

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ -1 & 0 & 2 \end{bmatrix} \text{ or } LU, LDU \text{ method}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ -1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -2 \\ 0 & 2 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2/3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -2 \\ 0 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -2 \\ 0 & 0 & \frac{4}{3} + 5 = \frac{11}{3} \end{bmatrix}$$

$$\text{or, } \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2/3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ -1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -2 \\ 0 & 0 & \frac{11}{3} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2/3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2/3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2/3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2/3 & 1 \end{bmatrix}^{-1}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}^{-1}$$

$$\left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2/3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \right\}^{-1} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ \frac{2}{3}(-2, 1, 0) \\ 1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2/3 & 1 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2/3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (1, 0, 0) \\ (2, 1, 0) \\ (-1, 0, 1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & -2/3 & 1 \end{bmatrix}$$

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$$A = LU,$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ -1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & -\frac{2}{3} & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & \frac{11}{3} \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -2 \\ 0 & 0 & \frac{11}{3} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & \frac{11}{3} \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2/3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= DU^*$$

$$A = LDU^*$$

$$|A| = |LDU^*|$$

$$|L| = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & -\frac{2}{3} & 1 \end{vmatrix} = 1$$

$$|U^*| = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 2/3 \\ 0 & 0 & 1 \end{vmatrix} = 1$$

$$|A| = \cancel{|A|} |LDU^*| = |L| |D| |U^*|$$

$$\therefore |A| = |L| |D| \cancel{|U^*|} = |D|$$

$$|D| = 1 \times (-3) \times \frac{11}{3} = -11.$$