Math 138 - January 4^{th} 2015

Review of Integration

Given a, b ϵ $\mathbb{R}$. a < b and n ϵ $\mathbb{N}$ we divide the interval [a, b] into n subintervals [x_{i-1}, x_i], i = 1 ... n of equal width $\Delta x = \frac{b-a}{n}$. amd choose sample points:

$$x_i * \epsilon [x_{i-1}, x_i], \text{ i = 1 ... n}$$

For a function f on the closed interval [a, b]. the definite integral of f from a to b is:
$\int_a^b f(x)dx = \lim n \to \infty \sum_{i=1}^n f(x_i*)\Delta x$

We say that f is integrable if the above limit exists and is independant of the sample points x_i*.
Recall that continuous functions are integrable.
Furthermore, $\lim n \to \infty \sum_{i=1}^n f(x_i*)\Delta x$ is the sum of the areas of the rectangles, and $\int_a^b f(x)dx$ is the area under a curve between two x values.

Fundamental Theorem of Calculus Suppose f is continous on [a,b].

I: If $g(x) = \int_a^x f(t)dt$, then g is differentiable and $g'(x) = f(x), x\epsilon$ [a, b]

II: $\int_a^b f(x)dx = F(b) - F(a)$ where F is **any** antiderivative of $f(x)$, i.e, $F'(x) = f(x), x$ ϵ [a, b]

Examples:
1) Find the derivative of g(x), where:
a) $g(x) = \int_1^x ln(1+t^2)dt$
b) $g(x) = \int_1^{3x+2} \frac{t}{1+t^3}dt$

Solution a): $g'(x) = ln(1+x^2)$ By FTC(I)
Solution b): Write $g(x) = f(u(x))$, where $f(u) = \int_1^u \frac{t}{1+t^3}dt$ and $u(x) = 3x+2$ By the chain rule and FTC(I),

$$\begin{aligned} g'(x) &= f'(u(x)) \cdot u'(x) \\ &= \end{aligned} \tag{1}$$