

An introduction to R in Personality Research

The First World Conference of Personality

William Revelle

Department of Psychology
Northwestern University
Evanston, Illinois USA



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UNIVERSITY

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Overview

1

▶ Part I: an introduction to R

- What is R
- A brief example
- Basic steps and graphics

2

▶ Part II: Using R for psychometrics

- Classical test theory
- Multivariate analysis
- Item Response Theory

3

▶ Part III: Structures, Objects, Functions

- The basic data structures
- Functions and objects
- Getting help
- Frequently used functions
- Writing your own functions



Outline of Part 1

1 What is R?

- Where did it come from, why use it?
- Installing R on your computer and adding packages
- Basic R capabilities: Calculation, Statistical tables, Graphics

2 A brief example

- A brief example of exploratory and confirmatory data analysis

3 Basic statistics and graphics

- 4 steps: read, explore, test, graph
- Basic descriptive and inferential statistics
 - t-test, ANOVA, χ^2
 - Linear Regression



R: Statistics for all of us

- ① What is it?
- ② Why use it?
- ③ Common (mis)perceptions of R
- ④ Examples for psychologists
 - graphical displays
 - basic statistics
 - advanced statistics
 - Although programming is easy in R, that is beyond the scope of today



R: What is it?

- ① R: An international collaboration
- ② R: The open source - public domain version of S+
- ③ R: Written by statistician (and all of us) for statisticians (and the rest of us)
- ④ R: Not just a statistics system, also an extensible language.
 - This means that as new statistics are developed they tend to appear in R far sooner than elsewhere.
 - R facilitates asking questions that have not already been asked.
- ⑤ R: encourages publications of "Reproducible Research"
 - integrate data, code, text into one document
 - Sweave and knitr



Statistical Programs for Psychologists

- General purpose programs

- R
- S+
- SAS
- SPSS
- STATA
- Systat

- Specialized programs

- Mx
- EQS
- AMOS
- LISREL
- MPlus
- Your favorite program



Statistical Programs for Psychologists

- General purpose programs

- R
- \$+
- \$A\$
- \$P\$
- \$TATA
- \$y\$tat

- Specialized programs

- Mx (OpenMx is part of R)
- EQ\$
- AMO\$
- LI\$REL
- MPi\$
- Your favorite program



R: A way of thinking

- “R is the lingua franca of statistical research. Work in all other languages should be discouraged.”
- “This is R. There is no if. Only how.”
- “Overall, SAS is about 11 years behind R and S-Plus in statistical capabilities (last year it was about 10 years behind) in my estimation.”
- “I quit using SAS in 1991 because my productivity jumped at least 20% within one month of using S-Plus.”

Taken from the R.-fortunes (selections from the R.-help list serve)



More fortunes

"You must realize that R is written by experts in statistics and statistical computing who, despite popular opinion, do not believe that everything in SAS and SPSS is worth copying. Some things done in such packages, which trace their roots back to the days of punched cards and magnetic tape when fitting a single linear model may take several days because your first 5 attempts failed due to syntax errors in the JCL or the SAS code, still reflect the approach of "give me every possible statistic that could be calculated from this model, whether or not it makes sense". The approach taken in R is different. The underlying assumption is that the user is thinking about the analysis while doing it. " (Douglas Bates, 2007)



R is open source, how can you trust it?

- Q: "When you use it [R], since it is written by so many authors, how do you know that the results are trustable?"
- A: "The R engine [...] is pretty well uniformly excellent code but you have to take my word for that. Actually, you don't. The whole engine is open source so, if you wish, you can check every line of it. If people were out to push dodgy software, this is not the way they'd go about it." (Bill Venables, 2004)



What is R?: Technically

- R is an open source implementation of S (S-Plus is a commercial implementation)
- R is available under GNU Copy-left
- The current version of R is 2.15.3
- R is a group project run by a core group of developers (with new releases ≈ semiannually)
- R 3.0 is to be released in April, 2013

(Adapted from Robert Gentleman)



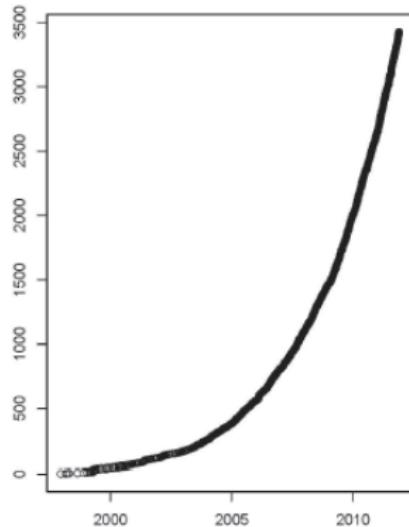
R: A brief history

- 1991-93: Ross Ihaka and Robert Gentleman begin work on R project at U. Auckland
- 1995: R available by ftp under the GPL
- 96-97: mailing list and R core group is formed
- 2000: John Chambers, designer of S joins the Rcore (wins a prize for best software from ACM for S)
- 2001-2011: Core team continues to improve base package with a new release every 6 months.
- Many others contribute “packages” to supplement the functionality for particular problems
 - 2003-04-01: 250 packages
 - 2004-10-01: 500 packages
 - 2007-04-12: 1,000 packages
 - 2009-10-04: 2,000 packages
 - 2011-05-12 3,000 packages
 - 2012-08-23 4,000 packages

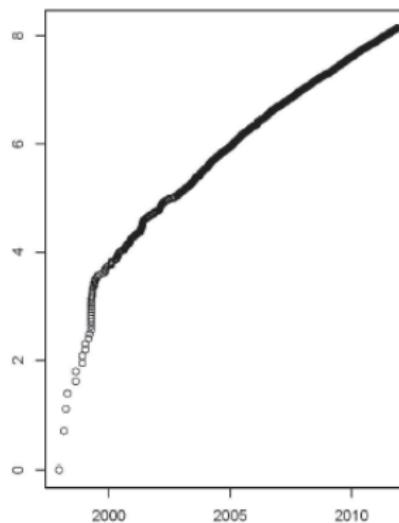


Has R grown too much? Exponential growth rate continues

Number of Active CRAN Packages



Log Number of Active CRAN Packages



Misconception: R is hard to use

① R doesn't have a GUI (Graphical User Interface)

- Partly true, many use syntax
- Partly not true, GUIs exist (e.g., R Commander, R-Studio)
- Quasi GUIs for Mac and PCs make syntax writing easier

② R syntax is hard to use

- Not really, unless you think an iPhone is hard to use
- Easier to give instructions of 1-4 lines of syntax rather than pictures of what menu to pull down.
- Keep a copy of your syntax, modify it for the next analysis.

③ R is not user friendly: A personological description of R

- R is introverted: it will tell you what you want to know if you ask, but not if you don't ask.
- R is conscientious: it wants commands to be correct.
- R is not agreeable: its error messages are at best cryptic.
- R is stable: it does not break down under stress.
- R is open: new ideas about statistics are easily developed.



Misconceptions: R is hard to learn

- With a brief web based tutorial

<http://personality-project.org/r>, 2nd and 3rd year undergraduates in psychological methods and personality research courses are using R for descriptive and inferential statistics and producing publication quality graphics.

- More and more psychology departments are using it for graduate and undergraduate instruction.

- R is easy to learn, hard to master

- R-help newsgroup is very supportive
- Multiple web based and pdf tutorials see (e.g.,
<http://www.r-project.org/>)
- Short courses using R for many applications

- Books and websites for SPSS and SAS users trying to learn R (e.g.,<http://oit.utk.edu/scc/RforSAS&SPSSusers.pdf> by Bob Muenchen).



Ok, how do I get it: Getting started with R

- ① Download from R Cran (<http://cran.r-project.org/>)
 - Choose appropriate operating system and download compiled R
- ② Install R (current version is 2.15.3) with 3.0 coming April 3
- ③ Start R
- ④ Add useful packages (just need to do this once)
 - `install.packages("ctv")` #this downloads the task view package
 - `library(ctv)` #this activates the ctv package
 - `install.views("Psychometrics")` #among others
 - Take a 5 minute break
- ⑤ Activate the package(s) you want to use today (e.g., *psych*)
 - `library(psych)` #necessary for most of today's examples
- ⑥ Use R
- ⑦ (See detailed tutorial at http://personality-project.org/r/psych/getting_started.pdf)



Go to the R.project.org

The R Project for Statistical Computing

About R

- [What is R?](#)
- [Contributors](#)
- [Screenshots](#)
- [What's new?](#)

Download, Packages

- [CRAN](#)

R Project

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Documentation

- [Manuals](#)
- [FAQs](#)
- [The R Journal](#)
- [Wiki](#)
- [Books](#)
- [Certification](#)
- [Other](#)

Getting Started:

- R is a free software environment for statistical computing and graphics. It compiles and runs on a wide variety of UNIX platforms, Windows and MacOS. To [download R](#), please choose your preferred [CRAN mirror](#).
- If you have questions about R like how to download and install the software, or what the license terms are, please read our [answers to frequently asked questions](#) before you send an email.



Installing R on your computer and adding packages

Go to the Comprehensive R Archive Network (CRAN)

The Comprehensive R Archive Network

[Download and Install R](#)

Precompiled binary distributions of the base system and contributed packages, **Windows and Mac** users most likely want one of these versions of R:

- [Download R for Linux](#)
- [Download R for \(Mac\) OS X](#)
- [Download R for Windows](#)

R is part of many Linux distributions, you should check with your Linux package management system in addition to the link above.

[Source Code for all Platforms](#)

Windows and Mac users most likely want to download the precompiled binaries listed in the upper box, not the source code. The sources have to be compiled before you can use them. If you do not know what this means, you probably do not want to do it!

- The latest release (2012-10-26, Trick or Treat): [R-2.15.2.tar.gz](#), read [what's new](#) in the latest version.
- Sources of [R alpha and beta releases](#) (daily snapshots, created only in time periods before a planned release).
- Daily snapshots of current patched and development versions are [available here](#). Please read about [new features and bug fixes](#) before filing corresponding feature requests or bug reports.
- Source code of older versions of R is [available here](#).
- Contributed extension [packages](#)

[Questions About R](#)

- If you have questions about R like how to download and install the software, or what the license terms are, please read our [answers to frequently asked questions](#) before you send an email.



Choose a mirror site near you

The R Project for Statistical Computing

www.r-project.org

Reader

Bill's Google Scholar Wikipedia DuckDuckGo News Google Maps RSeek.org win-builder CRAN Package

The R Project for Statistical Computing



CRAN Mirrors

The Comprehensive R Archive Network is available at the following URLs, please choose a location close to you. Some statistics on the status of the mirrors can be found here: [main page](#), [windows release](#), [windows old release](#).

O-Cloud

<http://cran.rstudio.com/>

Rstudio, automatic redirection to servers worldwide

Argentina

<http://mirror.feaglp.unlp.edu.ar/CRAN/>

Universidad Nacional de La Plata

<http://r.mirror.mendoza-conicet.gob.ar/>

CONICET Mendoza

Australia

<http://cran.csiro.au/>

CSIRO

<http://cran.ms.unimelb.edu.au/>

University of Melbourne

Austria

<http://cran.at.r-project.org/>

Wirtschaftsuniversitaet Wien

Belgium

<http://www.freestatistics.org/cran/>

K.U.Leuven Association

Brazil

<http://cran-r.c3sl.ufpr.br/>

Universidade Federal do Parana

<http://cran.fiocruz.br/>

Oswaldo Cruz Foundation, Rio de Janeiro

<http://www.vps.fimvz.usp.br/CRAN/>

University of Sao Paulo, Sao Paulo

<http://brieger.esalq.usp.br/CRAN/>

University of Sao Paulo, Piracicaba

Canada

<http://cran.stat.sfu.ca/>

Simon Fraser University, Burnaby

<http://mirror.its.dal.ca/cran/>

Dalhousie University, Halifax

<http://probability.ca/cran/>

University of Toronto

<http://cran.skazkaforyou.com/>

iWeb, Montreal

<http://cran.parentingamerica.com/>

iWeb, Montreal

Chile

<http://dirichlet.mat.puc.cl/>

Pontificia Universidad Catolica de Chile, Santiago

China

<http://ftp.ctex.org/mirrors/CRAN/>

CTEX.ORG

<http://mirror.bjtu.edu.cn/cran/>

Beijing Jiaotong University, Beijing

<http://cran.dataguru.cn>

Dataguru (a, Guangzhou

<http://mirrors.ustc.edu.cn/CRAN/>

University of Science and Technology of China

<http://mirrors.xmu.edu.cn/CRAN/>

Xiamen University

Colombia

<http://www.lanee.unal.edu.co/CRAN/>

National University of Colombia



Download and install the appropriate version – PC

The Comprehensive R Archive Network

http://cran.r-project.org/ Q R CRAN

Bill's Apple Yahoo! scholar.google.com Google Maps Wikipedia YouTube News (609) Popular CRAN Package

R for Windows

Subdirectories:

- [base](#) Binaries for base distribution (managed by Duncan Murdoch)
- [contrib](#) Binaries of contributed packages (managed by Uwe Ligges)

Please do not submit binaries to CRAN. Package developers might want to contact Duncan Murdoch or Uwe Ligges directly in case of questions / suggestions related to Windows binaries.

You may also want to read the [R FAQ](#) and [R for Windows FAQ](#).

Note: CRAN does some checks on these binaries for viruses, but cannot give guarantees. Use the normal precautions with downloaded executables.

CRAN
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↗

Download and install the appropriate version – Mac

The Comprehensive R Archive Network

cran.r-project.org

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The Comprehensive R Archive Network

R for Mac OS X

This directory contains binaries for a base distribution and packages to run on Mac OS X (release 10.5 and above). Mac OS 8.6 to 9.2 (and Mac OS X 10.1) are no longer supported but you can find the last supported release of R for these systems (which is R 1.7.1) [here](#). Releases for old Mac OS X systems (through Mac OS X 10.4) and PowerPC Macs can be found in the [old](#) directory.

Note: CRAN does not have Mac OS X systems and cannot check these binaries for viruses. Although we take precautions when assembling binaries, please use the normal precautions with downloaded executables.

R 2.15.2 "Trick or Treat" released on 2012/10/26

This binary distribution of R and the GUI supports Intel (32-bit and 64-bit) based Macs on Mac OS X 10.5 (Leopard) or higher.

Please check the MD5 checksum of the downloaded image to ensure that it has not been tampered with or corrupted during the mirroring process. For example type
`md5 R-2.15.2.pkg`
in the *Terminal* application to print the MD5 checksum for the R-2.15.2.pkg image.

Files:

R-2.15.2.pkg (latest version) **R 2.15.2** binary for Mac OS X 10.5 (Leopard) and higher, signed package.
MD5-hash: 8935aaefc90e522e7b1da487c50d0d3c
(ca. 64MB)

Contains R 2.15.2 framework, R.app GUI 1.53 in 32-bit and 64-bit for Intel Macs. The above file is an Installer package which can be installed by double-clicking. Depending on your browser, you may need to press the control key and click on this link to download the file.

This package only contains the R framework, 32-bit GUI (R.app) and 64-bit GUI (R64.app). For Tk/Tk libraries (needed if you want to use tktk) and GNU Fortran (needed if you want to compile packages from sources that contain FORTRAN code) please see [the tools directory](#).

Mac-GUI-1.53.tar.gz
MD5-hash: 039ab50b0bac01d024e612fb8d613d00

Sources for the R.app GUI 1.51 for Mac OS X. This file is only needed if you want to join the development of the GUI, it is not intended for regular users. Read the INSTALL file for further instructions.

Starting R on a PC

The screenshot shows the RGui application window titled "RGui". The menu bar includes "File", "Edit", "View", "Misc", "Packages", "Windows", and "Help". Below the menu is a toolbar with icons for file operations like Open, Save, Print, and Stop. The main area is labeled "R Console". The console output is as follows:

```
R version 2.13.0 (2011-04-13)
Copyright (C) 2011 The R Foundation for Statistical Computing
ISBN 3-900051-07-0
Platform: i386-pc-mingw32/i386 (32-bit)

R is free software and comes with ABSOLUTELY NO WARRANTY.
You are welcome to redistribute it under certain conditions.
Type 'license()' or 'licence()' for distribution details.

Natural language support but running in an English locale

R is a collaborative project with many contributors.
Type 'contributors()' for more information and
'citation()' on how to cite R or R packages in publications.

Type 'demo()' for some demos, 'help()' for on-line help, or
'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.
```

> |

Start up R and get ready to play (Mac version)

```
R version 2.15.2 (2012-10-26) -- "Trick or Treat"  
Copyright (C) 2012 The R Foundation for Statistical Computing  
ISBN 3-900051-07-0  
Platform: i386-apple-darwin9.8.0/i386 (32-bit)
```

R is free software and comes with ABSOLUTELY NO WARRANTY.
You are welcome to redistribute it under certain conditions.
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'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.

[R.app GUI 1.53 (6335) i386-apple-darwin9.8.0]

```
> # > is the prompt for all commands #is for comments
```



Annotated installation guide: don't type the >

```
> install.packages("ctv")
```

- Install the task view installer package. You might have to choose a “mirror” site.

```
> library(ctv)
```

- Make it active

```
> install.views("Psychometrics")
```

- Install all the packages in the “Psychometrics” task view. This will take a few minutes.

#or just install a few packages

```
> install.packages("psych")
```

- Or, just install one package (e.g., psych)

```
> install.packages("GPArotation")
```

- as well as a few suggested packages that add functionality for factor

```
> install.packages("MASS")
```

- rotation, multivariate normal distributions, etc.

```
> install.packages("mvtnorm")
```



Installing just the psych package



```
> install.packages("psych")
--- Please select a CRAN mirror for use in this session ---
trying URL 'http://cran.stat.ucla.edu/bin/windows/contrib/2.13/psych_1.0-97.zip'
Content type 'application/zip' length 1952216 bytes (1.9 Mb)
opened URL
downloaded 1.9 Mb
```

Or, install and use `ctv` package to load a task view on a PC

The screenshot shows the RGui - [R Console] window. It displays the R startup message, including copyright information, platform details, and license terms. A red arrow points from the text "Use the package menu to select a mirror" to the "Packages" menu item in the top menu bar. Another red arrow points from the text "CRAN mirror" to the highlighted text "CRAN mirror" in the terminal session output.

```
RGui - [R Console]
File Edit View Misc Packages Windows Help
[Icons]
Copyright (C) 2011 The R Foundation for Statistical Computing
ISBN 3-900051-07-0
Platform: i386-pc-mingw32/i386 (32-bit)

R is free software and comes with ABSOLUTELY NO WARRANTY.
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Type 'contributors()' for more information and
'citation()' on how to cite R or R packages in publications.

Type 'demo()' for some demos, 'help()' for on-line help, or
'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.

> install.packages("ctv")
--- Please select a CRAN mirror for use in this session ---
trying URL 'http://cran.stat.ucla.edu/bin/windows/contrib/2.13/ctv_0.7-2.zip'
Content type 'application/zip' length 298753 bytes (291 Kb)
opened URL
downloaded 291 Kb

package 'ctv' successfully unpacked and MD5 sums checked

The downloaded packages are in
  C:\users\revelle\Temp\RtmpwNzUtt\downloaded_packages
> library(ctv)
> |
```



Check the version number for R (should be $\geq 2.5.2$) and for psych ($\geq 1.3.2$)

```
> library(psych)
> sessionInfo()

R version 2.15.2 (2012-10-26)
Platform: i386-apple-darwin9.8.0/i386 (32-bit)

locale:
[1] en_US.UTF-8/en_US.UTF-8/en_US.UTF-8/C/en_US.UTF-8/en_US.UTF-8

attached base packages:
[1] stats      graphics   grDevices utils      datasets   methods    base

other attached packages:
[1] psych_1.3.2
```



R is extensible: The use of “packages”

- More than 4000 packages are available for R (and growing daily)
- Can search all packages that do a particular operation by using the `sos` package
 - `install.packages("sos")` #if you haven't already
 - `library(sos)` # make it active once you have it
 - `findFn("X")` #will search a web data base for all packages/functions that have "X"
 - `findFn("principal components analysis ")` #will return 1516 matches and reports the top 400
 - `findFn("Item Response Theory")` # will return 231 matches
 - `findFn("INDSCAL ")` # will return 7 matches.
- `install.packages("X")` will install a particular package (add it to your R library – you need to do this just once)
- `library(X)` #will make the package X available to use if it has been installed (and thus in your library)



A small subset of very useful packages

- General use

- core R
- MASS
- lattice
- lme4 (core)
- psych
- Zelig

- Special use

- ltm
- sem
- lavaan
- OpenMx
- GPArotation
- mvtnorm
- > 4000 known
- + ?

- General applications

- most descriptive and inferential stats
- Modern Applied Statistics with S
- Lattice or Trellis graphics
- Linear mixed-effects models
- Personality and psychometrics
- General purpose toolkit

- More specialized packages

- Latent Trait Model (IRT)
- SEM and CFA (one group)
- SEM and CFA (multiple groups)
- SEM and CFA (multiple groups +)
- Jennrich + Browne rotations
- Multivariate distributions
- Thousands of more packages on CRAN
- Code on webpages/journal articles



Basic R commands – remember don't enter the >

R is just a fancy calculator. Add, subtract, sum, products, group

```
> 2 + 2
```

```
[1] 4
```

```
> 3^4
```

```
[1] 81
```

```
> sum(1:10)
```

```
[1] 55
```

```
> prod(c(1, 2, 3, 5, 7))
```

```
[1] 210
```

It is also a statistics table (the normal distribution, the t distribution)

```
> pnorm(q = 1)
```

```
[1] 0.8413447
```

```
> pt(q = 2, df = 20)
```

```
[1] 0.9703672
```



R is a set of distributions. Don't buy a stats book with tables!

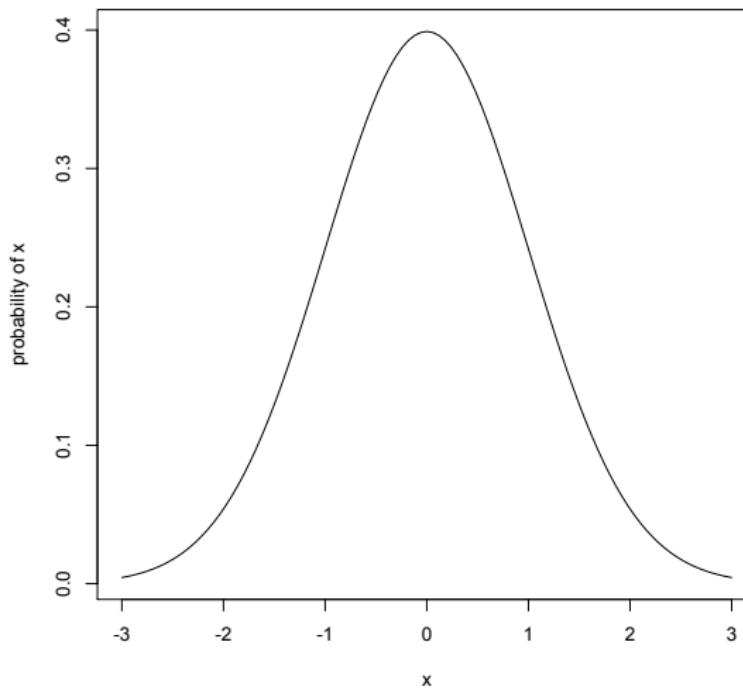
Table : To obtain the density, prefix with d , probability with p , quantiles with q and to generate random values with r . (e.g., the normal distribution may be chosen by using `dnorm`, `pnorm`, `qnorm`, or `rnorm`.)

Distribution	base name	P 1	P 2	P 3	example application
<i>Normal</i>	norm	mean	sigma		Most data
<i>Multivariate normal</i>	mvnorm	mean	r	sigma	Most data
<i>Log Normal</i>	Inorm	log mean	log sigma		income or reaction time
<i>Uniform</i>	unif	min	max		rectangular distributions
<i>Binomial</i>	binom	size	prob		Bernoulli trials (e.g. coin flips)
<i>Student's t</i>	t	df		nc	Finding significance of a t-test
<i>Multivariate t</i>	mvt	df	corr	nc	Multivariate applications
<i>Fisher's F</i>	f	df1	df2	nc	Testing for significance of F test
χ^2	chisq	df		nc	Testing for significance of χ^2
<i>Exponential</i>	exp	rate			Exponential decay
<i>Gamma</i>	gamma	shape	rate	scale	distribution theoryh
<i>Hypergeometric</i>	hyper	m	n	k	
<i>Logistic</i>	logis	location	scale		Item Response Theory
<i>Poisson</i>	pois	lambda			Count data
<i>Weibull</i>	weibull	shape	scale		Reaction time distributions
<i>Cauchy</i>	cauchy	location	scale	log	infinite variance!



R can draw distributions

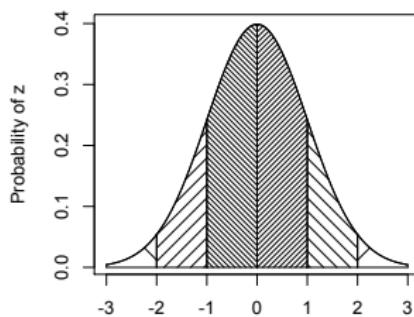
A normal curve



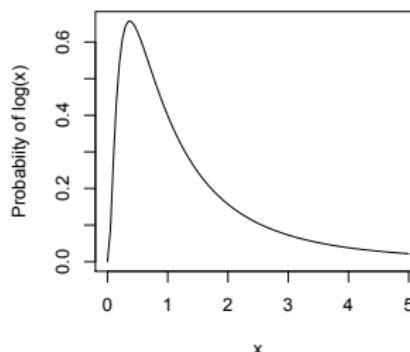
```
curve(dnorm(x),-3,3,ylab="probability of x",main="A normal curve")
```

R can draw more interesting distributions

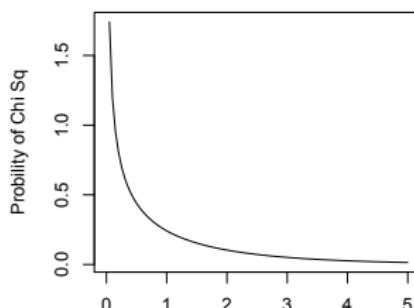
The normal curve



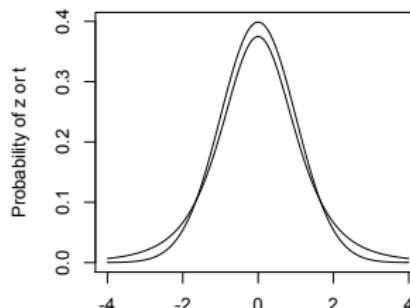
Log normal



Chi Square distribution



Normal and t with 4 df



R is also a graphics calculator

The first line draws the normal curve, the second prints the title, the next lines draw the cross hatching.

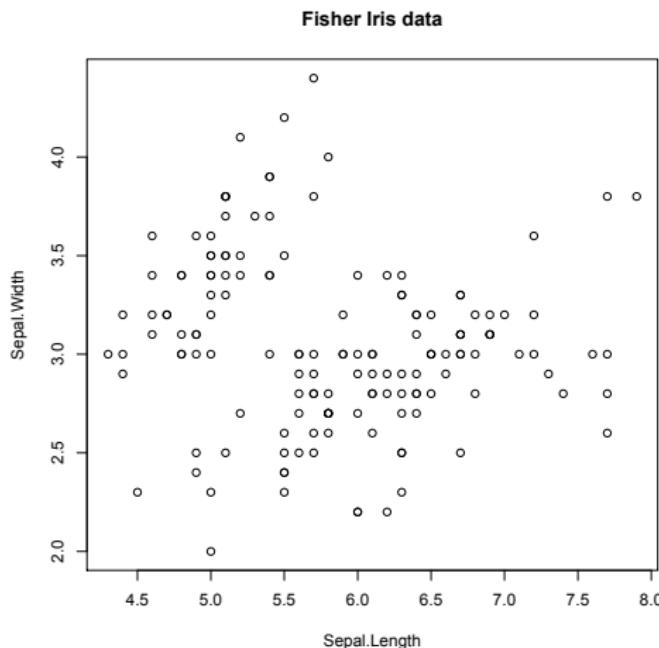
```

op <- par(mfrow=c(2,2))      #set up a 2 x 2 graph
curve(dnorm(x),-3,3,xlab="",ylab="Probability of z")
title(main="The normal curve",outer=FALSE)
xvals <- seq(-3,-2,length=100)
dvals <- dnorm(xvals)
polygon(c(xvals,rev(xvals)),c(rep(0,100),rev(dvals)),density=2,angle=-45)
xvals <- seq(-2,-1,length=100)
dvals <- dnorm(xvals)
polygon(c(xvals,rev(xvals)),c(rep(0,100),rev(dvals)),density=14,angle=45)
xvals <- seq(-1,-0,length=100)
dvals <- dnorm(xvals)
polygon(c(xvals,rev(xvals)),c(rep(0,100),rev(dvals)),density=34,angle=-45)
xvals <- seq(2,3,length=100)
dvals <- dnorm(xvals)
polygon(c(xvals,rev(xvals)),c(rep(0,100),rev(dvals)),density=2,angle=45)
xvals <- seq(1,2,length=100)
dvals <- dnorm(xvals)
polygon(c(xvals,rev(xvals)),c(rep(0,100),rev(dvals)),density=14,angle=-45)
xvals <- seq(0,1,length=100)
dvals <- dnorm(xvals)
polygon(c(xvals,rev(xvals)),c(rep(0,100),rev(dvals)),density=34,angle=45)
curve(dlnorm(x),0,5,ylab='Probability of log(x)',main='Log normal')
curve(dchisq(x,1),0,5,ylab='Probability of Chi Sq',xlab='Chi Sq',main='Chi Square distribution')
curve(dnorm(x),-4,4,ylab='Probability of z or t',xlab='z or t',main='Normal and t with 4 df')
curve(dt(x,4),add=TRUE)
op <- par(mfrow=c(1,1))

```



A simple scatter plot using plot shows Fisher's Iris data set

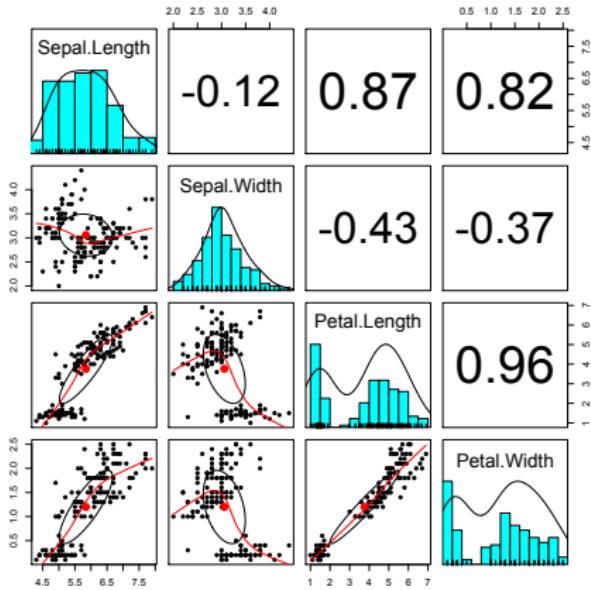


```
plot(iris[1:2], xlab="Sepal.Length", ylab="Sepal.Width",  
main="Fisher Iris data")
```



Basic R capabilities: Calculation, Statistical tables, Graphics

A scatter plot matrix with loess regression using pairs.panels shows more information than a simple scatter plot

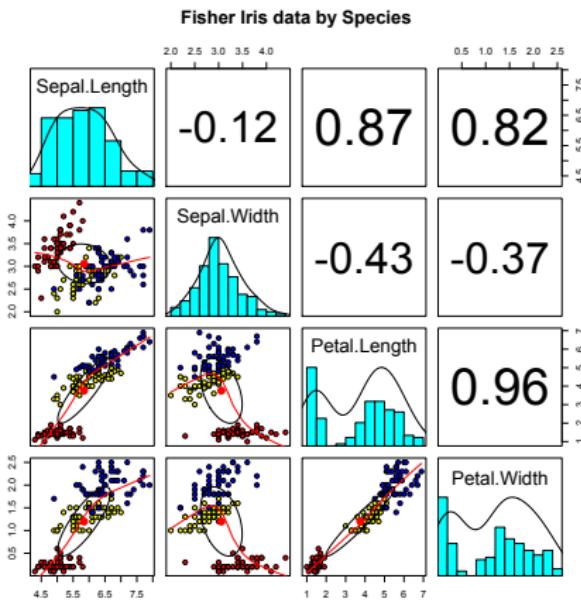


`pairs.panels(iris[1:4])`

- ① Correlations above the diagonal
- ② Diagonal shows histograms and densities
- ③ scatter plots below the diagonal with correlation ellipse
- ④ locally smoothed (loess) regressions for each pair



A better SPLOM with colors for groups using pairs.panels



- ➊ Correlations above the diagonal
- ➋ Diagonal shows histograms and densities
- ➌ scatter plots below the diagonal with correlation ellipse
- ➍ locally smoothed (loess) regressions for each pair
- ➎ optional color coding of grouping variables.

```
pairs.panels(iris[1:4], bg=c("red", "yellow", "blue")
[iris$Species], pch=21, main="Fisher Iris data by
Species")
```



A brief example of exploratory and confirmatory data analysis

A brief example with real data

- ① Get the data
- ② Descriptive statistics
 - Graphic
 - Numerical
- ③ Inferential statistics using the linear model
 - regressions
- ④ More graphic displays



Get the data and describe it

- ① First read the data, either from a built in data set, a local file, a remote file, or from the clipboard.
- ② Describe the data using the `describe` function from *psych*

```
> my.data <- sat.act #an example data file that is part of psych
#or
> file.name <- file.choose()      #look for it on your hard drive
#or
> file.name <-"http://personality-project.org/r/aps/sat.act.txt"
#now read it
> my.data <- read.table(file.name,header=TRUE)
#or
> my.data <- read.clipboard() #if you have copied the data to the clipboard
> describe(my.data) #report basic descriptive statistics
```

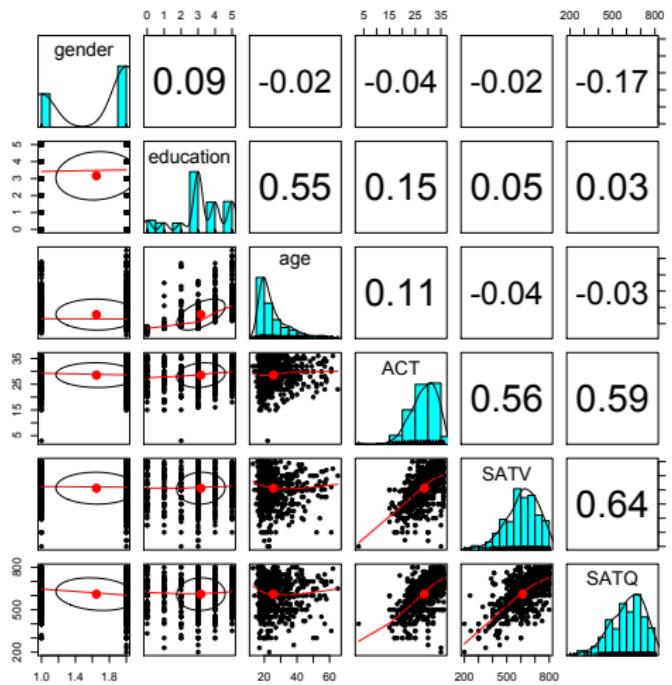
	var	n	mean	sd	median	trimmed	mad	min	max	range	skew	kurtosis
gender	1	700	1.65	0.48	2	1.68	0.00	1	2	1	-0.61	-1
education	2	700	3.16	1.43	3	3.31	1.48	0	5	5	-0.68	-0
age	3	700	25.59	9.50	22	23.86	5.93	13	65	52	1.64	2
ACT	4	700	28.55	4.82	29	28.84	4.45	3	36	33	-0.66	0
SATV	5	700	612.23	112.90	620	619.45	118.61	200	800	600	-0.64	0
SATQ	6	687	610.22	115.64	620	617.25	118.61	200	800	600	-0.59	0



A brief example of exploratory and confirmatory data analysis

Graphic display of data using pairs.panels

```
pairs.panels(my.data) #Note the outlier for ACT
```



Clean up the data using the scrub function

For the variable "ACT" make any value < 4 NA.

Then describe the results. Note that one case was dropped

```
> cleaned <- scrub(my.data, "ACT", min=4)
> describe(cleaned)
```

	var	n	mean	sd	median	trimmed	mad	min	max	range	skew	kurtosis
gender	1	700	1.65	0.48	2	1.68	0.00	1	2	1	-0.61	-1
education	2	700	3.16	1.43	3	3.31	1.48	0	5	5	-0.68	-0
age	3	700	25.59	9.50	22	23.86	5.93	13	65	52	1.64	2
ACT	4	699	28.58	4.73	29	28.85	4.45	15	36	21	-0.50	-0
SATV	5	700	612.23	112.90	620	619.45	118.61	200	800	600	-0.64	0
SATQ	6	687	610.22	115.64	620	617.25	118.61	200	800	600	-0.59	0



Find the correlations using cor

Specify that you want pairwise deletion. The default correlation is "pearson", other options include "spearman" and "kendall"

```
> cor(sat.act,use="pairwise")
```

	gender	education	age	ACT	SATV	SATQ
gender	1.00000000	0.08726909	-0.02085375	-0.03650344	-0.01884338	-0.16530333
education	0.08726909	1.00000000	0.54826952	0.15482888	0.04647692	0.03462572
age	-0.02085375	0.54826952	1.00000000	0.11054633	-0.04235393	-0.03394431
ACT	-0.03650344	0.15482888	0.11054633	1.00000000	0.56105620	0.58711216
SATV	-0.01884338	0.04647692	-0.04235393	0.56105620	1.00000000	0.64429994
SATQ	-0.16530333	0.03462572	-0.03394431	0.58711216	0.64429994	1.00000000

This is far more decimals than one wants, we should round the output. This is done by directly applying the round function.



Find the pairwise correlations, round to 2 decimals

Find the correlations with the `cor` function specifying pairwise deletion. Use the `round` function on the output.

```
> round(cor(cleaned,use="pairwise"),2)
```

	gender	education	age	ACT	SATV	SATQ
gender	1.00	0.09	-0.02	-0.05	-0.02	-0.17
education	0.09	1.00	0.55	0.15	0.05	0.03
age	-0.02	0.55	1.00	0.11	-0.04	-0.03
ACT	-0.05	0.15	0.11	1.00	0.55	0.59
SATV	-0.02	0.05	-0.04	0.55	1.00	0.64
SATQ	-0.17	0.03	-0.03	0.59	0.64	1.00



Yet another way: use the lowerCor function from psych

psych uses default values and displays that make sense for psychological research. These defaults can be overridden by specifying various choices. Note that the column labels have been automatically shortened to make for equal spacing.

```
> lowerCor(sat.act)
```

	gendr	edctn	age	ACT	SATV	SATQ
gender	1.00					
education	0.09	1.00				
age	-0.02	0.55	1.00			
ACT	-0.04	0.15	0.11	1.00		
SATV	-0.02	0.05	-0.04	0.56	1.00	
SATQ	-0.17	0.03	-0.03	0.59	0.64	1.00



Test the correlations for significance using corr.test

```
> corr.test(cleaned)
Call:corr.test(x = cleaned)
Correlation matrix
```

	gender	education	age	ACT	SATV	SATQ
gender	1.00	0.09	-0.02	-0.05	-0.02	-0.17
education	0.09	1.00	0.55	0.15	0.05	0.03
age	-0.02	0.55	1.00	0.11	-0.04	-0.03
ACT	-0.05	0.15	0.11	1.00	0.55	0.59
SATV	-0.02	0.05	-0.04	0.55	1.00	0.64
SATQ	-0.17	0.03	-0.03	0.59	0.64	1.00

Sample Size

	gender	education	age	ACT	SATV	SATQ
gender	700	700	700	699	700	687

...
SATQ 687 687 687 686 687 687

Probability values (Entries above the diagonal are adjusted for multiple tests.)

	gender	education	age	ACT	SATV	SATQ
gender	0.00	0.02	0.58	0.21	0.62	0.00
education	0.02	0.00	0.00	0.00	0.22	0.36
age	0.58	0.00	0.00	0.00	0.26	0.37
ACT	0.21	0.00	0.00	0.00	0.00	0.00
SATV	0.62	0.22	0.26	0.00	0.00	0.00
SATQ	0.00	0.36	0.37	0.00	0.00	0.00



A brief example of exploratory and confirmatory data analysis

Are education and gender independent? χ^2 Test of association

```
T <- with(my.data, table(gender, education))
```

```
> T
```

		education					
		0	1	2	3	4	5
gender		27	20	23	80	51	46
1	30	25	21	195	87	95	
2							

```
> chisq.test(T)
```

Pearson's Chi-squared test

data: T

X-squared = 16.0851, df = 5, p-value = 0.006605

- ① First create a table of associations

- Do this on our data (my.data)
- Use the “with” command to specify the data set

- ② Show the table

- ③ Apply χ^2 test



Multiple regression

- ① Use the sat.act data example
- ② Do the linear model
- ③ Summarize the results

```
mod1 <- lm(SATV ~ education + gender + SATQ, data=my.data)
> summary(mod1, digits=2)
```

Call:

```
lm(formula = SATV ~ education + gender + SATQ, data = my.data)
```

Residuals:

Min	1Q	Median	3Q	Max
-372.91	-49.08	2.30	53.68	251.93

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	180.87348	23.41019	7.726	3.96e-14 ***
education	1.24043	2.32361	0.534	0.59363
gender	20.69271	6.99651	2.958	0.00321 **
SATQ	0.64489	0.02891	22.309	< 2e-16 ***

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 86.24 on 683 degrees of freedom

(13 observations deleted due to missingness)

Multiple R-squared: 0.4231, Adjusted R-squared: 0.4205

F-statistic: 167 on 3 and 683 DF, p-value: < 2.2e-16



Zero center the data before examining interactions

In order to examine interactions using multiple regression, we must first “zero center” the data. This may be done using the `scale` function. By default, `scale` will standardize the variables. So to keep the original metric, we make the scaling parameter `FALSE`. Note that `scale` returns a matrix but that we will need a `data.frame` when we do the regression.

```
zsat <- data.frame(scale(my.data,scale=FALSE))
describe(zsat)
```

	var	n	mean	sd	median	trimmed	mad	min	max	range	skew
gender	1	700	0	0.48	0.35	0.04	0.00	-0.65	0.35	1	-0.61
education	2	700	0	1.43	-0.16	0.14	1.48	-3.16	1.84	5	-0.68
age	3	700	0	9.50	-3.59	-1.73	5.93	-12.59	39.41	52	1.64
ACT	4	700	0	4.82	0.45	0.30	4.45	-25.55	7.45	33	-0.66
SATV	5	700	0	112.90	7.77	7.22	118.61	-412.23	187.77	600	-0.64
SATQ	6	687	0	115.64	9.78	7.04	118.61	-410.22	189.78	600	-0.59



Zero center the data before examining interactions

```
> zsat <- data.frame(scale(my.data, scale=FALSE))
> mod2 <- lm(SATV ~ education * gender * SATQ, data=zsat)
> summary(mod2)
```

Call:

```
lm(formula = SATV ~ education * gender * SATQ, data = zsat)
```

Residuals:

Min	1Q	Median	3Q	Max
-372.53	-48.76	3.33	51.24	238.50

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	0.773576	3.304938	0.234	0.81500
education	2.517314	2.337889	1.077	0.28198
gender	18.485906	6.964694	2.654	0.00814 **
SATQ	0.620527	0.028925	21.453	< 2e-16 ***
education:gender	1.249926	4.759374	0.263	0.79292
education:SATQ	-0.101444	0.020100	-5.047	5.77e-07 ***
gender:SATQ	0.007339	0.060850	0.121	0.90404
education:gender:SATQ	0.035822	0.041192	0.870	0.38481

Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 . 1



Compare model 1 and model 2

Test the difference between the two linear models

```
> anova(mod1,mod2)
```

Analysis of Variance Table

Model 1: SATV ~ education + gender + SATQ

Model 2: SATV ~ education * gender * SATQ

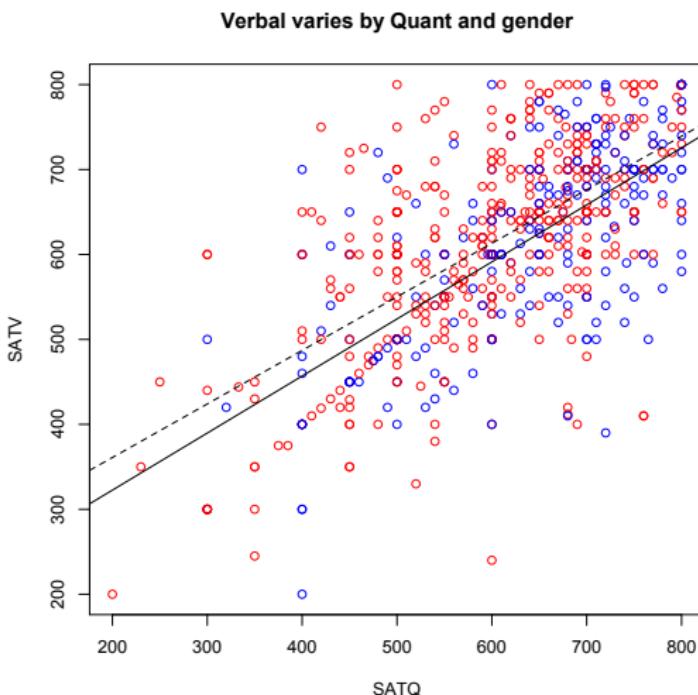
Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
1	683	5079984			
2	679	4870243	4	209742	7.3104 9.115e-06 ***
<hr/>					

Signif. codes: 0 ⚫***⚫ 0.001 ⚫**⚫ 0.01 ⚫*⚫ 0.05 ⚫.⚫ 0.1 ⚫



A brief example of exploratory and confirmatory data analysis

Show the regression lines by gender. Add color if desired.



First plot the points

```
> with(my.data, plot(SATV~SATQ,
  col=c("blue","red") [gender]))
```

Then draw two lines

```
> by(my.data,my.data$gender,
  function(x) abline
    (lm(SATV~SATQ,data=x),
     lty=c("solid","dashed"))
```

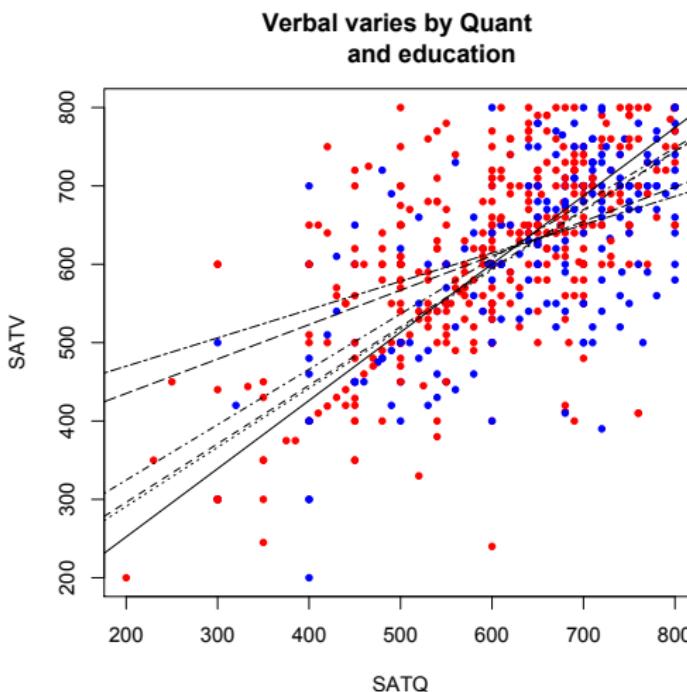
Add the title

```
> title("Verbal varies by Quant
  and gender")
```



A brief example of exploratory and confirmatory data analysis

Show the regression lines by education



Plot the points

```
> with(my.data, plot(SATV~SATQ,
  col=c("blue", "red") [gender]))
```

Add the regression lines

```
by(my.data, my.data$education,
  function(x) abline (lm(SATV~SATQ, data=x),
  lty=c("solid", "dashed", "dotted",
  "dotdash", "longdash",
  "twodash") [(x$education+1)]))
```

Add the title

```
> title("Verbal varies by Quant
  and education")
```



Using R for psychological statistics: Basic statistics

① Writing syntax

- For a single line, just type it
- Mistakes can be redone by using the up arrow key
- For longer code, use a text editor (built into some GUIs)

② Data entry

- Using built in data sets for examples
- Copying from another program (using the "clipboard")
- Reading a text or csv file
- Importing from SPSS or SAS
- Simulate it (using various simulation routines)

③ Descriptives

- Graphical displays
- Descriptive statistics
- Correlation

④ Inferential

- the t test
- the F test
- the linear model



Data entry overview

- ① Using built in data sets for examples
 - `data()` will list > 100 data sets in the `datasets` package as well as all sets in loaded packages.
 - Most packages have associated data sets used as examples
 - *psych* has > 40 example data sets
- ② Copying from another program
 - use copy and paste into R using `read.clipboard` and its variations
- ③ Reading a text or csv file
 - read a local or remote file
- ④ Importing from SPSS or SAS
- ⑤ Simulate it (using various simulation routines)



Examples of built in data sets from the psych package

```
> data(package="psych")
```

Bechtoldt

Seven data sets showing a bifactor solution.

Dwyer

8 cognitive variables used by Dwyer for an example

Reise

Seven data sets showing a bifactor solution.

all.income (income)

US family income from US census 2008

bfi

25 Personality items representing 5 factors

blot

Bond's Logical Operations Test - BLOT

burt

11 emotional variables from Burt (1915)

cities

Distances between 11 US cities

epi.bfi

13 personality scales from the Eysenck Personality and Big 5 inventory

flat (affect)

Two data sets of affect and arousal scores as a function of personality and movie conditions

galton

Galton's Mid parent child height data

income

US family income from US census 2008

iqitems

16 multiple choice IQ items

msq

75 mood items from the Motivational State Questionnaire
3896 participants

neo

NEO correlation matrix from the NEO_PI_R manual

sat.act

3 Measures of ability: SATV, SATQ, ACT

Thurstone

The classic Thurstone 9 variable problem

veg (vegetables)

Paired comparison of preferences for 9 vegetables



Reading data from another program –using the clipboard

- ① Read the data in your favorite spreadsheet or text editor
- ② Copy to the clipboard
- ③ Execute the appropriate `read.clipboard` function with or without various options specified

```
my.data <- read.clipboard()    #assumes headers and tab or space delimited
my.data <- read.clipboard.csv()  #assumes headers and comma delimited
my.data <- read.clipboard.tab()  #assumes headers and tab delimited
                                (e.g., from Excel)
my.data <- read.clipboard.lower() #read in a matrix given the lower
my.data <- read.clipboard.upper() # or upper off diagonal
my.data <- read.clipboard.fwf()   #read in data using a fixed format width
                                (see read.fwf for instructions)
```

- ④ `read.clipboard()` has default values for the most common cases and these do not need to be specified. Consult `?read.clipboard` for details.



Reading from a local or remote file

- ➊ Perhaps the standard way of reading in data is using the `read` command.
 - ➌ First must specify the location of the file
 - ➌ Can either type this in directly or use the `file.choose` function
 - ➌ The file name/location can be a remote URL

- ➋ Two examples of reading data

```
file.name <- file.choose() #this opens a window to allow you find the file
my.data <- read.table(file.name)
datafilename="http://personality-project.org/r/datasets/R.appendix1.data"
data.ex1=read.table(datafilename,header=TRUE)  #read the data into a table

> dim(data.ex1) #what are the dimensions of what we read?
[1] 18  2
> describe(data.ex1) #do the data look right?
      var   n   mean    sd median trimmed   mad min max range skew kurtosis
Dosage*     1 18  1.89  0.76      2    1.88 1.48    1    3      2 0.16 -1.16
Alertness   2 18 27.67  6.82     27   27.50 8.15    17   41      24 0.25  0.6
```



read a “foreign” file e.g., an SPSS sav file

`read.spss` reads a file stored by the SPSS save or export commands.

```
read.spss(file, use.value.labels = TRUE, to.data.frame = FALSE,  
         max.value.labels = Inf, trim.factor.names = FALSE,  
         trim_values = TRUE, reencode = NA, use.missing = to.data.frame)
```

`file` Character string: the name of the file or URL to read.

`use.value.labels` Convert variables with value labels into R factors with those levels?

`to.data.frame` return a data frame? Defaults to FALSE, probably should be TRUE in most cases.

`max.value.labels` Only variables with value labels and at most this many unique values will be converted to factors if `use.value.labels = TRUE`.

`trim.factor.names` Logical: trim trailing spaces from factor levels?

`trim_values` logical: should values and value labels have trailing spaces ignored when matching for `use.value.labels = TRUE`?

`use.missing` logical: should information on user-defined missing values be used to set the corresponding values to NA?



Simulate data

For many demonstration purposes, it is convenient to generate simulated data with a certain defined structure. The *psych* package has a number of built in simulation functions. Here are a few of them.

① Simulate various item structures

`sim.congeneric` A one factor congeneric measure model

`sim.items` A two factor structure with either simple structure or a circumplex structure.

`sim.rasch` Generate items for a one parameter IRT model.

`sim.irt` Generate items for a one-four parameter IRT Model

② Simulate various factor structures

`sim.simplex` Default is a four factor structure with a three time point simplex structure.

`sim.hierarchical` Default is 9 variables with three correlated factors.



Basic descriptive and inferential statistics

Get the data and look at it

Read in some data, look at the first and last few cases, and then get basic descriptive statistics. For this example, we will use a built in data set.

```
> my.data <- epi.bfi  
> headtail(my.data)
```

	epiE	epiS	epiImp	epilie	epiNeur	bfagree	bfcon	bfext	bfneur	bfopen	bdi	traitanx	stateanx
1	18	10	7	3	9	138	96	141	51	138	1	24	22
2	16	8	5	1	12	101	99	107	116	132	7	41	40
3	6	1	3	2	5	143	118	38	68	90	4	37	44
4	12	6	4	3	15	104	106	64	114	101	8	54	40
...
228	12	7	4	3	15	155	129	127	88	110	9	35	34
229	19	10	7	2	11	162	152	163	104	164	1	29	47
230	4	1	1	2	10	95	111	75	123	138	5	39	58
231	8	6	3	2	15	85	62	90	131	96	24	58	58

epi.bfi has 231 cases from two personality measures



Basic descriptive and inferential statistics

Now find the descriptive statistics for this data set

```
> describe(my.data)
```

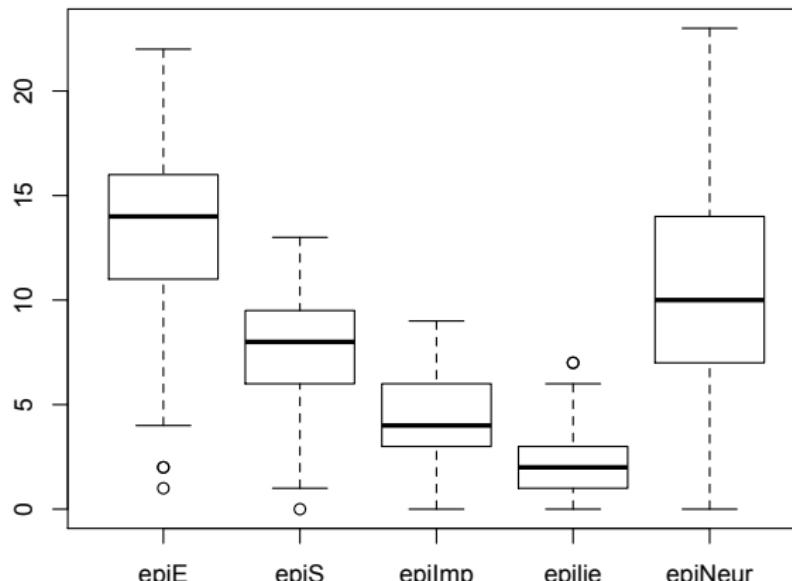
	var	n	mean	sd	median	trimmed	mad	min	max	range	skew	kurtosis	se
epiE	1	231	13.33	4.14	14	13.49	4.45	1	22	21	-0.33	-0.01	0.27
epiS	2	231	7.58	2.69	8	7.77	2.97	0	13	13	-0.57	0.04	0.18
epiImp	3	231	4.37	1.88	4	4.36	1.48	0	9	9	0.06	-0.59	0.12
epilie	4	231	2.38	1.50	2	2.27	1.48	0	7	7	0.66	0.30	0.10
epiNeur	5	231	10.41	4.90	10	10.39	4.45	0	23	23	0.06	-0.46	0.32
bfagree	6	231	125.00	18.14	126	125.26	17.79	74	167	93	-0.21	-0.22	1.19
bfcon	7	231	113.25	21.88	114	113.42	22.24	53	178	125	-0.02	0.29	1.44
bfext	8	231	102.18	26.45	104	102.99	22.24	8	168	160	-0.41	0.58	1.74
bfneur	9	231	87.97	23.34	90	87.70	23.72	34	152	118	0.07	-0.51	1.54
bfopen	10	231	123.43	20.51	125	123.78	20.76	73	173	100	-0.16	-0.11	1.35
bdi	11	231	6.78	5.78	6	5.97	4.45	0	27	27	1.29	1.60	0.38
traitanx	12	231	39.01	9.52	38	38.36	8.90	22	71	49	0.67	0.54	0.63
stateanx	13	231	39.85	11.48	38	38.92	10.38	21	79	58	0.72	0.04	0.76



Boxplots are a convenient descriptive device

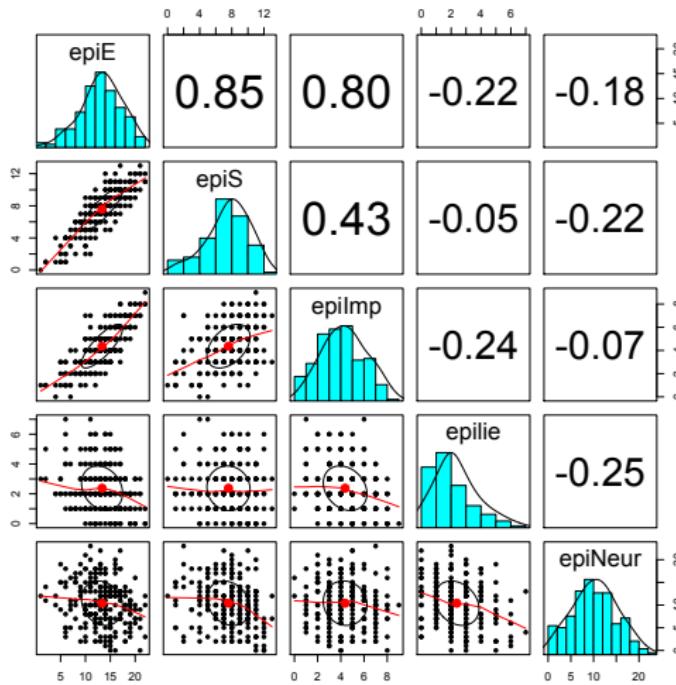
Show the Tukey “boxplot” for the Eysenck Personality Inventory
`boxplot(my.data[1:5]) #just the first 5 variables`

Boxplots of EPI scales



Basic descriptive and inferential statistics

Plot the scatter plot matrix (SPLOM) of the first 5 variables using the `pairs.panels` function



Use the `pairs.panels` function from *psych*

```
pairs.panels(my.data[1:5])
```



Find the correlations for this data set, round off to 2 decimal places

```
> round(cor(my.data, use = "pairwise"), 2)
```

	epiE	epiS	epiImp	epilie	epiNeur	bfagree	bfcon	bfext	bfneur	bfopen	
epiE	1.00	0.85	0.80	-0.22	-0.18	0.18	-0.11	0.54	-0.09	0.14	-0.01
epiS	0.85	1.00	0.43	-0.05	-0.22	0.20	0.05	0.58	-0.07	0.15	-0.01
epiImp	0.80	0.43	1.00	-0.24	-0.07	0.08	-0.24	0.35	-0.09	0.07	-0.01
epilie	-0.22	-0.05	-0.24	1.00	-0.25	0.17	0.23	-0.04	-0.22	-0.03	-0.01
epiNeur	-0.18	-0.22	-0.07	-0.25	1.00	-0.08	-0.13	-0.17	0.63	0.09	0.01
bfagree	0.18	0.20	0.08	0.17	-0.08	1.00	0.45	0.48	-0.04	0.39	-0.01
bfcon	-0.11	0.05	-0.24	0.23	-0.13	0.45	1.00	0.27	0.04	0.31	-0.01
bfext	0.54	0.58	0.35	-0.04	-0.17	0.48	0.27	1.00	0.04	0.46	-0.01
bfneur	-0.09	-0.07	-0.09	-0.22	0.63	-0.04	0.04	0.04	1.00	0.29	0.01
bfopen	0.14	0.15	0.07	-0.03	0.09	0.39	0.31	0.46	0.29	1.00	-0.01
bdi	-0.16	-0.13	-0.11	-0.20	0.58	-0.14	-0.18	-0.14	0.47	-0.08	1.00
traitanx	-0.23	-0.26	-0.12	-0.23	0.73	-0.31	-0.29	-0.39	0.59	-0.11	0.01
stateanx	-0.13	-0.12	-0.09	-0.15	0.49	-0.19	-0.14	-0.15	0.49	-0.04	0.01



Find the correlations using lowerCor

```
> lowerCor(my.data)
```

```
          epiE  epiS  epImp epili epiNr bfagr bfcon bfext bfner bfopn bdi   trtn
epiE      1.00
epiS      0.85  1.00
epiImp    0.80  0.43  1.00
epilie    -0.22 -0.05 -0.24  1.00
epiNeur   -0.18 -0.22 -0.07 -0.25  1.00
bfagree   0.18  0.20  0.08  0.17 -0.08  1.00
bfcon     -0.11  0.05 -0.24  0.23 -0.13  0.45  1.00
bfext     0.54  0.58  0.35 -0.04 -0.17  0.48  0.27  1.00
bfneur    -0.09 -0.07 -0.09 -0.22  0.63 -0.04  0.04  0.04  1.00
bfopen    0.14  0.15  0.07 -0.03  0.09  0.39  0.31  0.46  0.29  1.00
bdi       -0.16 -0.13 -0.11 -0.20  0.58 -0.14 -0.18 -0.14  0.47 -0.08  1.00
traitanx -0.23 -0.26 -0.12 -0.23  0.73 -0.31 -0.29 -0.39  0.59 -0.11  0.65  1.00
stateanx -0.13 -0.12 -0.09 -0.15  0.49 -0.19 -0.14 -0.15  0.49 -0.04  0.61  0.55
```



t.test demonstration with Student's data

```
> with(sleep,t.test(extra~group))
```

sleep

Welch Two Sample t-test

data: extra by group

t = -1.8608, df = 17.776, p-value = 0.07939

alternative hypothesis: true difference in means is not equal to zero

95 percent confidence interval:

-3.3654832 0.2054832

sample estimates:

mean in group 1 mean in group 2

0.75 2.33

extra group ID

1 0.7 1 1

2 -1.6 1 2

3 -0.2 1 3

4 -1.2 1 4

5 -0.1 1 5

6 3.4 1 6

7 3.7 1 7

... But the data were actually paired. Do it for a paired t-test

```
> with(sleep,t.test(extra~group,paired=TRUE))
```

13 1.1 2 3

Paired t-test

14 0.1 2 4

data: extra by group

15 -0.1 2 5

t = -4.0621, df = 9, p-value = 0.002833

16 4.4 2 6

alternative hypothesis: true difference in means is not equal to zero

17 5.5 2 7

95 percent confidence interval:

18 1.6 2 8

-2.4598858 -0.7001142

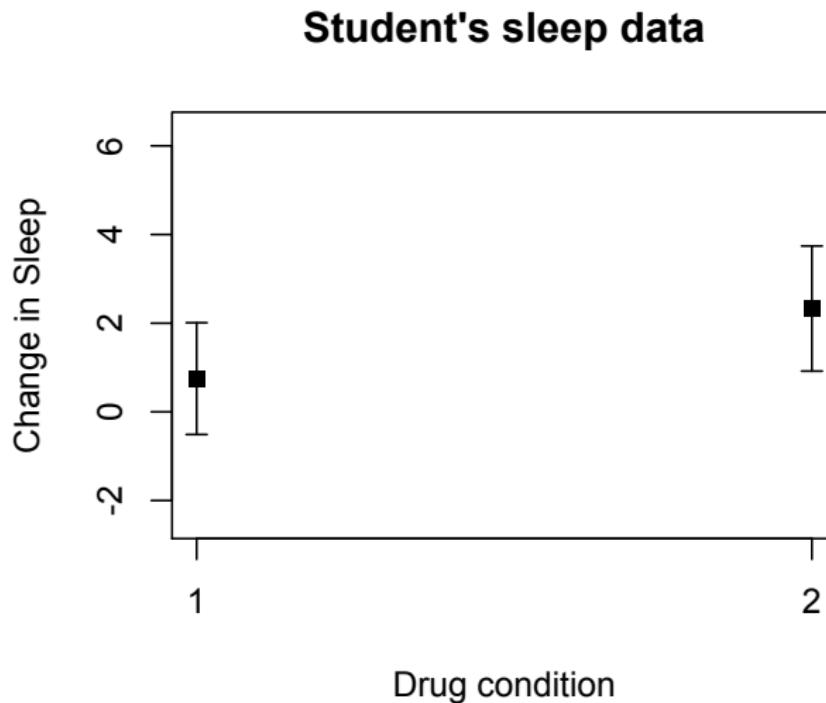
19 4.6 2 9

sample estimates:

20 3.4 2 10

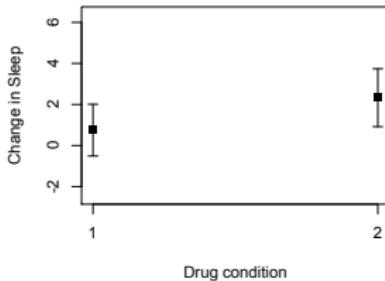
mean of the differences

Two ways of showing Student's t test data

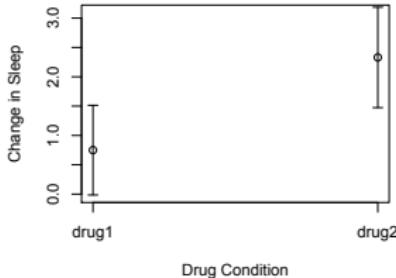


Two ways of showing Student's t test data

Student's sleep data



Student's paired sleep data



Use the `error.bars.by` and `error.bars` functions. Note that we need to change the data structure a little bit to get the within subject error bars.

```
> error.bars.by(sleep$extra,sleep$group,
  by.var=TRUE, lines=FALSE,
  ylab="Change in Sleep", xlab="Drug
  condition",main="Student's sleep data")
```

```
> error.bars(data.frame(drug1=sleep[1:10,1],
  drug2=sleep[11:20,1]), within=TRUE,
  ylab="Change in Sleep"
  ,xlab="Drug Condition",
  main="Student's paired sleep data")
```



Analysis of Variance

- ① aov is designed for balanced designs, and the results can be hard to interpret without balance: beware that missing values in the response(s) will likely lose the balance.
- ② If there are two or more error strata, the methods used are statistically inefficient without balance, and it may be better to use lme in package *nlme*.

```
datafilename="http://personality-project.org/R/datasets/R.appendix2.data"
data.ex2=read.table(datafilename,header=T)      #read the data into a table
data.ex2                                         #show the data
data.ex2                                         #show the data
Observation Gender Dosage Alertness
1           1     m     a      8
2           2     m     a     12
3           3     m     a     13
4           4     m     a     12
...
14          14    f     b     12
15          15    f     b     18
16          16    f     b     22
```



Analysis of Variance

- ① do the analysis of variances and the show the table of results

```
aov.ex2 = aov(Alertness~Gender*Dosage,data=data.ex2) #do the analysis of variance  
summary(aov.ex2) #show the summary table
```

```
> aov.ex2 = aov(Alertness~Gender*Dosage,data=data.ex2) #do the analysis of variance  
> summary(aov.ex2) #show the summary table
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Gender	1	76.562	76.562	2.9518	0.1115
Dosage	1	5.062	5.062	0.1952	0.6665
Gender:Dosage	1	0.063	0.063	0.0024	0.9617



Show the results table

```
> print(model.tables(aov.ex2, "means"), digits=3)
```

Residuals 12 311.250 25.938

Tables of means

Grand mean

14.0625

Gender

Gender

f m

16.25 11.88

Dosage

Dosage

a b

13.50 14.62

Gender:Dosage

Dosage

Gender a b

f 15.75 16.75

m 11.25 12.50



Analysis of Variance: Within subjects

- ① Somewhat more complicated because we need to convert “wide” data.frames to “long” or “narrow” data.frame.
- ② This can be done by using the stack function. Some data sets are already in the long format.
- ③ A detailed discussion of how to work with repeated measures designs is at
<http://personality-project.org/r/r.anova.html> and
at <http://personality-project.org/r>



Analysis of variance within subjects

```
> datafilename="http://personality-project.org/r/datasets/R.appendix5.data"
> data.ex5=read.table(datafilename,header=T)    #read the data into a table
> #data.ex5                                         #show the data
> aov.ex5 =
+ aov(Recall~(Task*Valence*Gender*Dosage)+Error(Subject/(Task*Valence))+ 
+ (Gender*Dosage),data.ex5)
> summary(aov.ex5)
```

Error: Subject

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Gender	1	542.26	542.26	5.6853	0.03449 *
Dosage	2	694.91	347.45	3.6429	0.05803 .
Gender:Dosage	2	70.80	35.40	0.3711	0.69760
Residuals	12	1144.56	95.38		
Signif. codes:	0	***	0.001	**	0.01
				0.05	0.1
				0.0	1

Error: Subject:Task

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Task	1	96.333	96.333	39.8621	3.868e-05 ***
Task:Gender	1	1.333	1.333	0.5517	0.4719
Task:Dosage	2	8.167	4.083	1.6897	0.2257
Task:Gender:Dosage	2	3.167	1.583	0.6552	0.5370
Residuals	12	29.000	2.417		
... (lots more)					



Multiple regression

- ① Use the sat.act data set from *psych*
- ② Do the linear model
- ③ Summarize the results

```
mod1 <- lm(SATV ~ education + gender + SATQ, data=sat.act)
> summary(mod1, digits=2)
```

Call:

```
lm(formula = SATV ~ education + gender + SATQ, data = sat.act)
```

Residuals:

Min	1Q	Median	3Q	Max
-372.91	-49.08	2.30	53.68	251.93

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	180.87348	23.41019	7.726	3.96e-14 ***
education	1.24043	2.32361	0.534	0.59363
gender	20.69271	6.99651	2.958	0.00321 **
SATQ	0.64489	0.02891	22.309	< 2e-16 ***

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 86.24 on 683 degrees of freedom

(13 observations deleted due to missingness)

Multiple R-squared: 0.4231, Adjusted R-squared: 0.4205

F-statistic: 167 on 3 and 683 DF, p-value: < 2.2e-16



Zero center the data before examining interactions

```
> zsat <- data.frame(scale(sat.act, scale=FALSE))
> mod2 <- lm(SATV ~ education * gender * SATQ, data=zsat)
> summary(mod2)
```

Call:

```
lm(formula = SATV ~ education * gender * SATQ, data = zsat)
```

Residuals:

Min	1Q	Median	3Q	Max
-372.53	-48.76	3.33	51.24	238.50

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	0.773576	3.304938	0.234	0.81500
education	2.517314	2.337889	1.077	0.28198
gender	18.485906	6.964694	2.654	0.00814 **
SATQ	0.620527	0.028925	21.453	< 2e-16 ***
education:gender	1.249926	4.759374	0.263	0.79292
education:SATQ	-0.101444	0.020100	-5.047	5.77e-07 ***
gender:SATQ	0.007339	0.060850	0.121	0.90404
education:gender:SATQ	0.035822	0.041192	0.870	0.38481

Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 0 1



Compare model 1 and model 2

Test the difference between the two linear models

```
> anova(mod1,mod2)
```

Analysis of Variance Table

Model 1: SATV ~ education + gender + SATQ

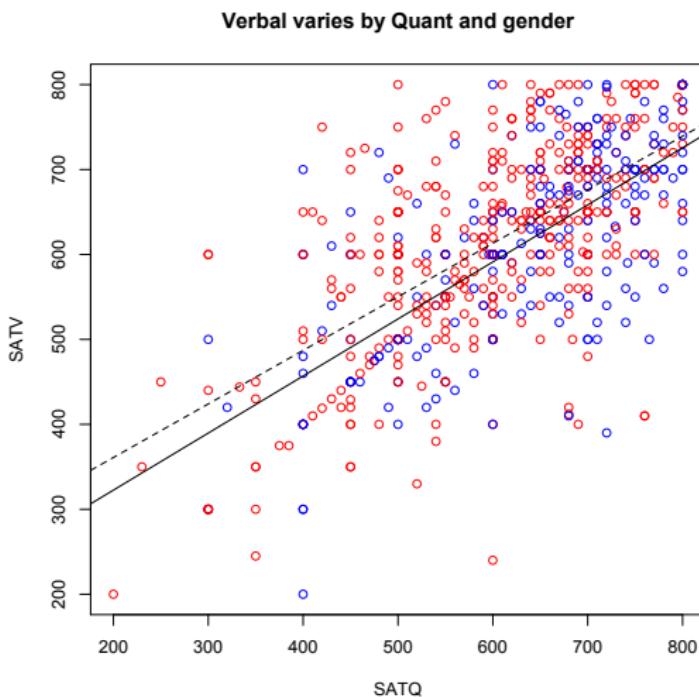
Model 2: SATV ~ education * gender * SATQ

Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
1	683	5079984			
2	679	4870243	4	209742	7.3104 9.115e-06 ***

Signif. codes: 0 ⚫***⚫ 0.001 ⚫**⚫ 0.01 ⚫*⚫ 0.05 ⚫.⚫ 0.1 ⚫



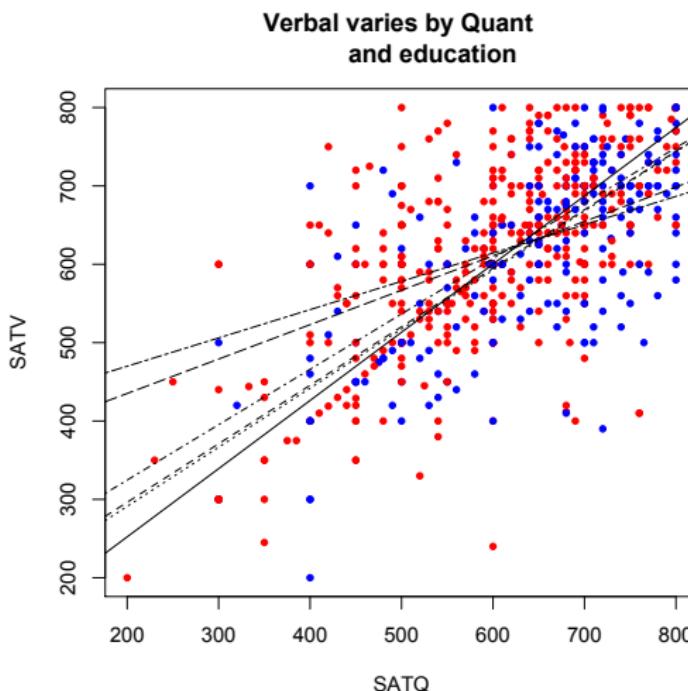
Show the regression lines by gender



```
> with(sat.act,plot(SATV~SATQ,  
+ col=c("blue","red")[gender]))  
> by(sat.act,sat.act$gender,  
+ function(x) abline  
+ (lm(SATV~SATQ,data=x),  
+ lty=c("solid","dashed"))  
> title("Verbal varies by Quant  
and gender")
```



Show the regression lines by education



```
# Show an interaction

> with(my.data,plot(SATV~SATQ,
+ col=c("blue","red")[gender]))
by(my.data,my.data$education,
function(x) abline (lm(SATV~SATQ,data=x),
lty=c("solid", "dashed", "dotted",
"dotdash", "longdash",
"twodash")[(x$education+1)]))

> title("Verbal varies by Quant
and education")
```



What is R?

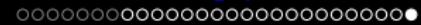


Basic descriptive and inferential statistics

A brief example



Basic statistics and graphics



► Part I: an introduction to R

► Part II: Using R for psychometrics

► Part III: Structures, Objects, Functions



Outline of Part II: Psychometrics and beyond

4 Psychometrics

- Classical Test measures of reliability
- Scoring a multiple choice test

5 Multivariate Analysis

- Factor Analysis
- Principal Components Analysis as an observed data model
- Cluster analysis of items
- Factor Extension and Set Correlation as ways of relating multiple domains

6 Structural Equation Modeling

- Confirmatory Factor Analysis
- Test invariance across groups

7 Item Response Theory

- Unifactorial IRT
- Multidimensional IRT



Classic theory estimates of reliability

① Scoring tests

`score.items` Score 1-n scales using a set of keys and finding the simple sum or average of items. Reversed items are indicated by -1

`score.multiple.choice` : Score multiple choice items by first converting to 0 or 1 and then proceeding to score the items.

② Alternative estimates of reliability

`alpha` α reliability of a single scale finds the average split half reliability. (some items may be reversed keyed).

`omega` ω_h reliability of a single scale estimates the general factor saturation of the test.

`guttman` Find the 6 Guttman reliability estimates



[Classical Test measures of reliability](#)

Using `score.items` to score 25 Big 5 items (taken from the bfi example)

```
#first create a list of items to score
> keys.list <- list(Agree=c(-1,2:5),Conscientious=c(6:8,-9,-10),Extraversion=c(-11,-12,13:15),
+ Neuroticism=c(16:20),Openness = c(21,-22,23,24,-25))
> keys <- make.keys(28,keys.list,item.labels=colnames(bfi)) #create the keys list
> bfi.scores <- score.items(keys,bfi) #use this list to score the items
> bfi.scores #show the statistics

Call: score.items(keys = keys, items = bfi)
```

(Unstandardized) Alpha:

	Agree	Conscientious	Extraversion	Neuroticism	Openness
alpha	0.7	0.72	0.76	0.81	0.6

Average item correlation:

	Agree	Conscientious	Extraversion	Neuroticism	Openness
average.r	0.32	0.34	0.39	0.46	0.23

Guttman 6* reliability:

	Agree	Conscientious	Extraversion	Neuroticism	Openness
Lambda.6	0.7	0.72	0.76	0.81	0.6

Scale intercorrelations corrected for attenuation

raw correlations below the diagonal, alpha on the diagonal
corrected correlations above the diagonal:

	Agree	Conscientious	Extraversion	Neuroticism	Openness
Agree	0.70	0.36	0.63	-0.245	0.23
Conscientious	0.26	0.72	0.35	-0.305	0.30
Extraversion	0.46	0.26	0.76	-0.284	0.32
Neuroticism	-0.18	-0.23	-0.22	0.812	-0.12
Openness	0.15	0.19	0.22	-0.086	0.60



score.items output, continued

Item by scale correlations:

corrected for item overlap and scale reliability

	Agree	Conscientious	Extraversion	Neuroticism	Openness
A1	-0.40	-0.06	-0.11	0.14	-0.14
A2	0.67	0.23	0.40	-0.07	0.17
A3	0.70	0.22	0.48	-0.11	0.17
A4	0.49	0.29	0.30	-0.14	0.01
A5	0.62	0.23	0.55	-0.23	0.18
C1	0.13	0.53	0.19	-0.08	0.28
C2	0.21	0.61	0.17	0.00	0.20
C3	0.21	0.54	0.14	-0.09	0.08
C4	-0.24	-0.66	-0.23	0.31	-0.23
C5	-0.26	-0.59	-0.29	0.36	-0.10
E1	-0.30	-0.06	-0.59	0.11	-0.16
E2	-0.39	-0.25	-0.70	0.34	-0.15
E3	0.44	0.20	0.60	-0.10	0.37
E4	0.51	0.23	0.68	-0.22	0.04
E5	0.34	0.40	0.55	-0.10	0.31
N1	-0.22	-0.21	-0.11	0.76	-0.12
N2	-0.22	-0.19	-0.12	0.74	-0.06
N3	-0.14	-0.20	-0.14	0.74	-0.03
N4	-0.22	-0.30	-0.39	0.62	-0.02
N5	-0.04	-0.14	-0.19	0.55	-0.18
O1	0.16	0.20	0.31	-0.09	0.52
O2	-0.01	-0.18	-0.07	0.19	-0.45
O3	0.26	0.20	0.42	-0.07	0.61
O4	0.06	-0.02	-0.10	0.21	0.32
O5	-0.09	-0.14	-0.11	0.11	-0.53
gender	0.25	0.11	0.12	0.14	-0.07
education	0.06	0.03	0.01	-0.06	0.13
age	0.22	0.14	0.07	-0.13	0.10



Comment about the output from a function

- ➊ Many functions produce far more output than you normally want
 - The package developer typically has decided what is most informative
 - This will be shown by displaying the function's output
 - Sometimes you will want to `summary(x)` the function output
- ➋ To access the other elements of the results you can
 - Look at the help file for the function where the output is discussed.
 - Or, just look at the `str` of the result.
- ➌ For instance, the `score.items` function reports frequently used statistics, but hides the actual scores.
 - To access an element of an object (e.g. the scores of `bfi.scores`), use the `$` or the `[[name]]` feature.



[Classical Test measures of reliability](#)

The structure of an object

```
> str(bfi.scores)

List of 13
 $ scores      : num [1:2800, 1:5] 20 20.2 19.8 20.6 20 ...
   ..- attr(*, "dimnames")=List of 2
   ... $ : chr [1:2800] "61617" "61618" "61620" "61621" ...
   ... $ : chr [1:5] "Agree" "Conscientious" "Extraversion" "Neuroticism" ...
 $ missing     : num [1:2800, 1:5] 0 0 0 0 0 0 0 0 0 0 ...
   ..- attr(*, "dimnames")=List of 2
   ... $ : chr [1:2800] "61617" "61618" "61620" "61621" ...
   ... $ : chr [1:5] "Agree" "Conscientious" "Extraversion" "Neuroticism" ...
 $ alpha       : num [1, 1:5] 0.701 0.725 0.76 0.812 0.597
   ..- attr(*, "dimnames")=List of 2
   ... $ : chr "alpha"
   ... $ : chr [1:5] "Agree" "Conscientious" "Extraversion" "Neuroticism" ...
 $ av.r        : num [1, 1:5] 0.319 0.345 0.387 0.463 0.228
   ..- attr(*, "dimnames")=List of 2
   ... $ : chr "average.r"
   ... $ : chr [1:5] "Agree" "Conscientious" "Extraversion" "Neuroticism" ...
 $ n.items     : Named num [1:5] 5 5 5 5 5
   ..- attr(*, "names")= chr [1:5] "Agree" "Conscientious" "Extraversion" "Neuroticism" ...
 $ item.cor    : num [1:28, 1:5] -0.577 0.726 0.759 0.654 0.686 ...
   ..- attr(*, "dimnames")=List of 2
   ... $ : chr [1:28] "A1" "A2" "A3" "A4" ...
   ... $ : chr [1:5] "Agree" "Conscientious" "Extraversion" "Neuroticism" ...
 $ cor          : num [1:5, 1:5] 1 0.258 0.462 -0.185 0.147 ...
   ..- attr(*, "dimnames")=List of 2
   ... $ : chr [1:5] "Agree" "Conscientious" "Extraversion" "Neuroticism" ...
   ... $ : chr [1:5] "Agree" "Conscientious" "Extraversion" "Neuroticism" ...
 $ corrected   : num [1:5, 1:5] 0.701 0.258 0.462 -0.185 0.147 ...
   ..- attr(*, "dimnames")=List of 2
   ... $ : chr [1:5] "Agree" "Conscientious" "Extraversion" "Neuroticism" ...
   ... $ : chr [1:5] "Agree" "Conscientious" "Extraversion" "Neuroticism" ...
```



Statistics on the scores from score.items

```
scores <- bfi.scores$scores      #one way to address them  
scores <- bfi.scores[["scores"]]      #another way to address the elements of a  
describe(scores)
```

	var	n	mean	sd	median	trimmed	mad	min	max	range	skew	kurt
Agree	1	2800	20.65	0.89	20.8	20.73	0.89	17.0	22	5.0	-0.77	-
Conscientious	2	2800	36.27	0.95	36.4	36.31	0.89	33.0	38	5.0	-0.41	-
Extraversion	3	2800	36.15	1.05	36.2	36.20	1.19	33.0	38	5.0	-0.48	-
Neuroticism	4	2800	3.16	1.19	3.0	3.13	1.19	1.0	6	5.0	0.22	-
Openness	5	2800	36.59	0.80	36.6	36.62	0.89	33.2	38	4.8	-0.34	-



Something is wrong with the scores!

The means for the scales look strange. This is because of the way items are reversed scored.

- ① `score.items` reverses items
 - to reverse, it subtracts item from $(\text{max} - \text{min}) + 1$
 - but for the bfi, the data include age and thus the max and min are incorrect.
- ② Can specify the maximum and minimum for the items to be used when reversing
 - (This is a reason to read the help file for each function!)
- ③ Reversing with the wrong minimum and maximum just affects the mean scores, not the scale reliabilities or intercorrelations



Score the items again, setting the min to 1, max to 6

Mean scores are in units of the items. If different scales have different number of items, this does not affect the mean scores, but does affect total scores.

```
bfi.scores <- score.items(keys,bfi,min=1,max=6)
describe(bfi.scores$scores)
```

	var	n	mean	sd	median	trimmed	mad	min	max	range	skew	kurtos
Agree	1	2800	4.65	0.89	4.8	4.73	0.89	1.0	6	5.0	-0.77	0.
Conscientious	2	2800	4.27	0.95	4.4	4.31	0.89	1.0	6	5.0	-0.41	-0.
Extraversion	3	2800	4.15	1.05	4.2	4.20	1.19	1.0	6	5.0	-0.48	-0.
Neuroticism	4	2800	3.16	1.19	3.0	3.13	1.19	1.0	6	5.0	0.22	-0.
Openness	5	2800	4.59	0.80	4.6	4.62	0.89	1.2	6	4.8	-0.34	-0.



Units of the scale

- ① Some people like to report scores as sum scores, others as mean scores
 - Sum scores are simple to find, but reflect the number of items on the scale. This can be confusing when comparing scores from alternative versions of a scale.
 - Mean scores are in the metric of the items.
- ② Different subfields of psychology seem to prefer one or the other. Many personality and clinical psychologists use total scores. Others of us prefer mean scores.
 - Beck Depression scores range from 0 - 60+
 - STAI Anxiety scores from 20-80
 - EPI extraversion from 0-24
- ③ But mean scores are more informative
- ④ `score.items` defaults to means, but will report totals if desired.
 - This is just one more example of the flexibility of functions.
 - As well as the need to read the help files!



Classical Test measures of reliability

Score for total scores

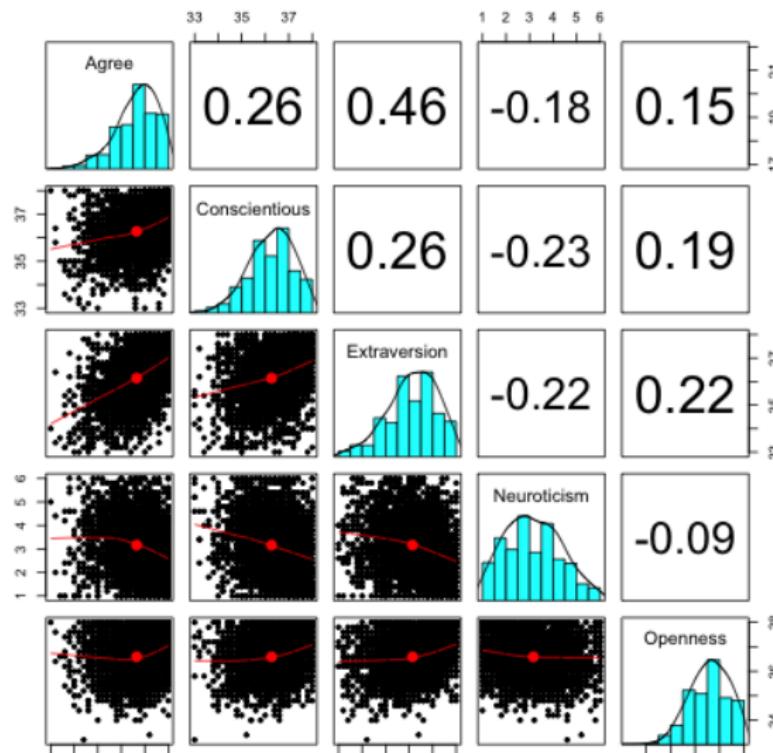
```
> bfi.totals <- score.items(keys,bfi,min=1,max=6,totals=TRUE)
> describe(bfi.totals[["scores"]])
```

	var	n	mean	sd	median	trimmed	mad	min	max	range	skew	kurtosis
Agree	1	2800	23.27	4.47	24	23.63	4.45	5	30	25	-0.77	0
Conscientious	2	2800	21.35	4.74	22	21.55	4.45	5	30	25	-0.41	-0
Extraversion	3	2800	20.73	5.27	21	21.02	5.93	5	30	25	-0.48	-0
Neuroticism	4	2800	15.81	5.93	15	15.64	5.93	5	30	25	0.22	-0
Openness	5	2800	22.95	4.02	23	23.10	4.45	6	30	24	-0.34	-0



Show the pairs.panels result of the big 5 scores

pairs.panels(scores)



[Scoring a multiple choice test](#)

Score a multiple score test

Using `score.multiple.choice` we can either just find item and scale statistics, or convert the items to correct/incorrect and then use other functions for further analysis.

```
data(iqitems)
iq.keys <- c(4,4,4, 6, 6,3,4,4, 5,2,2,4, 3,2,6,7) #what are the right answers
score.multiple.choice(iq.keys,iqitems) #get the item responses and alpha reliability
```

```
> score.multiple.choice(iq.keys,iqitems) #this just gives summary statistics
Call: score.multiple.choice(key = iq.keys, data = iqitems)
```

```
(Unstandardized) Alpha:
[1] 0.84
```

```
Average item correlation:
[1] 0.25
```

item statistics

	key	0	1	2	3	4	5	6	7	8	miss	r	n	mean	sd	skew	kurtosis	se
reason.4	4	0.05	0.05	0.11	0.10	0.64	0.03	0.02	0.00	0.00	0	0.59	1523	0.64	0.48	-0.58	-1.66	0.01
reason.16	4	0.04	0.06	0.08	0.10	0.70	0.01	0.00	0.00	0.00	0	0.53	1524	0.70	0.46	-0.86	-1.26	0.01
reason.17	4	0.05	0.03	0.05	0.03	0.70	0.03	0.11	0.00	0.00	0	0.59	1523	0.70	0.46	-0.86	-1.26	0.01
reason.19	6	0.04	0.02	0.13	0.03	0.06	0.10	0.62	0.00	0.00	0	0.56	1523	0.62	0.49	-0.47	-1.78	0.01
letter.7	6	0.05	0.01	0.05	0.03	0.11	0.14	0.60	0.00	0.00	0	0.58	1524	0.60	0.49	-0.41	-1.84	0.01
letter.33	3	0.06	0.10	0.13	0.57	0.04	0.09	0.02	0.00	0.00	0	0.56	1523	0.57	0.50	-0.29	-1.92	0.01
letter.34	4	0.04	0.09	0.07	0.11	0.61	0.05	0.02	0.00	0.00	0	0.59	1523	0.61	0.49	-0.46	-1.79	0.01
letter.58	4	0.06	0.14	0.09	0.09	0.44	0.16	0.01	0.00	0.00	0	0.58	1525	0.44	0.50	0.23	-1.95	0.01
matrix.45	5	0.04	0.01	0.06	0.14	0.18	0.53	0.04	0.00	0.00	0	0.51	1523	0.53	0.50	-0.10	-1.99	0.01
matrix.46	2	0.04	0.12	0.55	0.07	0.11	0.06	0.05	0.00	0.00	0	0.52	1524	0.55	0.50	-0.20	-1.96	0.01
matrix.47	2	0.04	0.05	0.61	0.07	0.11	0.06	0.06	0.00	0.00	0	0.55	1523	0.61	0.49	-0.47	-1.78	0.01
matrix.55	4	0.04	0.02	0.18	0.14	0.37	0.07	0.18	0.00	0.00	0	0.45	1524	0.37	0.48	0.52	-1.73	0.01



Scoring a multiple choice test

Convert the items to correct and incorrect

```
> iq.scrub <- scrub(iqitems,isvalue=0) #first get rid of the zero responses
> iq.tf <- score.multiple.choice(iq.keys,iq.scrub,score=FALSE) #convert to wrong (0) and correct (1) for analysis
```

	var	n	mean	sd	median	trimmed	mad	min	max	range	skew	kurtosis	se
reason.4	1	1442	0.68	0.47	1	0.72	0	0	1	1	-0.75	-1.44	0.01
reason.16	2	1463	0.73	0.45	1	0.78	0	0	1	1	-1.02	-0.96	0.01
reason.17	3	1440	0.74	0.44	1	0.80	0	0	1	1	-1.08	-0.84	0.01
reason.19	4	1456	0.64	0.48	1	0.68	0	0	1	1	-0.60	-1.64	0.01
letter.7	5	1441	0.63	0.48	1	0.67	0	0	1	1	-0.56	-1.69	0.01
letter.33	6	1438	0.61	0.49	1	0.63	0	0	1	1	-0.43	-1.82	0.01
letter.34	7	1455	0.64	0.48	1	0.68	0	0	1	1	-0.59	-1.65	0.01
letter.58	8	1438	0.47	0.50	0	0.46	0	0	1	1	0.12	-1.99	0.01
matrix.45	9	1458	0.55	0.50	1	0.56	0	0	1	1	-0.20	-1.96	0.01
matrix.46	10	1470	0.57	0.50	1	0.59	0	0	1	1	-0.28	-1.92	0.01
matrix.47	11	1465	0.64	0.48	1	0.67	0	0	1	1	-0.57	-1.67	0.01
matrix.55	12	1459	0.39	0.49	0	0.36	0	0	1	1	0.45	-1.80	0.01
rotate.3	13	1456	0.20	0.40	0	0.1`3	0	0	1	1	1.48	0.19	0.01
rotate.4	14	1460	0.22	0.42	0	0.15	0	0	1	1	1.34	-0.21	0.01
rotate.6	15	1456	0.31	0.46	0	0.27	0	0	1	1	0.80	-1.35	0.01
rotate.8	16	1460	0.19	0.39	0	0.12	0	0	1	1	1.55	0.41	0.01



Scoring a multiple choice test

Just give me alpha, I don't know any better

For the user who wants to know just the alpha of a set of items and is used to SPSS output, the alpha function is provided.

Better alternatives include the guttman function which provides more information.

```
alpha(iq.tf)
```

Reliability analysis
Call: alpha(x = iq.tf)

	raw_alpha	std.alpha	G6(smc)	average_r	mean	sd
	0.83	0.83	0.84	0.23	0.49	0.25

```
alpha(iq.tf)
```

Item statistics

	n	r	r.cor	r.drop	mean	sd
reason.4	1442	0.58	0.54	0.50	0.68	0.47
reason.16	1463	0.50	0.44	0.41	0.73	0.45
reason.17	1440	0.57	0.54	0.49	0.74	0.44
reason.19	1456	0.52	0.47	0.43	0.64	0.48
letter.7	1441	0.56	0.52	0.48	0.63	0.48
letter.33	1438	0.53	0.48	0.44	0.61	0.49
letter.34	1455	0.57	0.53	0.49	0.64	0.48
letter.58	1438	0.57	0.52	0.48	0.47	0.50
matrix.45	1458	0.48	0.42	0.38	0.55	0.50
matrix.46	1470	0.49	0.43	0.40	0.57	0.50
matrix.47	1465	0.52	0.47	0.43	0.64	0.48
matrix.55	1459	0.42	0.35	0.32	0.39	0.49
rotate.3	1456	0.54	0.51	0.44	0.20	0.40
rotate.4	1460	0.58	0.56	0.48	0.22	0.42
rotate.6	1456	0.56	0.53	0.46	0.31	0.46
rotate.8	1460	0.51	0.47	0.41	0.19	0.39



Multivariate data reduction and description

A recurring theme in personality research is the description of personality items (be they adjectives or short questions), in terms of a limited number of higher order dimensions. These are typically identified through factor analysis, principal components analysis, or cluster analysis. All of these procedures are straightforward in R.

- ① Exploratory factor analysis: a latent trait model
 - Items are assumed to represent the influence of unobserved (latent) variables.
 - Issues are the means of extraction, the number of factors to extract, the rotations to use, the estimation of factor scores.
 - Factor scores are *estimated*
- ② Confirmatory factor analysis: a latent trait model
 - (discussed under structural equation modeling) the typical model is one of a cluster structure with items loading on one and only one factor.
 - This assumption is probably not appropriate, and rotational techniques for complexity > 1 are available.



Multivariate data reduction and description: 2

- ① Principal Components analysis: an observed variable model
 - Components are defined as sums of observed variables.
 - Component scores may be calculated as weighted sums, not *estimated* as is necessary for factor scores.
 - Components include measurement error as part of the score.
- ② Cluster analysis, although usually applied to clustering of objects (people), may be applied to clustering of items.
 - Some algorithms take reliability into account (correct for attenuation), and thus implicitly become latent variable models.



There are several ways to do factor analysis in R

- ① factanal from core R
 - Maximum likelihood factor analysis
- ② fa and fa.poly from *psych* (replacing factor.pa, fa.wls)
 - data input = A correlation matrix or a raw data matrix. If raw data, the correlation matrix will be found using pairwise deletion.
 - factor method = factoring method fm="minres" will do a minimum residual (OLS), fm="wls" will do a weighted least squares (WLS) solution, fm="gls" does a generalized weighted least squares (GLS), fm="pa" will do the principal factor solution, fm="ml" will do a maximum likelihood factor analysis
 - rotation method = "none", "varimax", "quartimax", "bentlerT", and "geominT" are orthogonal rotations. "promax", "oblimin", "simplimax", "bentlerQ", and "geominQ" or "cluster" are possible rotations or transformations of the solution. The default is to do a oblimin transformation.
 - Confidence intervals may be found by bootstrapping multiple solutions.



The number of factors problem

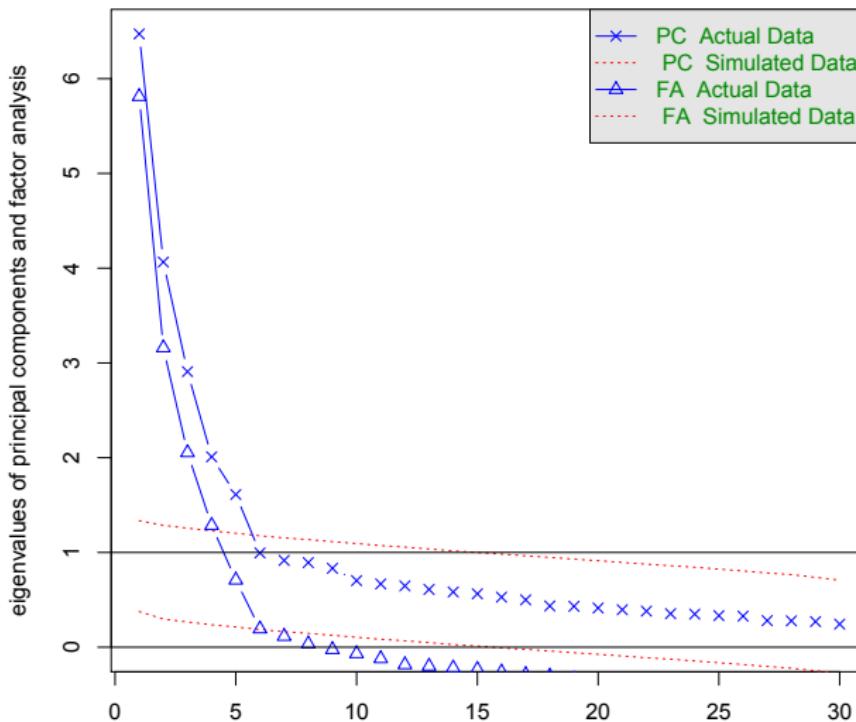
"It is easy to solve the number of factors problem, I do it everyday before breakfast. The problem is what is the right answer "
(attributed to Henry Kaiser)

- ① χ^2 tests (either of n factor solution or of change from n-1 to n factors)
 - Sensitive to sample size.
 - Larger samples have more significant factors
- ② Scree test
 - Generally good, sometimes hard to identify break in scree
- ③ Parallel analysis (compare to random data)
 - Factors and components give different solutions
- ④ Very Simple Structure
 - Works well with items of complexity 1 or 2
- ⑤ Minimum Average Partial
- ⑥ Eigen values > 1
 - Perhaps the uniformly agreed worst test



Parallel analysis of 30 NEO facets

Parallel analysis of 30 neo facets items



Very Simple Structure and Velicer's Map criterion

```
> VSS(bfi[1:25],title="Very Simple Structure of 25 Big 5 items")
```

Very Simple Structure of 25 Big 5 items

Call: VSS(x = bfi[1:25], title = "Very Simple Structure of 25 Big 5 items")

VSS complexity 1 achieves a maximum of 0.58 with 4 factors

VSS complexity 2 achieves a maximum of 0.74 with 4 factors

The Velicer MAP criterion achieves a minimum of 0.01 with 5 factors

Velicer MAP

```
[1] 0.02 0.02 0.02 0.02 0.01 0.02 0.02 0.02
```

Very Simple Structure Complexity 1

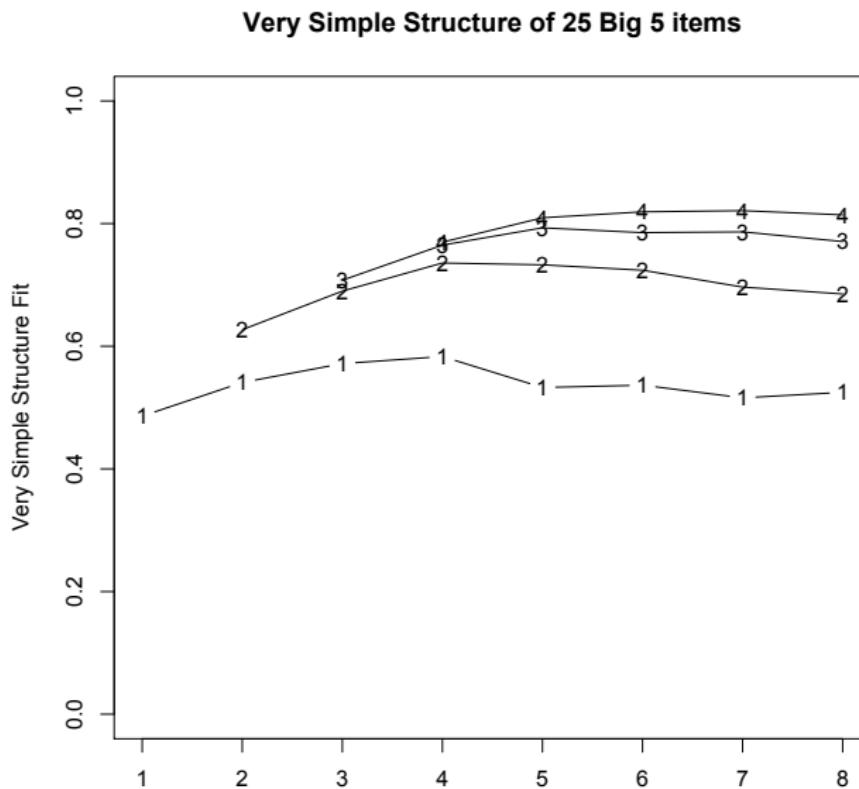
```
[1] 0.49 0.54 0.57 0.58 0.53 0.54 0.52 0.52
```

Very Simple Structure Complexity 2

```
[1] 0.00 0.63 0.69 0.74 0.73 0.72 0.70 0.69
```



Very Simple Structure and Velicer's Map



Factor Analysis

Factor analysis of Thurstone 9 variable problem

```
> f3 <- fa(Thurstone,3,n.obs=213) #we want a 3 factor solution, otherwise, use the defaults
> f3
```

Factor Analysis using method = minres
 Call: fa(r = Thurstone, nfactors = 3, n.obs = 213)
 Standardized loadings (pattern matrix) based upon correlation matrix

	MR1	MR2	MR3	h2	u2
Sentences	0.91	-0.04	0.04	0.82	0.18
Vocabulary	0.89	0.06	-0.03	0.84	0.16
Sent.Completion	0.83	0.04	0.00	0.73	0.27
First.Letters	0.00	0.86	0.00	0.73	0.27
4.Letter.Words	-0.01	0.74	0.10	0.63	0.37
Suffixes	0.18	0.63	-0.08	0.50	0.50
Letter.Series	0.03	-0.01	0.84	0.72	0.28
Pedigrees	0.37	-0.05	0.47	0.50	0.50
Letter.Group	-0.06	0.21	0.64	0.53	0.47

	MR1	MR2	MR3
SS loadings	2.64	1.86	1.50
Proportion Var	0.29	0.21	0.17
Cumulative Var	0.29	0.50	0.67
Proportion Explained	0.44	0.31	0.25
Cumulative Proportion	0.44	0.75	1.00

With factor correlations of

	MR1	MR2	MR3
MR1	1.00	0.59	0.54
MR2	0.59	1.00	0.52
MR3	0.54	0.52	1.00
...			



Factor analysis output, continued

Test of the hypothesis that 3 factors are sufficient.

The degrees of freedom for the null model are 36 and the objective function was 5.2 with Chi Square of 1081.97

The degrees of freedom for the model are 12 and the objective function was 0.01

The root mean square of the residuals is 0

The df corrected root mean square of the residuals is 0.01

The number of observations was 213 with Chi Square = 2.82 with prob < 1

Tucker Lewis Index of factoring reliability = 1.027

RMSEA index = 0 and the 90 % confidence intervals are 0 0.023

BIC = -61.51

Fit based upon off diagonal values = 1

Measures of factor score adequacy

MR1 MR2 MR3

Correlation of scores with factors 0.96 0.92 0.90

Multiple R square of scores with factors 0.93 0.85 0.81

Minimum correlation of possible factor scores 0.86 0.71 0.63



Bootstrapped confidence intervals

```
> f3 <- fa(Thurstone, 3, n.obs=213, n.iter=20) #to do bootstrapping
```

Coefficients and bootstrapped confidence intervals

	low	MR1	upper	low	MR2	upper	low	MR3	upper
Sentences	0.80	0.91	0.96	-0.10	-0.04	0.04	-0.02	0.04	0.13
Vocabulary	0.77	0.89	0.94	0.01	0.06	0.16	-0.10	-0.03	0.07
Sent.Completion	0.73	0.83	0.92	-0.06	0.04	0.11	-0.09	0.00	0.09
First.Letters	-0.06	0.00	0.10	0.68	0.86	0.93	-0.08	0.00	0.10
4.Letter.Words	-0.13	-0.01	0.10	0.58	0.74	0.84	0.03	0.10	0.21
Suffixes	0.00	0.18	0.34	0.49	0.63	0.76	-0.19	-0.08	0.03
Letter.Series	-0.04	0.03	0.12	-0.12	-0.01	0.11	0.53	0.84	0.96
Pedigrees	0.26	0.37	0.52	-0.17	-0.05	0.07	0.26	0.47	0.61
Letter.Group	-0.19	-0.06	0.05	0.07	0.21	0.35	0.43	0.64	0.79

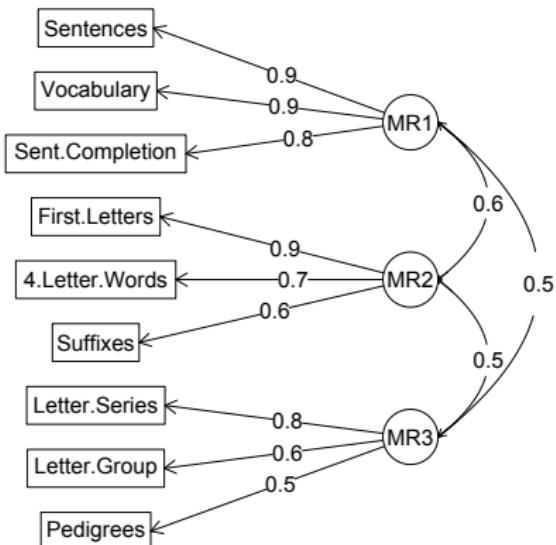
Interfactor correlations and bootstrapped confidence intervals

	lower	estimate	upper
1	0.39	0.59	0.63
2	0.34	0.54	0.59
3	0.32	0.52	0.56



The simple factor structure (pattern) may be shown graphically

Factor Analysis



Factor score estimates are found by default

- ① Because of issues of factor score indeterminacy, these are estimated factor scores.
- ② The correlation between these estimates and the factors is reported. ($R^2 = \text{diag}(\mathbf{WF})$)
- ③ There are multiple ways of estimating factor scores. All are based upon \mathbf{WX}' where \mathbf{C} is the covariance matrix of the raw scores (\mathbf{X}) and the \mathbf{W} matrix is found by
 - regression: $\mathbf{W} = \mathbf{F}'\mathbf{C}^{-1}$
 - Bartlett: $\mathbf{W} = \mathbf{U}^{-2}\mathbf{F}(\mathbf{F}'\mathbf{U}^{-2}\mathbf{F})^{-1}$.
 - TenBerge: let $\mathbf{L} = \mathbf{F}\Phi^{1/2}$, and $\mathbf{D} = \mathbf{R}^{1/2}\mathbf{L}(\mathbf{L}'\mathbf{C}^{-1}\mathbf{L})^{-1/2}$, then $\mathbf{W} = \mathbf{C}^{-1/2}\mathbf{D}\Phi^{1/2}$



Analyzing the higher order structure: the ω coefficients

- ① If items or scales intercorrelate, they in turn may be factored.
 - The effect of these higher order factors may be found on the lowest level variables and then removed from the first level factors.
 - The debate about the “general factor of personality” hinges on this method.
 - Higher order factors may be found using exploratory or confirmatory procedures.
- ② `omega` is an exploratory hierarchical factoring function to find
 - ω_h (hierarchical), an estimate of the general factor of a test
 - ω_t , an estimate of the reliable variance in a test
- ③ `omega.sem` will do a confirmatory analysis based upon the simple cluster structure found by `omega`
 - CFA solutions based upon a simple cluster structure will overestimate the general factor by not identifying all the cross loadings.



omega analysis of the Thurstone problem.

```
> omega(Thurstone, n.obs=213) #defaults to 3 factors
```

```
Call: omega(m = Thurstone, nfactors = 3, n.obs = 213)
```

Alpha: 0.89

G.6: 0.91

Omega Hierarchical: 0.74

Omega H asymptotic: 0.79

Omega Total 0.93

Schmid Leiman Factor loadings greater than 0.2

	g	F1*	F2*	F3*	h2	u2	p2
Sentences	0.71	0.57			0.82	0.18	0.61
Vocabulary	0.73	0.55			0.84	0.16	0.63
Sent.Completion	0.68	0.52			0.73	0.27	0.63
First.Letters	0.65		0.56		0.73	0.27	0.57
4.Letter.Words	0.62		0.49		0.63	0.37	0.61
Suffixes	0.56		0.41		0.50	0.50	0.63
Letter.Series	0.59			0.61	0.72	0.28	0.48
Pedigrees	0.58	0.23			0.34	0.50	0.50
Letter.Group	0.54			0.46	0.53	0.47	0.56

With eigenvalues of:

g	F1*	F2*	F3*
3.58	0.96	0.74	0.71



omega output continued

With eigenvalues of:

g	F1*	F2*	F3*
3.58	0.96	0.74	0.71

general/max 3.71 max/min = 1.35

mean percent general = 0.6 with sd = 0.05 and cv of 0.09

The degrees of freedom are 12 and the fit is 0.01

The number of observations was 213 with Chi Square = 2.82 with prob < 1

The root mean square of the residuals is 0

The df corrected root mean square of the residuals is 0.01

RMSEA index = 0 and the 90 % confidence intervals are 0 0.023

BIC = -61.51

Compare this with the adequacy of just a general factor and no group factors

The degrees of freedom for just the general factor are 27 and the fit is 1.48

The number of observations was 213 with Chi Square = 307.1 with prob < 2.8e-49

The root mean square of the residuals is 0.1

The df corrected root mean square of the residuals is 0.16

RMSEA index = 0.224 and the 90 % confidence intervals are 0.223 0.226

BIC = 162.35

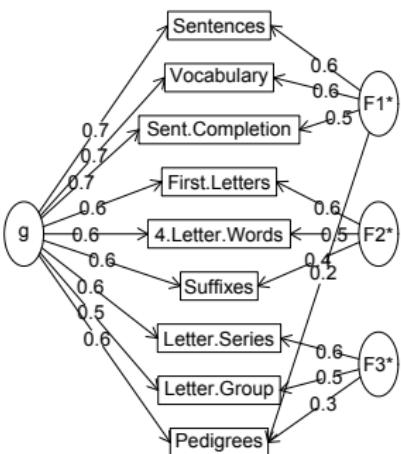
Measures of factor score adequacy

	g	F1*	F2*	F3*
Correlation of scores with factors	0.86	0.73	0.72	0.75
Multiple R square of scores with factors	0.74	0.54	0.52	0.56
Minimum correlation of factor score estimates	0.49	0.08	0.03	0.11

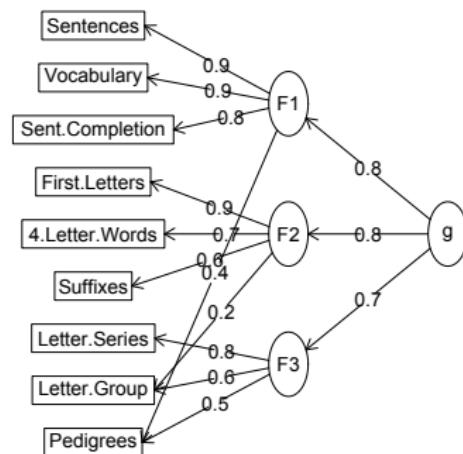


Two ways of viewing the higher order structure

Omega



Hierarchical (multilevel) Structure



Factor Analysis

Omega analysis of the iq data

```
> omega(iq.tf,4,title="Omega of ICAR 16 ability items) #specify 4 lower level factors
```

Omega of ICAR 16 ability items

Call: omega(m = iq.tf, nfactors = 4, title = "Omega of ICAR 16 ability items")

Alpha: 0.83

G.6: 0.84

Omega Hierarchical: 0.65

Omega H asymptotic: 0.76

Omega Total 0.86

Schmid Leiman Factor loadings greater than 0.2

	g	F1*	F2*	F3*	F4*	h2	u2	p2
reason.4	0.50			0.27		0.34	0.66	0.73
reason.16	0.42			0.21		0.23	0.77	0.76
reason.17	0.55			0.47		0.52	0.48	0.57
reason.19	0.44			0.21		0.25	0.75	0.77
letter.7	0.52	0.35				0.39	0.61	0.69
letter.33	0.46	0.30				0.31	0.69	0.70
letter.34	0.54	0.38				0.43	0.57	0.67
letter.58	0.47	0.20				0.28	0.72	0.78
matrix.45	0.40			0.66	0.59	0.41	0.27	
matrix.46	0.40			0.26	0.24	0.76	0.65	
matrix.47	0.42				0.23	0.77	0.79	
matrix.55	0.28			0.12	0.88	0.65		
rotate.3	0.36	0.61			0.50	0.50	0.26	
rotate.4	0.41	0.61			0.54	0.46	0.31	
rotate.6	0.40	0.49			0.41	0.59	0.39	
rotate.8	0.32	0.53			0.40	0.60	0.26	

With eigenvalues of:

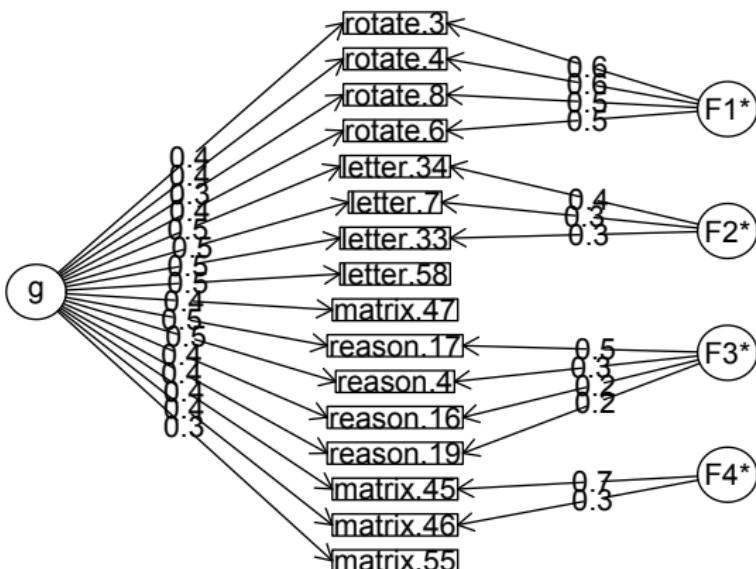
g	F1*	F2*	F3*	F4*
---	-----	-----	-----	-----

3.04	1.32	0.46	0.42	0.55
------	------	------	------	------



Omega of ICAR 16 ability items

Omega of ICAR 16 ability items



Principal Components Analysis as an observed data model

Principal Components Analysis is an observed data model

```
> principal(Thurstone,3,n.obs=213) #ask for 3 components
```

Principal Components Analysis

Call: principal(r = Thurstone, nfactors = 3, n.obs = 213)

Standardized loadings based upon correlation matrix

	RC1	RC2	RC3	h2	u2
Sentences	0.86	0.24	0.23	0.86	0.14
Vocabulary	0.85	0.31	0.19	0.86	0.14
Sent.Completion	0.85	0.26	0.19	0.83	0.17
First.Letters	0.23	0.82	0.23	0.78	0.22
4.Letter.Words	0.18	0.79	0.30	0.75	0.25
Suffixes	0.31	0.77	0.06	0.70	0.30
Letter.Series	0.25	0.16	0.83	0.78	0.22
Pedigrees	0.53	0.08	0.61	0.67	0.33
Letter.Group	0.10	0.31	0.80	0.75	0.25

	RC1	RC2	RC3
--	-----	-----	-----

SS loadings	2.73	2.25	1.99
-------------	------	------	------

Proportion Var	0.30	0.25	0.22
----------------	------	------	------

Cumulative Var	0.30	0.55	0.78
----------------	------	------	------

Test of the hypothesis that 3 factors are sufficient.

The degrees of freedom for the null model are 36 and the objective function was 127.9.

The degrees of freedom for the model are 12 and the objective function was 127.9.

The number of observations was 213 with Chi Square = 127.9 with prob < 1.6e-10.

Fit based upon off diagonal values = 0.98



Cluster analysis of items

Cluster analysis as an alternative to factor analysis and principal components analysis

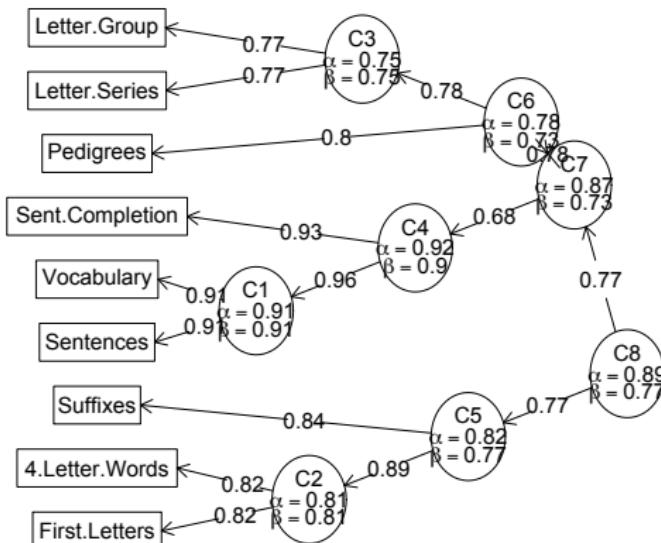
- ① An alternative to factor analysis for dimensional reduction is cluster analysis
 - The iclust algorithm was developed for clustering items based upon basic psychometric principals
- ② Procedure
 - ① Find the correlation matrix
 - ② Identify the most similar pair of items (correcting for attenuation)
 - ③ Combine them.
 - ④ Repeat steps 1-3 until β (the worst split half reliability) fails to increase.
 - ⑤ As an alternative, a specified number of clusters may be extracted.



A hierarchical cluster structure found by iclust

iclust(Thurstone)

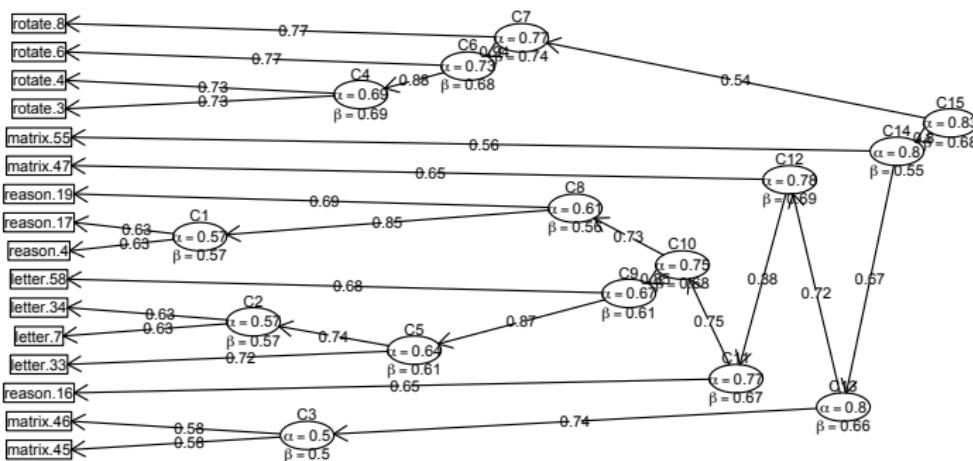
iclust



A hierarchical cluster structure of 16 ability items using iclust

iclust(iq.tf)

ICLUST of the ICAR 16 iq items

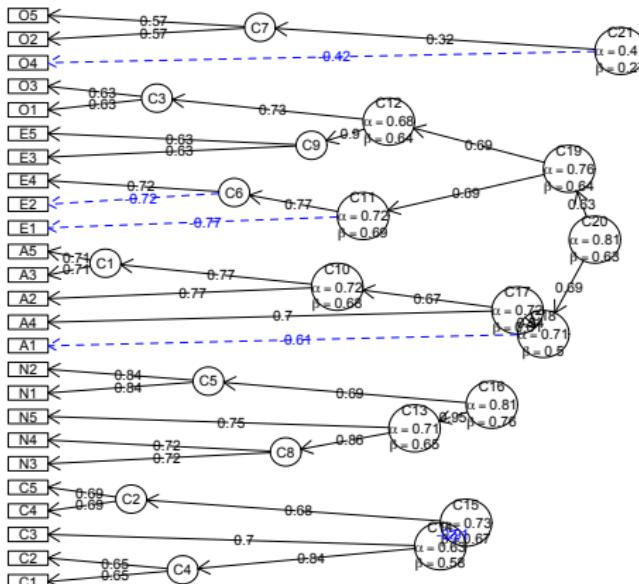


Cluster analysis of items

A hierarchical cluster structure of 25 Big 5 items found by iclust

`iclust(bfi[1:25])`

ICLUST of 25 personality items

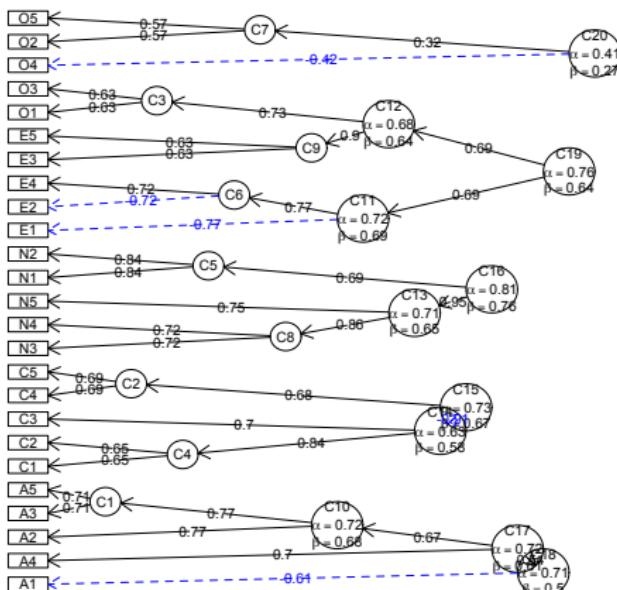




Cluster analysis of items

A hierarchical cluster structure of 25 Big 5 items found by iclust with a more strict criterion

ICLUST of 25 personality items -- stricter beta



Cluster analysis of items

ICLUST produces basic scale reliability information

```
> iclust(bfi[1:25],beta=2,title="ICLUST of 25 personality items -- stricter beta
```

```
CLUST (Item Cluster Analysis)
```

```
Call: iclust(r.mat = bfi[1:25], beta = 2, title = "ICLUST of 25 personality items")
```

Purified Alpha:

```
C16 C19 C18 C15 C20  
0.81 0.76 0.71 0.73 0.61
```

G6* reliability:

```
C16 C19 C18 C15 C20  
0.81 0.64 0.68 0.58 0.45
```

Original Beta:

```
C16 C19 C18 C15 C20  
0.76 0.64 0.50 0.67 0.27
```

Cluster size:

```
C16 C19 C18 C15 C20  
5 5 5 5 5
```



Cluster analysis of items

ICLUST output (continued) shows item by cluster loadings and cluster intercorrelations

Item by Cluster Structure matrix:

	C19	C18	C16	C15	C20
A1	-0.10	-0.39	0.14	0.05	0.13
A2	0.40	0.67	-0.07	-0.23	-0.19
....					
04	-0.10	0.06	0.21	0.00	-0.33
05	-0.11	-0.10	0.11	0.15	0.53

With eigenvalues of:

C19	C18	C16	C15	C20
3.6	3.1	3.0	2.6	1.9

Purified scale intercorrelations

reliabilities on diagonal

correlations corrected for attenuation above diagonal:

	C19	C18	C16	C15	C20
C19	0.76	0.64	-0.28	-0.36	-0.35
C18	0.47	0.71	-0.24	-0.35	-0.25
C16	-0.22	-0.18	0.81	0.29	0.11
C15	-0.27	-0.25	0.22	0.73	0.30
C20	-0.24	-0.16	0.07	0.20	0.61



Factor Extension and Set Correlation

- ① Originally developed by Dwyer for the case of having completed a factor analysis and then a new variable is introduced.
 - At the time, factoring was hard and time consuming
- ② May now be used to extend the factors from one domain into another domain.
 - Differs from SEM in that the factors are estimated in the first domain and are not changed with the addition of the second domain
- ③ Another technique for relating two domains is "Set Correlation" as discussed by Cohen, Cohen, Aiken and West.



Factor Extension and the structure of affect

- ① Consider the joint analysis of Energetic and Tense Arousal with Positive and Negative Affect
 - EA = "active" "alert" "aroused" -("sleepy" "tired" "drowsy")
 - TA = "anxious" "jittery" "nervous" -("calm" "relaxed" "at-ease")
 - PA = "happy" "pleased"
 - NA = "unhappy" "sad"
- ② What is the location of PA and NA in the EA/TA space
- ③ What is the structure of the joint space?
- ④ Use the data in the Motivational State Questionnaire (msq) data set.
 - 75 mood and arousal items given over 10 years to various participants (N=3896)



Basic commands for display and analysis

```
eata <- c("active", "alert", "aroused",
"sleepy", "tired", "drowsy",
"anxious", "jittery", "nervous",
"calm", "relaxed", "at-ease",
"happy", "pleased", "unhappy", "sad")
```

```
R <- lowerCor(msq[eata])
```

```
cor.plot(R, main="Arousal and Affect terms")
```

```
f.all <- fa(R, 2)
```

```
fe.all <- fa.extend(R, 2, 1:12, 13:16)
```

```
op <- par(mfrow=c(1, 2))
```

```
fa.plot(f.all, labels=rownames(R), ylim=c(-1,
xlim=c(-1, 1), title="FA combined")
```

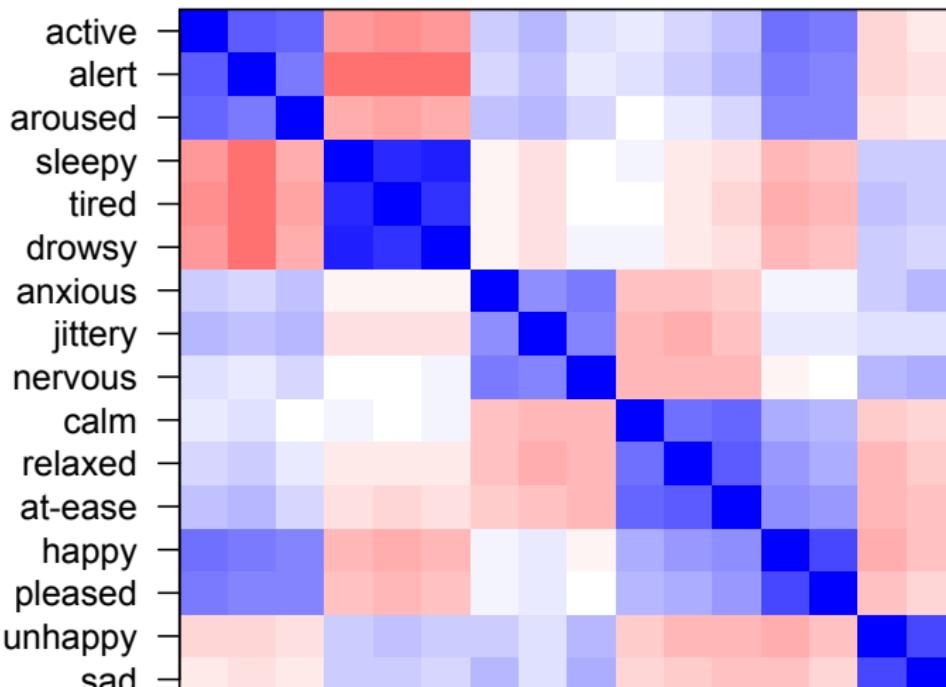
```
fa.plot(fe.all, labels=rownames(R), ylim=c(-1, 1),
xlim=c(-1, 1), title="Extend EA/TA")
```

- ➊ get the data
- ➋ find the correlations
- ➌ show the correlations graphically
- ➍ factor entire set
- ➎ factor EA/TA space – extend to PA/NA
- ➏ Display the results



A cor.plot of the data

Arousal and Affect terms



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ooooooo

Factor Extension and Set Correlation as ways of relating multiple domains

```
fa(r = R, nfactors = 2)
```

Factor Analysis using method = minres
 Call: fa(r = R, nfactors = 2)
 Standardized loadings (pattern matrix)

	MR1	MR2	h2	u2
active	-0.52	0.25	0.39	0.61
alert	-0.64	0.22	0.52	0.48
aroused	-0.46	0.16	0.27	0.73
sleepy	0.89	0.06	0.78	0.22
tired	0.86	0.01	0.73	0.27
drowsy	0.88	0.07	0.75	0.25
anxious	-0.21	-0.34	0.13	0.87
jittery	-0.31	-0.34	0.17	0.83
nervous	-0.15	-0.40	0.16	0.84
calm	0.18	0.67	0.43	0.57
relaxed	0.07	0.71	0.48	0.52
at-ease	0.00	0.74	0.55	0.45
happy	-0.30	0.59	0.51	0.49
pleased	-0.28	0.53	0.42	0.58
unhappy	0.14	-0.45	0.25	0.75
sad	0.11	-0.39	0.19	0.81

	MR1	MR2
SS loadings	3.65	3.07
Proportion Var	0.23	0.19
Cumulative Var	0.23	0.42
Proportion Explained	0.54	0.46
Cumulative Proportion	0.54	1.00

```
fa.extend(r = R, nfactors = 2, ov = 1:12, ev = 13:16)
```

Factor Analysis using method = minres
 Call: fa.extend(r = R, nfactors = 2, ov = 1:12, ev = 13:16)
 Standardized loadings (pattern matrix)

	MR1	MR2	h2	u2
active	-0.57	0.02	0.32	0.68
alert	-0.68	0.07	0.47	0.53
aroused	-0.49	-0.07	0.24	0.76
sleepy	0.88	0.01	0.78	0.22
tired	0.85	-0.01	0.73	0.27
drowsy	0.87	0.01	0.76	0.24
anxious	-0.14	-0.50	0.26	0.74
jittery	-0.23	-0.53	0.33	0.67
nervous	-0.07	-0.55	0.30	0.70
calm	0.04	0.68	0.46	0.54
relaxed	-0.08	0.69	0.49	0.51
at-ease	-0.15	0.69	0.51	0.49
happy	-0.49	0.32	0.36	0.64
pleased	-0.45	0.27	0.29	0.71
unhappy	0.22	-0.36	0.19	0.81
sad	0.17	-0.33	0.15	0.85

	MR1	MR2
SS loadings	3.95	2.69
Proportion Var	0.25	0.17
Cumulative Var	0.25	0.42
Proportion Explained	0.59	0.41
Cumulative Proportion	0.59	1.00

With factor correlations of

MR1	MR2	
MR1	1.00	-0.21
MR2	-0.21	1.00

With factor correlations of

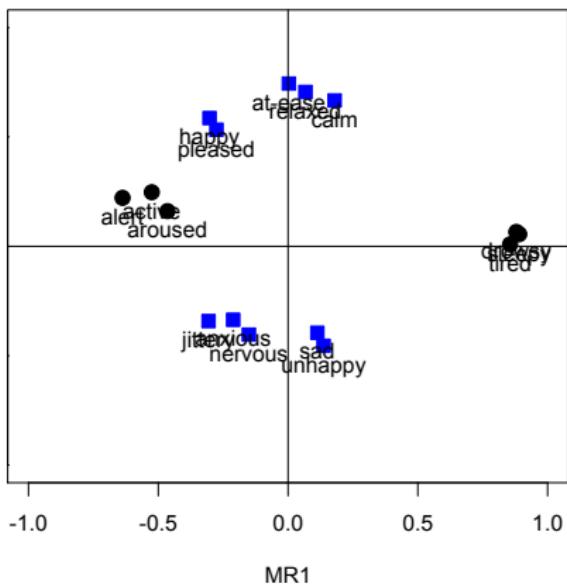
MR1	MR2	
MR1	1.00	-0.06
MR2	-0.06	1.00



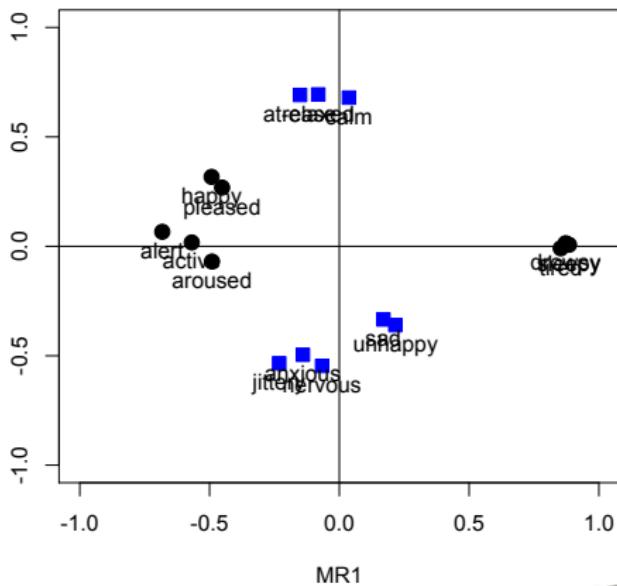
Factor Extension and Set Correlation as ways of relating multiple domains

A fa.plot of the two solutions

FA combined

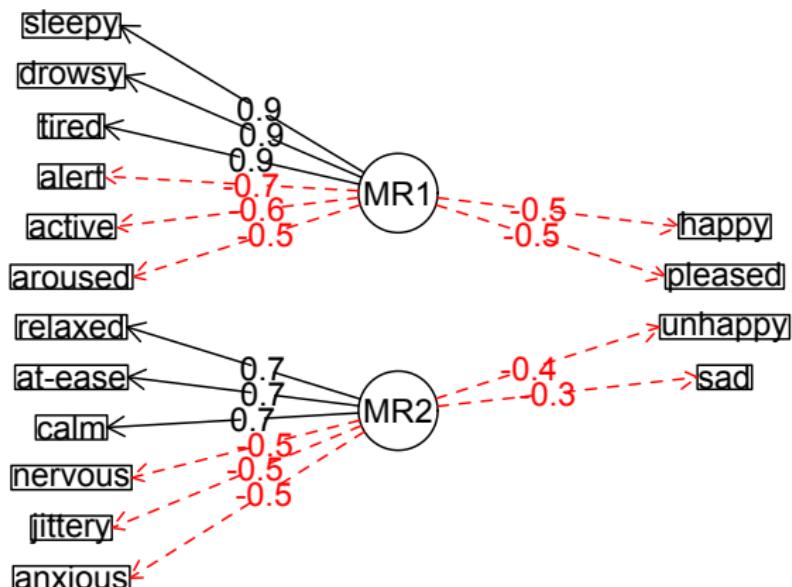


Extend EA/TA



Factor extension of Energetic and Tense Arousal to Affect

EA and TA factors extended to PA and NA



Set correlation is a generalized R^2 between two sets of variables

$R^2 = 1 - \prod (1 - \lambda_i^2)$ where λ_i^2 is the i th squared canonical correlation. Unfortunately, the R^2 is sensitive to one of the canonical correlations being very high. An alternative, T^2 , is the proportion of additive variance and is the average of the squared canonicals.

```
> set.cor(y=13:16,x=1:12,data=R)
```

```
Call: set.cor(y = 13:16, x = 1:12, data = R)
```

```
Multiple Regression from matrix input
```

```
Beta weights
```

	happy	pleased	unhappy	sad
active	0.28	0.25	-0.07	-0.02
alert	0.17	0.15	0.05	0.01
aroused	0.16	0.20	-0.05	-0.04
sleepy	0.04	0.05	0.03	0.08
tired	-0.03	-0.05	0.17	0.14
drowsy	0.01	0.03	0.00	-0.04
anxious	0.01	0.01	0.10	0.17
jittery	0.02	0.00	-0.04	-0.03
nervous	-0.01	0.01	0.19	0.20
calm	0.08	0.08	0.00	0.04
relaxed	0.13	0.10	-0.10	-0.06
at-ease	0.20	0.17	-0.12	-0.10

```
> set.cor(y=13:16,x=1:12,data=R)
```

```
Multiple R
```

happy	pleased	unhappy	sad
0.69	0.64	0.43	0.41

```
Multiple R2
```

happy	pleased	unhappy	sad
0.47	0.41	0.18	0.17

```
Various estimates of between set correlations
```

```
Squared Canonical Correlations
```

```
[1] 0.5187 0.1551 0.0095 0.0041
```

```
Chisq of canonical correlations
```

```
NULL
```

```
Average squared canonical correlation = 0.17
```

```
Cohen's Set Correlation R2 = 0.6
```



Structural Equation modeling packages

SEM packages allow for Confirmatory Factor Analysis as well as Structural modeling.

- ① sem (by John Fox and others)
 - uses RAM notation
 - does not handle multiple groups
 - does not seem to be actively developed
- ② lavaan (by Yves Rosseel and others)
 - Mimics as much as possible MPLUS output
 - Allows for multiple groups
 - Easy syntax
- ③ OpenMx (by Steve Bolker, Michael Neale, and others)
 - Open source and R version of Mx
 - Allows for multiple groups (and almost anything else)
 - Complicated syntax



lavaan analysis – from the example – output mimics MPlus

```
#The Holzinger and Swineford (1939) example
HS.model <- ' visual  =~ x1 + x2 + x3
              textual =~ x4 + x5 + x6
              speed   =~ x7 + x8 + x9 '

fit <- lavaan(HS.model, data=HolzingerSwineford1939,
               auto.var=TRUE, auto.fix.first=TRUE,
               auto.cov.lv.x=TRUE)
summary(fit, fit.measures=TRUE)

lavaan (0.4-7) converged normally after 35 iterations
```

Number of observations	301
Estimator	ML
Minimum Function Chi-square	85.306
Degrees of freedom	24
P-value	0.000

Chi-square test baseline model:

Minimum Function Chi-square	918.852
Degrees of freedom	36
P-value	0.00



Confirmatory Factor Analysis

lavaan example – continued

Full model versus baseline model:

Comparative Fit Index (CFI)	0.931
Tucker-Lewis Index (TLI)	0.896

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	-3737.745
Loglikelihood unrestricted model (H1)	-3695.092

Number of free parameters 21

Akaike (AIC) 7517.490

Bayesian (BIC) 7595.339

Sample-size adjusted Bayesian (BIC) 7528.739

Root Mean Square Error of Approximation:

RMSEA 0.092

90 Percent Confidence Interval 0.071 0.114

P-value RMSEA <= 0.05 0.001

Standardized Root Mean Square Residual:

SRMR 0.065

Parameter estimates:

Information	Expected		
Standard Errors	Standard		
Estimate	Std.err	Z-value	P(> z)

Latent variables:

visual =~				
x1	1.000			
x2	0.554	0.100	5.554	0.000
x3	0.729	0.109	6.685	0.000

textual =~				
x4	1.000			
x5	1.113	0.065	17.014	0.000
x6	0.926	0.055	16.703	0.000



Test invariance across groups

Using lavaan to examine measurement invariance – from the example

```
HW.model <- ' visual =~ x1 + x2 + x3
```

```
      textual =~ x4 + x5 + x6
```

```
      speed =~ x7 + x8 + x9 '
```

```
measurementInvariance(HW.model, data=HolzingerSwineford1939, group="school")
```

Measurement invariance tests:

Model 1: configural invariance:

chisq	df	pvalue	cfi	rmsea	bic
115.851	48.000	0.000	0.923	0.097	7604.094

chisq	df	pvalue	cfi	rmsea	bic
124.044	54.000	0.000	0.921	0.093	7578.043

Model 2: weak invariance (equal loadings):

chisq	df	pvalue	cfi	rmsea	bic
124.044	54.000	0.000	0.921	0.093	7578.043

[Model 1 versus model 2]

delta.chisq	delta.df	delta.p.value	delta.cfi
8.192	6.000	0.224	0.002

Model 3: strong invariance (equal loadings + intercepts):

chisq	df	pvalue	cfi	rmsea	bic
164.103	60.000	0.000	0.882	0.107	7686.588

[Model 1 versus model 3]

delta.chisq	delta.df	delta.p.value	delta.cfi
48.251	12.000	0.000	0.041

[Model 2 versus model 3]

delta.chisq	delta.df	delta.p.value	delta.cfi
40.059	6.000	0.000	0.038

Model 4: equal loadings + intercepts + means:

chisq	df	pvalue	cfi	rmsea	bic
204.605	63.000	0.000	0.854	0.122	7709.969

[Model 1 versus model 4]

delta.chisq	delta.df	delta.p.value	delta.cfi
88.754	15.000	0.000	0.069

[Model 3 versus model 4]

delta.chisq	delta.df	delta.p.value	delta.cfi
-------------	----------	---------------	-----------



Item Response Theory

- ① Said to be the “new psychometrics”, IRT combines item and person information
 - Several packages for IRT, including 1 parameter (Rasch) as well as 2 and 3 parameter models
 - These estimate the parameters using standard IRT approaches
- ② An alternative is to recognize that 2 parameter IRT models are just factor models applied to the *tetrachoric* or *polychoric* correlations.
 - That is, find the factor analysis loadings (λ_i) and the item endorsement frequencies expressed as normal deviates (τ_i) and then convert to IRT parameters
 - discrimination $\alpha = \frac{\lambda_i}{\sqrt{1-\lambda_i^2}}$
 - location (difficulty) $\delta = \frac{\tau_i}{\sqrt{1-\lambda_i^2}}$



Multiple packages to do Item Response Theory analysis

- ① *psych* uses a factor analytic procedure to estimate item discriminations and locations
 - look at examples for `irt.fa`
 - two example data sets: `iqitems` and `bfi`
- ② `irt.fa` finds either tetrachoric or polychoric correlation matrices
 - Returns normal factor analysis output as well as IRT parameters
 - Converts factor loadings to discriminations
 - Saves the tetrachoric/polychoric correlation matrix for faster reanalyses
- ③ `plot.irt` plots item information and item characteristic functions
- ④ Other packages include *ltm*, *MCMCpack* (for Markov chain Monte Carlo k-dimensional IRT models), and *irtoys* for interfacing with different packages.



IRT analysis of 16 iq items – dichotomous items

```
> iq.keys <- c(4,4,4, 6, 6,3,4,4, 5,2,2,4, 3,2,6,7)
> iq.tf <- score.multiple.choice(iq.keys,iq.scrub,score=FALSE) #convert to wrong
> iq.irt <- irt.fa(iq.tf)
> plot(iq.irt)
> iq.irt
```

Item Response Analysis using Factor Analysis

Call: irt.fa(x = iq.tf)

Item Response Analysis using Factor Analysis

Summary information by factor and item

Factor = 1

	-3	-2	-1	0	1	2	3
--	----	----	----	---	---	---	---

reason.4	0.05	0.24	0.64	0.53	0.16	0.03	0.01
----------	------	------	------	------	------	------	------

reason.16	0.08	0.22	0.38	0.31	0.14	0.05	0.01
-----------	------	------	------	------	------	------	------

...

letter.58	0.02	0.09	0.30	0.53	0.35	0.12	0.03
-----------	------	------	------	------	------	------	------

matrix.45	0.05	0.11	0.19	0.23	0.17	0.09	0.04
-----------	------	------	------	------	------	------	------

...

rotate.6	0.01	0.03	0.15	0.53	0.69	0.25	0.05
----------	------	------	------	------	------	------	------

rotate.8	0.00	0.02	0.08	0.29	0.59	0.41	0.13
----------	------	------	------	------	------	------	------

Test Info	0.67	2.11	4.73	5.83	5.28	2.55	0.69
-----------	------	------	------	------	------	------	------

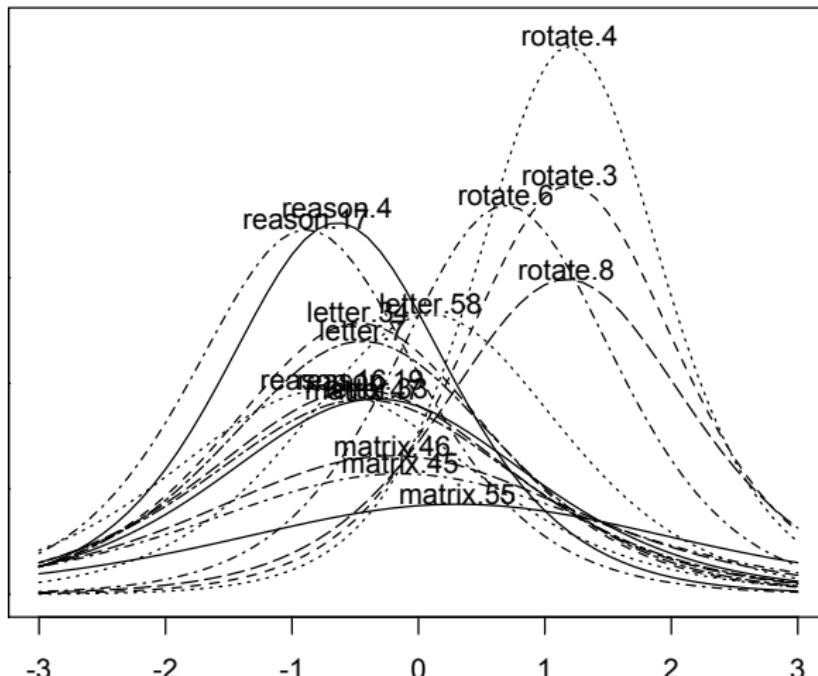
SEM	1.22	0.69	0.46	0.41	0.44	0.63	1.20
-----	------	------	------	------	------	------	------

Reliability	-0.49	0.53	0.79	0.83	0.81	0.61	-0.45
-------------	-------	------	------	------	------	------	-------



Item Response Information curves for 16 iq items

Item information for 16 ability items



Extending IRT to the multidimensional case

- ① By using a factor analytic approach, we can find IRT parameters for multiple factors
 - `irt.fa` will find multiple factors and then convert the highest loadings on each factor to IRT parameters
- ② One powerful advantage of IRT is that by displaying item information statistics, we can choose items that provide maximal information.
 - Area under the curve is reported for each item information curve.
 - Can also plot item characteristic curves, or test information curves.



IRT analysis of the first 15 bfi items – Polytomous items – this is time consuming the first time

```
> int.bfi <- int.fa(bfi[1:15],3) #save the results for a faster reanalysis
> int.bfi
```

Item Response Analysis using Factor Analysis

Call: int.fa(x = bfi[1:15], 3)

Item discrimination and location for factor MR2

	discrimination	location.1	location.2	location.3	location.4	location.5
A1	0.06	-0.44	0.32	0.74	1.23	1.89
...						

C1	0.77	-2.45	-1.74	-1.14	-0.26	1.00
----	------	-------	-------	-------	-------	------

C2	0.92	-2.52	-1.62	-1.03	-0.15	1.15
----	------	-------	-------	-------	-------	------

C3	0.72	-2.31	-1.45	-0.93	-0.03	1.18
----	------	-------	-------	-------	-------	------

C4	-0.95	-0.81	0.22	0.86	1.73	2.75
----	-------	-------	------	------	------	------

C5	-0.73	-1.13	-0.36	0.03	0.76	1.57
----	-------	-------	-------	------	------	------

E1	0.11	-0.71	-0.07	0.30	0.78	1.37
----	------	-------	-------	------	------	------

...						
-----	--	--	--	--	--	--

Item discrimination and location for factor MR3

	discrimination	location.1	location.2	location.3	location.4	location.5
A1	-0.62	-0.51	0.38	0.87	1.45	2.22

A2	1.02	-3.02	-2.19	-1.70	-0.68	0.69
----	------	-------	-------	-------	-------	------

A3	1.23	-2.93	-2.09	-1.52	-0.52	0.96
----	------	-------	-------	-------	-------	------

A4	0.51	-1.89	-1.30	-0.99	-0.43	0.25
----	------	-------	-------	-------	-------	------

A5	0.67	-2.44	-1.63	-1.11	-0.30	0.81
----	------	-------	-------	-------	-------	------

...						
-----	--	--	--	--	--	--

E5	0.05	-1.82	-1.21	-0.78	-0.15	0.77
----	------	-------	-------	-------	-------	------

Item discrimination and location for factor MR1

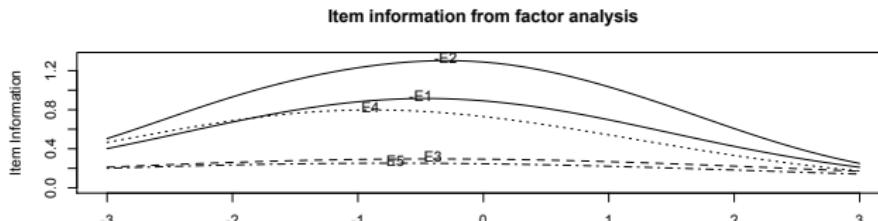
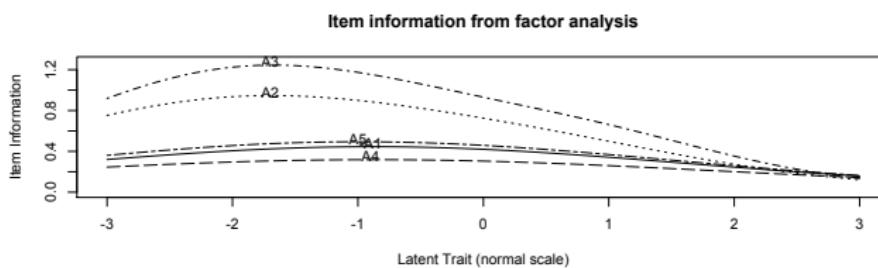
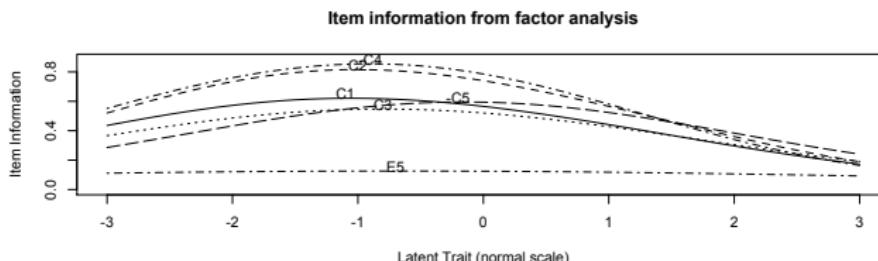
	discrimination	location.1	location.2	location.3	location.4	location.5
C5	-0.14	-0.92	-0.30	0.02	0.62	1.28

E1	-0.94	-0.97	-0.09	0.41	1.06	1.86
----	-------	-------	-------	------	------	------

E2	-1.25	-1.40	-0.27	0.22	1.18	2.13
----	-------	-------	-------	------	------	------



Plot the item information functions for the three factors



Further capabilities

- ① Very powerful graphics (e.g., lattice, networks, social networks)
- ② Multilevel models used for a variety of personality studies.
 - Structure of emotion within and between individuals
 - Longitudinal measures of change
 - Interpersonal relations
 - Eye tracking analysis
- ③ Simulations as a means of testing theory
- ④ Dynamic models
- ⑤ Data analysis that interests you, not someone else



[Multidimensional IRT](#)[▶ Part I: an introduction to R](#)[▶ Part II: Using R for psychometrics](#)[▶ Part III: Structures, Objects, Functions](#)

Outline of Part III: Basic R Commands

- 8 Data Structures
- 9 Objects and functions
- 10 Getting help
- 11 Frequently used functions
- 12 More on Functions
 - Writing your own function



A brief technical interlude

① Data structures

- The basic: scalers, vectors, matrices
- More advanced: data frames and lists
- Showing the data

② Getting the length, dimensions and structure of a data structure

- `length(x)`, `dim(x)`, `str(x)`

③ Objects and Functions

- Functions act upon objects
- Functions actually are objects themselves
- Getting help for a function or a package



The basic types of data structures

- ① Scalers (characters, integers, reals, complex)

```
> A <- 1
```

```
> B <- 2
```

- ② Vectors (of scalers, all of one type) have length

```
> C <- month.name[1:5]
```

```
> D <- 12:24
```

```
> length(D)
```

```
[1] 13
```

- ③ Matrices (all of one type) have dimensions

```
> E <- matrix(1:20, ncol = 4)
```

```
> dim(E)
```

```
[1] 5 4
```



Show values by entering the variable name

```
> A
```

```
[1] 1
```

```
> B
```

```
[1] 2
```

```
> C
```

```
[1] "January"  "February"  "March"      "April"       "May"
```

```
> D
```

```
[1] 12 13 14 15 16 17 18 19 20 21 22 23 24
```

```
> E
```

	[,1]	[,2]	[,3]	[,4]
[1,]	1	6	11	16
[2,]	2	7	12	17
[3,]	3	8	13	18
[4,]	4	9	14	19
[5,]	5	10	15	20



More complicated (and useful) types: Data frames and Lists

- ➊ Data frames are collections of vectors and may be of different type. They have two dimensions.

```
> E.df <- data.frame(names = C, values = c(31, 28, 31, 30, 31))  
> dim(E.df)
```

```
[1] 5 2
```

- ➋ Lists are collections of what ever you want. They have length, but do not have dimensions.

```
> F <- list(first = A, a.vector = C, a.matrix = E)  
> length(F)
```

```
[1] 3
```



Show values by entering the variable name

```
> E.df
```

	names	values
1	January	31
2	February	28
3	March	31
4	April	30
5	May	31

```
> F
```

```
$first  
[1] 1
```

```
$a.vector  
[1] "January" "February" "March" "April" "May"
```

```
$a.matrix  
[,1] [,2] [,3] [,4]  
[1,] 1 6 11 16  
[2,] 2 7 12 17  
[3,] 3 8 13 18  
[4,] 4 9 14 19  
[5,] 5 10 15 20
```



- ① To show the structure of a list, use `str`

```
> str(F)
```

List of 3

```
$ first    : num 1
$ a.vector: chr [1:5] "January" "February" "March" "April" ...
$ a.matrix: int [1:5, 1:4] 1 2 3 4 5 6 7 8 9 10 ...
```

- ② to address an element of a list, call it by name or number, to get a row or column of a matrix specify the row, column or both.

```
> F[[2]]
```

```
[1] "January"   "February"  "March"      "April"       "May"
```

```
> F[["a.matrix"]][, 2]
```

```
[1] 6 7 8 9 10
```

```
> F[["a.matrix"]][2, ]
```

```
[1] 2 7 12 17
```



Addressing the elements of a data.frame or matrix

Setting row and column names using paste

```
> E <- matrix(1:20, ncol = 4)
> colnames(E) <- paste("C", 1:ncol(E), sep = "")
> rownames(E) <- paste("R", 1:nrow(E), sep = "")
> E
```

	C1	C2	C3	C4
R1	1	6	11	16
R2	2	7	12	17
R3	3	8	13	18
R4	4	9	14	19
R5	5	10	15	20

```
> E["R2", ]
```

C1	C2	C3	C4
2	7	12	17

```
> E[, 3:4]
```

	C3	C4
R1	11	16
R2	12	17
R3	13	18
R4	14	19
R5	15	20



Objects and Functions

- ① R is a collection of Functions that act upon and return Objects
- ② Although most functions can act on an object and return an object ($a = f(b)$), some are binary operators
 - primitive arithmetic functions +, -, *, /, %*%,
 - logical functions <, >, ==, !=
- ③ Some functions do not return values
 - `print(x, digits=3)`
 - `summary(some object)`
- ④ But most useful functions act on an object and return a resulting object
 - this allows for extraordinary power because you can combine functions by making the output of one the input of the next.
 - The number of R functions is very large, for each package has introduced more functions, but for any one task, not many functions need to be learned.



Getting help

- ① All functions have a help menu
 - `help(the function)`
 - `? the function`
 - most function help pages have examples to show how to use the function
- ② Most packages have “vignettes” that give overviews of all the functions in the package and are somewhat more readable than the help for a specific function.
 - The examples are longer, somewhat more readable. (e.g., the vignette for *psych* is available either from the menu (Mac) or from <http://cran.r-project.org/web/packages/psych/vignettes/overview.pdf>)
- ③ To find a function in the entire R space, use `findFn` in the *sos* package.
- ④ Online tutorials (e.g., <http://Rpad.org> for a list of important commands, <http://personality-project.org/r>) for a tutorial for psychologists.
- ⑤ Online and hard copy books



A few of the most useful data manipulations functions (adapted from Rpad-refcard). Use ? for details

file.choose () find a file
file.choose (new=TRUE) create a new file
read.table (filename)
read.csv (filename) reads a comma separated file
read.delim (filename) reads a tab delimited file
c (...) combine arguments
from:to e.g., 4:8
seq (from,to, by)
rep (x,times) repeat x
gl (n,k,...) generate factor levels
matrix (x,nrow=,ncol=) create a matrix
dim (x) dimensions of x

data.frame (...) create a data frame
list (...) create a list
colnames (x)
rownames (x)
rbind (...) combine by rows
cbind (...) combine by columns
is.na (x) also is.null(x), is...
na.omit (x) ignore missing data
table (x)
merge (x,y)
as.matrix (x) convert to a matrix,
as.data.frame (x) convert to a data.frame
ls () show workspace
rm () remove variables from workspace



More useful statistical functions, Use ? for details

mean (x)
is.na (x) also `is.null(x)`, `is...`
na.omit (x) ignore missing data
sum (x)
rowSums (x) see also `colSums(x)`
min (x)
max (x)
range (x)
table (x)
summary (x) depends upon x
sd (x) standard deviation
cor (x) correlation
cov (x) covariance
solve (x) inverse of x
lm (y~x) linear model
aov (y~x) ANOVA

Selected functions from *psych* package
describe (x) descriptive stats
describe.by (x,y) descriptives by group
pairs.panels (x) SPLOM
error.bars (x) means + error bars
error.bars.by (x) Error bars by groups
fa (x) Factor analysis
iclust (x) Item cluster analysis
score.items (x) score multiple scales
score.multiple.choice (x) score multiple choice scales
alpha (x) Cronbach's alpha
omega (x) MacDonald's omega
irt.fa (x) Item response theory through factor analysis



More psych commands

Simulation functions

<code>sim</code>	a factor simplex
<code>sim.simplex</code>	an item simplex
<code>sim.item</code>	items with 2 dimensional simple structure
<code>sim.circ</code>	items in a circumplex structure
<code>sim.congeneric</code>	items for a congeneric measurement model
<code>sim.hierarchical</code>	items with a hierarchical factor structure
<code>sim.rasch</code>	Rasch items
<code>sim.irt</code>	1-4 parameter IRT items
<code>sim.structural</code>	a general structural model
<code>sim.anova</code>	for ANOVA and lm problems

Graphical displays of structure

<code>diagram</code>	a generic set of diagram tools
<code>fa.diagram</code>	Show a factor structure
<code>omega.diagram</code>	Show Schmid Leiman structures
<code>ICLUST.diagram</code>	draw a cluster tree
<code>plot.psych</code>	a generic call for various plots additional data displays
<code>error.crosses</code>	two way error bars
<code>biplot.psych</code>	Plot factors and scores on same graph
<code>draw.tetra</code>	Show a tetrachoric correlation
<code>scatter.hist</code>	scatter plot with histogram



Writing your own function

- ➊ At first, one just has a few lines of syntax that are repeatedly used
 - This could be any routine operation that you do
 - Probably hard coded and needing minor modifications each time.
- ➋ Think of making this into a short function
 - Specify the input parameters
 - Return either a single value, vector or matrix or return a list
- ➌ Test the function
 - Modify it a little to be more general
 - Perhaps specify a few default values
- ➍ Add this to your file of frequently used operations.
- ➎ To see how other functions work, just type in their name
 - Copy it to you text editor
 - Change a few lines
 - Paste it back into R (you must say the name <- function(...)



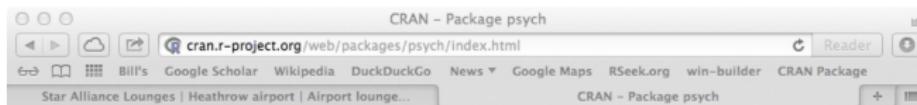
Writing functions is more typically “adapting” a function

- ➊ Many functions do almost what you want to do, but not quite.
 - Their defaults are not what you like
 - You might see a way of adding something
- ➋ Learn by reading other people’s code
 - Either directly from the console
 - Download the source from CRAN
- ➌ Try to understand what the person is doing
 - Styles differ
 - Use a style you like
 - Document your work
- ➍ If you find a bug
 - Write the package maintainer
 - Say what you did, what you expected, what you got
 - R is a community, be helpful



[Writing your own function](#)

Getting information about a package and its contents



psych: Procedures for Psychological, Psychometric, and Personality Research

A number of routines for personality, psychometrics and experimental psychology. Functions are primarily for scale construction using factor analysis, cluster analysis and reliability analysis, although others provide basic descriptive statistics. Item Response Theory is done using factor analysis of tetrachoric and polychoric correlations. Functions for simulating particular item and test structures are included. Several functions serve as a useful front end for structural equation modeling. Graphical displays of path diagrams, factor analysis and structural equation models are created using basic graphics. Some of the functions are written to support a book on psychometrics as well as publications in personality research. For more information, see the personality-project.org/r webpage.

Version: 1.3.2
 Suggests: [MASS](#), [GPArotation](#), [mvtnorm](#), [polycor](#), [sem](#), [lavaan](#), [Rcsdp](#), [graph](#), [Rgraphviz](#)
 Published: 2013-02-26
 Author: William Revelle
 Maintainer: William Revelle <revelle at northwestern.edu>
 License: [GPL \(≥ 2\)](#)
 URL: <http://personality-project.org/r>, <http://personality-project.org/r/psych.manual.pdf>
 NeedsCompilation: no
 Citation: [psych citation info](#)
 In views: [Psychometrics](#)
 CRAN checks: [psych results](#)

Downloads:

Package source: [psych_1.3.2.tar.gz](#)
 MacOS X binary: [psych_1.3.2.tgz](#)
 Windows binary: [psych_1.3.2.zip](#)
 Reference manual: [psych.pdf](#)
 Vignettes: [Overview of the psych package](#)
[input for sem](#)
 News/ChangeLog: [NEWS](#)
 Old sources: [psych archive](#)



A few final thoughts

① Topics not discussed

- Multilevel modeling is done in *multilevel*, *nlme*
- Graphics can be done in *lattice* (implementation of Trellis), or *ggobi*
- Network analysis in *sna* and *qgraph*
- Sweave allows for automatic report generation embedded in L^AT_EXor OpenOffice.

② R is a journey, you learn by doing but never master it

- R is merely a tool for helping us do better research
- R allows us to ask questions that we want to ask, not those that others have asked already

③ Warning: R can be addictive and lead to proselytizing.



[Writing your own function](#)

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