HW4

지능정보융합학과

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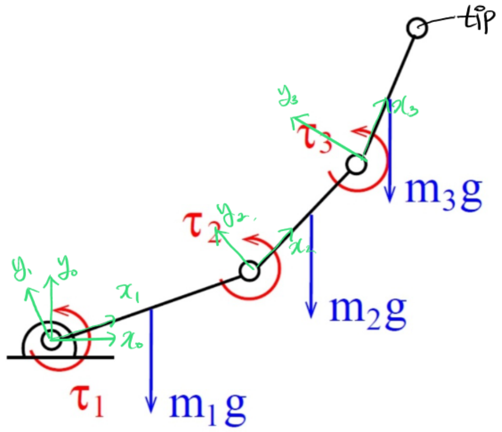
Singluarity

1. The following illustration shows a **planar** RRR manipulator.



The lengths of each link are l_1, l_2, and l_3, respectively. The center of mass of each link is at the middle point of the link.

(a) Compute the basic Jacobian, J_0, which is 6×3 matrix, for the end-effector. $\begin{pmatrix} v \\ \omega \end{pmatrix} = J_0 \dot{q}$

(b) When is this Jacobian singular?

(c) Explain the physical meaning of this singularity.

ⓐ $\boxed{\text{DH parameters}}$

	a_{i-1}	α_{i-1}	θ_i	d_i
1	0	0	θ_1	0
2	L_1	0	θ_2	0
3	L_2	0	θ_3	0

$\boxed{\text{Forward Kinematics}}$

$$ {}^{0}_{tip}T = {}^{0}_{1}T \, {}^{1}_{2}T \, {}^{2}_{3}T \, {}^{3}_{tip}T $$

$$ = \left[\begin{array}{ccc|c} C_{123} & -S_{123} & 0 & L_1C_1 + L_2C_{12} + L_3C_{123} \\ S_{123} & C_{123} & 0 & L_1S_1 + L_2S_{12} + L_3S_{123} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] $$

$$ {}^{0}J_0 = \begin{bmatrix} {}^{0}J_v \\ {}^{0}J_\omega \end{bmatrix} = \begin{bmatrix} \frac{\partial X_p}{\partial \theta_1} & \frac{\partial X_p}{\partial \theta_2} & \frac{\partial X_p}{\partial \theta_3} \\ {}^{0}Z_1 & {}^{0}Z_2 & {}^{0}Z_3 \end{bmatrix} $$

$$ = \left[\begin{array}{c|c|c} -L_1S_1 - L_2S_{12} - L_3S_{123} & -L_2S_{12} - L_3S_{123} & -L_3S_{123} \\ L_1C_1 + L_2C_{12} + L_3C_{123} & L_2C_{12} + L_3C_{123} & L_3C_{123} \\ 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{array}\right] $$

ans ... ⓐ

ⓑ End-effector의 ω_{xy} (angular velocity in the xy direction)와 V_z (linear velocity in the z direction)는 Configuration에 의해 구속되어 있으므로 ⓐ에서 구한 Jacobian은 이미 singular ans1 ... ⓑ

Consider Reduced Jacobian (IntroRobotics - Jacobian Characteristics 9p)

$$ \begin{pmatrix} V_x \\ V_y \\ \omega_z \end{pmatrix} = \left[\begin{array}{c|c|c} -L_1S_1 - L_2S_{12} - L_3S_{123} & -L_2S_{12} - L_3S_{123} & -L_3S_{123} \\ L_1C_1 + L_2C_{12} + L_3C_{123} & L_2C_{12} + L_3C_{123} & L_3C_{123} \\ 1 & 1 & 1 \end{array}\right] \begin{pmatrix} \dot\theta_1 \\ \dot\theta_2 \\ \dot\theta_3 \end{pmatrix} $$

$$ \begin{pmatrix} V_x \\ V_y \\ \omega_z \end{pmatrix} = {}^{0}J_{0,\text{Reduced}} \begin{pmatrix} \dot\theta_1 \\ \dot\theta_2 \\ \dot\theta_3 \end{pmatrix} $$

$$ \det\left({}^{0}J_{0,\text{Reduced}}\right) = L_1L_2\sin\theta_2 $$

If we consider V_x, V_y, ω_z of the end-effector (planar RRR manipulator),

Singularity at $\theta_2 = k\pi$ ans2 ... ⓑ

(where, $k = 0, 1, 2, \cdots$)

ⓒ

$$ {}^{2}J_{0,Reduced} = {}^{2}_{0}R \, {}^{0}J_{0,Reduced} $$

$$ = {}^{0}_{2}R^{T} \, {}^{0}J_{0,Reduced} $$

$$ = \begin{bmatrix} L_1S_2 - L_3S_3 & -L_3S_3 & -L_3S_3 \\ L_1C_2 + L_2 + L_3C_3 & L_2 + L_3C_3 & L_3C_3 \\ 1 & 1 & 1 \end{bmatrix} $$

$$ {}^{2}J_{0,Reduced}(\theta_2 = k\pi) = \begin{bmatrix} -L_3S_3 & -L_3S_3 & -L_3S_3 \\ L_1+L_2+L_3C_3 & L_2+L_3C_3 & L_3C_3 \\ 1 & 1 & 1 \end{bmatrix} $$

$$ \begin{bmatrix} \delta x \\ \delta y \\ \delta\theta \end{bmatrix} = \begin{bmatrix} -L_3S_3 & -L_3S_3 & -L_3S_3 \\ L_1+L_2+L_3C_3 & L_2+L_3C_3 & L_3C_3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \delta\theta_1 \\ \delta\theta_2 \\ \delta\theta_3 \end{bmatrix} $$

Row 1 and Row 3 : linear dependent ※ $Rank({}^{2}J_{0,Reduced}) = 2$

$$ \delta x = (-L_3S_3)(\delta\theta_1 + \delta\theta_2 + \delta\theta_3) $$
$$ \delta\theta = (\delta\theta_1 + \delta\theta_2 + \delta\theta_3) $$

$\therefore$ $\delta x = -L_3S_3\,\delta\theta$ 라는 Constraint이 발생
즉, 주어진 joint configuration으로 end-effector operational space의 δx와 $\delta\theta$를 independent 하게 제어할 수 없음.

ans… ⓒ