)}((1-p)\Pi\mathbf{s}^{(1)} + p\mathbf{1}) \end{aligned}$$

2 Poisson-gamma belief network



Let K_t be the number of hidden units at layer t. Define random variables:

- $\mathbf{x}^{(T)} \sim Pois(-\mathbf{\Phi}^{(T)}\boldsymbol{\theta}^{(T)} \ln(1 - p^{(T)}))$
- $\mathbf{x}^{(t)} \in \mathbb{Z}^{K_{t-1}}$, where $x_v^{(t)} = \sum_{k=1}^{K_{t-2}} z_{vk}$, for $t = 1, \ldots, T-1$
- $\mathbf{m}^{(t)}|\mathbf{x}^{(t)} \sim SumLog(\mathbf{x}^{(t)}, p^{(t+1)})$, where $m_k = \sum_{l=1}^{x_k} n_{kl}$
- $\mathbf{Z}^{(t)}|\mathbf{m}^{(t)} \sim Mult(\mathbf{m}^{(t)}, {\Phi^{(t)}}^T)$

and let $F^{(t)}$, $G^{(t)}$, and g be the PGFs of $\mathbf{x}^{(t)}$, $\mathbf{m}^{(t)}$, and n_{kl}, respectively.

$$\begin{aligned}
F^{(T)}(\mathbf{s}^{(T)}) &= \prod_k e^{\lambda_k(s_k-1)}, \text{ since independent} \\
G^{(T)}(\mathbf{t}^{(T)}) &= F^{(T)}(g(t_1^{(T)}), \ldots, g(t_{K_{T-1}}^{(T)})) \\
F^{(T-1)}(\mathbf{s}^{(T-1)}) &= G^{(T)}({\Phi^{(T)}}^T \mathbf{s}^{(T-1)})
\end{aligned}$$