$$A=\begin{bmatrix}1 & 3 & 1\\ 2 & 1 & 1\\ 1 & 3 & 3\end{bmatrix}$$ $A = LDU$ 로 분해

$$\begin{bmatrix}1 & 0 & 0\\ -2 & 1 & 0\\ -1 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 3 & 1\\ 2 & 1 & 1\\ 1 & 3 & 3\end{bmatrix}=\begin{bmatrix}1 & 3 & 1\\ -2(1,3,1)+(2,1,1)\\ -1(1,3,1)+(1,3,3)\end{bmatrix}$$

$$=\begin{bmatrix}1 & 3 & 1\\ 0 & -5 & -1\\ 0 & 0 & 2\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & -5 & 0\\ 0 & 0 & 2\end{bmatrix}\begin{bmatrix}1 & 3 & 1\\ 0 & 1 & \frac{1}{5}\\ 0 & 0 & 1\end{bmatrix}$$

$|A| = 1\times(-5)\times 2 = -10$

$$\begin{bmatrix}1 & 0 & 0\\ -2 & 1 & 0\\ -1 & 0 & 1\end{bmatrix}^{-1}=\begin{bmatrix}1 & 0 & 0\\ 2 & 1 & 0\\ 1 & 0 & 1\end{bmatrix}$$ (확인!)

$$A=\begin{bmatrix}1 & 0 & 0\\ 2 & 1 & 0\\ 1 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & -5 & 0\\ 0 & 0 & 2\end{bmatrix}\begin{bmatrix}1 & 3 & 1\\ 0 & 1 & \frac{1}{5}\\ 0 & 0 & 1\end{bmatrix}$$

$= LDU$

$$\begin{bmatrix} 1 & 3 & 1 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 1 & 3 & 3 & 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 3 & 1 & 1 & 0 & 0 \\ -2 & -6 & -2 & -2 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ -1 & -3 & -1 & -1 & 0 & 0 \\ 1 & 3 & 3 & 0 & 0 & 1 \end{bmatrix}$$

맨위에서

$$\Rightarrow \begin{bmatrix} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & -5 & -1 & -2 & 1 & 0 \\ 0 & 0 & 2 & -1 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1/5 & 2/5 & -1/5 & 0 \\ 0 & 0 & 1 & -1/2 & 0 & 1/2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & -3 & -\frac{3}{5} & -\frac{6}{5} & \frac{3}{5} & 0 \\ 0 & 1 & \frac{1}{5} & \frac{2}{5} & -\frac{1}{5} & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & \frac{2}{5} & -\frac{1}{5} & \frac{3}{5} & 0 \\ 0 & 1 & \frac{1}{5} & \frac{2}{5} & -\frac{1}{5} & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & \frac{2}{5} & -\frac{1}{5} & \frac{3}{5} & 0 \\ 0 & 0 & -\frac{2}{5} & \frac{1}{5} & 0 & -\frac{1}{5} \\ 0 & 1 & \frac{1}{5} & \frac{2}{5} & -\frac{1}{5} & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & \frac{3}{5} & -\frac{1}{5} \\ 0 & 1 & \frac{1}{5} & \frac{2}{5} & -\frac{1}{5} & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

기본행연산을 확장한 행렬 전체에 적용.
역행렬을 찾는 방법

$$\Rightarrow \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 0 & \frac{3}{5} & -\frac{1}{5} \\ 0 & 1 & \frac{1}{5} & \frac{2}{5} & -\frac{1}{5} & 0 \\ 0 & 0 & -\frac{1}{5} & \frac{1}{10} & 0 & -\frac{1}{10} \\ 0 & 0 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} \end{array}\right] \Rightarrow \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 0 & \frac{3}{5} & -\frac{1}{5} \\ 0 & 1 & 0 & \frac{5}{10} & -\frac{1}{5} & -\frac{1}{10} \\ 0 & 0 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 0 & \frac{6}{10} & -\frac{2}{10} \\ 0 & 1 & 0 & \frac{5}{10} & -\frac{2}{10} & -\frac{1}{10} \\ 0 & 0 & 1 & -\frac{5}{10} & 0 & \frac{5}{10} \end{array}\right]$$

$$\therefore A^{-1} = \frac{1}{10}\begin{bmatrix} 0 & 6 & -2 \\ 5 & -2 & -1 \\ -5 & 0 & 5 \end{bmatrix}$$

확인 $\frac{1}{10}\begin{bmatrix} 0 & 6 & -2 \\ 5 & -2 & -1 \\ -5 & 0 & 5 \end{bmatrix}\begin{bmatrix} 1 & 3 & 1 \\ 2 & 1 & 1 \\ 1 & 3 & 3 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 12 & 6 & 6 \\ -2 & -6 & -6 \\ 5 & 15 & 5 \\ -4 & -2 & -2 \\ -1 & -3 & -3 \\ -5 & -15 & -5 \\ 5 & 15 & 15 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix} = I$

(3)