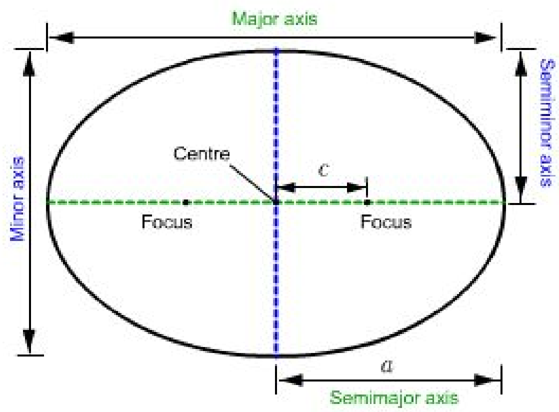
Physics of Orbital Motion

Recall: Properties of an Ellipse
- **Semimajor axis:** Average distance of a body from the orbit's focus
- **Orbit focus:** Location of the centre of mass of the system

Semimajor axis $= a$
Distance from centre to focus $= c$
Eccentricity $= e$

$$e = \frac{c}{a}$$



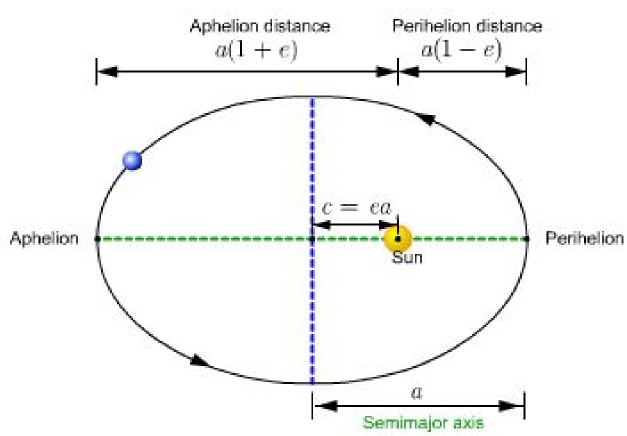
More Ellipse properties:

$$r_{\min} = a(1 - e)$$

$$r_{\max} = a(1 + e)$$

$$r_{\min} + r_{\max} = 2a$$

$$e = \frac{c}{a}$$



Where rmin = Perihelion, rmax = Aphelion

Centre of Mass
- An object's orbit is an ellipse, around the centre of mass
- The location of the centre of mass depends on relative masses
- If the system has equal masses, the centre of mass is in the middle b/w the masses

- For the Solar System, the mass of the Sun is much greater than the mass of the planets, so centre of mass is inside the Sun

$m_1 a_1 = m_2 a_2$

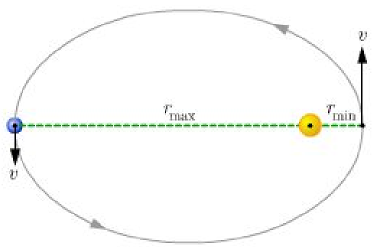
A more general form of Kepler's Third Law, $P^2(\text{years}) = a^3(\text{AU})$, is:

$\left(m_1 + m_2\right) P^2 = a^3 = \left(a_1 + a_2\right)^3$

<u>Conservation of Energy</u>

$Total\ orbital\ energy = Gravitational\ potential\ energy + Kinetic\ energy$

- With no external forces, total orbital energy is constant
- Gravitational potential energy increases as distance increases
- Orbital kinetic energy increases as distance decreases ($0.5mv^2$)



<u>Conservation of Angular Momentum</u>

$Angular\ momentum = mass \times speed \times distance$

$$L = m \times v \times r$$

- L: "Spin" or angular momentum
  - Conserved quantity
  - Constant with no external forces
  - If distance changes, speed must change

<u>Kepler's Laws Derived from Newton's Equations in *Celestial Mechanics*</u>

- We can use conservation of momentum, centripetal forces + gravity to get a generalized **Kepler's Third Law**

$$P^2 = \left[\frac{4\pi^2}{G\left(m_1 + m_2\right)}\right] a^3$$

- Another derived formula

$$v^2 = G\left(m_1 + m_2\right)\left(\frac{2}{r} - \frac{1}{a}\right)$$

When it's a circle (eccentricity = 0):

$$v_{circ} = \sqrt{\frac{GM}{r}}$$

Summary of Kepler:

- Kepler's:

$$P^2 = a^3$$

- More general:

$$(m_1 + m_2)P^2 = a^3 = (a_1 + a_2)^3$$

  - Units are: Solar masses, distance in AU, time in years.

- Most general:

$$(m_1 + m_2)P^2 = \frac{4\pi^2}{G}(a_1 + a_2)^3$$

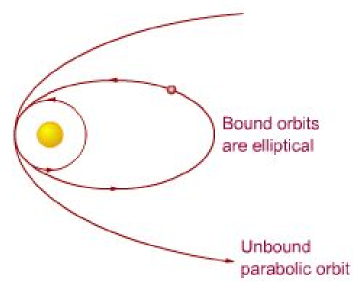
  - Units are: kg, m, s.
  - Semimajor axis for each object around centre of mass is given.

Orbitals and Energy

- Bound orbit ($0 < e < 1$): $v < v_{esc}$

$$v_{esc} = \sqrt{2}v_{circ} = \sqrt{\frac{2GM}{r}}$$

- Unbound orbit : $v \geq v_{esc}$

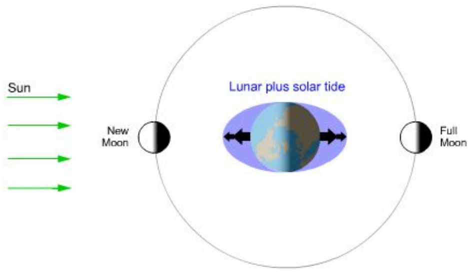


Tides

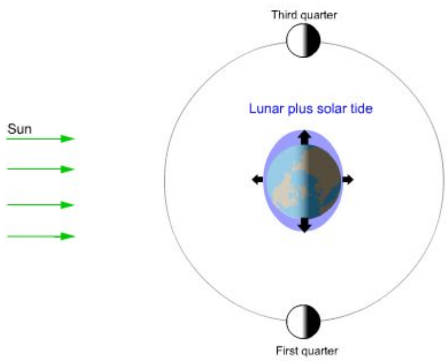
- Caused by differential forces across a body
- Force of gravy is inversely proportional to the square of the distance between the centres of the two masses
  - Newton's Universal Theory of Gravitation

$$F_g = G\frac{m_1 m_2}{d^2}$$

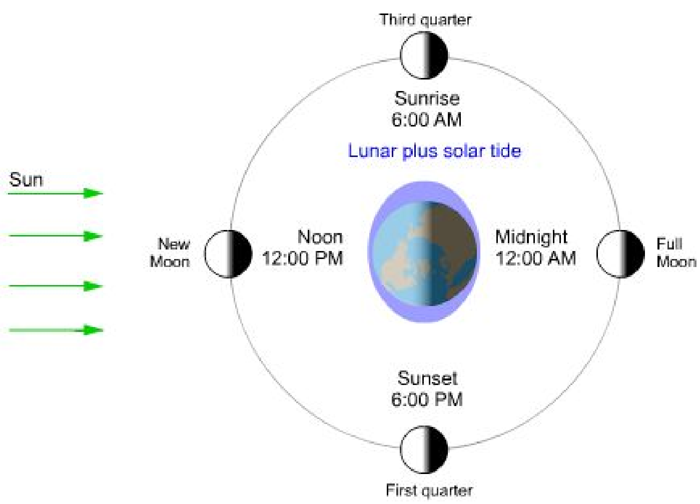
- Earth has gravitational attraction to Moon
  - Side facing Moon has strong attraction, but the side opposite it does not
    - This difference "pulls" Earth in different directions, causing tidal bulges



- Spring tides - highest + lowest tides

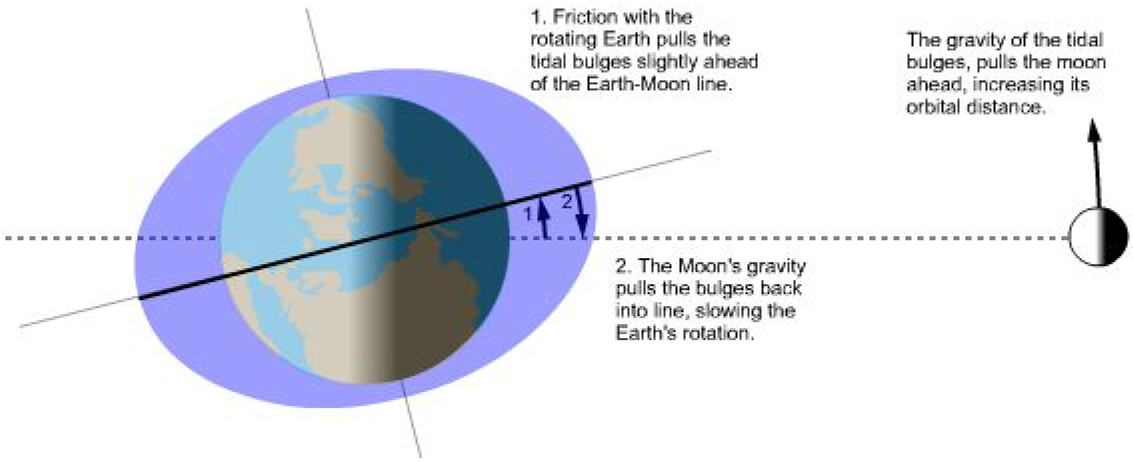


- Neap tides - highest tides are lowest, vice versa



Tidal Friction

- Slows Earth's rotation (Increases Earth-Moon distance)



Other Tidal effects:

- Tidal forces are important wherever differential gravitational forces are large enough
- Ex:
    - Planetary rings
    - Jovian moons
- Binary stars:
    - Stars in close pairs, large envelopes
- Vicinity of black holes
- Galaxies
    - Tidal streams from encounters
    - Warped disks

Orbital Effects:

- Orbits can be altered by an exchange of energy
    - Ex: Collisions, near encounters (multibody or tidal effects)
        - Comets, Jupiter, spacecraft, star clusters